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Estimation of signal and noise for a whole-body photon counting research CT system

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Abstract

Photon-counting CT (PCCT) may yield potential value for many clinical applications due to its relative immunity to electronic noise, increased geometric efficiency relative to current scintillating detectors, and the ability to resolve energy information about the detected photons. However, there are a large number of parameters that require optimization, particularly the energy thresholds configurations. Fast and accurate estimation of signal and noise in PCCT can benefit the optimization of acquisition parameters for specific diagnostic tasks. Based on the acquisition parameters and detector response of our research PCCT system, we derived mathematical models for both signal and noise. The signal model took the tube spectrum, beam filtration, object attenuation, water beam hardening, and detector response into account. The noise model considered the relationship between noise and radiation dose, as well as the propagation of noise as threshold data are subtracted to yield energy bin data. To determine the absolute noise value, a noise look-up table (LUT) was acquired using a limited number of calibration scans. The noise estimation algorithm then used the noise LUT to estimate noise for scans with a variety of combination of energy thresholds, dose levels, and object attenuation. Validation of the estimation algorithms was performed on our whole-body research PCCT system using semi-anthropomorphic water phantoms and solutions of calcium and iodine. The algorithms achieved accurate estimation of signal and noise for a variety of scanning parameter combinations. The proposed method can be used to optimize energy thresholds configuration for many clinical applications of PCCT.

Introduction

Recent advances in photon-counting CT (PCCT) may yield potential value for many clinical applications. Energy resolving photon-counting detectors (PCD) allow simultaneous multi-energy measurements that may benefit material differentiation and quantification in X-ray CT. Additionally, its better geometric dose efficiency and noise properties can enhance image CNR or reduce the usage of radiation dose and contrast agent. The performance of PCCT highly depends on the selection of scanning parameters, especially the energy thresholds. Therefore, the acquisition parameters for a PCCT scan need be optimized to maximize the SNR for different tasks. Due to the huge number of possible combination of scanning parameters, particularly the selection of the energy thresholds, it is impractical to perform experiments that cover the whole parameter space. Although full simulation of PCCT scans with physical detector response can be performed, the data processing and reconstruction can be time-consuming and thus impractical for the optimization purposes. A

fast and accurate estimation of signal and noise in PCCT is needed to optimize the scan protocol before a clinical exam. The purpose of this work was to propose and validate a quick way to estimate both the signal and noise in images acquired from our research PCCT system using a combination of energy thresholds, objects, and radiation dose levels so that scanning parameters can be optimized to achieve the highest SNR.

Methods

(1) Physics of PCCT acquisition

PCD can suffer from non-ideal detector responses (e.g. charge sharing, Compton scattering, and k-escape, etc.) so that the detected x-ray spectrum (pulse spectrum) can be different from the incident spectrum. The detector response data represents the energy transfer probability, $D(E', E)$, for a photon of energy E being detected above an energy thresholds E' . With known detector response data, the pulse spectrum corresponding to an energy threshold E' can be expressed as:

$$S_P(E') = \int_0^{kVp} S_T(E) \cdot D(E', E) dE \quad (1)$$

Here S_T represents the transmitted spectrum after the scanning object and S_P the pulse spectrum. kVp is the tube potential. Hence, the photon count between two energy thresholds (E'_{Th_L} and E'_{Th_H}) is

$$\int_{E'_{Th_L}}^{E'_{Th_H}} \int_0^{kVp} S_T(E) \cdot D(E', E) dE dE' \quad (2)$$

By switching the order of integration, the corresponding transmitted spectrum detected within $[E'_{Th_L}, E'_{Th_H}]$ is

$$S_T(E) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' \quad (3)$$

(2) Signal model

Like in conventional CT, the signal, i.e. the effective linear attenuation coefficient (LAC), can be derived from Eq. 3:

$$\mu_{eff} = \frac{\int_0^{kVp} S_T(E) \cdot \mu(E) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' dE}{\int_0^{kVp} S_T(E) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' dE} = \int_0^{kVp} w(E) \cdot \mu(E) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' dE \quad (4)$$

Here, $w(E)$ is the system weighting function. For a mixture of M basis material,

$$\mu_{eff} = \sum_{i=1}^M \hat{\rho}_i \int_0^{kVp} w(E) \cdot \left(\frac{\mu}{\rho} \right)_i(E) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' dE \quad (5)$$

where $(\mu/\rho)_i$ and $\hat{\rho}_i$ are the mass attenuation coefficients (MAC) and density of basis material i . When water beam hardening correction is performed, the water component in Eq. 5 should be replaced by the corresponding water MAC. In Eq. 5, $w(E)$ can be derived from the tube spectrum, beam filtration, and material attenuation. For simplicity, here we use water equivalent diameter (WED) to approximate the effective attenuation of the object, therefore:

