Optimization of rotational arc station parameter optimized radiation therapy

P. Dong and B. Ungun

Department of Radiation Oncology, Stanford University, Stanford, California 94305

S. Boyd

Department of Electrical Engineering, Stanford University, Stanford, California 94305

L. Xing^{a)}

Department of Radiation Oncology, Stanford University, Stanford, California 94305

and Department of Electrical Engineering, Stanford University, Stanford, California 94305

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Purpose: To develop a fast optimization method for station parameter optimized radiation therapy (SPORT) and show that SPORT is capable of matching VMAT in both plan quality and delivery efficiency by using three clinical cases of different disease sites.

Methods: The angular space from 0° to 360° was divided into 180 station points (SPs). A candidate aperture was assigned to each of the SPs based on the calculation results using a column generation algorithm. The weights of the apertures were then obtained by optimizing the objective function using a state-of-the-art GPU based proximal operator graph solver. To avoid being trapped in a local minimum in beamlet-based aperture selection using the gradient descent algorithm, a stochastic gradient descent was employed here. Apertures with zero or low weight were thrown out. To find out whether there was room to further improve the plan by adding more apertures or SPs, the authors repeated the above procedure with consideration of the existing dose distribution from the last iteration. At the end of the second iteration, the weights of all the apertures were reoptimized, including those of the first iteration. The above procedure was repeated until the plan could not be improved any further. The optimization technique was assessed by using three clinical cases (prostate, head and neck, and brain) with the results compared to that obtained using conventional VMAT in terms of dosimetric properties, treatment time, and total MU.

Results: Marked dosimetric quality improvement was demonstrated in the SPORT plans for all three studied cases. For the prostate case, the volume of the 50% prescription dose was decreased by 22% for the rectum and 6% for the bladder. For the head and neck case, SPORT improved the mean dose for the left and right parotids by 15% each. The maximum dose was lowered from 72.7 to 71.7 Gy for the mandible, and from 30.7 to 27.3 Gy for the spinal cord. The mean dose for the pharynx and larynx was reduced by 8% and 6%, respectively. For the brain case, the doses to the eyes, chiasm, and inner ears were all improved. SPORT shortened the treatment time by ~1 min for the prostate case, ~0.5 min for brain case, and ~0.2 min for the head and neck case.

Conclusions: The dosimetric quality and delivery efficiency presented here indicate that SPORT is an intriguing alternative treatment modality. With the widespread adoption of digital linac, SPORT should lead to improved patient care in the future. © 2016 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4960000>]

Key words: SPORT, VMAT, optimization, inverse planning, IMRT

1. INTRODUCTION

Digital linacs with important new features have emerged for clinical use.^{1,2} A distinctive feature of digital linac is that the geometric/mechanic parameters characterizing radiation delivery such as the gantry angle, collimator angle, and couch angle are discretized and can be controlled easily in a programmable fashion. The fundamental unit is a station point (SP) or control point, which defines a discretized space-MU point in digital linac delivery. In the emerging digital era of radiation therapy (RT), the status of all the digital

components is checked every ~10 ms by the central control unit, making it possible to deliver a treatment station by station with effective modulation of station parameters. Different from previous generation(s) of linacs, a motion trajectory of delivery in space-MU can be realized by algorithmically concatenating a collection of SPs with controlled and simultaneous motion of potentially all accelerator's mechanical variables.

In practice, however, existing RT schemes do not fully take the above digital control capabilities into consideration. Therefore, how to leverage these new developments to

FIG. 1. Aperture selection using beamlets' desirability scores. Left: 20×20 beamlets with desirability scores ranging from -1.5 to 1.5 . A positive score promotes the selection of the beamlet. Right: An aperture formed by the MLC leaves guarding a subgroup of contiguous beamlets with the largest sum of scores for each row.

overcome many of the limitations of current RT techniques becomes an urgent issue. Given that the fundamental unit in digital linac is SP, it is fair to say that the next-generation of RT will be all about the optimization of station parameters, which we call station parameter optimized radiation therapy (SPORT).³ VMAT (Ref. 4) and IMRT represent two special, and often nonoptimal, cases of SPORT. The premise of SPORT is that it will enable us to realize the enormous potential of digital linacs through optimal weighting and spatial distribution of the SPs (including noncoplanar, nonisocentric distributions, and even multiple isocenters).

