Passivity of memristor-based BAM neural networks with different memductance and uncertain delays

R. Anbuviithya¹ · K. Mathiyalagan² · R. Sakthivel^{3,4} · P. Prakash⁵

Received: 22 July 2015 / Accepted: 13 April 2016 / Published online: 27 April 2016
© Springer Science+Business Media Dordrecht 2016

Abstract This paper addresses the passivity problem for a class of memristor-based bidirectional associate memory (BAM) neural networks with uncertain time-varying delays. In particular, the proposed memristive BAM neural networks is formulated with two different types of memductance functions. By constructing proper Lyapunov–Krasovskii functional and using differential inclusions theory, a new set of sufficient condition is obtained in terms of linear matrix inequalities which guarantee the passivity criteria for the considered neural networks. Finally, two numerical examples are given to illustrate the effectiveness of the proposed theoretical results.

Keywords Memristor · BAM neural networks · Passivity · Linear matrix inequality · Uncertain delay

Introduction

Neural networks have been paid much attention for the past two decades since they have wide applications in the fields of signal processing, biological sciences, associate memories,

combinational optimization, automatic control and so on. Recently, as a special kind of neural networks namely memristor-based neural networks have attracted a lot of attention. Memristor which is considered to be the fourth passive circuit element which was first predicted by Chua (1971) and its first passive implementation realized by the Hewlett-Packard research team in 2008 (Strukov et al. 2008). The new circuit element of memristor shares many properties of resistor and shares the same unit of measurement (ohm) (Cantley et al. 2012; Zhang et al. 2012). It is proved that the memristor exhibits the feature of pinched hysteresis the same as the neurons in the human brain and because of this feature, several researchers construct the memristor-based neural networks to emulate the human brain through replacing the resistor by the memristor. Subsequently, the study of the dynamics of memristor-based neural networks has been reported in Kim et al. (2012), Pershin and Ventra (2012), Sun et al. (2013), Wang et al. (2014), Wen et al. (2012), Zhang et al. (2013), Yang et al. (2015), Li and Cao (2015), Chandrasekar et al. (2014), Hu et al. (2015), Kumar et al. (2016) and Yang et al. (2014) for the purpose of achieving its successful applications in real-world problems. Moreover, Zhang et al. (2015) adopted nonsmooth analysis and control theory to handle memristive neural networks with discontinuous right-hand side to ensure the ultimate boundedness and global exponential attractivity of the memristor based neural networks in the sense of Filippov solutions.

The stability analysis for a class of two-layer hetero associative neural networks, called bidirectional associate memory neural networks has played an important role because of its widespread applications in many fields of science and engineering. BAM neural networks were originally introduced in Kosko (1987), it practically displays many nice properties due to its special structure of connection weights. The problem of delay-dependent exponential passivity

---

✉ R. Sakthivel
krsakthivel@yahoo.com

¹ Department of Mathematics, National Institute of Technology, Tiruchirappalli 620 015, India

² Department of Mathematics, Anna University-Regional Centre, Coimbatore 641 047, India

³ Department of Mathematics, Sri Ramakrishna Institute of Technology, Coimbatore 641 010, Tamil Nadu, India

⁴ Department of Mathematics, Sungkyunkwan University, Suwon 440-746, The Republic of Korea

⁵ Department of Mathematics, Periyar University, Salem 636 011, India

analysis for BAM neural networks with time-varying delays is investigated in Du et al. (2013). Under the framework of Filippov solutions, Cai and Huang (2014) investigated the viability and dissipativity of solutions for memristive BAM neural networks by using the matrix measure approach and generalized Halanay inequalities.

Among all the dynamical systems, the time-delay systems have been a topic of great interest over the past decades. There are many kinds of delays in dynamic systems, such as constant delays, time-varying delays, switched time-delays, mixed time-delays, distributed time-delays, mode-dependent time-delays and so on. However, in reality, time-delay is frequently encountered in neural networks which is often a source of instability, divergence and oscillations. The problem of delay-dependent robust exponential stability in mean square for a class of uncertain stochastic Hopfield neural networks with discrete and distributed time-varying delays has been studied in Li et al. (2008). Recently, Sakthivel et al. (2015) studied the state estimation for BAM neural networks with mixed delays which includes a constant delay in the leakage term, time-varying discrete delay and constant distributed delay and Cao and Wan (2014), discussed a single inertial BAM neural network with time-varying delays and external disturbance inputs. Further, many important results on the dynamical behaviors for time delayed neural networks have been reported in Ji et al. (2014), Kwon et al. (2014), Li et al. (2009, 2014), Sakthivel et al. (2012), Cao et al. (2013) and Qi et al. (2014). Another major problem in real-world systems is uncertainty which is unavoidable and exists in many practical systems, it also the source for instability and deterioration of performance of system (Roh and Oh 1999). Specifically, the uncertainty may include modeling error, parameter perturbations, nonlinear approximation errors and external disturbances. The robust non-fragile H_∞ control problem has been investigated for uncertain stochastic time-delay systems containing sector nonlinearities in Tian et al. (2007). So it is important to study about the time delay in the presence of uncertainties.

On the other hand, an essential property in circuits and system theory is passivity or positive realness which is applicable to the properties of hybrid matrices of various classes of networks, analysis of the stability of dynamical

systems, inverse problem of linear optimal control, Popov criterion and spectral factorization by algebra (Anderson and Vongpanitlerd 1973). The main aim of passivity theory is that the passive properties of system can keep the system internally stable. Recently, the problem of passivity analysis for various kind of neural networks has been investigated (see Hu et al. 2013; Lou and Cui 2007; Sakthivel et al. 2011; Song and Cao 2012; Zhu et al. 2010; Ali et al. 2016; Rakkiyappan et al. 2015 and references therein). In Wen et al. (2013), the problem of exponential passivity for memristor-based recurrent neural networks with time-varying delays has been investigated. More recently, the exponential passivity analysis for memristor recurrent neural networks with constant time delays is studied in Wu and Zeng (2014). A preliminary study concerning the dynamic flows in memristor-based wavelet neural networks with continuous feedback functions and discontinuous feedback functions in the presence of different memductance functions has been reported in Wu et al. (2013). However, to the best of our knowledge, the research on delay-dependent passivity analysis of memristor-based BAM neural networks is still an open problem that deserves further investigation.

Motivated by the above discussions, in this paper, we investigate the passivity of memristive BAM neural networks with uncertain differentiable time delays. Moreover, in this paper we deal with two different types of memductance functions, in which memristive neural networks may display different types of dynamic features. A new type of Lyapunov functional is constructed to derive the novel delay-dependent passivity conditions. The results are established in the framework of LMIs and that can be solved using available numerical software. Finally, numerical examples are given to illustrate the usefulness of the proposed results.

Problem formulation and preliminaries

Model description

In this paper, we consider a general class of BAM neural networks based on the Kirchhoff's current law, the i -th and j -th subsystems are described by the following system of equations;

$$\begin{cases} \dot{p}_i(t) = -p_i(t) + \frac{1}{\bar{C}_i} \sum_{j=1}^m \frac{f_j(q_j(t))}{\mathcal{R}_{ji}} \times \text{sgn}_{ji} + \frac{1}{\bar{C}_i} \sum_{j=1}^m \frac{f_j(q_j(t - \tau(t)))}{\mathcal{F}_{ji}} \times \text{sgn}_{ji} + \frac{\mathcal{I}_i(t)}{\bar{C}_i}, \\ \dot{q}_j(t) = -q_j(t) + \frac{1}{\bar{C}_j} \sum_{i=1}^n \frac{g_i(p_i(t))}{\tilde{\mathcal{R}}_{ij}} \times \text{sgn}_{ij} + \frac{1}{\bar{C}_j} \sum_{i=1}^n \frac{g_i(p_i(t - \tau(t)))}{\tilde{\mathcal{F}}_{ij}} \times \text{sgn}_{ij} + \frac{\mathcal{J}_j(t)}{\bar{C}_j}, \end{cases} \quad t \geq 0, \quad (1)$$

where $\text{sgin}_{ji} = \text{sgin}_{ij} = 1$ for $i \neq j$ and -1 for $i = j$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$; $p_i(t)$ is the voltage of the capacitor C_i , $q_j(t)$ is the voltage of the capacitor $\tilde{C}_j$; $\mathcal{R}_{ji}$ denotes the resistor between the continuous feedback function $f_j(q_j(t))$ and $p_i(t)$; $\mathcal{F}_{ji}$ denotes the resistor between the continuous feedback function $f_j(q_j(t - \tau(t)))$ and $p_i(t)$; $\tilde{\mathcal{R}}_{ij}$ denotes the resistor between the continuous feedback function $g_i(p_i(t))$ and $q_j(t)$; $\tilde{\mathcal{F}}_{ij}$ denotes the resistor between the continuous feedback function $g_i(p_i(t - \tau(t)))$ and $q_j(t)$; $\mathcal{I}_i(t)$ denotes the continuous external input on the i th unit; $\mathcal{J}_j(t)$ denotes the continuous external input on the j th unit and the non negative continuous variable $\tau(t)$ corresponds to the transmission time-varying delay that satisfies $0 \leq \tau(t) = \tau + \Delta_\tau(t)$, where $\tau \geq 0$ is a known upper bound, delay term $\tau(t)$ is assumed to be differentiable and satisfies $\dot{\tau}(t) \leq \mu$, where μ is the positive constant. Also, $|\Delta_\tau(t)|$ denotes the uncertain element in time-varying delay which satisfies $|\Delta_\tau(t)| \leq \delta_\tau$, where δ_τ is a positive constant.

