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Prescribed intensity design for extended sources in three-dimensional rotational geometry

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Abstract

Regulating the intensity distribution of an extended source to produce a prescribed illumination in three-dimensional (3D) rotationally symmetric geometry remains a challenging issue in illumination design. In this Letter, we present an effective method focusing on creating prescribed intensity designs for extended sources. By this method, a prescribed 3D intensity design is first converted into a two-dimensional intensity design for the extended source, a new approach is used to calculate the initial patch to generate a more stable design, and then a feedback strategy is employed to improve the performance of the aspherical lens in 3D rotational geometry. Three examples are presented to demonstrate the effectiveness of the proposed method in terms of performance and capacity for tackling complex designs.

Prescribed intensity/illuminance design for extended light sources in three-dimensional (3D) geometry, which aims to regulate the intensity distribution of an extended source to produce a prescribed illumination by a means of some optical surfaces, is a classical and challenging issue in illumination design [1]. Since the étendue of an extended source is nonzero, the influence of the source size on the performance of an illumination system usually cannot be ignored in a compact design. Consequently, those design methods, which rely on point source or parallel beam assumption, are not able to generate good designs. In the past two decades, several methods were developed to solve the prescribed illumination problem [2–5]. These methods allow effective control on the light distribution of the source and achieve good performance in 3D translational geometry. However, 3D light distribution control becomes notably less effective in creating 3D rotationally symmetric lens element by applying a rotation to a two-dimensional (2D) lens profile achieved by these methods, due to the inadequate control of skew rays [1]. A few illuminance compensation approaches have been developed for 3D rotational (or freeform) designs of extended source [6–8]. The key to these methods is iteratively using illuminance compensation to improve the performance of a design created by a zero-étendue method. Due to the nature of an extended source, energy waste caused by total internal reflection often cannot be avoided, and these methods are less effective in designs where the ratio of the distance between the lens and the source to the

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size of the source (i.e., H/d) is less than 3 [9], as shown in Fig. 1. The method presented in Ref. [10] was developed for 3D rotational design of extended Lambertian sources. However, this method might be challenging to implement because both ray tracing and optimization are required for the calculation of each data point, and usually hundreds of data points are needed to obtain a satisfactory design. Thus, these methods mentioned above may not be the best choice in 3D rotational design.

Usually, a prescribed illuminance design can be converted into a prescribed intensity design as long as the influence of the lens size on the optical performance can be ignored. Thus, we only focus on the prescribed intensity design in this Letter, and propose a better option for 3D rotational design. The design procedure of the method is shown in Fig. 2. Given a prescribed intensity, first we perform a 2D design for the extended source with the prescribed intensity and analyze the 3D performance of the aspherical lens by ray-tracing technique. Then, a feedback strategy will be employed to further improve the optical performance if the optical performance cannot meet the requirements. As the rest of this report will show, the proposed method has the performance and capacity to quickly and easily tackle complex designs.

Let $I(\beta)$ be the prescribed intensity distribution with an angular range between $\beta_{\min}$ β $\beta_{\max}$ (here, $\beta_{\min} = -\beta_{\max}$). Suppose the extended source is a Lambertian disk source with a diameter of d and an angular range of emission between $\theta_{\min}$ θ $\theta_{\max}$ ($\theta_{\min} = -\theta_{\max}$). Assume the light source is surrounded by a medium of refractive index n . Figure 3(a) illustrates the 2D intensity design in the meridian plane, where $W(\beta)$ represents the apparent projected width of outgoing rays. Assume two edge rays, 3 and 4, are parallel to each other with an emission angle β_r , then $W(\beta_r)$ can be understood as the distance between these two edge rays, as shown in Fig. 3(a). Since S_1S_2 is a Lambertian source, the luminance, L , of the light source is a constant. Then, we have $I(\beta) = L \times W(\beta)$ in a loss-less system and know that the total energy of the incident beam should equal that of the outgoing beam, that is

$$\int_{\theta_{\min}}^{\theta_{\max}} L d \cos \theta d\theta = \int_{\beta_{\min}}^{\beta_{\max}} I(\beta) d\beta. \quad (1)$$

