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A geometric calibration method for inverse geometry computed tomography using P-matrices

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Abstract

Accurate and artifact free reconstruction of tomographic images requires precise knowledge of the imaging system geometry. This work proposes a novel projection matrix (P-matrix) based calibration method to enable C-arm inverse geometry CT (IGCT). The method is evaluated for scanning-beam digital x-ray (SBDX), a C-arm mounted inverse geometry fluoroscopic technology. A helical configuration of fiducials is imaged at each gantry angle in a rotational acquisition. For each gantry angle, digital tomosynthesis is performed at multiple planes and a composite image analogous to a cone-beam projection is generated from the plane stack. The geometry of the C-arm, source array, and detector array is determined at each angle by constructing a parameterized 3D-to-2D projection matrix that minimizes the sum-of-squared deviations between measured and projected fiducial coordinates. Simulations were used to evaluate calibration performance with translations and rotations of the source and detector. In a geometry with 1 mm translation of the central ray relative to the axis-of-rotation and 1 degree yaw of the detector and source arrays, the maximum error in the recovered translational parameters was 0.4 mm and maximum error in the rotation parameter was 0.02 degrees. The relative root-mean-square error in a reconstruction of a numerical thorax phantom was 0.4% using the calibration method, versus 7.7% without calibration. Changes in source-detector-distance were the most challenging to estimate. Reconstruction of experimental SBDX data using the proposed method eliminated double contour artifacts present in a non-calibrated reconstruction. The proposed IGCT geometric calibration method reduces image artifacts when uncertainties exist in system geometry.

Keywords

Inverse geometry; scanning beam digital x-ray; C-arm calibration; projection matrix; computed tomography

1. INTRODUCTION

Scanning-beam digital x-ray (SBDX) is a low-dose inverse geometry fluoroscopic technology designed for cardiac interventions (Fig. 1A). The inverse geometry design provides SBDX with an inherent real-time tomosynthesis capability that has been exploited for a number of applications including frame-by-frame 3D tracking of high-contrast objects

such as cardiac catheters, calibration-free vessel measurements for device sizing, and stereoscopic fluoroscopy.

Recently, the feasibility of CT-based 3D anatomic mapping with SBDX was demonstrated through numerical simulations. In practical C-arm based inverse geometry CT, uncertainties in the imaging geometry may exist due to manufacturing tolerances, non-ideal source and detector alignment, or C-arm deflection during rotation. An incorrect mapping between the assumed 3D object coordinate system and the projection acquisition system has been shown to degrade spatial resolution and can introduce image artifacts. This work proposes a new gantry calibration method for IGCT that is motivated by the projection matrix (P-matrix) technique often used in cone-beam CT (CBCT). The ability of the method to recover parameters describing SBDX system geometry is examined using numerical simulations. Finally, experimental SBDX data acquired with a rotating phantom stage is used to demonstrate that the proposed method reduces image artifacts caused by geometric uncertainty.

2. METHODS

2.1 SBDX image reconstruction

SBDX uses an electromagnetically scanned electron beam incident upon a large-area transmission style tungsten target (Figure 1). The electron beam is raster scanned over a 71 by 71 array of source focal spot positions every 1/15 s. A multi-hole collimator defines a series of narrow overlapping x-ray beams convergent upon a CdTe photon-counting detector. The geometric relationship among the narrow beam projections is constrained by the precise and rigid geometry of the SBDX collimator and the fixed detector position. The SBDX system geometry for this study is summarized in Table 1.

SBDX has an inherent tomosynthesis capability due to the use of inverse geometry beam scanning. A live display analogous to conventional 15 fps fluoroscopy is generated using a GPU-based real-time image reconstructor. Each displayed 2D image frame is generated through a two stage reconstruction procedure. First, shift-and-add digital tomosynthesis is performed to generate a stack of 32 single plane images with 5 mm plane spacing (Fig. 1A). As shown in Fig. 1(B), the pixel centers for the stack of tomosynthesis images are defined such that a fixed pixel position (e.g. row 100, column 100) in the stack corresponds to a ray originating at the detector center. Next, a gradient filtering procedure is applied to each of the single-plane images to identify local regions of high sharpness and contrast. The final 2D “composite” image is then formed by selecting, for each pixel position, the pixel value from the single-plane image with highest contrast and sharpness. Due to the geometry of the tomosynthesis pixel centers and the compositing procedure, the final composite image can be viewed as an inverted “virtual” cone-beam projection of the in-focus objects in the patient volume (see Fig. 1(C)). A virtual SBDX projection originates at the center of the detector and falls on the source plane. The pitch of the virtual detector elements at the source plane is 0.23 mm.