(A1) For $i \in \{1, 2, \dots, n\}$ and $j \in \{1, 2, \dots, m\}$, the activation functions f_j and g_i are continuous function, for all p_1, p_2, q_1 and $q_2 \in \mathbb{R}$, there exist positive numbers e_j and l_i such that

$$0 \leq \frac{f_j(q_1) - f_j(q_2)}{q_1 - q_2} \leq e_j, \quad f_j(0) = 0, \quad q_1 \neq q_2,$$

$$0 \leq \frac{g_i(p_1) - g_i(p_2)}{p_1 - p_2} \leq l_i, \quad g_i(0) = 0, \quad p_1 \neq p_2.$$

Also, it follows from (1) that

$$\begin{cases} \dot{p}_i(t) = -p_i(t) + \sum_{j=1}^m a_{ji}^* f_j(q_j(t)) + \sum_{j=1}^m b_{ji}^* f_j(q_j(t - \tau(t))) + u_i(t), \\ \dot{q}_j(t) = -q_j(t) + \sum_{i=1}^n c_{ij}^* f_i(p_i(t)) + \sum_{i=1}^n d_{ij}^* f_i(p_i(t - \tau(t))) + v_j(t). \end{cases} \quad (2)$$

where $a_{ji}^* = \frac{\text{sgin}_{ji}}{C_i \mathcal{R}_{ji}}$, $b_{ji}^* = \frac{\text{sgin}_{ji}}{C_i \mathcal{F}_{ji}}$, $c_{ij}^* = \frac{\text{sgin}_{ij}}{C_j \tilde{\mathcal{R}}_{ij}}$, $d_{ij}^* = \frac{\text{sgin}_{ij}}{C_j \tilde{\mathcal{F}}_{ij}}$, $u_i(t) = \frac{\mathcal{I}_i(t)}{C_i}$, $v_j(t) = \frac{\mathcal{J}_j(t)}{C_j}$.

By replacing the resistors $\mathcal{R}_{ji}$, $\mathcal{F}_{ij}$, $\tilde{\mathcal{R}}_{ij}$ and $\tilde{\mathcal{F}}_{ij}$ in the primitive BAM neural networks (1) or (2) with memristors, whose memductances are W_{ji} , M_{ji} , $\tilde{W}_{ij}$ and $\tilde{M}_{ij}$ respectively, then we can construct the memristor-based BAM neural networks in the following form

$$\begin{cases} \dot{p}_i(t) = -p_i(t) + \sum_{j=1}^m a_{ji} f_j(q_j(t)) + \sum_{j=1}^m b_{ji} f_j(q_j(t - \tau(t))) + u_i(t), \\ \dot{q}_j(t) = -q_j(t) + \sum_{i=1}^n c_{ij} g_i(p_i(t)) + \sum_{i=1}^n d_{ij} g_i(p_i(t - \tau(t))) + v_j(t), \end{cases} \quad (3)$$

where $a_{ji}(p_i(t)) = \frac{W_{ji}}{C_i} \times \text{sgin}_{ji}$, $b_{ji}(p_i(t)) = \frac{M_{ji}}{C_i} \times \text{sgin}_{ji}$, $c_{ij}(q_j(t)) = \frac{\tilde{W}_{ij}}{C_j} \times \text{sgin}_{ij}$, $d_{ij}(q_j(t)) = \frac{\tilde{M}_{ij}}{C_j} \times \text{sgin}_{ij}$. Taking the past results of MRNNs into consideration, under different

pinched hysteresis loops the evolutionary tendency or process of memristive systems evolves into different forms. It is generally known that the pinched hysteresis loop is due to the nonlinear relationship between the memristance current and voltage. As two typical memductance functions, in this paper, we discuss the following two cases.

Case 1 The memductance functions are given by

$$W_{ji} = \begin{cases} a'_{ji}, & |\sigma_{ji}| < l_{ji}, \\ b'_{ji}, & |\sigma_{ji}| > l_{ji}, \end{cases} \quad M_{ji} = \begin{cases} \bar{a}'_{ji}, & |\bar{\sigma}_{ji}| < \bar{l}_{ji}, \\ \bar{b}'_{ji}, & |\bar{\sigma}_{ji}| > \bar{l}_{ji}, \end{cases}$$

$$\tilde{W}_{ij} = \begin{cases} c'_{ij}, & |\tilde{\sigma}_{ij}| < \tilde{l}'_{ij}, \\ d'_{ij}, & |\tilde{\sigma}_{ij}| > \tilde{l}'_{ij}, \end{cases} \quad \tilde{M}_{ij} = \begin{cases} \bar{c}'_{ij}, & |\tilde{\sigma}_{ij}| < \bar{\tilde{l}}'_{ij}, \\ \bar{d}'_{ij}, & |\tilde{\sigma}_{ij}| > \bar{\tilde{l}}'_{ij}, \end{cases}$$

where a'_{ji} , b'_{ji} , $\bar{a}'_{ji}$, $\bar{b}'_{ji}$, c'_{ij} , d'_{ij} , $\bar{c}'_{ij}$, $\bar{d}'_{ij}$, l_{ji} , $\bar{l}_{ji}$, $\tilde{l}'_{ij}$ and $\bar{\tilde{l}}'_{ij}$ are constants for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$. Further, σ_{ji} , $\bar{\sigma}_{ji}$, $\tilde{\sigma}_{ij}$ and $\bar{\tilde{\sigma}}_{ij}$ denote magnetic fluxes corresponding to memristors W_{ji} , M_{ji} , $\tilde{W}_{ij}$ and $\tilde{M}_{ij}$ respectively.

Case 2 We consider a state-dependent continuous system. Here, the memductance functions are given by

$$W_{ji} = c_{ji} + 3e_{ji}\sigma_{ji}^2 \quad \text{and} \quad M_{ji} = \bar{c}_{ji} + 3\bar{e}_{ji}\bar{\sigma}_{ji}^2$$

$$\tilde{W}_{ij} = c'_{ij} + 3e'_{ij}\tilde{\sigma}_{ij}^2 \quad \text{and} \quad \tilde{M}_{ij} = \bar{c}'_{ij} + 3\bar{e}'_{ij}\bar{\tilde{\sigma}}_{ij}^2$$

where c_{ji} , e_{ji} , $\bar{c}_{ji}$, $\bar{e}_{ji}$, c'_{ij} , e'_{ij} , $\bar{c}'_{ij}$ and $\bar{e}'_{ij}$ are constants for $i = 1, \dots, n$ and $j = 1, \dots, m$.

Remark 2.1 It is well known that memristive neural networks may display different types of dynamic features in the presence of different memductance functions, i.e., state dependent switched system and continuous system. Although the analytical method is based on two different theory architectures, the proposed criteria is very similar. The unified form of criterion is an effective methodology of enhancing the proposed criterion to be easily applied to different situations.

Also, by the features of memristor given in Case 1 and Case 2, the following two cases will happen.

Case 1a By Case 1, $a_{ji}(p_i(t))$, $b_{ji}(p_i(t))$, $c_{ij}(q_j(t))$ and $d_{ij}(q_j(t))$ are switching jumps and

$$a_{ji}(p_i(t)) = \begin{cases} \hat{a}_{ji}, & |p_i(t)| < \Lambda_i \\ \tilde{a}_{ji}, & |p_i(t)| > \Lambda_i, \end{cases} \quad b_{ji}(p_i(t)) = \begin{cases} \hat{b}_{ji}, & |p_i(t)| < \Lambda_i \\ \tilde{b}_{ji}, & |p_i(t)| > \Lambda_i, \end{cases}$$

$$c_{ij}(q_j(t)) = \begin{cases} \hat{c}_{ij}, & |q_j(t)| < \tilde{\Lambda}_j \\ \tilde{c}_{ij}, & |q_j(t)| > \tilde{\Lambda}_j, \end{cases} \quad d_{ij}(q_j(t)) = \begin{cases} \hat{d}_{ij}, & |q_j(t)| < \tilde{\Lambda}_j \\ \tilde{d}_{ij}, & |q_j(t)| > \tilde{\Lambda}_j. \end{cases}$$

in which the switching jumps Λ_i and $\tilde{\Lambda}_j$ are all non-negative, a_{ji} , b_{ji} , c_{ij} and d_{ij} are known constant numbers.

According to the above analysis, $a_{ji}(p_i(t))$, $b_{ji}(p_i(t))$, $c_{ij}(q_j(t))$ and $d_{ij}(q_j(t))$ are varying according to the states $p_i(t)$, $p_i(t - \tau(t))$, $q_j(t)$ and $q_j(t - \tau(t))$ of the system, respectively. It is clear that $a_{ji}(p_i)$ and $b_{ji}(p_i)$ (similarly, $c_{ij}(q_j)$ and $d_{ij}(q_j)$) are discontinuous or piecewise

continuous functions with at most two points of discontinuity $\pm\Lambda_i$ ($\pm\tilde{\Lambda}_j$). Meanwhile, the discontinuous functions $a_{ji}(p_i)$ and $b_{ji}(p_i)$ (similarly, $c_{ij}(q_j)$ and $d_{ij}(q_j)$) are undefined at the points of discontinuity $\pm\Lambda_i$ ($\pm\tilde{\Lambda}_j$). Let $\bar{co}[\mathbb{E}]$ denote the closure of the convex hull of given set $\mathbb{E}$. Obviously, we can obtain that

$$\bar{co}[a_{ji}(p_i)] = \begin{cases} \hat{a}_{ji}, & |p_i| < \Lambda_i, \\ [\underline{a}_{ji}, \bar{a}_{ji}], & |p_i| = \Lambda_i, \\ \check{a}_{ji}, & |p_i| > \Lambda_i, \end{cases} \quad \bar{co}[b_{ji}(p_i)] = \begin{cases} \hat{b}_{ji}, & |p_i| < \Lambda_i, \\ [\underline{b}_{ji}, \bar{b}_{ji}], & |p_i| = \Lambda_i, \\ \check{b}_{ji}, & |p_i| > \Lambda_i, \end{cases}$$