Since we need to control the edge rays emitted from S_1 and S_2 , an initial patch is required before we start to design such a sequential optical surface [1]. A common way to find an initial patch is to employ polynomial fitting techniques, such as a second-order polynomial [1]. Here, we present an alternative method to design the initial patch, and the comparisons between this new method and the polynomial fitting techniques given at the end of this Letter demonstrate this new method is more stable. In Fig. 4(a), C_1C_2 represents the initial patch. Here, C_2 is a mirror point of C_1 about the z axis. It is required that the two edge rays, S_1C_1 and S_2C_2 , should be refracted by the initial patch to take the resulting direction angle $\beta = 0^\circ$. Since the distance between the rays 1 and 2 equals $W(0)$, the x -coordinate of C_1 equals $W(0)/2$. Then, C_1 and C_2 can be defined when h , which denotes the z -coordinate of C_1 , is preset. An edge ray emitted from S_2 passes through an arbitrary point P_i on C_1C_2 and takes the resulting direction angle β_i . To guarantee a successful design, it is required that $\beta_{i-1} < \beta_i$. Here, β_{i-1} is the direction angle of the outgoing ray at the previous point P_{i-1} , as shown in Fig. 4(a). Suppose the direction angle of the outgoing ray β can be represented

mathematically by a monotonically increasing function $\beta = f(x)$. That is, $d\beta/dx > 0$ on $[-W(0)/2, W(0)/2]$. To achieve this goal, the new method defines a set of data points on $C_1 C_2$ and $S_1 S_2$, respectively, which are equally spaced along the x axis. The initial patch is then obtained by enforcing that the refracted rays of the incident rays $Q_i P_i$ ($i = 0, 1, \dots, N$) by the initial patch take the direction $\beta = 0^\circ$, as shown in Fig. 4(b).

When the initial patch is obtained, we can calculate the rest of the lens profile. Take the calculation of point P_{N+i} as an example, as shown in Fig. 3(a). Trace an edge ray, $S_2 P_i$, and calculate the direction angle β_i of its outgoing ray, ray 3. Find a ray which is emitted from S_1 and is just the one (ray 4) fulfilling that the distance between the two parallel rays 3 and 4 equals the prescribed width, $W(\beta_j)$. Then, calculate the new point P_{N+i} of the lens profile as the intersection point between the tangent line of its previous point P_{N+i-1} and ray 4, and further calculate the normal vector $\mathbf{N}_{N+i}$ by application of the Snell's law. Perform the whole calculation until the direction angle of the incident ray from S_1 satisfies $\theta > \theta_{\max}$. In Fig. 3(b), M is the right end point of the lens profile. Assume the direction angle of the incident ray $S_1 M$ equals $\theta_{\max}$, and that of its outgoing ray, ray 6, is β_m . We calculate the incident ray $S_2 P_j$ with a direction angle θ_j , which also takes the resulting direction angle β_m after refraction of the lens. Since only one surface is used here, we have $\theta_j < \theta_{\max}$ with no more degrees of freedom to guarantee the prescribed projected width for those rays between $\theta_j < \theta_{\max}$ emitted from S_2 . It means some incident rays from S_2 cannot be well controlled, and we have $\beta_m < \beta_{\max}$ (here, we define β_m as the maximum effective angle we can obtain). Consequently, an arbitrary ray emitted from S_2 with a direction angle between $\theta_j < \theta_{\max}$ will take a resulting direction angle between $\beta_m < \beta_k$. (β_k is the direction angle of the ray 7. Usually, $\beta_k > \beta_{\max}$.) Note that although a more stable method is used here to find a better initial patch, the 2D design may still be strongly determined by the z -coordinate of C_1 when $\theta_{\max} < 90^\circ$. Sometimes, the outgoing rays may still not propagate in a manner we want, as shown in Fig. 5. Thus, it is critical to select an appropriate value of h to ensure all the incident rays emitted from the interior of the source can be well controlled.

When the 2D design is ready, the 3D model of the aspherical lens is generated by applying the rotation to the lens profile. Usually, good agreement cannot be achieved between the prescribed intensity and the actual one produced by the 3D model due to skew rays. Thus, an intensity feedback strategy is employed here in order to improve the 3D performance of the design. Figure 6(a) shows how to define an intensity feedback function. As stated above, the maximum effective angle of 2D design satisfies $\beta_m < \beta_{\max}$. Besides, the maximum effective angle of 3D design, β_C , might be different from β_m due to skew rays (usually, $\beta_C < \beta_m < \beta_{\max}$). Inevitably, there will be a region of abrupt intensity change near β_C , as shown in Fig. 6(a). Since this region cannot be eliminated due to the limitation of one single surface, a design can be considered acceptable as long as good agreement is achieved for the region within $[0, \beta_C]$. To reduce the influence of this region on the design, we set a cut-off line and calculate the feedback function on $[0, \beta_C]$. Let $I_i(\beta)$ denote the prescribed intensity and $I_j(\beta)$ be the actual 3D intensity obtained from the j th iteration. The feedback function is defined by

$$\eta_i(\beta) = I_t(\beta) / I_i(\beta), \quad 0 \leq \beta \leq \beta_C. \quad (2)$$

