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Multidirectional and Topography-based Dynamic-scale Varifold Representations with Application to Matching Developing Cortical Surfaces

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Abstract

The human cerebral cortex is marked by great complexity as well as substantial dynamic changes during early postnatal development. To obtain a fairly comprehensive picture of its age-induced and/or disorder-related cortical changes, one needs to match cortical surfaces to one another, while maximizing their *anatomical alignment*. Methods that geodesically shoot surfaces into one another as currents (a distribution of *oriented* normals) and varifolds (a distribution of *non-oriented* normals) provide an elegant Riemannian framework for *generic* surface matching and reliable statistical analysis. However, both conventional current and varifold matching methods have two key limitations. First, they only use the normals of the surface to measure its geometry and guide the warping process, which overlooks the importance of the orientations of the inherently convoluted cortical sulcal and gyral folds. Second, the ‘conversion’ of a surface into a current or a varifold operates at a *fixed* scale under which geometric surface details will be neglected, which ignores the *dynamic scales* of cortical foldings. To overcome these limitations and improve varifold-based cortical surface registration, we propose two different strategies. The first strategy decomposes each cortical surface into its normal and tangent varifold representations, by integrating principal curvature direction field into the varifold matching framework, thus providing rich information of the orientation of cortical folding and better characterization of the complex cortical geometry. The second strategy explores the informative cortical geometric features to perform a *dynamic-scale* measurement of the cortical surface that depends on the local surface topography (e.g., principal curvature), thereby we introduce the concept of a *topography-based dynamic-scale varifold*. We tested the proposed varifold variants for registering 12 pairs of dynamically developing cortical surfaces from 0 to 6 months of age. Both variants improved the matching accuracy in terms of closeness to the target surface and the goodness of alignment with regional anatomical boundaries, when compared with three state-of-the-art methods: (1) diffeomorphic spectral matching, (2) conventional current-based surface matching, and (3) conventional varifold-based surface matching.

Keywords

Multidirectional varifold representation; dynamic scale varifold metric; cortical surface matching; surface topography; surface registration; brain development

1. Introduction

Studying cerebral cortex development across large-scale imaging datasets stretched the frontiers of our understanding of neurodevelopment and brain disorders (Dubois et al., 2008b,a; 2014; Drobetz et al., 2014; Li et al., 2015; Iyer et al., 2015; Ming et al., 2015; Tremblay and Deschamps, 2015; Yan et al., 2015). Notably, these studies require a fundamental step which consists in accurately and meaningfully “linking” these highly convoluted surfaces to one another. This is referred to as *registration* or *matching* in medical image analysis, which founds atlasing, group comparison and statistical analysis of regional growth in a population of subjects. Due to the remarkable convolution and inter-subject variability of cortical foldings, volume-based warping typically produced poorly aligned sulcal and gyral folds (Thompson and Toga, 1996). More sophisticated volume-based cortical registration methods exploited the geometry of the surface and defined local sulcal features to guide the harmonic mapping between cortical hemispheres mapped into a unit square (Joshi et al., 2005, 2007). However, Anticevic *et al.* demonstrated in (Anticevic et al., 2008) the superiority of surface-based registration over volume-based registration for aligning cortical sulci. Indeed, cortical surface-based registration can better align the convoluted and variable cortical folding as it better exploits the topology and topography of the cortex during registration (Essen et al., 2012).

Early leading cortical surface matching tools such as Freesurfer (Fischl et al., 1999) and Spherical Demons (Yeo et al., 2010) were based on geometric features such as cortical curvature and sulcal depth to drive the warping of spherical surfaces. More recently, a multimodal surface matching framework was proposed in (Robinson et al., 2014), where they adapt the discrete Markov random field to spherical surface registration, benefitting from multivariate features. However, all these methods (Fischl et al., 1999; Yeo et al., 2010; Robinson et al., 2014) do not directly operate on the cortical surface, as they map each cortical hemisphere onto a sphere and then register them in the spherical space, which inevitably introduces distortion to surface metrics. Recent solutions that target *exact* matching relied on a spectral representation of the geometric properties of the surface, where *smooth* correspondence was generated between spatially smooth low-frequency harmonics (or surface face vibration modes) (Lombaert et al., 2011, 2013a). As an extension to this work, Lombaert *et al.* incorporated more local geometric features in an exact surface matching framework, which estimated a *diffeomorphic* correspondence map *via* a simple closest neighbor search in the surface spectral domain (Lombaert et al., 2013b). Its accuracy measured up to the performance of Freesurfer (Fischl et al., 1999) and Spherical Demons (Yeo et al., 2010).

On the other hand, *inexact* surface matching methods based on geodesically shooting one surface into another present a spatially consistent way for both establishing diffeomorphic

correspondences between shapes and measuring their dissimilarity. In (Vaillant and Glaunes, 2005; Durrleman et al., 2009), the *current metric* laid groundwork for developing generic diffeomorphic surface registration and regression models without the need to establish the point-to-point surface landmark correspondence on the longitudinal shapes. One of the key strengths of this mathematical model is that it allows to measure the dissimilarity between complex shapes of different dimensions such as distributions of unlabelled points (e.g., anatomical landmarks), curves (e.g., fiber tracts) and surfaces (e.g., cortices); and thereby to *simultaneously and consistently* track local deformations in a set of *multi-dimensional shapes* within the powerful large diffeomorphic deformation metric mapping (LD-DMM) framework (Trouvé, 1998; Dupuis and Grenander, 1998). This allows to perform statistics on the surface and its diffeomorphic deformation as diffeomorphisms facilitate further statistical analysis and atlas building (Gori et al., 2013). One drawback of the current-based shape representation model is that it annihilates the sum of two shapes with opposing normals. Recently, Charon *et al.* (Charon and Trouvé, 2013) solved this problem by proposing the use of the *varifold metric*—a variant of the current metric—for matching shapes with inconsistent orientations. Surfaces are encoded as a set of non-oriented normals, which are embedded into a space endowed with the varifold dissimilarity metric. Most importantly, varifold-based shape matching is robustly and easily extendable to multimodal imaging (e.g., white matter fibers (derived from DTI) encoded as 2D varifolds), thus one could effectively embed any shape of any given dimension into a common space of distributions, where they can be deformed, matched and compared (e.g., a set of anatomical shape complexes (Durrleman et al., 2014)). Besides, surface representation as a varifold is robust to mesh imperfections such as holes, spikes, inconsistent orientation or irregular meshing (Durrleman et al., 2014).