$$\bar{co}[c_{ij}(q_j)] = \begin{cases} \hat{c}_{ij}, & |q_j| < \tilde{\Lambda}_j, \\ [\underline{c}_{ij}, \bar{c}_{ij}], & |q_j| = \tilde{\Lambda}_j, \\ \check{c}_{ij}, & |q_j| > \tilde{\Lambda}_j, \end{cases} \quad \bar{co}[d_{ij}(q_j)] = \begin{cases} \hat{d}_{ij}, & |q_j| < \tilde{\Lambda}_j, \\ [\underline{d}_{ij}, \bar{d}_{ij}], & |q_j| = \tilde{\Lambda}_j, \\ \check{d}_{ij}, & |q_j| > \tilde{\Lambda}_j, \end{cases}$$

where $\bar{a}_{ji} = \max\{\hat{a}_{ji}, \check{a}_{ji}\}$, $\underline{a}_{ji} = \min\{\hat{a}_{ji}, \check{a}_{ji}\}$, $\bar{b}_{ji} = \max\{\hat{b}_{ji}, \check{b}_{ji}\}$, $\underline{b}_{ji} = \min\{\hat{b}_{ji}, \check{b}_{ji}\}$, $\bar{c}_{ij} = \max\{\hat{c}_{ij}, \check{c}_{ij}\}$, $\underline{c}_{ij} = \min\{\hat{c}_{ij}, \check{c}_{ij}\}$, for $i = 1, \dots, n$ and $j = 1, \dots, m$. That is to say $\bar{co}[a_{ji}(p_i)]$ (or $\bar{co}[b_{ji}(p_i)]$) and $\bar{co}[c_{ij}(q_j)]$ (or $\bar{co}[d_{ij}(q_j)]$) are an intervals with non-empty interior when $a_{ji}(p_i)$ (or $b_{ji}(p_i)$) and $c_{ij}(q_j)$ (or $d_{ij}(q_j)$) are discontinuous at $|p_i| = \Lambda_i$ and $|q_j| = \tilde{\Lambda}_j$ respectively, while $\bar{co}[a_{ji}(p_i)] = \{a_{ji}(p_i)\}$ (or $\bar{co}[b_{ji}(p_i)] = \{b_{ji}(p_i)\}$) and $\bar{co}[c_{ij}(q_j)] = \{c_{ij}(q_j)\}$ (or $\bar{co}[d_{ij}(q_j)] = \{d_{ij}(q_j)\}$) are singleton sets when $a_{ji}(p_i)$ (or $b_{ji}(p_i)$) and $c_{ij}(q_j)$ (or $d_{ij}(q_j)$) are continuous at p_i and q_i , respectively. Therefore, this BAM neural network based on memristors is a state-dependent switched system.

Case 2a In the Case 2, $a_{ji}(p_i(t))$, $b_{ji}(p_i(t))$, $c_{ij}(q_j(t))$ and $d_{ij}(q_j(t))$ are continuous functions and for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$,

$$\underline{\Gamma}_{ji} \leq a_{ji}(p_i(t)) \leq \bar{\Gamma}_{ji}, \quad \underline{\Upsilon}_{ji} \leq b_{ji}(p_i(t)) \leq \bar{\Upsilon}_{ji},$$

$$\underline{\Xi}_{ij} \leq c_{ij}(q_j(t)) \leq \bar{\Xi}_{ij}, \quad \underline{\Phi}_{ij} \leq d_{ij}(q_j(t)) \leq \bar{\Phi}_{ij},$$

where the jumps $\underline{\Gamma}_{ji}$, $\underline{\Upsilon}_{ji}$, $\underline{\Xi}_{ij}$, $\underline{\Phi}_{ij}$, $\bar{\Gamma}_{ji}$, $\bar{\Upsilon}_{ji}$, $\bar{\Xi}_{ij}$ and $\bar{\Phi}_{ij}$ are constants.

By considering **Case 1**, system (3) becomes

or equivalently, there exist $\mathcal{A}_{ji}(t) \in \bar{co}[a_{ji}(p_i(t))]$, $\mathcal{B}_{ji}(t) \in \bar{co}[b_{ji}(p_i(t))]$, $\mathcal{C}'_{ij}(t) \in \bar{co}[c_{ij}(q_j(t))]$ and $\mathcal{D}'_{ij}(t) \in \bar{co}[d_{ij}(q_j(t))]$, such that

$$\begin{cases} \dot{p}(t) = -p(t) + A(t)f(q(t)) + B(t)f(q(t - \tau(t))) + u(t), \\ \dot{q}(t) = -q(t) + C(t)g(p(t)) + D(t)g(p(t - \tau(t))) + v(t), \end{cases} \quad (5)$$

where $A(t) = [(\mathcal{A}_{ji}(t))_{n \times n}]^T$, $B(t) = [(\mathcal{B}_{ji}(t))_{n \times n}]^T$, $C(t) = [(\mathcal{C}'_{ij}(t))_{m \times m}]^T$, $D(t) = [(\mathcal{D}'_{ij}(t))_{m \times m}]^T$, $p(t) = (p_1(t), \dots, p_n(t))^T$, $q(t) = (q_1(t), \dots, q_m(t))^T$, $f(q(t)) = [f(q_1(t)), \dots, f(q_m(t))]^T$, $g(p(t)) = [g(p_1(t)), \dots, g(p_n(t))]^T$.

Notations

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense (see Filippov 1988). Let $\mathfrak{Z}([-\tau, 0], \mathbb{R}^n)$ be the Bannach Space of continuous functions $\phi : [-\tau, 0] \rightarrow \mathbb{R}^n$ with the norm $\|\phi\| = \sup_{s \in [-\tau, 0]} |\phi(s)|$. $\mathbb{R}^n$ denotes the n -dimensional space with the scalar product $\langle \cdot, \cdot \rangle$ and the vector norm $\|\cdot\|$. $\langle \cdot, \cdot \rangle$ represents the interval. $co\{\tilde{\Pi}, \hat{\Pi}\}$ denotes closure of the convex hull generated by real numbers $\tilde{\Pi}$

$$\begin{cases} \dot{p}_i(t) = -p_i(t) + \sum_{j=1}^m \bar{co}[a_{ji}(p_i(t))]f_j(q_j(t)) + \sum_{j=1}^m \bar{co}[b_{ji}(p_i(t))]f_j(q_j(t - \tau(t))) + u_i(t), \\ \dot{q}_j(t) = -q_j(t) + \sum_{i=1}^n \bar{co}[c_{ij}(q_j(t))]g_i(p_i(t)) + \sum_{i=1}^n \bar{co}[d_{ij}(q_j(t))]g_i(p_i(t - \tau(t))) + v_j(t), \end{cases} \quad (4)$$

and $\widehat{\Pi}$ or real matrices $\widetilde{\Pi}$ and $\widehat{\Pi}$. Let $\bar{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\bar{b}_{ij} = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\underline{b}_{ij} = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $\tilde{a}_{ij} = \max\{\hat{a}_{ij}\}$, $\tilde{b}_{ij} = \max\{\hat{b}_{ij}\}$, for $i, j = 1, 2, \dots, n$. $P > 0$ means that the matrix P is a real symmetric positive definite matrix, whereas $P < 0$ indicates a negative-definite matrix. I and 0 represent the identity matrix and zero matrix with compatible dimension. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry. $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix.

Now we present a definition and Lemma which will be used in driving our main results.

Definition 2.2 Memristive BAM networks (5) is said to be robustly passive, if there exists a scalar $\gamma > 0$ such that

$$2 \int_0^{t_x} [f^T(q(s)) \quad g^T(p(s))] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \geq -\gamma \int_0^{t_x} [u^T(s) \quad v^T(s)] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds,$$

for all $t_x \geq 0$ and for all solution of (5) with $p(0) = 0$ and $q(0) = 0$.

Lemma 2.3 (Fang and Park 2013) For any matrix $R \in \mathcal{R}^{n \times n}$, $R = R^T > 0$, scalars $a < b$, vector $\omega : [a, b] \mapsto \mathcal{R}^n$ such that the integration concerned is well defined, then

$$\begin{aligned} (b-a) \int_a^b \omega^T(s) R \omega(s) ds &\geq \left(\int_a^b \omega(s) ds \right)^T R \left(\int_a^b \omega(s) ds \right), \\ \frac{(b-a)^2}{2} \int_a^b \int_{t+\theta}^t \omega^T(s) R \omega(s) ds d\theta &\geq \left(\int_a^b \int_{t+\theta}^t \omega(s) ds d\theta \right)^T R \left(\int_a^b \int_{t+\theta}^t \omega(s) ds d\theta \right), \\ \frac{(b-a)^3}{6} \int_a^b \int_{\theta_1}^b \int_{t+\theta_2}^t \omega^T(s) R \omega(s) ds d\theta_2 d\theta_1 &\geq \left(\int_a^b \int_{\theta_1}^b \int_{t+\theta_2}^t \omega(s) ds d\theta_2 d\theta_1 \right)^T R \left(\int_a^b \int_{\theta_1}^b \int_{t+\theta_2}^t \omega(s) ds d\theta_2 d\theta_1 \right). \end{aligned}$$

Main results

In this section, we present our main results. More precisely, by constructing a novel LKF together with LMI technique, a set of condition is derived to make memristive BAM neural networks (1) with the conditions in Case 1 and Case 2 robustly passive. In particular, first we obtain a set of sufficient condition in terms of LMIs for the passivity of the state-dependent switched memristive BAM neural networks with uncertain delays, which has nonsmooth dynamics. Then the results can be extended to

study the passivity of the state-dependent continuous memristive BAM neural networks.

Passivity criteria for Case 1a

The following theorem presents a sufficient condition for obtaining passivity of the state-dependent switched memristive BAM neural networks with uncertain delays.