2.2 Geometric calibration

Since the SBDX composite image can be viewed as a virtual cone-beam projection, a projection matrix approach can be used to estimate the geometric parameters of the SBDX system for calibration. The 3×4 projection matrix $\mathbf{P}$ maps a point (x, y, z) in the 3D object frame-of-reference to homogeneous coordinates (u, v) on the virtual detector plane:

$$[\lambda u, \lambda v, \lambda]^T = \mathbf{P}[\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{1}]^T \quad (1)$$

The unknown projection matrix $\mathbf{P}$ is written as a product of three matrices $\mathbf{P} = \mathbf{KRT}$. Here $\mathbf{K}$ describes the virtual projection geometry's intrinsic parameters, $\mathbf{R}$ is a rotation matrix, and $\mathbf{T}$ is a translation matrix. $\mathbf{K}$ depends on source-detector-distance (SDD), pitch between virtual detector elements (s_p), and the coordinates (u_o, v_o) which define the piercing point on the virtual detector. $\mathbf{R}$ depends on three angles $(\theta_x, \theta_y, \theta_z)$ describing the rotations about the three principal axes. $\mathbf{T}$ depends on the location of the virtual source point (x_s, y_s, z_s) in the 3D object coordinate system. Denoting $c_j = \cos(\theta_j)$ and $s_j = \sin(\theta_j)$, the projection matrix $\mathbf{P}$ is given by:

$$\mathbf{P} = \begin{bmatrix} -u_o & SDD/s_p & 0 \\ -v_o & 0 & SDD/s_p \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_x & s_x \\ 0 & -s_x & c_x \end{bmatrix} \begin{bmatrix} c_y & 0 & -s_y \\ 0 & 1 & 0 \\ s_y & 0 & c_y \end{bmatrix} \begin{bmatrix} c_z & s_z & 0 \\ -s_z & c_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_s \\ 0 & 1 & 0 & -y_s \\ 0 & 0 & 1 & -z_s \end{bmatrix} \quad (2)$$

The $\mathbf{P}$ matrix is parameterized by a vector, ξ , consisting of nine elements, $\xi = [SDD, u_o, v_o, \theta_x, \theta_y, \theta_z, x_s, y_s, z_s]$. The pitch between virtual detector elements, s_p , equals 0.23 mm. A calibration phantom containing a known helical configuration of N high contrast point-like markers is then imaged (Fig. 2A). The virtual detector coordinates (u_i, v_i) of the projections of the markers are then determined using a center-of-mass technique. The geometric parameters describing the IGCT system are estimated by minimizing the sum-of-squared differences between the measured positions of the markers (u_i, v_i) and the $\mathbf{P}$ -matrix-projected marker detector coordinates $(u_i(\xi), v_i(\xi))$:

$$\hat{\xi} = \arg \min_{\xi} \frac{1}{N} \sum_{i=1}^N [(u_i(\xi) - u_i)^2 + (v_i(\xi) - v_i)^2] \quad (3)$$

The optimization was performed using a Quasi-Newton method, with initial parameters set to the nominal system geometry described in Table 1.

2.3 Simulations

The performance of the proposed geometric calibration method was evaluated through numerical simulations of SBDX CT data acquisition with simulated deviations from the nominal SBDX geometry. An SBDX CT data acquisition consisting of 210 view angles

uniformly distributed over 210 degrees was simulated for a known helical configuration of 30 spherical steel fiducials. The helix phantom was centered at iso-center with its long axis aligned with the axis-of-rotation (z-axis; see Fig. 2A). The SBDX source and detector were translated 1 mm in the +y direction and the source and detector arrays were rotated 1 degree about the x-axis to mimic a hypothetical system misalignment. For each gantry orientation, a composite image (Fig. 2C) was reconstructed and a 3×4 projection matrix $\mathbf{P}$ was derived.