However, the conventional varifold matching framework developed in (Charon and Trouvé, 2013; Durrleman et al., 2014) does not consider the principal curvature direction of the deforming surface, whereas this represents a key feature of the convoluted cortical surface as it encodes the local orientation of sulcal and gyral folds that marked previous work on the cortex (Boucher et al., 2009; Li et al., 2010; Boucher et al., 2011). Furthermore, using the conventional varifold metric to measure surfaces and estimate distance between them operates at a *fixed* scale under which geometric surface details (e.g., bumps) will be overlooked, thus ignoring the (spatially-varying) scales of cortical foldings. Indeed, the width of cortical folds and their orientations change during development and diseases as demonstrated in (Boucher et al., 2009, 2011), hence we refer to the changing cortical scales as ‘dynamic’ with regard to location in space. Therefore, to better capture the cortical surface geometry, one could integrate the folds *scales* and *orientations* into the measurement metric—which is at the heart of this work. Herein, we propose two different variants to further improve the conventional varifold-based surface matching method introduced in (Charon and Trouvé, 2013) and (Durrleman et al., 2014). For the first variant, we add a novel *multidirectional* varifold surface representation encoded by its principal curvature direction, which will be combined to its normal varifold representation to solve a variational problem for shape matching. For the second variant, we propose a novel *topography-based dynamic-scale varifold metric* that measures the surface at a *dynamic scale* that spatially varies with the surface topography (e.g., principal curvature). Finally, we compare the

accuracy of the proposed variants for varifold-matching improvement with: (1) diffeomorphic spectral cortical matching (Lombaert et al., 2013b), (2) conventional current-based surface matching (Durrleman et al., 2009), and (3) conventional varifold-based surface matching methods (Charon and Trouvé, 2013; Durrleman et al., 2014) in terms of geometric concordance between target and warped shapes and also the alignment between the boundaries of cortical regions. The proposed varifold variants can also be transferred to current-based matching frameworks. Of note, a preliminary version of this manuscript was presented at MICCAI 2015 (Rekik et al., 2015c). This submission was substantially improved and offers new contributions in the following aspects: (1) introducing the *topography-based dynamic-scale varifold metric*, (2) demonstrating the outperformance of the proposed variants with respect to several state-of-the-art methods, (3) inter-variant performance comparisons, and (4) a more detailed discussion and future directions.

2. Varifold-based surface matching

Geometric measure theory provides powerful tools to build dissimilarity metrics between shapes represented as measures without requiring point-to-point correspondences. More recently, the approach of varifolds was introduced in geometric measure theory and adopted to solve shape matching problems in (Charon and Trouvé, 2013; Durrleman et al., 2014) to overcome orientation issues in current theory (Durrleman et al., 2009). Using measures makes it much more convenient to define metrics without using parametrizations. We first introduce the key ingredients of representing a shape as a varifold and performing pair-wise varifold matching.

Measuring a surface as a varifold

Measuring a surface S as a varifold is based on embedding the surface space into a Reproducing Kernel Hilbert Space (RKHS) E , where it is encoded using a set of its *nonoriented* unit normals $\overleftarrow{n(x)}$ attached at each of its vertices x (Fig. 1). This kernel-based embedding allows to define a proper distance between different embedded surfaces. The nonoriented vectors that encode the surface are defined as elements of the space of non-oriented tangent spaces $G_d(E)$ (Grassman manifold). In the case of surfaces, G_d is defined as the quotient of the unit sphere $\mathbb{S}^2$ by two group elements $\{\pm Id_{\mathbb{R}^3}\}$, where $Id_{\mathbb{R}^3}$ is the space of unit identity 3D vectors. An element u in this quotient space $G_d(E)$ belongs to the class of equivalent elements $(u, u/|u|, -u/|u|)$. Any surface is thereby represented as a distribution of non-oriented spaces tangent to each of its vertices and spread out in the embedding space E .

A varifold surface is measured in a similar way that we measure a current surface, except that the reproducing positive Gaussian kernel k_e spanning the space E is *augmented* by a linear continuous a Cauchy-Binet kernel k_t on the Grassman manifold $G_d(E)$, which leads to ‘annihilating’ the orientation of the normals, thereby producing a nonoriented measurement of the surface (Charon and Trouvé, 2013). In a continuous setting, a varifold is defined as a

continuous linear form that integrates a test field $\omega \in W$: $S(\omega) = \int_{\Omega_S} \omega(x, \overleftarrow{n(x)}) |n(x)| dx$. More specifically, the reproducing kernel k on the space of varifolds is the tensor product of

kernels on E and on $G_d(E)$: $k = k_e \otimes k_r$. Hence, in this kernel-based embedding, the vector space W is defined as a RKHS on the square-integrable space $C_0(E \times G_d(E))$. In particular, for $x, y \in E$ and $u, \vec{v} \in G_d(E)$, the varifold kernel is defined as