Theorem 3.1 Assume that (A1) holds. For given positive scalars τ and μ , the memristive BAM neural network (1) is robustly passive, if there exist matrices $P_1 > 0, P_2 > 0$, $R = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0$, $Z_r > 0$ ($r = 1, 2, \dots, 6$), positive diagonal matrices L , E and $S_p > 0$ ($p = 1, 2$), $G_s > 0$ ($s = 1, 2, \dots, 6$) and matrices N_1, N_2 such that the following LMI holds:

$$\Omega = [\Omega_{k,l}]_{22 \times 22} < 0, \quad (6)$$

where

$$\begin{aligned} \Omega_{1,1} &= \tau R_1 + L S_1 L - \frac{R_3}{(\tau + \delta_\tau)} + \frac{\tau^2}{2} Z_1 - 2 Z_3 \\ &\quad - \frac{\tau Z_3}{(\tau + \delta_\tau)} + \frac{\tau^3}{6} Z_5 - L G_1 L, \\ \Omega_{1,2} &= \frac{(1-\mu)}{(\tau + \delta_\tau)} R_3 + \frac{\tau(1-\mu)}{(\tau + \delta_\tau)} Z_3, \quad \Omega_{1,4} = P_1 + \tau R_2 \\ &\quad - N_1^T, \quad \Omega_{1,7} = -R_2^T \frac{1}{(\tau + \delta_\tau)} + \frac{2 Z_3}{(\tau + \delta_\tau)}, \end{aligned}$$

$$\begin{aligned} \Omega_{1,15} &= G_2^T L^T, \quad \Omega_{2,2} = -(1-\mu) L S_1 L - \frac{(1-\mu)^2}{\tau} R_3 \\ &\quad - R_3 \frac{(1-\mu)^2}{\tau + \delta_\tau} - 2 Z_3 - \frac{\tau(1-\mu)^2}{\tau + \delta_\tau} Z_3, \\ \Omega_{2,3} &= \frac{(1-\mu)}{\tau} R_3, \quad \Omega_{2,7} = \frac{(1-\mu) R_2^T}{\tau + \delta_\tau}, \quad \Omega_{2,8} = \frac{2 Z_3}{\tau} \\ &\quad - \frac{(1-\mu) R_2^T}{\tau}, \quad \Omega_{2,16} = -G_3^T L^T, \quad \Omega_{3,3} = -\frac{R_3}{\tau}, \\ \Omega_{3,8} &= \frac{R_2}{\tau}, \quad \Omega_{4,4} = \tau R_3 + \frac{\tau^2 Z_3}{2} - 2 N_1, \quad \Omega_{4,5} = N_1 A, \end{aligned}$$

$$\begin{aligned}
\Omega_{4,6} &= N_1 B, \quad \Omega_{4,21} = N_1, \quad \Omega_{5,5} = -S_2 - G_4 - 2G_5, \\
\Omega_{5,11} &= G_5^T E^T, \quad \Omega_{6,6} = (1 - \mu)S_2 - 2G_6, \quad \Omega_{6,12} = G_6^T E^T, \\
\Omega_{7,7} &= -\frac{R_1}{\tau + \delta_\tau} - \frac{2Z_3}{(\tau + \delta_\tau)^2}, \quad \Omega_{9,9} = -\frac{2Z_1}{\tau^2}, \\
\Omega_{8,8} &= -\frac{R_1}{\tau} - \frac{2Z_3}{\tau^2}, \quad \Omega_{10,10} = -\frac{6Z_5}{\tau^3}, \\
\Omega_{11,11} &= \tau Q_1 + ES_2 E - \frac{Q_3}{(\tau + \delta_\tau)} + \frac{\tau^2}{2} Z_2 - 2Z_4 - \frac{\tau Z_4}{(\tau + \delta_\tau)} \\
&\quad + \frac{\tau^3 Z_6}{6} - EG_4 E, \quad \Omega_{11,12} = \frac{(1 - \mu)}{(\tau + \delta_\tau)} Q_3 + \frac{\tau(1 - \mu)}{(\tau + \delta_\tau)} Z_4, \\
\Omega_{11,14} &= P_2 + \tau Q_2 - N_2^T, \quad \Omega_{11,17} = -\frac{R_2^T}{(\tau + \delta_\tau)} \\
&\quad + \frac{2Z_4}{(\tau + \delta_\tau)}, \quad \Omega_{12,12} = -(1 - \mu)ES_2 E - Q_3 \frac{(1 - \mu)^2}{\tau} \\
&\quad - R_3 \frac{(1 - \mu)^2}{\tau + \delta_\tau} - 2Z_4 - \frac{\tau(1 - \mu)^2}{\tau + \delta_\tau} Z_4, \\
\Omega_{12,13} &= \frac{(1 - \mu)}{\tau} Q_3, \quad \Omega_{12,17} = Q_2^T \frac{(1 - \mu)}{\tau + \delta_\tau}, \\
\Omega_{12,18} &= \frac{2Z_4}{\tau} - \frac{(1 - \mu)Q_2^T}{\tau}, \quad \Omega_{13,13} = -\frac{Q_3}{\tau}, \quad \Omega_{13,18} = \frac{Q_2^T}{\tau}, \\
\Omega_{14,14} &= \tau Q_3 + \frac{\tau^2 Z_4}{2} - 2N_2, \quad \Omega_{14,15} = N_2 C, \quad \Omega_{14,16} = N_2 D, \\
\Omega_{14,22} &= N_2, \quad \Omega_{15,15} = -S_1 - G_1 - 2G_2, \\
\Omega_{16,16} &= (1 - \mu)S_1 - 2G_3, \quad \Omega_{17,17} = -\frac{Q_1}{\tau + \delta_\tau} - \frac{2Z_4}{(\tau + \delta_\tau)^2}, \\
\Omega_{18,18} &= -\frac{Q_1}{\tau} - \frac{2Z_4}{\tau^2} \Omega_{19,19} = -\frac{2Z_2}{\tau^2}, \quad \Omega_{20,20} = -\frac{6Z_6}{\tau^3}, \\
\Omega_{21,21} &= -\gamma I, \quad \Omega_{22,22} = -\gamma I \text{ and the other terms are zero.}
\end{aligned}$$

Proof Consider the following Lyapunov–Krasovskii functional:

$$V(t, p(t), q(t)) = \sum_{i=1}^5 V_i(t, p(t), q(t)), \quad (7)$$

where $V_1(t, p(t), q(t)) = p^T(t)P_1 p(t) + q^T(t)P_2 q(t)$,

$$\begin{aligned}
V_2(t, p(t), q(t)) &= \int_{t-\tau(t)}^t [p^T(s)LS_1 Lp(s) - f^T(q(s))S_2 f(q(s))] ds \\
&\quad + \int_{t-\tau(t)}^t [q^T(s)ES_2 Eq(s) - g^T(p(s))S_1 g(p(s))] ds \\
V_3(t, p(t), q(t)) &= \int_{-\tau}^0 \int_{t+\theta}^t \lambda_1^T(s) R \lambda_1(s) ds d\theta \\
&\quad + \int_{-\tau}^0 \int_{t+\theta}^t \lambda_2^T(s) Q \lambda_2(s) ds d\theta,
\end{aligned}$$

$$\begin{aligned}
V_4(t, p(t), q(t)) &= \int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t p^T(s) Z_1 p(s) ds d\theta_2 d\theta_1 \\
&\quad + \int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t q^T(s) Z_2 q(s) ds d\theta_2 d\theta_1
\end{aligned}$$

$$\begin{aligned}
&\quad + \int_{-\tau}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{p}^T(s) Z_3 \dot{p}(s) ds d\lambda d\theta \\
&\quad + \int_{-\tau}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{q}^T(s) Z_4 \dot{q}(s) ds d\lambda d\theta, \\
V_5(t, p(t), q(t)) &= \int_{-\tau}^0 \int_{\theta_1}^0 \int_{\theta_2}^0 \int_{t+\theta_3}^t p^T(s) Z_5 p(s) ds d\theta_3 d\theta_2 d\theta_1 \\
&\quad + \int_{-\tau}^0 \int_{\theta_1}^0 \int_{\theta_2}^0 \int_{t+\theta_3}^t q^T(s) Z_6 q(s) ds d\theta_3 d\theta_2 d\theta_1,
\end{aligned}$$

with $\lambda_1^T(t) = [p^T(t) \dot{p}^T(t)]$, $\lambda_2^T(t) = [q^T(t) \dot{q}^T(t)]$. Taking the time derivative $V(t, p(t), q(t))$, we have

$$\dot{V}_1(t, p(t), q(t)) = 2p^T(t)P_1 \dot{p}(t) + 2q^T(t)P_2 \dot{q}(t), \quad (8)$$