$$w(E) = \frac{S_0(E) \cdot \exp(-\mu_w(E) \cdot WED)}{\int_0^{kVp} S_0(E) \cdot \exp(-\mu_w(E) \cdot WED) \cdot \int_{E'_{Th_L}}^{E'_{Th_H}} D(E', E) dE' dE} \quad (6)$$

where $S_0(E)$ and $\mu_w(E)$ are incident spectrum and water LAC, respectively. In this study, the WED (in cm) was directly calculated from the reconstructed PCCT images using the following formula:

$$WED = 2 \sqrt{\sum_{i \in I} (CT_i + 1000) / 1000 \times A / \pi} \quad (7)$$

where i represents the index of pixel in image I and A is the pixel area in cm^2 . Hence, the signal model (figure 1) took the tube spectrum, beam filtration, scanning object, water beam hardening, and detector response into account.

(3) Noise Model

The noise model in PCCT images was derived assuming Poisson photon statistics. PCCT energy bin projection data are acquired by measuring the attenuation of X-rays between two energy thresholds, Th_L and Th_H .

$$P_{ThL}^{ThH} = \ln \left(\frac{N_{0_{ThL}} - N_{0_{ThH}}}{N_{ThL} - N_{ThH}} \right) \quad (8)$$

where $N_{0_{ThL}}$ is the counts above the low threshold in the air scan and N_{ThL} is the corresponding counts transmitted through the scanning object. The noise variance in the projection depends on both of these count values and can be determined by noise propagation analysis. For subtraction performed between counts measured from the same detector pixel and same exposure, this is straightforward. For the chess mode, where different detector pixels are subtracted, the noise propagation analysis, neglecting the covariance term, yields a noise variance of:

$$\left(\sigma_{ThL}^{ThH} \right)^2 = \frac{(N_{ThL} + N_{ThH})}{(N_{ThL} - N_{ThH})^2} \quad (9)$$

The estimation of absolute noise level in CT images requires a calibration because it is affected by a variety of factors, such as acquisition protocol, data preprocessing steps, and reconstruction parameters. For the estimation of local noise variance at the center of an object scanned at the iso-center, an inversely proportional relationship between noise and dose (photon counts) can be approximate by^{1, 2}:

$$\sigma_I^2 = k/N \quad (10)$$

For simplification, we assume that the conversion factor k is constant for all PCCT images. By combining Eq. 9 and 10,

$$\sigma_{I_{ThL}}^{ThH} = \sqrt{\frac{\sigma_{I_{ThL}}^2 + \sigma_{I_{ThH}}^2}{(\sigma_{I_{ThL}}^2 - \sigma_{I_{ThH}}^2)^2}} \times \sigma_{I_{ThL}} \times \sigma_{I_{ThH}} \quad (11)$$

Hence, the noise in bin images can be quantitatively derived from the noise in threshold images. Noise in threshold images, however, has no such dependence and can be measured independently.

For the noise estimation, a single measurement for each threshold can be sufficient. Once the noise measurement is complete for each threshold available on the scanner, the noise in bin images can be predicted for a vast number of possible thresholds configurations.

Consequently, a noise look-up table $\sigma_{I,0_{Th}}$ can be acquired for a certain object with all available thresholds that only requires a limited number of scans. The noise LUT can then be fed into the noise estimation algorithm (Eq. 11) for noise estimates.

The noise LUT acquired at a certain attenuation (WED_0) and dose level (mAs_0) can also be used for the noise estimation of scans at a different radiation dose (mAs_1) or attenuation (WED_1).

$$\sigma_{I,1}^{ThH} = \sqrt{\frac{mAs_0}{mAs_1}} \times e^{(\mu_{w,eff} \times (WED_1 - WED_0))} \times \sigma_{I,0}^{ThH} \quad (12)$$

An example was given in figure 2 to illustrate how the noise estimation algorithm works.

(4) PCCT experiments details (system, phantom, acquisition parameters)

We performed a validation of the proposed algorithms using data acquired on a research whole-body PCCT (Siemens Healthcare)^{3, 4}. The acquisition parameters used were: 140 kV, 1s rotation time, 2304 views per rotation. A chess pattern read-out mode was used so that 4 measurements were acquired simultaneously in a single scan, each one associated with a different energy threshold. Energy bin images were derived by subtracting the photon counts detected by two adjacent detector pixels prior to the logarithm step in the data pre-processing. The energy threshold and bin projection data were reconstructed to create PCCT images using a quantitative medium smooth body kernel (D30), 200 mm reconstruction FOV, and an image thickness of 2mm. The energy thresholds configurations (in keVs) used in this study were list in table 1:

(a) Signal estimation—Validation of the signal estimation algorithm was performed using scans of calcium and iodine water solutions. Eight solutions of different compositions were scanned. A head protocol was used to scan samples in a 20-cm semi-anthropomorphic water tank (WED: 18.8 cm) with configurations A–D at 440mAs. A body protocol was used to scan solution samples in a 35-cm diameter water tank (WED: 32.3 cm) with configuration A and four different mAs values: 400, 200, 100, and 50. An in-house FDK reconstruction was performed to generate images for the LAC measurements. Signal was estimated using Eq. (4).