$k((x, \vec{u}), (y, \vec{v})) = k_e(x, y) \left(\frac{u^T v}{|u||v|} \right)^2$, where $k_e(x, y) = \exp(-|x - y|^2 / \sigma_e^2)$ is a *fixed* Gaussian scalar kernel that decays at a rate σ_e . Although measuring a surface as a varifold does not require any parameterization; however, it largely depends on the kernel parameter σ_e , which defines the scale at which we examine the surface geometric details. In other words, the smaller this scale the more geometric details will be captured when measuring the surface using the varifold metric, and the larger it is the more geometric details will be neglected (e.g., very narrow cortical folds). The space of varifold is then defined as the *dual* space W^* (i.e., the space of linear mappings from W into $\mathbb{R}$), where any varifold in W^* is defined as: $\omega(x, u) = \delta_{(x, u)}(\omega) = \langle k((x, u), \cdot), \omega \rangle_W$, with $\delta_{(x, u)}$ denoting a Dirac varifold that associates to any ω its evaluation $\omega(x, u)$. In a more discrete setting where a surface S is composed of N faces (triangles), the varifold integral can be approximated into a discrete sum of Dirac varifolds parameterized by the positions x_i of the centers of its faces and their corresponding normals: $S = \sum_{i=1}^N \delta_{(x_i, \vec{n}_i)}$. More importantly, this succinct surface representation nicely allows to define dissimilarity measures between pairs of shapes $S = \sum_i \delta_{(x_i, \vec{n}_i)}$ and $S' = \sum_j \delta_{(x'_j, \vec{n}'_j)}$, as we endow the varifold space with a dot-product:

$$\langle S, S' \rangle_{W^*} = \sum_i \sum_j k_e(x_i, x'_j) \frac{(n_i^T n'_j)^2}{|n_i||n'_j|} \quad (1)$$

It is quite obvious that this conventional varifold-based dot product relies on a *unidirectional* representation of the surface where only the normal direction to the surface is considered.

Geodesic shooting of varifolds for registration

Measuring distances between varifolds lays the foundation for shooting a source varifold S_0 onto a target varifold S_1 , as it is formulated as a distance minimization problem while constraining the deformation trajectories from source to target to be smooth, invertible and geodesic (optimal). Using a Hamiltonian approach, this geodesic shooting of one surface onto another is parametrized by the initial deformation momenta p_0 , which evolves through a conservation equation and allows to fully recover the deformation trajectories for any timepoint t within the deformation time interval $[0, 1]$ (Miller et al., 2015). These geodesic trajectories are the solution of the following ordinary differential flow equation based on a LDDMM formulation:

$$\begin{cases} \frac{d\phi_t(x)}{dt} = v_t \circ \phi_t(x), & t \in [0, 1] \\ \phi_0 = Id_{\mathbb{R}^3}. \end{cases} \quad (2)$$

where ϕ_t is the diffeomorphism which acts on each face center x of S_0 to deform into $S_1 = \phi_1 \cdot S_0$. As in (Durrleman et al., 2014), a set of optimal *control points* $\{c_k\}_{k=1,\dots,N_c}$ were used to guide the surface warping process. These are estimated within a solid variational formulation along with the initial deformation momenta and the displaced positions of the moving surface vertices. The velocity $v_\lambda(x)$ at any point $x \in E$ is defined as the sum of convolutions between scalar functions k_V located at the set of control points and the estimated deformation momenta $\{p_k\}$: $v(x) = \sum_{k=1}^{N_c} k_V(x, c_k)p_k$. Thus, the right-invariant metric in V is defined as $\int_0^1 |v_t|_V^2 dt = \int_0^1 p(t)^T k_V(c(t), c(t)) p(t) dt$. Notably, the deformation space V is also defined as a RHKS that is densely spanned by a reproducing Gaussian kernel k_V which decays at a fixed rate σ_V . The role of this kernel bandwidth σ_V is to define the deformation scale below which vertices of the surface will remain unmoved (*i.e.*, no deformation).

Unidirectional varifold-based registration functional

Through minimizing the following energy functional, we estimate the optimal initial deformation momenta, optimal control points and optimal warped vertices' positions:

$$J = \frac{1}{2} \int_0^1 |v_t|_V^2 dt + \gamma \|\phi_1^v \cdot S_0 - S_1\|_{W^*}^2 \quad (3)$$

The energy functional comprises two key terms. The first energy term forces the warping trajectory to be smooth through minimizing the squared norm of the time-dependent velocity field v_t . The second term makes the trajectory end close enough to the target surface through minimizing the difference between the warped and the ground truth surfaces. The parameter γ defines the trade-off between these terms. The objective functional J is minimized through a typical conjugate gradient descent algorithm as in (Durrleman et al., 2014).

3. Proposed strategies for improving varifold-based cortical surface matching

3.1. Surface multidirectional varifold representation

To better guide the geometric varifold warping, we augment the varifold-based unidirectional surface representation by adding a new direction: the principal curvature direction κ . This engenders a novel multidirectional representation of the surface: $\mathcal{S}(\omega) = S^{\vec{n}}(\omega) + S^{\vec{\kappa}}(\omega)$, where $S^{\vec{n}}(\omega) = \int_{\Omega_S} \omega(x, \vec{n}(x)) |\vec{n}(x)| dx$ and $S^{\vec{\kappa}}(\omega) = \int_{\Omega_S} \omega(x, \vec{\kappa}(x)) |\vec{\kappa}(x)| dx$.

Decomposing the surface into both of these 'orthogonal' components means that, instead of *reading* the shape of a surface in one direction using only the set of normals associated with its faces, we perform an additional tangential reading in the principal curvature direction; thereby collecting more geometric features that will be warped when registering surfaces (Fig. 1). However, unlike the computation of surface normals, the estimation of the principal direction is challenging as it might be noisy at flat cortical areas where both minimum and maximum principal curvatures are very small (Fig. 2). To solve this problem, we first

compute the principal directions and curvature derivatives using an efficient finite difference method (Almgren, 2004). Second, we adopt a robust method developed in (Li et al., 2009) to estimate smooth principal curvature direction that uniformly point toward the direction parallel to its folds. These are estimated through solving the variational diffusion equation: $E_d = \int_{\Omega_S} \|\nabla \kappa(x)\|^2 + |\tau(x)| \times \|\kappa(x) - \eta(x)\|^2 dx$ with respect to $\kappa(x) \cdot \eta(x) = 0$; where η denotes the original principal curvature direction, κ represents the diffused principal curvature direction, and $\tau(x)$ is set to the principal curvature value at each vertex x . This denoises the noisy tangent field at the ambiguous at cortical regions, while still preserving the original informative tangent field at highly bended regions. Hence, this diffusion process allows to generates a smooth tangential varifold-based surface representation, providing rich information of cortical folding.