$$\begin{aligned}
\dot{V}_2(t, p(t), q(t)) &= p^T(t)LS_1 Lp(t) - f^T(q(t))S_2 f(q(t)) \\
&\quad + q^T(t)ES_2 Eq(t) - g^T(p(t))S_1 g(p(t)) \\
&\quad + (1 - \mu) \left[-p^T(t - \tau(t))LS_1 Lp(t - \tau(t)) \right. \\
&\quad + f^T(q(t - \tau(t)))S_2 f(q(t - \tau(t))) \\
&\quad - q^T(t - \tau(t))ES_2 Eq(t - \tau(t)) \\
&\quad \left. + g^T(p(t - \tau(t)))S_1 g(p(t - \tau(t))) \right], \quad (9)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(t, p(t), q(t)) &\leq \tau p^T(t)R_1 p(t) + 2\tau p^T(t)R_2 \dot{p}(t) \\
&\quad + \tau \dot{p}^T(t)R_3 \dot{p}(t) + \tau q^T(t)Q_1 q(t) + 2\tau q^T(t)Q_2 \dot{q}(t) \\
&\quad + \tau \dot{q}^T(t)Q_3 \dot{q}(t) - \frac{1}{\tau} \left(\int_{t-\tau}^{t-\tau(t)} p(s) ds \right)^T \\
&\quad R_1 \left(\int_{t-\tau}^{t-\tau(t)} p(s) ds \right) - \frac{1}{\tau} \mathcal{O}_1^T R_3 \mathcal{O}_1 - \frac{2}{\tau} \left(\int_{t-\tau}^{t-\tau(t)} p(s) ds \right)^T \\
&\quad R_2 \mathcal{O}_1 - \frac{1}{\tau + \delta_\tau} \left(\int_{t-\tau(t)}^t p(s) ds \right)^T R_1 \left(\int_{t-\tau(t)}^t p(s) ds \right) \\
&\quad - \frac{2}{\tau + \delta_\tau} \left(\int_{t-\tau(t)}^t p(s) ds \right)^T \\
&\quad R_2 \mathcal{O}_2 - \frac{1}{\tau + \delta_\tau} \mathcal{O}_2^T R_3 \mathcal{O}_2 - \frac{2}{\tau} \left(\int_{t-\tau}^{t-\tau(t)} q(s) ds \right)^T \\
&\quad Q_2 \mathcal{O}_3 - \frac{1}{\tau} \mathcal{O}_3^T Q_3 \mathcal{O}_3 - \frac{1}{\tau} \left(\int_{t-\tau}^{t-\tau(t)} q(s) ds \right)^T \\
&\quad Q_1 \left(\int_{t-\tau}^{t-\tau(t)} q(s) ds \right) - \frac{1}{\tau + \delta_\tau} \left(\int_{t-\tau(t)}^t q(s) ds \right)^T \\
&\quad Q_1 \left(\int_{t-\tau(t)}^t q(s) ds \right) - \frac{2}{\tau + \delta_\tau} \left(\int_{t-\tau(t)}^t q(s) ds \right)^T \\
&\quad Q_2 \mathcal{O}_4 - \frac{1}{\tau + \delta_\tau} \mathcal{O}_4^T Q_3 \mathcal{O}_4,
\end{aligned}$$

(10)

$$\begin{aligned}
\dot{V}_4(t, p(t), q(t)) &\leq \frac{\tau^2}{2} p^T(t) Z_1 p(t) + \frac{\tau^2}{2} q^T(t) Z_2 q(t) \\
&\quad + \frac{\tau^2}{2} \dot{p}^T(t) Z_3 \dot{p}(t) + \frac{\tau^2}{2} \dot{q}^T(t) Z_4 \dot{q}(t) \\
&\quad - \frac{2}{\tau^2} \left(\int_{-\tau}^0 \int_{t+\theta}^t p(s) ds d\theta \right)^T Z_1 \left(\int_{-\tau}^0 \int_{t+\theta}^t p(s) ds d\theta \right) \\
&\quad - \frac{2}{\tau^2} \left(\int_{-\tau}^0 \int_{t+\theta}^t q(s) ds d\theta \right)^T Z_2 \left(\int_{-\tau}^0 \int_{t+\theta}^t q(s) ds d\theta \right) \\
&\quad - \frac{2}{(\tau + \delta_\tau)^2} \mathcal{I}_1^T Z_3 \mathcal{I}_1 - \frac{2}{\tau^2} \mathcal{I}_2^T Z_3 \mathcal{I}_2 \\
&\quad - \frac{2}{(\tau + \delta_\tau)^2} \mathcal{I}_3^T Z_4 \mathcal{I}_3 - \frac{2}{\tau^2} \mathcal{I}_4^T Z_4 \mathcal{I}_4 - \mathcal{I}_5^T Z_4 \mathcal{I}_5 - \mathcal{I}_6^T Z_4 \mathcal{I}_6,
\end{aligned} \quad (11)$$

$$\begin{aligned}
\dot{V}_5(t, p(t), q(t)) &\leq \frac{\tau^3}{6} p^T(t) Z_5 p(t) + \frac{\tau^3}{6} q^T(t) Z_6 q(t) \\
&\quad - \frac{6}{\tau^3} \mathcal{I}_7^T Z_5 \mathcal{I}_7 - \frac{6}{\tau^3} \mathcal{I}_7^T Z_6 \mathcal{I}_7,
\end{aligned} \quad (12)$$

where $\mathcal{O}_1 = [(1 - \mu)p(t - \tau(t)) - p(t - \tau)]$, $\mathcal{O}_2 = [p(t) - (1 - \mu)p(t - \tau(t))]$, $\mathcal{O}_3 = [(1 - \mu)q(t - \tau(t)) - q(t - \tau)]$, $\mathcal{O}_4 = [q(t) - (1 - \mu)q(t - \tau(t))]$, $\mathcal{I}_1 = [(\tau + \delta_\tau)p^T(t) - \int_{t-\tau(t)}^t p^T(s) ds]$, $\mathcal{I}_2 = [\tau p^T(t - \tau(t)) - \int_{t-\tau}^{t-\tau(t)} p^T(s) ds]$, $\mathcal{I}_3 = [(\tau + \delta_\tau)q^T(t) - \int_{t-\tau(t)}^t q^T(s) ds]$, $\mathcal{I}_4 = [\tau q^T(t - \tau(t)) - \int_{t-\tau}^{t-\tau(t)} q^T(s) ds]$, $\mathcal{I}_5 = [p^T(t) - (1 - \mu)p^T(t - \tau(t))]$, $\mathcal{I}_6 = [q^T(t) - (1 - \mu)q^T(t - \tau(t))]$, $\mathcal{I}_7 = \int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t p^T(s) ds d\theta_2 d\theta_1$, and $\mathcal{I}_8 = \int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t q^T(s) ds d\theta_2 d\theta_1$.

For any matrices N_1, N_2 with appropriate dimensions the following equalities hold:

$$\begin{aligned}
2p^T(t)N_1[-p(t) + Af(q(t)) + Bf(q(t - \tau(t)) \\
+ u(t) - \dot{p}(t)] = 0,
\end{aligned} \quad (13)$$

$$\begin{aligned}
2\dot{q}^T(t)N_2[-q(t) + Cg(p(t)) + Dg(p(t - \tau(t)) \\
+ v(t) - \dot{q}(t)] = 0.
\end{aligned} \quad (14)$$

Further, for positive diagonal matrices G_1, G_2, G_3, G_4, G_5 and G_6 , using the assumption (A1), we have

$$p^T(t)LG_1Lp(t) - g^T(p(t))G_1g(p(t)) \geq 0, \quad (15)$$

$$2g^T(p(t))G_2(Lp(t) - g(p(t))) \geq 0, \quad (16)$$

$$2g^T(p(t - \tau(t)))G_3(Lp(t - \tau(t)) - g(p(t - \tau(t)))) \geq 0, \quad (17)$$

$$q^T(t)EG_4Eq(t) - f^T(q(t))G_4f(q(t)) \geq 0, \quad (18)$$

$$2f^T(q(t))G_5(Eq(t) - f(q(t))) \geq 0, \quad (19)$$

$$2f^T(q(t - \tau(t)))G_6(Eq(t - \tau(t)) - f(q(t - \tau(t)))) \geq 0. \quad (20)$$

Then using (8)–(20), we get

$$\begin{aligned}
\dot{V}(t, p(t), q(t)) - 2[f^T(q(t))g^T(p(t))] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \\
- \gamma[u^T(t)v^T(t)] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \leq \zeta^T(t)\Omega\zeta(t),
\end{aligned} \quad (21)$$

Therefore we conclude that, if the LMI (6) holds, then

$$\begin{aligned}
\text{where } \zeta^T(t) = &\begin{bmatrix} p^T(t) & p^T(t - \tau(t)) & p^T(t - \tau) & \dot{p}^T(t) & f^T(q(t)) & f^T(q(t - \tau(t))) & \left(\int_{t-\tau(t)}^t p(s) ds \right)^T \\ & \left(\int_{t-\tau}^{t-\tau(t)} p(s) ds \right)^T & \left(\int_{-\tau}^0 \int_{t+\theta}^t p(s) ds d\theta \right)^T & \left(\int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t p(s) ds d\theta_2 d\theta_1 \right)^T & q^T(t) \\ & q^T(t - \tau(t)) & q^T(t - \tau) & \dot{q}^T(t) & g^T(p(t)) & g^T(p(t - \tau(t))) & \left(\int_{t-\tau(t)}^t q(s) ds \right)^T \\ & \left(\int_{t-\tau}^{t-\tau(t)} q(s) ds \right)^T & \left(\int_{-\tau}^0 \int_{t+\theta}^t q(s) ds d\theta \right)^T & \left(\int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t q(s) ds d\theta_2 d\theta_1 \right)^T & u^T(t) & v^T(t) \end{bmatrix}.
\end{aligned}$$

$$\begin{aligned}
\dot{V}(t, p(t), q(t)) - 2[f^T(q(t))g^T(p(t))] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \\
- \gamma[u^T(t)v^T(t)] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \leq 0.
\end{aligned} \quad (22)$$

By integrating the Eq. (22) over the time period $[0, t_x]$, we have

$$\begin{aligned}
& 2 \int_0^{t_x} [f^T(q(s)) \ g^T(p(s))] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \geq V(t_x, p(t_x), q(t_x)) \\
& \quad - V(0, p(0), q(0)) \\
& \quad - \gamma \int_0^{t_x} [u^T(s) \ v^T(s)] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\
& \geq -\gamma \int_0^{t_x} [u^T(s) \ v^T(s)] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds
\end{aligned} \quad (23)$$

for $p(0) = 0$, $q(0) = 0$, we have $V(0, p(0), q(0)) = 0$. Thus, it follows from Definition 2.2 that the memristive BAM neural network is robustly passive. Hence the proof is completed.

Passivity criteria for Case 2a

In this subsection, we prove the passivity criteria for the state-dependent continuous memristive BAM neural networks via following theorem.