The CT data acquisition scheme was then repeated for a numerical thorax phantom. Reconstruction was performed using a gridded filtered back projection algorithm (gFBP) with and without geometric calibration to assess the proposed method's ability to reduce image artifacts. In order to quantify the accuracy of the reconstruction with and without calibration, the relative root-mean-squared-error (rRMSE) error was calculated versus a gFBP reconstruction of the thorax phantom accounting for the known deviations in system geometry. The rRMSE is defined as:

$$rRMSE(x) = \sqrt{\frac{1}{N} \sum_i^N \left(\frac{x_i - x_i^{ref}}{\max(x^{ref}) - \min(x^{ref})} \right)^2} \quad (4)$$

2.4 Sensitivity analysis

The sensitivity of the method to uncertainties in SDD, translations and rotations was investigated. The nominal SBDX system geometry parameters defined in Table 1 were perturbed by varying amounts. The SDD was varied from 1495 mm to 1505 mm with the SAD held constant at 450 mm. The SAD was varied from 445 mm to 455 mm with the SDD held constant at 1500 mm. The SBDX source and detector were translated 1 mm and 5 mm in the y-direction. The SBDX source and detector were translated 1 mm and 5 mm in the z-direction. Note that translations about the x-axis are equivalent to variations in SAD and SDD. To evaluate the sensitivity of the method to rotations, the SBDX source and detector were rotated 1 degree and 5 degrees about the x-axis. Finally, the SBDX source and detector were rotated 1 degree and 5 degrees about the y-axis. For each scenario, the proposed calibration method was used to estimate the system geometry and compared versus the known perturbations. The mean error and standard deviation in the estimated geometric parameters were determined for 210 gantry view angles evenly distributed over 210 degrees.

2.5 Experimental validation

The proposed calibration method was tested using experimental projection data acquired with SBDX. The SBDX gantry was rotated 90 degrees to a lateral angulation and a helix phantom was placed on a rotating stage at iso-center (Fig. 3A). The calibration phantom was constructed out of poly(methyl methacrylate) (PMMA) and consists of a known helical configuration of 41 spherical steel fiducials. The diameter of the fiducials was 1/16 inch. The angular pitch and increment between fiducials along the z-axis were 22.5 degrees and 0.15 inch respectively. The phantom contains a 3/32 inch diameter reference fiducial which is larger than the other fiducials. The larger fiducial can be used to relate the known 3D fiducial coordinates to the 2D projection coordinates using connected component analysis.

Only fiducials appearing in the field-of-view are used for calibration. The outer diameter of the phantom measured 4 inches and the wall thickness was 3/8 inch.

The helix phantom was imaged using a step-and-shoot rotation technique to acquire 210 view angles over a 210 degree short-scan acquisition. Imaging was performed at 100 kV tube potential, 30 mA peak tube current (14% full power), with a 71×71 , 15 frames/sec scanning technique. A total of 5 frames were integrated to reduce image noise at each angle. A 0.5 mm Cu plate and 7 cm of acrylic were placed in the beam before the helix phantom to simulate typical beam hardening encountered during patient imaging. For each view angle, a composite image was reconstructed and a 3×4 projection matrix $\mathbf{P}$ was derived.

The helix phantom was then replaced with a custom-made plastic phantom designed to resemble a left atrium. The hollow atrium was filled with 3 ml iohexol contrast agent (Omnipaque 350, GE Healthcare, Waukesha, WI, USA) diluted with 88 ml of water and placed in an 85 mm diameter cylinder of deionized water that fits entirely within the SBDX field-of-view (Fig. 3B). SBDX projection data was then acquired using the same step-and-shoot technique and imaging technique used for the calibration phantom. The chamber phantom was reconstructed using gFBP with and without geometric calibration. Table 2 presents the rebinning algorithm parameters used for reconstruction.