We would like to point out that k_e can be defined differently for each direction such as different rates of decay. As for the current work, we use the same kernel to reduce the number of parameters for empirical tuning. More importantly, the proposed multidirectional inner product may unravel richer *generic* mathematical theoretical properties of the multidirectional varifold space for multidimensional shapes, which is currently beyond the scope of this paper that focuses on cortical surface registration.

Multidirectional varifold-based registration functional—To estimate the initial deformation momentum attached to each optimal control point, we now transport the multidirectional varifold by the estimated diffeomorphism ϕ_1^v , through the minimization of the following new energy functional using the numerical scheme proposed in (Durrleman et al., 2014):

$$J_{\text{multidirectional}} = \frac{1}{2} \int_0^1 |v_t|_V^2 dt + \gamma_n \|\phi_1^v \cdot S_0^{\overleftarrow{n}} - S_1^{\overleftarrow{n}}\|_{W^*}^2 + \gamma_\kappa \|\phi_1^v \cdot S_0^{\overleftarrow{\kappa}} - S_1^{\overleftarrow{\kappa}}\|_{W^*}^2 \quad (4)$$

Note that the fidelity-to-data term in this functional (Eq. 4) comprises two weighted terms: the first one is for measuring the surface in the conventional varifold space using its normal directions, and the second one uses its principal curvature directions. By minimizing this energy functional, we define the geodesics that warp one surface onto another by decreasing the distance between the normal varifold representations of both surfaces and their respective tangential representations. Of note, this work focuses on practical applications of the proposed multidirectional varifold representation for cortical surface registration. Although a theoretical proof of convergence as the one developed for functional varifolds in (Charlier and Trouvé, 2014, 2015) would be of interest to demonstrate, this sidesteps the main focus of our study.

3.2. Surface dynamic-scale varifold representation

Measuring a surface as a varifold and estimating distances between varifolds highly depend on the choice of the reproducing kernel k_e spanning the varifold space W^* . Previously, current-based and varifold-based surface representation and deformation frameworks (Vaillant and Glaunes, 2005; Durrleman et al., 2008, 2009, 2012; Charon and Trouvé, 2013; Durrleman et al., 2014) used *only static* reproducing Gaussian kernels k_e with a *fixed*

bandwidth σ_e defining a scale under which local measurement of the surface will be considered as noise. In this paper, we introduce the concept of a *dynamic* kernel that allows to measure the geometry of the surface at different bandwidths (or scales) that vary locally according to *any* scalar field. Considering the dynamic scales of the cortical surface foldings demonstrated in Fig. 3, one would better capture the surface topography when varying the measurement bandwidth according to principal curvature value, which reflects the topographic cortical properties. When using a static reproducing kernel (Fig 4-a), at a fixed location x , only the closest x_k vertices to x will have the strongest contribution to the varifold measurement performed at x , as the bandwidth σ_e is fixed. However, when we vary the bandwidth according to a topography-derived scalar field, e.g., maximum principal curvature τ_1 or minimum principal curvature τ_2 fields, the convolutions computed at all surface vertices x_k with respect to x will capture different surface properties as illustrated in Fig 4. Notably, each time we change the reproducing kernel formula, we define a new metric that induces a novel dot product on the varifold space W^* between two surfaces $\langle S, S' \rangle_{W^*}$, thereby changing the way we measure distances (or similarities) between surfaces.

With the *dynamic* reproducing kernel, not only the closest vertices to the location of measurement x will influence how we ‘read’ the surface, but also the local topography of the surface (the breadth of its hills and valleys). To illustrate the impact of using different dynamic kernels (both symmetric and non-symmetric) on the induced local convolution at x , we propose three different kernels that capture different properties of the surface:

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Dynamic kernel I (Fig 4-b): $k_e^{\tau_2}(x, x_k) = \exp\left(\frac{-\|x - x_k\|^2}{f(\tau_2(x_k))^2}\right)$, where f is defined as: $f(\tau_2(x_k)) = 10 - 8\tau_2(x_k)$, which is a linear normalization process

that would bring the original curvature values into a range of $[1, \frac{60 \times \sigma_v}{100}]$. This is simply achieved through solving the linear function $f(x) = ax + b$, with $a, b \in \mathbb{R}$, such that the minimum curvature value maps to a value of 1 and the

maximum curvature maps to a value of $\frac{60 \times \sigma_v}{100}$. Indeed, as pointed out in (Durrleman et al., 2008, 2009, 2012), the shape kernel bandwidth should be smaller than the deformation kernel bandwidth. So here, with a 60% upper limit, the deformation process will still be reliable. We define a dynamic kernel $k_e^{\tau_2}$ that uses a small bandwidth in highly curved areas and a large bandwidth in at areas, thereby allowing the local measurement to decrease the scale at which we look at small cortical foldings and to increase that scale for larger foldings. Figure 4-b clearly shows the more diffusive property of the proposed dynamic kernel in relatively at areas (i.e. small τ_2) that are quite far (in the Euclidean sense) from the measurement vertex x .

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Dynamic kernel II (Fig 4-c): $k_e^{\tau_1}(x, x_k) = \exp\left(\frac{-\|x - x_k\|^2}{\tau_1(x)^2}\right)$, with $\tau_1(x) \geq 0$. In case $\tau_1(x)$ is zero, we set its value to be very close to zero. This kernel allows to locally measure the surface at a specific vertex x while exploring the topography of the neighborhood and attributing high convolution values along

the principal orientation of the gyral crests and sulcal valleys. This is quite intuitive since the selected bandwidth is the maximum principal curvature τ_1 (Fig 3).