Then, the target intensity distribution used for the $(i+1)$ th iteration to design the lens profile is given by

$$I_{t(i+1)}(\beta) = I_t(\beta) \times \prod_{j=0}^i \eta_j(\beta), \quad 0 \leq \beta \leq \beta_C. \quad (3)$$

After that, the intensity distribution on $[\beta_C, \beta_{\max}]$ can be calculated by applying interpolation. With this new target intensity on $[0, \beta_{\max}]$, we can update the 2D design and perform 3D ray-tracing for the aspherical lens. Here, we employ the fractional RMS to quantify the difference between the actual intensity and the prescribed one:

$$\text{RMS} = \sqrt{\frac{1}{M_1} \sum_{k=1}^{M_1} \left(\frac{I_{ak} - I_{tk}}{I_{tk}} \right)^2}, \quad (4)$$

where M_1 is the number of the sample points, I_{tk} is the target intensity of the k th point defined by the prescribed intensity, and I_{ak} is the actual intensity of the k th point. Suppose the iteration stops when $\text{RMS} < \text{Tol}$. Here, Tol is a predefined value measuring the tolerance for the difference between the prescribed and obtained distributions. A smaller value of RMS represents less difference (of course, a better agreement) between the actual intensity and the prescribed one. Since the initial point of the iterative design is obtained from a 2D design of the extended source, the initial point can usually be close enough to the solution. That means the convergence of the proposed method can always be guaranteed. This characteristic will be proved by the examples.

The rest of the paper will present three examples to illustrate the elegance of this method for prescribed intensity design. In the first example, the prescribed intensity is a piecewise function given in Eq. (5):

$$I(\beta) = \begin{cases} -\beta + K_1, & 0 \leq \beta \leq \beta_{\max}/2 \\ K_1 - \beta_{\max}/2, & \beta_{\max}/2 \leq \beta \leq \beta_{\max} \end{cases}. \quad (5)$$

The value of K_1 should satisfy Eq. (1). $d = 1$ mm for the three examples, and the design parameters are given in Table 1. We assume the luminance of the light source, $L = 1$. The initial 3D performance of the design given in Fig. 6(b) shows clearly the actual intensity deviates significantly from the prescribed one. RMS of the initial design equals 0.0650 on $[0, \beta_C]$. Then, the feedback strategy is employed to improve the performance. Five million rays are traced during each iteration, and it only takes about 14 min to meet the stopping criterion by using a computer with a 3.60 GHz Intel(R) Core(TM) i7-4790 CPU. The convergence of the design given in Fig. 6(c) tells us the iteration stops after four 4 iterations, and RMS decreases from 0.0650 to 0.0046. From Fig. 6(b) we see a pretty good agreement

is achieved between the actual intensity and the target one. Figure 6(d) shows the obvious difference between the initial and final profiles of the aspherical lens.

The second example is also a complex design, as defined by Eq. (6):

$$I(\beta) = \begin{cases} K_2/\cos\beta, & 0 \leq \beta \leq \beta_{\max}/2 \\ K_2/\cos(\beta_{\max}/2), & \beta_{\max}/2 \leq \beta \leq \beta_{\max} \end{cases} \quad (6)$$

Similarly, the value of K_2 is calculated by the energy conservation. From Fig. 7(a) we see the initial design deviates significantly from the target, with $\text{RMS} = 0.0380$ at the beginning. After three iterations, we get a better design with $\text{RMS} = 0.002$, as shown in Fig. 6(c). It only takes about 12 min to meet the stopping criterion. The intensity of the final design is given in Fig. 7(a). Obviously, good agreement is achieved between the actual intensity and the prescribed one. From Fig. 7(b) we see the initial and final lens profiles are quite different.