The increased degrees of freedom in digital linac present a hurdle for treatment planning. A major challenge is how to optimize the angular distribution of the SPs or control points. In current VMAT, the SPs are generally distributed uniformly over a specific angular span, which may under or oversample the beams in angles. A variety of recent publications provided innovative solutions. Li and Xing emphasized the importance of nonuniform SP distribution and used a heuristic angular modulation index (MI) to identify the angles which potentially benefit from adding more SPs.⁵ Kim *et al.*⁶ proposed to use beams-eye-view dosimetric (BEVD) as guidance for beam selection and total-variation regularization term as aperture generation method. This method requires a large amount of memory as all the beamlet dose deposition matrices need to be available for optimization. Zarepisheh *et al.*⁷ integrated three optimization techniques consisting of column generation, subgradient method, and pattern search to tackle this large scale optimization problem. However, the authors stopped at ~ 36 SPs per plan, which are too few for a rotational arc delivery. Another technique called FusionArc⁸ improved original VMAT plans by inserting 3–7 IMRT beams according to the gradient of the objective function. The IMRT beams in their implementation are beamlet based and an additional sequencing step is required.

The aim of this work is to present a practical approach for SPORT optimization based on and improved upon previous methods. An effective technique for SPs selection and aperture

generation is proposed for coplanar arc SPORT treatment. The technique is applied to three clinical cases (prostate, head and neck, and brain), representing different optimization complexities and challenges, and the results are compared with conventional VMAT planning.

FIG. 2. Flowchart of the SPORT optimization process: 1 and 2: For each of the 180 candidate station points, one candidate aperture is produced by the column generation method. 3: All those candidate apertures are added into the POGS solver to solve for the optimal weight for each aperture. Note that for some of the apertures, the optimal weight will be zero. 4: Decide if the optimized plan is clinically acceptable. 5 and 6: The gradient information is sent back to step 1 and 2, helping generate the next set of apertures. 7 and 8: The apertures with nonzero weights calculated in step 3 are combined into the candidate apertures from step 1 and 2, while the apertures with zero weights are eliminated.

FIG. 3. Dose distribution of SPORT and VMAT for the prostate case.

2. METHODS

The master optimization problem is to select a group of SPs which satisfies a set of clinical goals within delivery restrictions such as treatment time, monitor units, and avoidance of collision. This is commonly formulated as

$$\text{Min}_x f(d_i) \quad \text{subject to } d_i = \sum_{\alpha} D_{\alpha i} x_{\alpha} \quad x \geq 0, \quad (1)$$

where d_i represents the dose to voxel i expressed as the sum of the product of $D_{\alpha i}$, dose deposited to voxel i from station α , and x_{α} , the fluence of station α . The objective function f takes the form of a weighted linear function.⁹ It consists of four parts: (1) Max (min) dose of OARs (PTV); (2) mean dose of OARs and PTV; (3) dose volume constraints modeled as piecewise linear functions;^{10,11} and (4) maximum dose constraints for OARs. For the weights associated with items (1)–(3), we choose initial values in the range of 1–10 and allow these weights to be manually adjusted during the optimization. Typically a low weight for the PTV objective promotes greater OAR sparing for early iterations, while a high weight allows more PTV coverage later. Another term in the objective function is the total station weight $\sum x_{\alpha}$, that is, the total monitor units, which plays a critical role in limiting the scattering dose and indirectly controlling the aperture sizes.

Equation (1) is a nondeterministic polynomial-time hard (NP-hard) optimization problem¹² if the total number of SPs

FIG. 5. Dose distribution of SPORT and VMAT for the head and neck case in the transverse and coronal plane.

is constrained to a fixed value, which is the case in this study. Column generation^{13,14} is an appealing solution to solve this problem. Briefly, for each beam direction, we calculated a desirability score for all the beamlets, according to

$$\sum_i \pi_i D_{\alpha i}, \quad \text{where } \pi_i = \frac{\partial f(d_i)}{\partial d_i}, \quad (2)$$

where π_i measures the per unit change of the objective function if the dose to the voxel i is increased and the summation describes the unit change if the beamlet intensity is increased. Row by row, in the direction of MLC leaf movement, we find the contiguous subgroup of beamlets which has the largest sum of π_i . This is a maximum subarray problem that can be readily solved by Kadane's algorithm,¹⁵ which can be found in Men *et al.*¹³ Figure 1 illustrates the introduced method. After the apertures for all angles

FIG. 4. DVH for the prostate case.