- *Symmetric dynamic kernel III* (Fig 4–d): Of note, the two aforementioned kernels are not symmetric as for any x and $y \in E$, $k_e^{\tau_2}(x, y) \neq k_e^{\tau_2}(y, x)$ and $k_e^{\tau_1}(x, y) \neq k_e^{\tau_1}(y, x)$. In this case, the RKHS will be equipped with a *symmetric dynamic* kernel with a spatially varying width such that: $k_e^{\tau_1, sym}(x, y) = k_e^{\tau_1, sym}(y, x)$.

$$k_e^{\tau_1, sym}(x, x_k) = \begin{cases} \exp\left(\frac{-\|x - x_k\|^2}{(|\tau_1(x)| - |\tau_1(x_k)|)^2}\right) & \text{if } x \neq x_k \\ 1 & \text{otherwise} \end{cases} \quad (5)$$

This kernel balances a trade-off between proximity to the measurement vertex and the similarity to its topography (in this case represented by the maximum principal curvature). In the illustrated case, the valleys of the cortical surface surrounding vertex x have large convolution values. We notice that the dynamic kernel **II** $k_e^{\tau_1}$ peaks at flat areas on the cortical surface (which demonstrates its dynamic topography), however, the symmetric dynamic kernel $k_e^{\tau_1, sym}$ displays a smoother appearance.

When using the varifold metric not to only measure surfaces but to also match them, one would preferably choose a kernel whose bandwidth becomes large at flat areas and decreases in highly folded areas. Furthermore, the reproducing kernel needs to be symmetric; thus, we selected $k_e^{\tau_2}$ kernel to define the *dynamic-scale* varifold matching for cortical surfaces. For the sake of demonstration and to show the diversity of measurements that one can make on a surface based on how we define the spatially changing kernel rate of decay, we included the dynamic kernel **II** and the symmetric dynamic kernel **III**, which may be more suited for other shape analysis applications and more rigorous mathematical analysis. We would like to note that throughout all our experiments, the curvature topographic field is computed at the center of each surface triangle by taking the mean of absolute curvature values at the 3 vertices of each triangular mesh.

Remark 1—We would like to note that using the *non-symmetric* dynamic kernels **I** and **II** will necessitate to define a new mathematical test space W to embed the ‘new’ varifold in, since RKHS revolves around symmetric kernels (e.g., the symmetric dynamic kernel **III**). These non-symmetric dynamic kernels may unravel different geometric characteristics of shapes as illustrated in Fig 4.

4. Results

There is no ground truth available for perfect brain surface matching. However, spectral diffeomorphic matching (SDM) (Lombaert et al., 2013b) has been demonstrated to provide highly accurate anatomical cortical matchings with a performance that measures up to the reliable cortical surface matching methods Freesurfer (Fischl et al., 1999) and Spherical

Demons (Yeo et al., 2010). Therefore, we consider SDM as a key benchmark for comparison. To further boost up its performance, we used the maximum principal curvature as an extra dimension to the surface spectral representation as it can better guide the registration (Lombaert et al., 2013b). We also compare the proposed mutidirectional and dynamic-scale varifold variants with the conventional current-based surface matching (Durrleman et al., 2009) and the conventional varifold-based surface matching (Charon and Trouné, 2013; Durrleman et al., 2014). For evaluation, we used two criteria: a vertex-wise distance between the warped and the target surfaces, and the surface Dice index overlap D

between two surfaces S and S' ($D(S, S') = \frac{2|S \cap S'|}{|S \cup S'|}$) for 35 anatomical regions of interest defined in (Desikan et al., 2006) and parcellated using the infant-specific method described in (Li et al., 2014c). We detail below the results for each evaluation criterion.

Image processing

All MR images at all the acquisition timepoints were preprocessed using an infant-specific framework developed in (Dai et al., 2013; Li et al., 2014b,c,d), including (1) the removal of the skull (Shi et al., 2012), followed by the removal of the cerebellum and brain stem by registering an atlas to each subject (Shen and Davatzikos, 2002; Wu et al., 2006); (2) intensity inhomogeneity correction using N3 method (Sled et al., 1998); (3) rigid alignment of each image to the age-specific infant brain atlas (Shi et al., 2011); (4) longitudinal tissue segmentation of infant brain MR images into white matter (WM), gray matter (GM), and cerebrospinal fluid (CSF) using a longitudinally consistent level-set-based segmentation method (Wang et al., 2013); and (5) filling the cortex insides and splitting the brain into left and right hemispheres.

Cortical surface reconstruction

For each segmented image, we reconstructed the inner cortical surface for each hemisphere using a deformable surface method (Li et al., 2012, 2014a). In our case, the number of faces is fixed to 81920 and the number of vertices to 40962, which provides a high-resolution mesh for capturing infant cortical topography. In particular, we corrected the topological and geometric defects in the WM and tessellated the cortical surface as a triangular face to guarantee a spherical topology for each hemisphere as proposed in (Li et al., 2012, 2014a). Ultimately, each cortical hemisphere was parcellated using the robust framework developed in (Li et al., 2014c).

Data and parameters setting

We evaluated the proposed framework on cortical surfaces of 12 infants, each with MRI scans acquired at around birth and 6 months of age. We empirically fixed the current and varifold parameters at the same values for all infants: $\sigma_e = 8mm$, $\sigma_v = 20mm$ and $\gamma = 0.1$. For the novel varifold matching framework, we set $\gamma_n = 0.1$ and $\gamma_k = 0.2$ to assign more weight to the fidelity-to-data term depending on the surface principal curvature direction.