Theorem 3.2 Assume that (A1) holds. For given positive scalars τ and μ , the memristive BAM neural network (1) is robustly passive, if there exist matrices $P_1 > 0$, $P_2 > 0$, $R = \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} > 0$, $Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} > 0$, $Z_r > 0$ ($r = 1, 2, \dots, 6$), positive diagonal matrices L , E , $S_p > 0$ ($p = 1, 2$), $G_s > 0$ ($s = 1, 2, \dots, 6$) and matrices N_1 , N_2 such that the following LMI holds:

$$\hat{\Omega} = [\hat{\Omega}_{k,l}]_{18 \times 18} < 0, \quad (24)$$

where

$$\begin{aligned}
\hat{\Omega}_{1,1} &= \tau R_1 - \frac{R_3}{(\tau + \delta_\tau)} + \frac{\tau^2}{2} Z_1 - 2Z_3 - \frac{\tau Z_3}{(\tau + \delta_\tau)} + \frac{\tau^3}{6} Z_5, \\
\hat{\Omega}_{1,2} &= \frac{(1-\mu)}{(\tau + \delta_\tau)} R_3 + \frac{\tau(1-\mu)}{(\tau + \delta_\tau)} Z_3, \hat{\Omega}_{1,4} = P_1 + \tau R_2 - N_1^T, \\
\hat{\Omega}_{1,5} &= -R_2^T \frac{1}{(\tau + \delta_\tau)} + \frac{2Z_3}{(\tau + \delta_\tau)}, \hat{\Omega}_{1,12} = (N_2 \Xi L)^T, \\
\hat{\Omega}_{2,2} &= -\frac{(1-\mu)^2}{\tau} R_3 - R_3 \frac{(1-\mu)^2}{\tau + \delta_\tau} - 2Z_3 - \frac{\tau(1-\mu)^2}{\tau + \delta_\tau} Z_3, \\
\hat{\Omega}_{2,3} &= \frac{(1-\mu)}{\tau} R_3, \hat{\Omega}_{2,5} = \frac{(1-\mu)R_2^T}{\tau + \delta_\tau}, \hat{\Omega}_{2,6} = \frac{2Z_3}{\tau} \\
&\quad - \frac{2(1-\mu)R_2^T}{\tau}, \hat{\Omega}_{2,12} = (N_2 \Phi L)^T, \\
\hat{\Omega}_{3,3} &= -\frac{R_3}{\tau}, \hat{\Omega}_{3,6} = \frac{R_2}{\tau}, \hat{\Omega}_{4,4} = \tau R_3 + \frac{\tau^2 Z_3}{2} - 2N_1, \\
\hat{\Omega}_{4,9} &= N_1 \Gamma E, \hat{\Omega}_{4,10} = (N_1 \Upsilon E)^T, \hat{\Omega}_{4,17} = N_1, \\
\hat{\Omega}_{5,5} &= -\frac{R_1}{\tau + \delta_\tau} - \frac{2Z_3}{(\tau + \delta_\tau)^2}, \hat{\Omega}_{6,6} = -\frac{R_1}{\tau} - \frac{2Z_3}{\tau^2},
\end{aligned}$$

$$\begin{aligned}
\hat{\Omega}_{7,7} &= -\frac{2Z_1}{\tau^2}, \hat{\Omega}_{8,8} = -\frac{6Z_5}{\tau^3}, \hat{\Omega}_{9,9} = \tau Q_1 - \frac{Q_3}{(\tau + \delta_\tau)} \\
&\quad + \frac{\tau^2}{2} Z_2 - 2Z_4 - \frac{\tau Z_4}{(\tau + \delta_\tau)} + \frac{\tau^3}{6} Z_6, \\
\hat{\Omega}_{9,10} &= \frac{(1-\mu)}{(\tau + \delta_\tau)} Q_3 + \frac{\tau(1-\mu)}{(\tau + \delta_\tau)} Z_4, \hat{\Omega}_{9,12} = P_2 + \tau Q_2 - N_2^T, \\
\hat{\Omega}_{9,13} &= -Q_2^T \frac{1}{(\tau + \delta_\tau)} + Z_4 \frac{2}{(\tau + \delta_\tau)}, \\
\hat{\Omega}_{10,10} &= -Q_3 \frac{(1-\mu)^2}{\tau} - Q_3 \frac{(1-\mu)^2}{\tau + \delta_\tau} \\
&\quad - 2Z_4 - \frac{\tau(1-\mu)^2}{\tau + \delta_\tau} Z_4, \hat{\Omega}_{10,11} = \frac{(1-\mu)}{\tau} Q_3, \\
\hat{\Omega}_{10,13} &= Q_2^T \frac{(1-\mu)}{\tau + \delta_\tau}, \hat{\Omega}_{10,14} = \frac{2Z_4}{\tau} - \frac{(1-\mu)Q_2^T}{\tau}, \\
\hat{\Omega}_{11,11} &= -\frac{Q_3}{\tau}, \hat{\Omega}_{11,14} = \frac{Q_2^T}{\tau}, \hat{\Omega}_{12,12} = \tau Q_3 + \frac{\tau^2 Z_4}{2} \\
&\quad - 2N_2, \hat{\Omega}_{12,18} = N_2, \hat{\Omega}_{13,13} = \frac{-Q_1}{\tau + \delta_\tau} - \frac{2Z_4}{(\tau + \delta_\tau)^2}, \\
\hat{\Omega}_{14,14} &= \frac{-Q_1}{\tau} - \frac{2Z_4}{\tau^2}, \hat{\Omega}_{15,15} = -\frac{2Z_2}{\tau^2}, \hat{\Omega}_{16,16} = -\frac{6Z_6}{\tau^3}, \\
\hat{\Omega}_{17,17} &= -\gamma, \hat{\Omega}_{18,18} = -\gamma
\end{aligned}$$

and the remaining terms are zero.

Proof By (3), it can be easily seen that

$$\begin{cases} \dot{p}_i(t) \in -p_i(t) + \sum_{j=1}^m \Gamma_{ji} |f_j(q_j(t))| + \sum_{j=1}^m \Upsilon_{ji} |f_j(q_j(t - \tau(t)))| + u_i(t), \\ \dot{q}_j(t) \in -q_j(t) + \sum_{i=1}^n \Xi_{ij} |g_i(p_i(t))| + \sum_{i=1}^n \Phi_{ij} |g_i(p_i(t - \tau(t)))| + v_j(t). \end{cases} \quad (25)$$

Transform (25) into the compact form as follows:

$$\begin{cases} \dot{p}(t) \leq -p(t) + \Gamma |f(q(t))| + \Upsilon |f(q(t - \tau(t)))| + u(t), \\ \dot{q}(t) \leq -q(t) + \Xi |g(p(t))| + \Phi |g(p(t - \tau(t)))| + v(t). \end{cases} \quad (26)$$

By taking the LKF as in Theorem 3.1 and considering Case 2 along with Case 2a, we have

$$\begin{aligned}
\dot{V}_2(t, p(t), q(t)) &= p^T(t) L S_1 L p(t) - f^T(q(t)) S_2 f(q(t)) \\
&\quad + q^T(t) E S_2 E q(t) - g^T(p(t)) S_1 g(p(t)) \\
&\quad + (1-\mu)[-p^T(t - \tau(t)) L S_1 L p(t - \tau(t)) \\
&\quad + f^T(q(t - \tau(t))) S_2 f(q(t - \tau(t))) \\
&\quad - q^T(t - \tau(t)) L_2 S_2 L_2 q(t - \tau(t)) \\
&\quad + g^T(p(t - \tau(t))) S_1 g(p(t - \tau(t)))], \\
&\leq p^T(t) L S_1 L p(t) - q^T(t) E S_2 E q(t) \\
&\quad + q^T(t) E S_2 E q(t) - p^T(t) L S_1 L p(t) \\
&\quad + (1-\mu)[-p^T(t - \tau(t)) L S_1 L p(t - \tau(t))
\end{aligned}$$

$$\begin{aligned}
& + q^T(t - \tau(t))ES_2Eq(t - \tau(t)) \\
& - q^T(t - \tau(t))ES_2Eq(t - \tau(t)) \\
& + p^T(t - \tau(t))LS_1Lp(t - \tau(t)) \quad (27)
\end{aligned}$$

and other time derivatives of $V_i(t, p(t), q(t))$, $i = 1, 3, 4, 5$ are same as in Theorem 3.1. On the other hand, for any matrices N_1 , N_2 of appropriate dimensions the following equalities hold:

$$2\dot{p}^T(t)N_1[-p(t) + \Gamma Eq(t) + \Upsilon Eq(t - \tau(t) + u(t) - \dot{p}(t))] = 0, \quad (28)$$

$$2\dot{q}^T(t)N_2[-q(t) + \Xi Lp(t) + \Phi Lp(t - \tau(t) + v(t) - \dot{q}(t))] = 0. \quad (29)$$

Then it is easy to get

$$\begin{aligned}
& \dot{V}(t, p(t), q(t)) - 2[f^T(q(t))g^T(p(t))]\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \\
& - \gamma[u^T(t) \quad v^T(t)]\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \leq \zeta^T(t)\hat{\Omega}\zeta'(t), \quad (30)
\end{aligned}$$

where

$$\begin{aligned}
\zeta^T(t) = & \left[p^T(t) \quad p^T(t - \tau(t)) \quad p^T(t - \tau) \quad \dot{p}^T(t) \quad \int_{t-\tau(t)}^t p^T(s)ds \quad \int_{t-\tau}^{t-\tau(t)} p^T(s)ds \quad \left(\int_{-\tau}^0 \int_{t+\theta}^t p(s)dsd\theta \right)^T \right. \\
& \left(\int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t p(s)dsd\theta_2d\theta_1 \right)^T \quad q^T(t) \quad q^T(t - \tau(t)) \quad q^T(t - \tau) \quad \dot{q}^T(t) \quad \int_{t-\tau(t)}^t q^T(s)ds \\
& \left. \int_{t-\tau}^{t-\tau(t)} q^T(s)ds \quad \left(\int_{-\tau}^0 \int_{t+\theta}^t q(s)dsd\theta \right)^T \quad \left(\int_{-\tau}^0 \int_{\theta_1}^0 \int_{t+\theta_2}^t q(s)dsd\theta_2d\theta_1 \right)^T \quad u^T(t) \quad v^T(t) \right].
\end{aligned}$$