3. RESULTS

3.1 Simulations

The geometric parameters corresponding to the simulated SBDX CT data acquisition were determined at each view angle. The difference between the extracted value and the true value was computed for each parameter of ξ . Table 3 summarizes the mean error and standard deviation in error versus gantry angle, for each parameter. Over all angles, the maximum error in a rotation parameter ($\theta_x, \theta_y, \theta_z$) was less than 0.02 degrees. The maximum error in the virtual source point (x_s, y_s, z_s) was 0.4 mm, and the maximum error in SDD was -0.13 mm. Errors in the (u_o, v_o) coordinates were less than the dimension of a virtual detector element.

A miniature thorax phantom fully enclosed within the SBDX 140 mm field-of-view was used to investigate geometry-calibration-related reconstruction artifacts without the presence of confounding truncation artifacts. The numerical thorax phantom was reconstructed without (Fig. 4A) and with (Fig. 4B) geometric calibration using gFBP. Figure 4(A) shows significant artifacts and distortion of structures caused by the failure to account for the translation and rotation of the system geometry. These artifacts were removed in the image reconstructed with the proposed calibration technique. The rRMSE was calculated versus an image reconstructed using gFBP with exactly known geometry. The rRMSE was 0.4% using the proposed geometric calibration method, versus 7.7% without geometric calibration.

3.2 Sensitivity analysis

Tables 4, 5 and 6 summarize geometry parameter recovery for additional deviations from the nominal geometry. A total of 210 gantry view angles were evaluated for each case.

Translations of the source and detector arrays of +1 mm and +5 mm along the y-axis and z-

axis were considered (Table 4). The average errors in parameters describing the virtual source point and SDD were less than or equal to 0.1 mm. The average errors observed in parameters describing rotations of the source and detector arrays were 0.01 degree.

Next, the impact of 1 degree and 5 degree rotations of the source and detector arrays about the x-axis and y-axis were examined (Table 5). The average errors observed in parameters describing rotations of the source and detector array were on the order of 0.01 degrees. A 1.8 mm average error in the estimated SDD value was observed when $\theta_y = +5$ degrees. Potential techniques to improve estimates of the SDD parameter are discussed in section 4 of this paper.

When the SDD was varied from 1495 to 1505 mm with the source to axis-of-rotation held constant, the mean errors in the estimated SDD were 4.3 and -4.8 mm (Table 6). When the SAD was varied from 445 mm to 455 mm with the source-detector-distance held constant, the mean errors in the estimated SAD value were -2.0 and 2.2 mm, respectively. Thus, the SDD and SAD were the most challenging parameters to estimate accurately.

The piercing point coordinates (u_o , v_o) were estimated with average errors less than or equal to 0.1 mm for each of the cases considered in Tables 4–6. Errors in the (u_o , v_o) coordinates were less than the dimension of a virtual detector element.

3.3 Experimental validation

The estimated geometric parameters for the bench-top SBDX set-up describing SDD, piercing point (u_o , v_o) and rotation (θ_x , θ_y) are presented in Table 7, averaged across view angle. The mean estimated SDD was 1500.0 mm, compared to a nominal value of 1500.0 mm. The proposed calibration method estimated the source and detector rotation about the x-axis to be 0.1 degrees and rotation about the y-axis to be -0.3 degrees.

The estimated view angle θ_z is plotted versus the view index in Figure 5. The view index corresponds to the 210 step-and-shoot angular positions. A linear regression was performed to determine the relationship between view index and view angle. The slope of the linear regression line was 1.0 degree per view index and the intercept was 0.0 degree. The coefficient of determination, R^2 , equals 1.0. The slope of the regression line is equivalent to the angular increment between view indices which was in agreement with the programmed motion controller increment of 1.0 ± 0.05 degree per view index.

The exact geometric parameters of the system (e.g. the exact distance between the tungsten target and the surface of the CdTe detector) were not measured directly due to the invasive and destructive nature of such measurements. However, the performance of the proposed method can be assessed qualitatively by comparing reconstructed CT images without (Fig. 6A and Fig. 6C) and with geometric calibration (Fig. 6B and 6D). Double contour artifacts and object blurring are present for the image reconstructed without geometric calibration. The artifacts are reduced in the image reconstructed with geometric calibration.