Benchmark with four state-of-the-art methods

We compared the results of the proposed matching strategies with: (1) diffeomorphic spectral cortical matching where we used the maximum principal curvature as a feature to refine the registration as pinpointed in (Lombaert et al., 2013b) to boost up its performance, (2) current-based surface matching (Durrleman et al., 2009), and (3) original varifold-based surface matching methods (Charon and Trouvé, 2013; Durrleman et al., 2014). Table 1 demonstrates that both dynamic-scale and multidirectional varifold matching methods achieve better performance than the three state-of-the-art methods *w.r.t* both evaluation criteria: anatomical alignment quantified by the mean Dice index and geometric concordance quantified by Euclidean mean distance error. Our performance has improved with a rate similar to the Dice overlap ratios reported in (Lombaert et al., 2013b) when compared with FreeSurfer and Spherical Demons. This improvement is notably visible in Fig. 5 where we display the mean Dice overlap between warped and target boundaries of 35 anatomical regions averaged across all subjects. Concordantly, Fig. 6 displays the mean Euclidean distance error between the warped and the target 35 boundaries of anatomical regions averaged across all infants and demonstrates that both proposed varifold matching strategies improved the matching accuracy. The matching performance gradually ameliorates from SDM method (Fig. 5-a) to the conventional current-based and varifold-based surface registration methods (Durrleman et al., 2014; Charon and Trouvé, 2013) (Fig. 5-b-c), better refines the alignment accuracy with the proposed dynamic-scale varifold matching (Fig. 5-d) and reaches its apex for the multidirectional varifold method (Fig. 5-e). Additionally, both unidirectional varifold matching methods using solely the principal curvature direction (*i.e.*, $\gamma_n = 0$) or the normal direction (*i.e.*, $\gamma_k = 0$) perform similarly with no statistical significance.

We also performed paired t-tests on the 5% significance level of the results obtained for each of these methods with respect to the conventional varifold matching method. The outperformance of the proposed variants were statistically significant ($p\text{-value} < 0.05$), with the statistical significance peaking when comparing the multidirectional varifold with the conventional one ($p\text{-value} < 0.001$). Overall, both dynamic-scale and multidirectional varifold matching methods significantly improved the matching accuracy over the presented state-of-the-art methods with the multidirectional variant striking the best performance. The proposed varifold representation strategies for cortical matching also significantly outperformed other methods in some highly variable and folded cortical regions (e.g., lateral orbito-frontal cortex and superior parietal cortex for mean Dice overlap in Fig. 7 and medial orbito-frontal cortex and superior frontal gyrus for mean Euclidean surface distance error in Fig. 8).

Insights into the prospect of multidirectional and dynamic-scale varifold representations unification

For the sake of investigation, we also evaluated the cortical surface matching accuracy while *simultaneously* measuring its geometry using a multidirectional dynamic-scale varifold representation. However, directly unifying these approaches decreased the cortical surface registration accuracy (mean Dice index = 87.74 ± 2.97) when compared with the conventional varifold representation (= 88.09 ± 1.38) (Table 1). Hence, we proposed to use an *adaptive* simultaneous combination of both multiscale and multidirectional surface

representation. Indeed, since the rich topographic information is already encoded in the dynamic kernel $k_e^{\tau_2}$, one wouldn't need to overcrowd the surface measurement along the diffused principal curvature direction by convolving it with this topographic dynamic kernel. In addition, the principal curvature direction vector field needs to be corrected and is not as stable as the normal vector field; thus, we measure S^{κ} using the static kernel k_e with fixed σ_e while measuring $S^{\tilde{\eta}}$ using the dynamic kernel $k_e^{\tau_2}$. Hence, the dynamic-scale varifold measurement is only performed along the normal direction. Subsequently, a potential mutiscale multidirectional varifold metric between two surfaces S and S' can be defined as:

$$\langle S, S' \rangle = \frac{1}{2} \sum_i \sum_j k_e^{\tau_2}(c_i, c'_j) \frac{(n_i^T n'_j)^2}{|n_i| |n'_j|} + \frac{1}{2} \sum_i \sum_j k_e(c_i, c'_j) \frac{(\kappa_i^T \kappa'_j)^2}{|\kappa_i| |\kappa'_j|}.$$

The mean Dice overlap across subjects (88.69 ± 1.45) using this merged dynamic-scale multidirectional varifold representation improved the registration accuracy when compared with the conventional varifold metric (88.09 ± 1.38); however, it did not outperform the respectively proposed dynamic-scale and multidirectional varifold variants (Table 1).

5. Discussion

Brain cortical surface analysis has motivated the development of several cortical matching methods to compare cortical morphology/functions and perform statistical analysis across populations of subjects in the hope of advancing our understanding of brain workings and development. Traditionally, geodesic shape matching models do not target exact matching, however, one would want to maximize the anatomical alignment between the warped and the target surfaces. In this paper, we presented two different varifold-based strategies to improve the conventional geodesic varifold-based shape registration method developed in (Charon and Trouvé, 2013; Durrleman et al., 2014). Both proposed dynamic-scale varifold metric and multidirectional varifold-based surface representations defined novel tools to measure distances between surfaces, and thereby influencing the performance of LDDMM-rooted surface deformation method developed in (Charon and Trouvé, 2013; Durrleman et al., 2014). Notably, the proposed framework capitalize on a rich topographic dynamic-scale and orthogonal representation of the deforming surface, applied for the first time to developing infant brains. Both proposed variants significantly improved the matching performance over several state-of-the-art surface registration methods, which would enable building better cortical surface atlases in the future, as well as lead to a better examination of subjects with abnormal cortical development.