Therefore, we conclude that, if the LMI holds, then

$$\begin{aligned}
& \dot{V}(t, p(t), q(t)) - 2[f^T(q(t))g^T(p(t))]\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \\
& - \gamma[u^T(t) \quad v^T(t)]\begin{bmatrix} u(t) \\ v(t) \end{bmatrix} \leq 0. \quad (31)
\end{aligned}$$

By integrating above the equation, with respect to t over the time period $[0, t_x]$, we have

$$\begin{aligned}
& 2 \int_0^{t_x} [f^T(q(s))g^T(p(s))]\begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \geq \\
& - \gamma \int_0^{t_x} [u^T(s) \quad v^T(s)]\begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \quad (32) \\
& \geq - \gamma \int_0^{t_x} [u^T(s) \quad v^T(s)]\begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds
\end{aligned}$$

for $p(0) = 0, q(0) = 0$, we have $V(0, p(0), q(0)) = 0$. This completes the proof. $\square$

Remark 3.3 It is noted that, the asymptotical stability conditions for system (3) if $u(t) = 0$ and $v(t) = 0$ can be easily derived from Theorems 3.1 and 3.2. From this, it proves the well known result that the passivity is a higher abstraction level of stability.

Remark 3.4 It is worth noting that the memristive neural networks is a hybrid system can be used in the wider scope. From the logical analysis of mathematical reasoning, we can see that the obtained results have a certain robustness, which may also be capable of tackling the external distractions. Further, it is noticed that, Wu and Zeng (2014) discussed the exponential passivity results for memristive neural networks in terms of linear matrix inequalities. In this work we have considered different types of memductance functions and obtained the passivity criteria for continuous and switching system. Therefore, the obtained results in this paper are different from the existing ones. Moreover, the method of this paper may be applied for analyzing other classes of

complex switched nonlinear systems in the presense of different memductance functions.

Numerical example

In this section, two numerical examples are given to demonstrate the effectiveness and applicability of the proposed results. It is worth pointing that the memristive neural networks (3) with different memductance functions is either a state-dependent switched system or a state-dependent continuous system, which is the generalization of those for conventional neural networks. In this section, we will discuss both the above two cases. First, let us consider the state dependent switched memristive BAM neural networks as we discussed in Case 1.

Example 1 Consider the following memristive BAM neural networks,

$$\begin{cases} \dot{p}_i(t) = -p_i(t) + \sum_{j=1}^2 a_{ji}(p_i(t))f_i(q_j(t)) + \sum_{j=1}^2 b_{ji}(p_i(t))f_i(q_j(t - \tau(t))) + u_i(t), \\ \dot{q}_j(t) = -q_j(t) + \sum_{i=1}^2 c_{ij}(q_j(t))g_j(p_i(t)) + \sum_{i=1}^2 d_{ij}(q_j(t))g_j(p_i(t - \tau(t))) + v_j(t), \end{cases} \quad i, j = 1, 2,$$

where

$$\begin{aligned}
a_{11}(p_1(t)) &= \begin{cases} -0.6, & |p_1(t)| < 1, \\ 0.6, & |p_1(t)| > 1, \end{cases} & a_{12}(p_1(t)) &= \begin{cases} -0.5, & |p_1(t)| < 1, \\ 0.5, & |p_1(t)| > 1, \end{cases} \\
a_{21}(p_2(t)) &= \begin{cases} -0.7, & |p_2(t)| < 1, \\ 0.7, & |p_2(t)| > 1, \end{cases} & a_{22}(p_2(t)) &= \begin{cases} -0.5, & |p_2(t)| < 1, \\ 0.5, & |p_2(t)| > 1, \end{cases} \\
b_{11}(p_1(t)) &= \begin{cases} -0.4, & |p_1(t)| < 1, \\ 0.4, & |p_1(t)| > 1, \end{cases} & b_{12}(p_1(t)) &= \begin{cases} -0.5, & |p_1(t)| < 1, \\ 0.5, & |p_1(t)| > 1, \end{cases} \\
b_{21}(p_2(t)) &= \begin{cases} -0.3, & |p_2(t)| < 1, \\ 0.3, & |p_2(t)| > 1, \end{cases} & b_{22}(p_2(t)) &= \begin{cases} -0.5, & |p_2(t)| < 1, \\ 0.5, & |p_2(t)| > 1, \end{cases} \\
c_{11}(q_1(t)) &= \begin{cases} -0.6, & |q_1(t)| < 1, \\ 0.6, & |q_1(t)| > 1, \end{cases} & c_{12}(q_1(t)) &= \begin{cases} -0.5, & |q_1(t)| < 1, \\ 0.5, & |q_1(t)| > 1, \end{cases} \\
c_{21}(q_2(t)) &= \begin{cases} -0.7, & |q_2(t)| < 1, \\ 0.7, & |q_2(t)| > 1, \end{cases} & c_{22}(q_2(t)) &= \begin{cases} -0.5, & |q_2(t)| < 1, \\ 0.5, & |q_2(t)| > 1, \end{cases} \\
d_{11}(q_1(t)) &= \begin{cases} -0.4, & |q_1(t)| < 1, \\ 0.4, & |q_1(t)| > 1, \end{cases} & d_{12}(q_1(t)) &= \begin{cases} -0.5, & |q_1(t)| < 1, \\ 0.5, & |q_1(t)| > 1, \end{cases} \\
d_{21}(q_2(t)) &= \begin{cases} -0.3, & |q_2(t)| < 1, \\ 0.3, & |q_2(t)| > 1, \end{cases} & d_{22}(q_2(t)) &= \begin{cases} -0.5, & |q_2(t)| < 1, \\ 0.5, & |q_2(t)| > 1, \end{cases}
\end{aligned} \tag{33}$$

with $\tau = 1$, $\delta_\tau = 0.04$, $\mu = 0.5$, $g(\rho) = g_1(\cdot) = g_2(\cdot) = \max\{\rho, 0\}$ and $f(\rho) = f_1(\cdot) = f_2(\cdot) = \max\{\rho, 0\}$, $\rho \in \mathbb{R}$. Then there exist the matrices of the following,

$$\begin{aligned}
A &= \begin{bmatrix} -0.6 & 0.5 \\ -0.7 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -0.4 & 0.5 \\ -0.3 & 0.4 \end{bmatrix}, \\
C &= \begin{bmatrix} -0.5 & 0.4 \\ -0.6 & 0.5 \end{bmatrix}, \quad D = \begin{bmatrix} -0.3 & 0.4 \\ -0.08 & 0.3 \end{bmatrix}.
\end{aligned}$$

Now by solving the LMI (6) by available numerical software, we can obtain a feasible solution to guarantee the robust passivity of the considered memristive BAM networks. For a better demonstration of the results, we choose

the external inputs $u(t) = v(t) = 0$ and for different initial values of the state variables $(p_1(t), p_2(t))$ and $(q_1(t), q_2(t))$, the trajectories of the state variables are shown in Fig. 1. The simulation results reveal that both $p(t)$ and $q(t)$ are converging to the equilibrium point zero, so we conclude that the considered memristive BAM neural networks are internally robustly stable. Hence it can be concluded that it is robustly passive.

Next, we consider the state dependent continuous system as we discussed in Case 2.

Example 2 Consider a two-dimensional memristive BAM neural network model

$$\begin{cases}
\dot{p}_1(t) = -p_1(t) + a_{11}(p_1(t))f_1(q_1(t)) + a_{12}(p_1(t))f_2(q_2(t)) \\
\quad + b_{11}(p_1(t))f_1(q_1(t - \tau(t))) + b_{12}(p_1(t))f_2(q_2(t - \tau(t))) + u_1(t), \\
\dot{p}_2(t) = -p_2(t) + a_{21}(p_2(t))f_1(q_1(t)) + a_{22}(p_2(t))f_2(q_2(t)) \\
\quad + b_{21}(p_2(t))f_1(q_1(t - \tau(t))) + b_{22}(p_2(t))f_2(q_2(t - \tau(t))) + u_2(t), \\
\dot{q}_1(t) = -q_1(t) + c_{11}(q_1(t))g_1(p_1(t)) + c_{12}(q_1(t))g_2(p_2(t)) \\
\quad + d_{11}(q_1(t))g_1(p_1(t - \tau(t))) + d_{12}(q_1(t))g_2(p_2(t - \tau(t))) + v_1(t), \\
\dot{q}_2(t) = -q_2(t) + c_{21}(q_2(t))g_1(p_1(t)) + c_{22}(q_2(t))g_2(p_2(t)) \\
\quad + d_{21}(q_2(t))g_1(p_1(t - \tau(t))) + d_{22}(q_2(t))g_2(p_2(t - \tau(t))) + v_2(t),
\end{cases} \quad (34)$$

$$\begin{aligned}
a_{11} &= 0.5 \sin(p_1(t)), & a_{12} &= -0.3 \cos(p_1(t)), & a_{21} &= 0.2 \sin(p_2(t)), & a_{22} &= 0.3 \cos(p_2(t)), \\
b_{11} &= 0.6 \sin(p_1(t)), & b_{12} &= 0.4 \cos(p_1(t)), & b_{21} &= 0.2 \sin(p_2(t)), & b_{22} &= 0.3 \cos(p_2(t)), \\
c_{11} &= 0.5 \sin(q_1(t)), & c_{12} &= -0.3 \cos(q_1(t)), & c_{21} &= 0.2 \sin(q_2(t)), & c_{22} &= 0.3 \cos(q_2(t)), \\
d_{11} &= 0.6 \sin(q_1(t)), & d_{12} &= 0.3 \cos(q_1(t)), & d_{21} &= 0.2 \sin(q_2(t)), & d_{22} &= 0.3 \cos(q_2(t)), \\
f(\rho) &= f_1(\cdot) = f_2(\cdot) = \tanh(\rho), & g(\rho) &= g_1(\cdot) = g_2(\cdot) = \tanh(\rho).
\end{aligned}$$

Fig. 1 State trajectories of system (33) for Case 1

For $\tau = 1$, $\mu = 0.6$ and $\delta_\tau = 0.03$, by solving the LMI in Theorem 3.2, we can obtain a feasible solution which are not provided here due to page constraint and it easy to see that the considered memristive neural network is robustly passive. Further, we present the simulation results in the absence of external inputs. For random initial values, the state trajectories are as shown in

Fig. 2. Further, the simulation results reveal that both $p(t)$ and $q(t)$ are converging to the equilibrium point zero which means that the considered memristive BAM neural network is internally stable. It is concluded from the simulation results that the memristive BAM neural networks with the two different types of memductance functions is passive.