FIG. 6. DVH for the head and neck case.

FIG. 7. DVH for the head and neck case.

were determined, we optimized the objective function to find the weight of each aperture. To avoid being trapped in a local minimum in beamlet-based aperture selection using the gradient descent algorithm, we employed stochastic gradient descent (SGD),¹⁶ where a 50% noise term was added to the absolute value of the gradient π_i without changing the sign. This modification helped the solver to escape from local minima, and lead to a faster convergence. The aperture(s) that has a weight close to zero was thrown away and replaced with new generated apertures in the next iteration. Up to this point, the optimization was simply aperture based optimization or VMAT optimization. To find out whether there was room for further improvement in the plan by adding more apertures or SPs, we repeated the above procedure again with consideration of the existing dose distribution from the last iteration. At the end of the second iteration, we reoptimized the weights of all the apertures, including those of the first iteration. The above procedure repeated until the plan could not be improved any further. Figure 2 illustrates the whole optimization process.

The final total number of apertures for both VMAT and SPORT was constrained to a user defined number, e.g., 180 in our study. Without such constraint, the algorithm would tend to pick smaller apertures including only a couple of beamlets, resulting in a large total MU and

FIG. 8. Dose distribution of SPORT and VMAT for the brain case.

FIG. 9. DVH for the brain case.

slowing down the dose delivery significantly. Additionally, the use of the same number of permissible (180) apertures ensures that the comparison between VMAT and SPORT is fair, as the plan quality and deliver efficiency may likely be affected by the number of apertures in the treatment.

For each planning study, we calculated the dose deposition matrix for 180 beams from the angle of 0° to 359° , consisting of typically 200–900 beamlets per gantry angle. In SPORT, multiple apertures were allowed per angle to achieve fine angle resolution of SP distribution. For example, if there were five apertures selected in an angular sector, this sector would be divided into five subsectors during arc sequencing under the assumption that the dosimetric change is negligible when the SPs are angularly varied within one degree.¹⁷

The aperture weights were optimized with proximal operator graph solver (POGS),^{18,19} an open sourced solver with GPU acceleration support for convex optimization of the form

FIG. 10. DVH for the brain case.

TABLE I. Delivery time and total monitor units estimated for single arc SPORT and VMAT with 2 Gy of dose delivery.

Site	VMAT (min)	SPORT (min)	VMAT (MU)	SPORT (MU)
Prostate	4.85	3.78	762	791
HN	5.96	5.78	632	643
Brain	2.92	2.36	841	892

$$\text{minimize } g_1(y) + g_2(x) \text{ subject to } y = Ax, \quad (3)$$

where g_1 and g_2 are convex and can take on the values $R \cup \{\infty\}$. In our specific problem, $y = Ax$ represents the dose y calculated from the dose deposition matrix A and aperture MU x . g_1 computes the mean, max, and min of the dose y and g_2 sums the MU of each aperture. It is beyond the scope of this paper to introduce this solver, however, we would like to discuss the utilization of different step size (penalty parameter) ρ for early and later iterations. A large value of ρ tends to encourage primal feasibility, while a small value tends to encourage dual feasibility.²⁰ In other words, with a small step size, the solution can reach a minimum quickly at the expense of violating certain dose constraints and the stability of the solution, whereas a large fixed step size leads to slow convergence with all dose constraints satisfied. Here, we used a fixed step size for each iteration that increased as more apertures were included in the plan. In this way, the early iterations allowed the objective function to decrease quickly and enabled the algorithm to traverse more of the solution space. The step size increased at later iterations and ensured the convergence of the optimization.

In order to achieve a fair comparison between SPORT and VMAT, we used the same procedure to generate both plans and kept the objective function weightings as same values without any parameter tuning and plan tweaking afterward, with the only difference being that SPORT allows multiple apertures at each SP.