4. DISCUSSION

This paper presents a novel geometric calibration method for inverse geometry CT. The method was evaluated for the SBDX system through numerical simulations and experimentally acquired projection data. For both scenarios, the proposed method reduced image artifacts observed in reconstructions performed without calibration. The rRMSE was reduced from 7.7% to 0.4% for the reconstruction of a numerical thorax phantom without and with geometric calibration, respectively.

The calibration method presented in this paper was inspired by the **P**-matrix approach used in conventional CT. It was demonstrated that the SBDX composite image is analogous to a virtual cone-beam projection originating at the center of the detector array. This was exploited to estimate geometric parameters by parameterizing the SBDX system properties used to construct the projection matrix. A limitation of this approach is that the 2D detector array is reduced to a single point referred to as the virtual source point. As a result, the proposed calibration technique is based on the assumption that rotations or translations of detector and source arrays occur in unison. Nonetheless, experimental data demonstrated the proposed method's ability to reduce artifacts caused by geometric uncertainties. Future work will investigate extending the **P**-matrix approach presented here to use a stereoscopic imaging method to estimate rotations and translations of the source and detector arrays independent of one another.

The sensitivity of the method to uncertainties in SDD, translations and rotations were examined through numerical simulations. The sensitivity analysis showed that uncertainties in parameters describing translation or rotation could be estimated with average errors on the order of 0.1 mm and 0.01 degrees. The most challenging parameter to estimate accurately was the SDD. The proposed method considered only information contained in the single multi-plane composite image during calibration. Future work could investigate using the 3D information contained in the tomosynthetic plane stack to reduce errors observed in the SDD parameter estimation. This initial experimental study used a rotating phantom stage to mimic C-arm rotation with precisely known rotation increments. However the SBDX source and detector are mounted to a C-arm with rotational capability. Future work will include applying the new calibration procedure to projection data acquired with a C-arm rotation of the SBDX source and detector arrays.

5. CONCLUSIONS

A method for geometric calibration of inverse geometry CT imaging systems using a **P**-matrix approach was demonstrated. The proposed calibration method was shown to suppress or remove image artifacts due to uncertainties in the imaging system geometry through simulations and a bench-top set-up with a rotating stage. Future work will apply and evaluate the proposed method using phantom data acquired in SBDX C-arm rotational scans. The development of geometric calibration techniques is an important step towards developing C-arm computed tomography for SBDX.

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Fig. 1.

A: SBDX performs digital tomosynthesis. **B:** Detector images are backprojected along the ray connecting each source position to the center of the detector. **C:** The composite image is analogous to a 2D virtual projection image.

Fig. 2.

A: SBDX CT acquisition consisting of source scanning and gantry rotation about the z-axis.

B: Matching virtual cone-beam projection geometry. **C:** Example composite image (virtual

projection) of the helix phantom.

Fig. 3.

A: SBDX CT data acquisition was simulated by rotating the C-arm to a horizontal position and rotating the imaging object. The helix calibration object is shown. **B:** A custom-made anatomic chamber model placed in an 85 mm diameter water cylinder.

Fig. 4.
A thorax phantom reconstructed without (A) and with (B) geometric calibration.

Fig. 5.

The estimated view angle (θ_z) is plotted versus the view angle index.

Fig. 6. SBDX projection data reconstructed without geometric calibration (A) shows double contour artifacts and blurring of object structures. The artifacts are eliminated when geometric calibration is used during image reconstruction (B). Sagittal slices without (C) and with (D) geometric calibration are presented. Display window is $[0.01, 0.03] \text{ mm}^{-1}$.

Table 1**SBDX system geometry**

Source-detector-distance (SDD)	1500 mm
Source-axis-distance (SAD)	450 mm
Focal spot positions	71×71
Focal spot pitch	2.3×2.3 mm
Native detector array	320×160
Native detector element pitch	0.33 mm
Detector bin mode	2×2

Table 2

Gridded FBP reconstruction parameters.