Improving cortical surface registration can be approached along two fundamental search lines. First, how to measure the surface? Second, how to deform it? In this paper, we mainly focused on developing a rich varifold metric to better measure the geometric properties of the cortical surface. To the best of our knowledge, we proposed the first spatially-varying varifold metric based on a topographic scalar field, and which omits the need to empirically tune the bandwidth of the reproducing kernel σ_e , influencing the structure of the varifold space. Now looking into the shape deformation analysis literature, a few studies used the concept of dynamic-scale kernels, but in the space of diffeomorphic deformations V , where

the used kernels belong to the RKHS V . For instance, (Risser et al., 2010) proposed a simultaneous fine and coarse smoothing of the estimated deformations in the LDDMM framework. Their key idea consists in registering images at several scales simultaneously through defining a kernel k_V as a weighted sum of Gaussian kernels with various fixed bandwidths σ_V . Hence, the resulting deformation trajectory is a summation of dynamic-scale deformations. Although our proposed variant operates in the varifold shape space (not the deformation space), our dynamic-scale varifold metric differs from the concept introduced in (Risser et al., 2010) as we don't need to fix multiple weights associated with coarse-to-fine Gaussian kernels that produce multiple coarse-to-fine shape deformations. Instead, we directly produce one varifold shape at different local scales (without the additional need for averaging or concatenation). This is achieved through tapping into the richness of the surface topography to derive a meaningful scalar field that *systematically* defines a new metric at each surface mesh. A more sophisticated dynamic kernel bundle for LDDMM was introduced in (Sommer et al., 2011, 2013) where they decouple the scales to allow the estimation of different deformation momenta at different scales, thereby enabling the algorithm to select the appropriate deformation at each scale individually. Recently, (Schmah et al., 2013) introduced the mathematical concept of spatially-varying metrics in LDDMM where *a priori* information on the deformation intensity helped design a set of deformation regularization Gaussian kernels, each associated with a specific region in the deforming object. All these methods (Risser et al., 2010; Sommer et al., 2011, 2013; Schmah et al., 2013) differ from ours as they focus on deformation scales and we focus on shape representation scales based on a topographic surface scalar field. It is also worth noting that although we constrained the bandwidth of the varifold kernel to be dynamic in space (i.e., only varying with space), one could also make it vary with time such that the guiding static field $\tau_1(x, t)$ can be updated at each timestep, thus becoming dynamic in both space and time dimensions (i.e., spatiotemporal).

The ability to describe the shape at precisely the right *scales* and in the most meaningful *directions* promises to allow a better statistical shape-based analysis and comparison incorporating scale and direction information. Interestingly, the multidirectional varifold representation outperformed the dynamic-scale varifold representation for infant surface registration. This can be explained by the heavy contribution of the new direction into the proposed energy functional (Eq 4). Furthermore, the proposed dynamic-scale varifold metric and multidirectional varifold-based shape representations are generic and can be extended to complex high-dimensional multishapes as formulated in (Arguillre et al., 2015). Additionally, the dynamic-scale varifold representation can also be used to represent low-dimensional shapes (e.g. points or lines), where the scale will spatially vary with an input scalar field defined in the space embedding the data. This can be flexibly adapted to the problem to solve and the kind of data that is used. We also proposed various topography-based dynamic-scale varifold metrics in (Fig. 1) to intuitively illustrate of the influence of using multiple kernels on locally measuring the cortical surfaces as varifolds. One could use these metrics to study a population of cortical shapes or other topographically rich shapes and examine how these metrics capture the morphometric shape change in space and time from different perspectives. Indeed, each of these varifold metrics capture a specific geometric property of the surface as explained in Section 3.2. It is worth mentioning that we

tested the proposed non-symmetric dynamic kernel **II** and the symmetric dynamic-scale varifold kernel but they did not improve the registration accuracy –which is quite expected since we want the scale to characterize the width of the folds. Having said that, we would like to point out that although we proposed one definition for the symmetric kernel, there are several ways for defining classes of symmetric kernels. Our interest was to introduce the core idea of a *symmetric* kernel that can churn out more sophisticated definitions for symmetric kernels that can be used in registration and other machine-learning applications. However, we would like to draw attention to the role of non-symmetric kernels (e.g., **I** and **II**) in capturing different topographic properties of surfaces, which can be explored in future shape analysis frameworks (see Remark **1**).

Moreover, our preliminary findings on the potential of unifying the multidirectional varifold representation with the dynamic-scale varifold representation showed that whether directly or adaptively unifying them as proposed in the last experiment of the Results section did not outperform solely using either of the proposed dynamic-scale or multidirectional varifold metrics. Hence, our future work would include developing a *coupled multidirectional dynamic-scale* varifold representation for improving surface registration, which would potentially need devising a more robust and refined strategy for combining both metrics with theoretical rigor –which is beyond the focus of this work. We would also like to point out that the conventional varifold-based method is computationally expensive as it takes around 30 *mins* to converge using the high-resolution meshing of the input surfaces and the computational time doubles when using one of the proposed variants. When compared to SDM, it is much slower. However, the main focus of this paper was not the computational time but the performance and introducing new concepts that may inspire researchers to rigorously derive novel theories or more sophisticated matching tools. As an additional note related to our previous work (Rekik et al., 2015c), we did not use topographic control points that were automatically placed on very highs and depths of the cortical surface as suggested in (Rekik et al., 2015c), since they did not improve the registration if used without being combined with the multidirectional variant. Hence, the control points were also part of the energy functional optimization problem as in the conventional varifold optimization setting (Durrleman et al., 2014).

As another future direction, since shape deformation also frequently occurs at different scales, one can easily integrate the proposed varifold representation metrics into a dynamic deformation kernel in a LDDMM variational framework. Additionally, as an alternative to spatially varying the varifold kernel based on an input scalar field, one can *learn* the appropriate kernel bandwidth that will maximize the cortical alignment as proposed in (Vialard and Risser, 2014) for learning a kernel matrix for estimating smoother deformations during 4D brain image registration. Our proposed varifold representation would also smoothly fit into the rigorous geometric setting for constrained geodesic shape evolution (Arguillre et al., 2015). Another potential application for our method would include spatiotemporal shape regression where shapes are successively morphed onto one another while enforcing temporal smoothness as in (Schwartz et al., 2015). One could also use the proposed varifold variants to better improve the performance of spatiotemporal shape

evolution prediction models as in (Rekik et al., 2015b,a), since shape regression is a fundamental step in building these learning-based models.