FIG. 11. Gantry speed variation for all six single arc plans.

FIG. 12. Dose distribution of single-arc SPORT and dual-arc VMAT for a prostate case.

Furthermore, we constrained the leaf traveling time between adjacent apertures to 1 s during the early optimization iterations. After the solution converges to an ideal distribution at a later optimization stage, this constraint is then tightened to 0.4 s, giving the optimizer opportunities to replace apertures causing longer leaf traveling with ones that are more similar to adjacent apertures, thus leading to a shortening of the total delivery time by ~15%.

The dosimetry comparison between VMAT and SPORT was performed for three different disease sites: a prostate, head and neck, and brain case. The dose calculation for all three cases was carried out using CERR's QIB algorithm,²¹ with a dose grid of $4 \times 4 \times 1.5 \text{ mm}^3$ and MLC leaf size of 2.5 mm for the brain case, and 5 mm for the other two cases. The candidate apertures were generated from all 180 angles with a 2° spacing. Both treatment techniques made use of 180 angular points in the final plan, the difference being that multiple apertures were allowed at an angle in SPORT.

3. RESULTS

Figures 3–10 present the DVH and dose distribution data for the three cases. Marked dose sparing is observed for bladder and rectum in the prostate SPORT treatment with similar PTV coverage for both VMAT and SPORT (Figs. 3 and 4). The volume at 50% of prescription dose is decreased by

FIG. 13. DVH for the prostate patient with dual-arc VMAT and single-arc SPORT.

FIG. 14. Dose distribution of single-arc SPORT and dual-arc VMAT for a head and neck case.

22% for rectum and 6% for bladder. Both VMAT and SPORT achieve satisfactory multilevel (5) PTV coverage for the head and neck case (Figs. 5–7); SPORT improves the mean dose of the left and right parotids by 15%. The maximum dose is lowered from 72.7 to 71.7 Gy for the mandible, and from 30.7 to 27.3 Gy for the spinal cord. The mean dose for pharynx and larynx is improved by 8% and 6%, respectively. For the brain case with a small target ($\sim 10 \text{ cm}^3$) and simple geometry, no difference is observed in PTV coverage and the doses to brain and brain stem; however, dose sparing for eyes, chiasm, and inner ears is improved in SPORT plan. The conformity indices for SPORT and VMAT are essentially the same for three cases. To compare the low dose spread between the two treatment modalities, we calculated the volumes of the patient body receiving at least 10% (V10) and 20% (V20) of the prescription dose. The SPORT improved the prostate V10, brain V10, head and neck V20, and prostate V20 by 0.7%, 1%, 3.5%, and 1.4% compared with VMAT, which had a 0.5% better head and neck V10 and a 1.5% better brain V20.

Table I shows the delivery time for the three cases, estimated by taking the longest of the time spent on gantry rotation (max $v_g = 6 \text{ deg/s}$), dose delivery (max $d = 600 \text{ MU/min}$),

FIG. 15. DVH for the H and N patient with dual-arc VMAT and single-arc SPORT.

FIG. 16. DVH for the H and N patient with dual-arc VMAT and single-arc SPORT.

and MLC leaf movement (max $v_l = 2.5 \text{ cm/s}$) when the linac moves from one SP to the next according to the following equation:

$$\sum_{i=1}^p \max \left(\frac{\Delta\theta}{v_g}, \frac{\Delta\text{MU}}{d}, \frac{\max l_j}{v_l} \right), \quad (4)$$

where p denotes the number of SPs, $\Delta\theta$ for the gantry angle difference between two stations, ΔMU for the MUs delivered, and l_j for the leaf bank movement distance. After the dwell time for each SP is calculated by Eq. (4), the actual dose rate, gantry speed, and leaf movement speed will be obtained with one of the three reaching the maximum value and the other two lower than the maximum values, thus achieving the fastest delivery possible. For example, if the leaf movement is the limiting factor, then the MLC will move at the maximum speed of 2.5 cm/s , while the gantry rotation speed and dose rate are adjusted to lower values accordingly. Clinically this is handled internally by the digital linac.