Number of view angles over 180 degrees	480
Rays per 2D view [columns $\times$ rows]	1120 $\times$ 290
2D view sampling pitch	0.125 mm $\times$ 0.5 mm
Radial kernel	1.4 mm $\times$ 2.0 mm
Angular kernel	2.0 degree $\times$ 2.0 degree

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Mean, standard deviation, and maximum errors in estimated geometric parameters for simulated 1 mm translation in y_s and a 1 degree rotation in θ_x .

Table 3

	x_s	y_s (mm)	z_s	θ_x	θ_y (degrees)	θ_z	SDD	u_0 (mm)	v_0
Mean Error	0.0	0.0	0.1	2.4×10^{-4}	-6.0×10^{-3}	-2.0×10^{-3}	0.0	0.0	0.0
Std. Dev.	0.1	0.1	0.1	4.0×10^{-3}	6.0×10^{-3}	6.0×10^{-3}	4.1×10^{-2}	2.0×10^{-4}	2.3×10^{-4}
Max Error	0.3	-0.3	0.4	-1.2×10^{-2}	-2.0×10^{-2}	-1.6×10^{-2}	-1.3×10^{-1}	-6.4×10^{-4}	-6.8×10^{-4}

Table 4

Mean and standard deviation of errors in estimated geometric parameters, for different deviations from the nominal system geometry.

Deviation from nominal parameter	x_s	y_s (mm)	z_s	θ_x	θ_y (degrees)	θ_z	SDD	u_o (mm)	v_o
$y_s = +5$ mm	0.0 ± 0.1	0.0 ± 0.1	0.1 ± 0.1	$0.0 \pm 3.4 \times 10^{-3}$	$0.0 \pm 6.6 \times 10^{-3}$	$0.0 \pm 6.6 \times 10^{-3}$	$0.0 \pm 4.5 \times 10^{-2}$	$0.0 \pm 4.4 \times 10^{-4}$	$0.0 \pm 3.0 \times 10^{-4}$
$y_s = +1$ mm	0.0 ± 0.1	0.0 ± 0.2	0.1 ± 0.1	$0.0 \pm 3.6 \times 10^{-3}$	$0.0 \pm 6.4 \times 10^{-3}$	$0.0 \pm 1.1 \times 10^{-2}$	$0.0 \pm 4.3 \times 10^{-2}$	$0.0 \pm 2.1 \times 10^{-4}$	$0.0 \pm 1.7 \times 10^{-4}$
$z_s = +5$ mm	0.0 ± 0.1	0.0 ± 0.1	0.0 ± 0.1	$0.0 \pm 3.7 \times 10^{-3}$	$0.0 \pm 5.0 \times 10^{-3}$	$0.0 \pm 7.8 \times 10^{-3}$	$0.0 \pm 4.5 \times 10^{-2}$	$0.0 \pm 2.0 \times 10^{-4}$	$0.0 \pm 8.8 \times 10^{-4}$
$z_s = +1$ mm	0.0 ± 0.1	0.0 ± 0.1	0.1 ± 0.1	$0.0 \pm 3.3 \times 10^{-3}$	$0.0 \pm 5.5 \times 10^{-3}$	$0.0 \pm 7.2 \times 10^{-3}$	$0.0 \pm 4.7 \times 10^{-2}$	$0.0 \pm 1.9 \times 10^{-4}$	$0.0 \pm 2.2 \times 10^{-4}$

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Table 5

Mean and standard deviation of errors in estimated geometric parameters, for rotational deviations from the nominal system geometry.