(b) Noise estimation—The noise estimation algorithm was verified using scans of water phantoms. All scans were performed twice to generate difference images for noise measurements. The noise level was determined in a circular ROI of 6 cm radius centered at the iso-center. First a noise calibration LUT was generated by performing 20 scans of a circular 20 cm water phantom at 440mAs so that all selectable thresholds (low threshold: 20~50 keV; upper threshlds: 50~90 keV) on the prototype PCCT were sampled at least once. Then test scans with three sampled configurations (A, B, and E: none of them was used in the noise calibration LUT scans.) and four dose levels (mAs: 400, 200, 100, and 50) were acquired separately. Noise estimation based on the noise LUT was performed for test scanning parameters and compared with the noise measurements. To evaluate the estimation accuracy for objects with varying size, a body scan was performed to measure noise in a 25cm semi-anthropomorphic water tank (WED: 23.8 cm) using configurations A at 400 and 200 mAs. Noise was estimated using Eq. (11) and (12).

Results

(a) Signal estimation

The accuracy of signal estimation was verified using data acquired on the research PCCT system. As shown in Table 2, for all eight samples scanned using configuration A with a head protocol, the algorithm predicted the LAC value with errors up to 11% in the low energy bins, but less than 5% in the high energy bins and all energy threshold images.

The estimation algorithm performed equally well for head scans with the other tested threshold configurations. Table 3 lists the average estimation error for all samples for the four configurations.

Table 4 lists average estimation error for body scans in the 35cm water tank using configuration A at four dose levels. The accuracy of LAC is still well maintained except for at very low radiation dose due to photon starvation.

(b) Noise estimation

Table 5 lists the estimation error for scans using the head protocol. For all three threshold configurations, the noise estimation algorithm based on the noise LUT predicted the absolute image noise accurately. For configuration A, average errors were ~0.9 HU (3.6%), with errors up to 4.8 HU (10.3%) at the lowest dose level. The performance was the same for configurations B and E, except for energy bin 1, which had a narrow energy width.

For a very narrow bin width and/or low radiation dose (bolded and underlined data in table 5, where errors exceeded 10%), photon starvation may occur so that the assumptions for the noise model are no longer held. The accuracy of noise estimation is degraded and image quality suffers from high noise and artifacts.

The noise estimation also worked for the body scans of the 25cm water phantom using the configuration A (Table 6). The higher estimation error in *ThH2* (85~140 keV) and Bin1 (25~45keV) at 200 mAs might be caused by the low photon statistics.

Conclusions/New and Breakthrough Work

We have developed an effective framework that is based on simple mathematics (and hence is very fast compared to a full Monte Carlo simulation) to estimate the signal and absolute noise level in PCCT images. The signal model takes into account tube spectrum, detector response, object attenuation, and beam hardening. The noise model was derived based on noise propagation analysis and a noise calibration look-up-table. We validated the estimation accuracy using data acquired on a prototype PCCT system. Having estimated both signal and noise, the optimization of the energy thresholds for a variety of clinically relevant tasks is currently under way.

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Figure 1.
Framework of the signal (LAC) estimation

Figure 2.
Framework of the noise estimation

Table 1

The energy thresholds configurations used for the validation experiments.

Configuration	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
A	25	45	65	85	25 ~ 45	45 ~ 65	65 ~ 85
B	20	25	57	77	20 ~ 25	25 ~ 57	57 ~ 77
C	20	31	55	90	20 ~ 31	31 ~ 55	55 ~ 90
D	20	39	64	89	20 ~ 39	39 ~ 64	64 ~ 89
E	20	35	57	77	20 ~ 35	35 ~ 57	57 ~ 77

The percent error of LAC estimation for all eight samples measured using the head protocol and configuration A.

Table 2

Sample #	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
Ca5	-1.31%	-1.86%	-3.27%	-4.07%	5.83%	0.32%	-2.30%
Ca6	-1.37%	-1.81%	-2.69%	-3.15%	3.65%	-0.43%	-2.08%
ICa1	-0.64%	-1.26%	-3.87%	-4.74%	10.75%	2.38%	-2.53%
ICa2	-0.44%	-1.05%	-3.10%	-3.95%	8.78%	1.69%	-2.02%
ICa3	-0.75%	-1.33%	-3.43%	-4.15%	8.76%	1.49%	-2.38%
ICa4	-0.47%	-1.21%	-3.72%	-4.69%	9.61%	2.06%	-2.55%
I5	-1.33%	-1.76%	-3.43%	-3.70%	5.87%	0.50%	-2.84%
I6	-1.50%	-1.78%	-2.86%	-3.17%	3.74%	-0.26%	-2.37%

Table 3

The average percent error of LAC estimation for all eight samples measured using the head protocol with four configurations.