SPORT shortens the treatment time by $\sim 1 \text{ min}$ for the prostate case, $\sim 0.5 \text{ min}$ for the brain case, and $\sim 0.2 \text{ min}$ for the head and neck case, respectively. Figure 11 clearly shows that SPORT increases the gantry speed to the maximum when sweeping through unselected SPs to save treatment time, which VMAT is incapable of because SPs are uniformly distributed and cannot be skipped. As seen in the histogram (Fig. 18), the SPORT plan has a larger percentage of small apertures compared with VMAT plan in each case, which directly translates to a modest increase in total monitor units

TABLE II. Treatment time and total monitor units of single-arc SPORT and dual-arc VMAT plans with the delivery of 2 Gy dose.

Site	VMAT (MU)	SPORT (MU)	VMAT (min)	SPORT (min)
Prostate	748	711	3.22	2.59
H&N	865	722	3.59	2.83

FIG. 17. The MU plus the number of apertures versus beam direction for three SPORT plans in a stacked bar plot. The height of the bar shows the total MU for that direction and stacked color bar represents the mu for each aperture belonging to that direction, with each color illustrating a unique aperture that belongs to that direction.

delivered for SPORT plans (Table I). With multiple apertures allowed for a SP, the SPORT solver has more freedom in picking apertures with less MLC travel, resulting in shorter delivery time.

Tradeoff between delivery time and plan quality can be further achieved in VMAT by introducing dual arcs or SPORT by limiting the number of apertures. Figures 12–16 show the DVH and dose distribution for the dual-arc VMAT plan and single-arc SPORT plan with 100 apertures. For the prostate plan, SPORT and VMAT plans are similar with SPORT sparing the rectum better with a mean dose of 8.4 vs 8.8 Gy for the VMAT. For the head and neck case, the SPORT plan renders lower dose to parotid, larynx and pharynx, and higher dose to spinal cord and mandible, while overdosing PTV56 slightly. While saving 5% and 20% in monitor units for the prostate and head and neck cases, the SPORT plans cut back delivery time by 0.6 and 0.8 min as shown in Table II.

4. DISCUSSION

Historically, there are a number of intensity modulation techniques to deliver high doses of radiation to the tumor while sparing the surrounding critical structures, including fixed-gantry IMRT (typically, with 5–10 beams) and VMAT (typically with 1–3 arcs). Each of these techniques captures certain aspect(s) of the desirable features of SPORT, but compromises in either dose distribution or delivery efficiency. Indeed, conventional IMRT often does not possess sufficient angular sampling required to spatially shape the doses for complicated clinical cases, whereas VMAT does not provide the desired beam intensity modulation in some or all directions and thus requires multiple arcs. The new treatment modality, SPORT, is achieved by increasing the angular sampling (including noncoplanar sampling) of radiation beams while eliminating dispensable segments of the incident fields. The method takes advantage of desirable features of both VMAT

FIG. 18. Histogram of the aperture sizes (cm^2) in each aperture.

FIG. 19. Histogram of aperture sizes (cm^2) in each aperture for SPORT plans: Objective function including total MU (blue) and not (red). (See color online version.)

and IMRT, thus it presents a truly optimal RT scheme with uncompromised angular sampling, beam modulation, delivery efficiency, and possibly energy and collimator modulation²² when going from one gantry angle to another.

SPORT treatment planning is to select an ensemble of SPs from the vast pool of candidate SPs so that the resultant dose distribution is optimal. In this work, we have developed a computationally efficient algorithm for SPORT plan optimization and shown that rotational arc SPORT matches VMAT in both plan quality and delivery efficiency. A salient feature of SPORT implementation here is that multiple apertures are allowed at a gantry angle when deemed beneficial by the greedy algorithm based on column generation¹³ and stochastic gradient descent.¹⁶ The number of apertures for the complicated head and neck case can be as many as six at some gantry angles, whereas for the simple prostate and brain cases, only one aperture appears for most angles (Fig. 17). The added apertures in our SPORT tend to be smaller ones, as shown in Fig. 18. We note that treatment time—especially MLC travel time—can be directly computed and compared with VMAT treatment in our system.

If not constrained, MUs of some apertures can be excessive in SPORT/VMAT optimization. The effectiveness of MU

suppression by introducing a total MU term in the objective function is illustrated in Fig. 19. The number of larger apertures is doubled or tripled after including this term.