Deviation from nominal parameter	x_s	y_s (mm)	z_s	θ_x	θ_y (degrees)	θ_z	SDD	u_0 (mm)	v_0
$\theta_x = +5$ degrees	0.0 ± 0.1	0.0 ± 0.1	0.1 ± 0.1	$0.0 \pm 4.5 \times 10^{-3}$	$0.0 \pm 6.7 \times 10^{-3}$	$0.0 \pm 5.7 \times 10^{-3}$	$0.0 \pm 3.8 \times 10^{-2}$	$0.0 \pm 1.6 \times 10^{-4}$	$0.0 \pm 2.0 \times 10^{-4}$
$\theta_x = +1$ degrees	0.1 ± 1.0	-0.4 ± 1.9	0.1 ± 0.1	$0.0 \pm 4.4 \times 10^{-3}$	$0.0 \pm 3.8 \times 10^{-2}$	$0.0 \pm 1.6 \times 10^{-2}$	$-0.7 \pm 3.0 \times 10^{-0}$	$0.0 \pm 4.0 \times 10^{-1}$	$0.1 \pm 9.8 \times 10^{-1}$
$\theta_y = +5$ degrees	-0.2 ± 1.0	0.6 ± 0.6	0.0 ± 0.1	$0.0 \pm 4.5 \times 10^{-3}$	$0.0 \pm 7.0 \times 10^{-3}$	$0.0 \pm 9.9 \times 10^{-3}$	$1.8 \pm 9.1 \times 10^{-2}$	$0.0 \pm 6.3 \times 10^{-3}$	$0.0 \pm 1.2 \times 10^{-2}$
$\theta_y = +1$ degrees	0.0 ± 0.1	0.0 ± 0.1	0.1 ± 0.1	$0.0 \pm 4.8 \times 10^{-3}$	$0.0 \pm 6.3 \times 10^{-3}$	$0.0 \pm 6.7 \times 10^{-3}$	$0.1 \pm 4.4 \times 10^{-2}$	$0.0 \pm 7.8 \times 10^{-4}$	$0.0 \pm 2.8 \times 10^{-3}$

Mean and standard deviation of errors in estimated geometric parameters, for deviations in source-detector-distance (SDD) and source to axis-of-rotation (SAD) from the nominal system geometry.

Table 6

Deviation from nominal parameter	x_s	y_s (mm)	z_s	θ_x	θ_y (degrees)	θ_z	SDD	u_0 (mm)	v_0
SDD = 1505 mm	0.6 ± 2.9	-1.6 ± 2.3	0.0 ± 0.1	$0.0 \pm 3.3 \times 10^{-3}$	$0. \pm 1.2 \times 10^{-2}$	$0.0 \pm 2.0 \times 10^{-2}$	-4.8 ± 3.4	$0.0 \pm 5.0 \times 10^{-1}$	$0.0 \pm 2.9 \times 10^{-1}$
SDD = 1495mm	-0.4 ± 3.1	1.5 ± 2.3	0.1 ± 0.1	$0.0 \pm 3.9 \times 10^{-3}$	$0.0 \pm 1.0 \times 10^{-2}$	$0.0 \pm 8.8 \times 10^{-3}$	4.3 ± 4.1	$0.0 \pm 1.5 \times 10^{-1}$	$0.0 \pm 1.8 \times 10^{-1}$
SAD = 445 mm	0.3 ± 1.4	-0.7 ± 1.3	0.0 ± 0.1	$0.0 \pm 3.7 \times 10^{-3}$	$0.0 \pm 8.4 \times 10^{-3}$	$0.0 \pm 2.0 \times 10^{-2}$	-2.0 ± 2.2	$0.0 \pm 5.0 \times 10^{-1}$	$0.0 \pm 1.9 \times 10^{-1}$
SAD = 455 mm	-0.1 ± 1.6	0.8 ± 1.1	0.1 ± 0.1	$0.0 \pm 3.6 \times 10^{-3}$	$0.0 \pm 8.8 \times 10^{-3}$	$0.0 \pm 8.9 \times 10^{-3}$	2.2 ± 2.0	$0.0 \pm 1.4 \times 10^{-1}$	$0.0 \pm 1.8 \times 10^{-1}$

Table 7

Estimated geometric parameters (SDD, θ_x , θ_y , u_o , v_o) for the bench-top SBDX set-up.

Parameter	Value
SDD (mm)	1500.0 ± 0.2
θ_x (degrees)	0.14 ± 0.4
θ_y (degrees)	-0.25 ± 0.2
u_o	355.5 ± 0.0
v_o	355.5 ± 0.0