Configuration	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
A	-0.98%	-1.51%	-3.29%	-3.95%	7.12%	0.97%	-2.38%
B	-1.20%	-1.07%	-2.70%	-3.79%	-0.46%	4.73%	-1.45%
C	-1.23%	-0.97%	-2.51%	-4.25%	1.16%	5.44%	-1.64%
D	-1.24%	-1.17%	-3.42%	-4.25%	3.81%	2.13%	-2.64%

The average percent error of LAC estimation for all six samples measured using the body protocol and configuration A.

Table 4

mAs	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
400	0.79%	-2.41%	0.57%	-2.95%	4.81%	2.28%	-2.38%
200	1.38%	-1.96%	0.86%	-2.46%	-1.43%	-3.44%	-4.93%
100	1.64%	-1.99%	0.83%	-2.84%	-7.76%	-11.00%	-11.09%
50	1.64%	-2.15%	0.80%	-5.48%	-12.26%	-15.12%	-18.02%

The absolute and percent error of noise estimation for scans of the 20cm water phantom using the head protocol.

Table 5

Configuration	mAs	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
A	400	0.05 (1.04%)	0.05 (0.85%)	0.05 (0.55%)	0.06 (0.42%)	0.41 (2.28%)	-1.35 (-7.67%)	-1.67 (-9.33%)
	200	0.13 (1.93%)	0.14 (1.62%)	0.14 (1.21%)	0.32 (1.69%)	0.68 (2.71%)	-2.00 (-8.00%)	-2.31 (-9.18%)
	100	0.27 (2.85%)	0.26 (2.13%)	0.15 (0.93%)	0.85 (3.25%)	1.86 (5.39%)	-2.00 (-5.79%)	-2.41 (-6.92%)
	50	0.26 (1.91%)	0.51 (3.03%)	0.59 (2.53%)	2.22 (6.18%)	<u>4.79</u> <u>(10.26%)</u>	0.40 (0.87%)	-0.08 (-0.18%)
B	400	0.06 (1.30%)	0.04 (0.76%)	0.03 (0.47%)	0.05 (0.50%)	<u>16.53</u> <u>(38.20%)</u>	-0.25 (-2.10%)	-1.14 (-7.03%)
	200	0.14 (2.21%)	0.12 (1.79%)	0.18 (1.78%)	-0.12 (-0.77%)	<u>25.76</u> <u>(43.78%)</u>	-0.11 (-0.64%)	-1.60 (-7.01%)
	100	0.24 (2.59%)	0.22 (2.24%)	0.24 (1.73%)	0.28 (1.33%)	<u>-41.23</u> <u>(-25.63%)</u>	-0.11 (-0.49%)	-1.71 (-5.40%)
	50	0.39 (3.02%)	0.37 (2.75%)	0.42 (2.10%)	1.36 (4.64%)	<u>-190.93</u> <u>(-53.02%)</u>	0.68 (2.15%)	0.29 (0.70%)
E	400	0.06 (1.22%)	0.16 (2.95%)	0.05 (0.78%)	-0.01 (-0.12%)	<u>1.81</u> <u>(10.14%)</u>	0.46 (2.45%)	-0.95 (-5.93%)
	200	0.17 (2.63%)	0.31 (4.02%)	0.12 (1.25%)	-0.04 (-0.28%)	<u>2.74</u> <u>(10.95%)</u>	0.48 (1.81%)	-1.53 (-6.70%)
	100	0.27 (2.96%)	0.40 (3.69%)	0.30 (2.15%)	0.25 (1.15%)	<u>4.32</u> <u>(12.37%)</u>	1.57 (4.28%)	-1.49 (-4.73%)
	50	0.40 (3.15%)	0.70 (4.57%)	0.60 (3.07%)	1.38 (4.69%)	<u>8.99</u> <u>(19.30%)</u>	<u>5.84</u> <u>(12.16%)</u>	0.73 (1.76%)

Table 6

The absolute and percent error of noise estimation for the 25cm phantom scans using the body protocol and configuration A.

mAs	ThL1	ThL2	ThH1	ThH2	Bin1	Bin2	Bin3
400	0.52 (7.49%)	0.66 (7.81%)	0.96 (8.37%)	1.81 (9.84%)	2.01 (7.47%)	-2.19 (-8.26%)	-2.22 (-8.60%)
200	0.85 (8.74%)	1.05 (8.88%)	1.40 (8.60%)	3.35 (13.28%)	4.07 (11.06%)	-2.53 (-6.85%)	-2.18 (-6.12%)