Previous column generation method^{13,14} limits the optimizer to one new aperture per iteration. The proposed technique here generates a new set of apertures, each for a gantry angle, at each iteration. The ultimate usefulness of the assigned apertures is determined by the optimization solver. Apertures with large fluence are kept and small fluence is thrown away. We found that this decreases the optimization time by five–ten-fold. The performance of the optimization is further improved by introducing a certain randomness in the gradient of the objection function (π_i) in the selection of the candidate apertures, which helps to prevent the optimization from being trapped at a local minimum like the red dotted SPORT (CG) line in Fig. 20. Our SGD based method works very well by experimenting and comparing with the classic CG. As a greedy algorithm utilizing the steepest direction of descent, classic CG method can be trapped in some local points such as a local minimum or saddle point. For high dimensional nonconvex problems such as the direct aperture optimization, we hypothesize that local minimum points are very rare, since it is hard to construct a high dimensional trap, within which the objective function along every dimension

FIG. 20. Objective function value versus iteration numbers in a log scale for the head and neck case. Column generation: CG. Stochastic gradient descent: SGD.

FIG. 21. Illustration for the CG (black line plot) method and SCG (red plot) on a saddle surface. (See color online version.)

decreases at the same time. It is far more common to have saddle points. As illustrated in Fig. 21 for a 3D case, the black line plots the optimization path the greedy algorithm takes on a saddle surface ($z = x^2 - y^2$) in the steepest gradient direction. When it reaches the saddle point (black point), the optimization is trapped, since the gradient is zero in every direction. However, if we add some noise in the gradient, the optimization can escape the saddle point in a few iterations as shown by the red line plot. The machine learning community routinely utilizes this concept to optimize large scale learning problems.¹⁶

The speed up is also made possible by the GPU parallelized POGS solver, capable of finishing an iteration within a second using a GTX titan X GPU card and i7 4790 CPU with 32 GB of memory. To reach an optimal plan, it takes 54, 33, and 16 s for the head and neck case, prostate case, and brain case without any weight tweaking. The step size ρ in POGS is tunable during optimization. We use smaller values (nominally 0.1) at early iterations and larger values (nominally 5) at a later stage, similar to the method of global search and local refinement, commonly used in the VMAT optimization publications. In the early phase, an approximate solution can be reached within several iterations. This makes it possible for users to see the plan quality trade off in seconds after changing the relative weightings between the OARs and PTV. Once the user settles on a set of weights, the optimization process will proceed to the later phase and find an optimal dose distribution.

SPORT utilized a slightly larger percentage of smaller apertures compared with VMAT, which may raise concerns about increased dosimetric uncertainties for small PTV targets, introduced by charge particle disequilibrium, detector size, positioning, etc. Those apertures sizes are similar to what typical SRS/SRT would use²³ and similar techniques and precautions could be readily applied to SPORT treatment to decrease the uncertainties.^{24,25} In the cases that small apertures should be absolutely avoided, we can control the candidate aperture sizes directly in steps 1 and 2 (Fig. 1) of the optimization process, thus eliminating any uncertainties associated with smaller apertures. The large acceleration and deceleration of the gantry head in the SPORT could become a source of dosimetric error and needs to be investigated further during simulated machine delivery. If it is proven to be a problem, stricter gantry speed variation constraints can be easily enforced in the optimization steps 1 and 2.

5. CONCLUSION

An effective optimization technique has been introduced for SPORT and VMAT optimization. The technique combines the column generation method, stochastic gradient descent, and GPU-based computation, and is particularly suitable for large-scale SPORT optimization problem. The performance of the proposed technique has been demonstrated by using three clinical cases.

Our preliminary results showed that SPORT provides a way for us to effectively utilize the technical capacity of digital

linacs and generates plans that match or improve conventional VMAT. Further systematic comparison with more cases in the future will be useful to firmly establish SPORT as a viable approach for radiation therapy.

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CONFLICT OF INTEREST DISCLOSURE

The authors have no conflicts of interest to disclose.

^{a)}Author to whom correspondence should be addressed. Electronic mail: lei@stanford.edu

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