In the third example, we change $\theta_{\max}$ from 24° to 90° . In this case, the situation discussed in Fig. 5 will not happen here. The prescribed intensity is governed by

$$I(\beta) = -\beta/2 + K, \quad 0 \leq \beta \leq \beta_{\max}. \quad (7)$$

Similarly, the value of K_3 can be calculated by applying the energy conservation. We have $\text{RMS} = 0.0211$ for the initial design which is given in Fig. 7(c). This figure clearly shows that the agreement between the actual intensity and the target one is not that good. After five iterations, the performance is significantly improved with RMS decreasing from 0.0211 to 0.0044, as shown in Fig. 6(c). It takes about 20 min to meet the stopping criterion. The initial and final profiles are given in Fig. 7(d). This figure tells us the two profiles are quite different. And we have $H/d = 2.672$ in this design. From these three examples, we can see the proposed method has the performance and capacity to easily tackle complex designs. And, the elapsed time in these examples indicates the proposed method is quite efficient.

As introduced above, we propose an alternative method to design the initial patch. Here, we make some comparisons between this method and the polynomial fitting techniques. During the last iteration of the 1st example, a second-order polynomial ($z = -0.0621x^2 + 3.0209$) and a fourth-order polynomial ($z = -0.3802x^4 + 0.194x^2 + 2.9778$) are, respectively, used as an initial patch. Figure 8(a) shows the function $\beta = f(x)$, and Fig. 8(b) gives the obtained intensity distribution. Since $\beta = f(x)$ is not a monotone increasing function for the fourth-order polynomial, its performance is not good at all. This figure also shows that the results obtained from the second-order polynomial ($\text{RMS} = 0.0041$) and the proposed method ($\text{RMS} = 0.0046$) are almost the same. Further, a second-order polynomial ($z = -0.2345x^2 + 2.6788$) is used as an initial patch during the last iteration in the 3rd example. From Figs. 8(c) and 8(d), we see some obvious difference between the two methods. $\text{RMS} = 0.0129$ for the second-order polynomial and $\text{RMS} = 0.0044$ for the new method. Obviously, the design obtained from the new method is better, and this method is more stable than the polynomial fitting techniques.

This work presents a method to solve the problem of prescribed intensity design for extended sources in 3D rotational geometry. The examples perfectly show that the method has the performance and capacity to quickly and easily tackle complex designs. Besides, the presented 2D design process can also be employed to solve the problem of prescribed intensity design for extended source in 3D translational geometry.

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Fig. 1.

Lens is designed by a zero-étendue method. Here, $d = S_1 S_2$ and $H/d = 2.6$. Some rays emitted from the edge points S_1 and S_2 of the light source are totally reflected internally on the surface.

Fig. 2.
Design procedure of the proposed method.

Fig. 4.
 (a) $\beta = f(x)$ must be a monotone increasing function. (b) Calculation of the initial patch.

Fig. 5.

Influence of h on the 2D design. The rays r_i ($i = 1, 2, 3$) have the maximum direction angle $\theta_{\max}$ from the source, and those rays l_i ($i = 1, 2, 3$) have the minimum direction angle $\theta_{\min}$. The edge ray principle says the outgoing ray r_2 will take the direction angle $\beta = \beta_k$, and the ray l_1 will take the direction angle $\beta = -\beta_k$. That is, all the outgoing rays from the interior of the source will be confined between the two rays. (a) Total internal reflection occurs when $h = 1$ mm. (b) The outgoing rays from the interior are not confined between r_2 and l_1 when $h = 1.2$ mm. (3) The outgoing rays from inner points are well controlled when $h = 3.5$ mm.

Fig. 6.

(a) Illustration of defining feedback function. (b) The initial and final designs. (c) Fast convergence of the method. (d) Difference between the initial and final profiles.

Fig. 7.

(a) The initial and final designs in the 2nd example. (b) Difference between the initial and final profiles. (c) The initial and final designs in the 3rd example. (d) Difference between the initial and final lens profiles.

Fig. 8.

(a) $\beta = f(x)$ and (b) the intensity distribution in the 1st example. (c) $\beta = f(x)$ and (d) the intensity distribution in the 3rd example.

Table 1

Design Parameters

Design	h (mm)	N	n	$\theta_{\max}$ ($^{\circ}$)	$\beta_{\max}$ ($^{\circ}$)	Tol
1	3	398	1.59	24	40	0.005
2	3	398	1.59	24	40	0.005
3	2.5	398	1.59	90	60	0.005