Fig. 2 State trajectories of system (33) for Case 2

Conclusions

The passivity problem for memristive BAM neural networks with uncertain input delays has been studied. Moreover, two types of memductance functions and uncertain time delays are considered to model the BAM networks. By constructing suitable LKFs and using LMI approach with Jensen's inequalities, new delay-dependent conditions are established in terms of LMIs to guarantee the passivity criteria for the memristive BAM neural networks. Finally, numerical examples are provided to show the validity and potentiality of the developed theoretical results.

Acknowledgments The work of R. Anbuviya was supported by the Department of Science and Technology, Government of India, New Delhi through the Grant No. DST/INSPIRE Fellowship/2011-IF110718.

References

- Ali M Syed, Saravanakumar R, Cao J (2016) New passivity criteria for memristor-based neutral-type stochastic BAM neural networks with mixed time-varying delays. *Neurocomputing* 171:1533–1547
- Anderson BDO, Vongpanitlerd S (1973) *Network analysis synthesis—a modern systems theory approach*. Prentice Hall, Englewood Cliffs
- Cai Z, Huang L (2014) Functional differential inclusions and dynamic behaviors for memristor-based BAM neural networks with time-varying delays. *Commun Nonlinear Sci Numer Simul* 19:1279–1300
- Cantley KD, Subramaniam A, Stiegler HJ, Chapman RA, Vogel EM (2012) Neural learning circuits utilizing nano-crystalline silicon transistors and memristors. *IEEE Trans Neural Netw Learn Syst* 23:565–573
- Cao J, Alofi A, Al-Mazrooei A, Elaiw A (2013) Synchronization of switched interval networks and applications to chaotic neural networks. *Abstr Appl Anal* 2013:1–13
- Cao J, Wan Y (2014) Matrix measure strategies for stability and synchronization of inertial BAM neural network with time delays. *Neural Netw* 53:165–172
- Chandrasekar A, Rakkiyappan R, Cao J, Lakshmanan S (2014) Synchronization of memristor-based recurrent neural networks with two delay components based on second-order reciprocally convex approach. *Neural Netw* 57:79–93
- Chua LO (1971) Memristor—the missing circuit element. *IEEE Trans Circuit Theory* 18:507–519
- Du Y, Zhong S, Zhou N, Nie L, Wang W (2013) Exponential passivity of BAM neural networks with time-varying delays. *Appl Math Comput* 221:727–740
- Fang M, Park JH (2013) A multiple integral approach to stability of neutral time-delay systems. *Appl Math Comput* 224:714–718
- Filippov AF (1988) *Differential equations with discontinuous right-hand sides*. Springer, Dordrecht
- Hu M, Cao J, Yang Y, Hu A (2013) Passivity analysis for switched generalized neural networks with time-varying delay and uncertain output. *IMA J Math Control Inf* 30:407–422
- Hu J, Cao J, Alofi A, Mazrooei AA, Elaiw A (2015) Pinning synchronization of coupled inertial delayed neural networks. *Cogn Neurodynamics* 9:341–350
- Ji MD, He Y, Zhang CK, Wu M (2014) Novel stability criteria for recurrent neural networks with time-varying delay. *Neurocomputing* 138:383–391
- Kim H, Sah MP, Yang CJ, Roska T, Chua LO (2012) Neural synaptic weighting with a pulse-based memristor circuit. *IEEE Trans Circuits Syst I Regul Pap* 59:148–158
- Kosko B (1987) Adaptive bidirectional associative memories. *Appl Opt* 26:4947–4960

- Kumar RS, Sugumaran G, Raja R, Zhu Q, Raja UK (2016) New stability criterion of neural networks with leakage delays and impulses: a piecewise delay method. *Cogn Neurodynamics* 10:85–98
- Kwon OM, Park MJ, Park Ju H, Lee SM, Cha EJ (2014) On stability analysis for neural networks with interval time-varying delays via some new augmented Lyapunov–Krasovskii functional. *Commun Nonlinear Sci Numer Simul* 19:3184–3201
- Li H, Chen B, Zhou Q, Fang S (2008) Robust exponential stability for uncertain stochastic neural networks with discrete and distributed time-varying delays. *Phys Lett A* 372:3385–3394
- Li H, Chen B, Zhou Q, Qian W (2009) Robust stability for uncertain delayed fuzzy Hopfield neural networks with Markovian jumping parameters. *IEEE Trans Syst Man Cybern B* 39:94–102
- Li B, Shen H, Song X, Zhao J (2014) Robust exponential H_∞ control for uncertain time-varying delay systems with input saturation: a Markov jump model approach. *Appl Math Comput* 237:190–202
- Li N, Cao J (2015) New synchronization criteria for memristor-based networks: adaptive control and feedback control schemes. *Neural Netw* 61:1–9
- Lou XY, Cui BT (2007) Passivity analysis of integro-differential neural networks with time-varying delays. *Neurocomputing* 70:1071–1078
- Pershin YV, Ventra MD (2012) Experimental demonstration of associative memory with memristive neural networks. *Neural Netw* 23:881–886
- Qi J, Li C, Huang T (2014) Stability of delayed memristive neural networks with time-varying impulses. *Cogn Neurodynamics* 8:429–436
- Rakkiyappan R, Chandrasekar A, Cao J (2015) Passivity and passification of memristor-based recurrent neural networks with additive time-varying delays. *IEEE Trans Neural Netw Learn Syst* 26:2043–2057
- Roh YH, Oh JH (1999) Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation. *Automatica* 35:1861–1865
- Sakthivel R, Mathiyalagan K, Marshal Anthoni S (2011) Design of a passification controller for uncertain fuzzy Hopfield neural networks with time-varying delays. *Phys Scr* 84:045024
- Sakthivel R, Mathiyalagan K, Marshal Anthoni S (2012) Robust H_∞ control for uncertain discrete-time stochastic neural networks with time-varying delays. *IET Control Theory Appl* 6:1220–1228
- Sakthivel R, Vadivel P, Mathiyalagan K, Arunkumar A, Sivachitra M (2015) Design of state estimator for bidirectional associative memory neural networks with leakage delays. *Inf Sci* 296:263–274
- Song Q, Cao J (2012) Passivity of uncertain neural networks with both leakage delay and time-varying delay. *Nonlinear Dyn* 67:1695–1707
- Strukov DB, Snider GS, Stewart GR, Williams RS (2008) The missing memristor found. *Nature* 453:80–83
- Sun JW, Shen Y, Yin Q, Xu CJ (2013) Compound synchronization of four memristor chaotic oscillator systems and secure communication. *Chaos* 23:013140
- Tian X, Xie L, Chen Y (2007) Robust non-fragile H_∞ control for uncertain time delayed stochastic systems with sector constraints. *IEEE Int Conf Control Autom* 1852–1856. doi:[10.1109/ICCA.2007.4376683](https://doi.org/10.1109/ICCA.2007.4376683)
- Wang X, Li C, Huang T (2014) Delay-dependent robust stability and stabilization of uncertain memristive delay neural networks. *Neurocomputing* 140:155–161
- Wen S, Zeng Z, Huang T (2012) Adaptive synchronization of memristor-based Chua's circuits. *Phys Lett A* 376:2775–2780
- Wen SP, Zeng ZG, Huang TW, Chen YR (2013) Passivity analysis of memristor-based recurrent neural networks with time-varying delays. *J Frankl Inst* 350:2354–2370
- Wu AL, Zeng ZG, Xiao J (2013) Dynamic evolution evoked by external inputs in memristor-based wavelet neural networks with different memductance functions. *Adv Differ Equ* 258:1–14
- Wu AL, Zeng ZG (2014) Exponential passivity of memristive neural networks with time delays. *Neural Netw* 49:11–18
- Yang X, Cao J, Yu W (2014) Exponential synchronization of memristive Cohen–Grossberg neural networks with mixed delays. *Cogn Neurodynamics* 8:239–249
- Yang X, Cao J, Qiu J (2015) pth moment exponential stochastic synchronization of coupled memristor-based neural networks with mixed delays via delayed impulsive control. *Neural Netw* 65:80–91
- Zhang GD, Shen Y, Sun JW (2012) Global exponential stability of a class of memristor-based recurrent neural networks with time-varying delays. *Neurocomputing* 18:149–154
- Zhang GD, Shen Y, Wang LM (2013) Global anti-synchronization of a class of chaotic memristive neural networks with time-varying delays. *Neural Netw* 46:1–8
- Zhang G, Shen Y, Xu C (2015) Global exponential stability in a Lagrange sense for memristive recurrent neural networks with time-varying delays. *Neurocomputing* 149:1330–1336
- Zhu S, Shen Y, Chen GC (2010) Exponential passivity of neural networks with time-varying delay and uncertainty. *Phys Lett A* 375:136–142