6. Conclusion

We have presented the first dynamic-scale and multidirectional varifold representations for cortical surface registration. The proposed metrics are generic and can be extended to other multidimensional shapes. Additionally, the proposed surface matching framework is very general and permits to model and solve far more general shape *topography-based* deformation problems. Both devised multidirectional and dynamic-scale metrics significantly outperformed different state-of-the-art methods. As the reproducing kernels k_e and k_v , respectively, yield a unique definition for both varifold shape representation space W^* and deformation space V , our future work would include a coupled dynamic-scale shape registration at both the representation and the deformation levels. Furthermore, learning at which scale we measure local geometric details of the shape and at which scale we deform it in a correlated way would be of great interest in reducing the number of empirical parameters to rely on during the shape warping process.

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Figure 1. Cortical surface representations

(a) The surface is represented using its oriented normals located at the centers of its faces (*i.e.*, a current) or (b) using its nonoriented normals (*i.e.*, a varifold). (d) We propose to represent the surface S as a sum of two directional varifolds (S^n and S^κ): one generated by its nonoriented normals (b) and the other by the nonoriented principal curvature direction (c).

Figure 2. Principal curvature direction diffusion

To correct the noisy principal curvature direction field in ambiguous flat areas, we diffuse it around reliable and informative sulcalgyral lines by solving the variational diffusion equation as proposed in (Li et al., 2009).

Figure 3. Multiscale cortical folding

The developing brain cortex has different folds with different *orientations* at different *scales*. The widths of the black arrows demonstrate the variability of cortical folding scales measured as the folds' widths in the direction across them.

Figure 4. Intuitive illustration of the influence of using multiple kernels on locally measuring the cortical surfaces as varifolds

(a) At a fixed vertex x , we visualize the convolution value at all x_k vertices on the surface. k_e represents the conventional static reproducing Gaussian kernel. (b) We display the result of the convolution using a dynamic kernel $k_e^{\tau_2}$ with a topographically varying bandwidth that depends on the minimum principal curvature τ_2 . The yellow arrows point to the diffusiveness of the proposed kernel. (c) A different dynamic kernel $k_e^{\tau_1}$ whose bandwidth is selected as the maximum principal curvature τ_1 . This allows to trace neighboring narrow foldings to x with maximum principal curvature τ_1 —which expands quite further on the cortical surface than the rest of the introduced kernels (top row, lateral cortical view). (d) We introduce a symmetric kernel such that for any vertices x and y on the surface $k_e^{\tau_1, sym}(x, y) = k_e^{\tau_1, sym}(y, x)$. We notice that the convolution values peak at proximal vertices x_k to vertex x (in the Euclidean sense) with dissimilar absolute maximum principal curvature values (red regions).

Figure 5. Mean Dice overlap between warped and target parcellated left hemispheres into 35 anatomical regions, averaged across all subjects

Clearly, our proposed dynamic-scale (d) and mutidirectional (e) varifold-based matching methods *significantly* outperform spectral diffeomorphic matching and the conventional varifold and current-based matching methods as displayed in both top lateral and bottom medial cortical views (p-value = 0.001 for dynamic-scale *vs.* conventional varifold and p-value = 0.0003 for mutidirectional *vs.* conventional varifold). We number in the figure some anatomical areas where the improvement is visually striking: (1) lateral occipital cortex, (2) inferior parietal cortex, (3) postcentral and precentral gyri, (4) rostral middle frontal gyrus, (5) insula cortex, (6) superior frontal gyrus, (7) paracentral lobule, (8) precuneus cortex, and (9) lingual gyrus.

Figure 6. Mean Euclidean distance error (mm) between the warped and the target surfaces in 35 anatomical regions, averaged across all subjects

Our proposed varifold variants (c–d) clearly outperform the conventional current and varifold-based matching methods. The improvement was statistically significant with a p-value < 0.0001 for the dynamic-scale variant and 0.002 for the multidirectional variant. We number in the figure some key anatomical areas where the improvement is visibly striking: (1) lateral occipital cortex, (2) bank of superior temporal sulcus, (3) postcentral gyrus, (4) caudal middle frontal gyrus, (5) pars triangularis, (6) medial orbito-frontal cortex, (7) superior frontal gyrus, (8) paracentral lobule, and (9) lingual gyrus.

Figure 7. Anatomical alignment quantified by Mean Dice overlap between the warped and target surfaces averaged across all subjects in six highly folded cortical regions

Both proposed dynamic-scale and multidirectional varifolds metrics improved the conventional varifold-based surface matching and outperformed both Spectral Diffeomorphic Matching (SDM) and the conventional current-based surface matching methods.

Figure 8. Mean Euclidean distance error (mm) between the warped and the target surfaces in six highly folded cortical regions, averaged across all subjects
 Notably, both proposed dynamic-scale and multidirectional varifold variants displayed the least geometric closeness error.

Table 1

Matching accuracy for twelve 0-to-6 month cortical surfaces.

12 infants	Euclidean distance between warped and target surfaces (mm)	Average Dice Index over 35 ROIs (%)
Spectral diffeomorphic exact matching	–	86.59 ± 0.53
Current-based surface matching	0.68 ± 0.74	86.66 ± 1.15
Conventional varifold-based surface matching (normals)	0.61 ± 0.66	88.09 ± 1.38
Unidirectional varifold-based surface matching (curvature)	0.71 ± 0.18	88.29 ± 2.14
Topography-based dynamic-scale varifold-based surface matching	0.60 ± 0.64	88.85 ± 1.35
Multidirectional varifold-based surface matching	0.56 ± 0.60	89.32 ± 1.65