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Brain Morphometry on Congenital Hand Deformities based on Teichmüller Space Theory

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Abstract

Congenital Hand Deformities (CHD) are usually occurred between fourth and eighth week after the embryo is formed. Failure of the transformation from arm bud cells to upper limb can lead to an abnormal appearing/functioning upper extremity which is presented at birth. Some causes are linked to genetics while others are affected by the environment, and the rest have remained unknown. CHD patients develop prehension through the use of their hands, which affect the brain as time passes. In recent years, CHD have gain increasing attention and researches have been conducted on CHD, both surgically and psychologically. However, the impacts of CHD on brain structure are not well-understood so far. Here, we propose a novel approach to apply Teichmüller space theory and conformal welding method to study brain morphometry in CHD patients. Conformal welding signature reflects the geometric relations among different functional areas on the cortex surface, which is intrinsic to the Riemannian metric, invariant under conformal deformation, and encodes complete information of the functional area boundaries. The computational algorithm is based on discrete surface Ricci flow, which has theoretic guarantees for the existence and uniqueness of the solutions. In practice, discrete Ricci flow is equivalent to a convex optimization problem, therefore has high numerically stability. In this paper, we compute the signatures of contours on general 3D surfaces with surface Ricci flow method, which encodes both global and local surface contour information. Then we evaluated the signatures of pre-central and post-central gyrus on healthy control and CHD subjects for analyzing brain cortical morphometry. Preliminary experimental results from 3D MRI data of CHD/control data demonstrate the effectiveness of our method. The statistical comparison between left and right brain gives us a better understanding on brain morphometry of subjects with Congenital Hand Deformities, in particular, missing the distal part of the upper limb.

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Keywords

Congenital Hand Deformities; Brain Morphometry; Surface Ricci Flow; Teichmüller Space Theory; Brain MRI

1. Introduction

Congenital anomalies affect 1% to 2% of newborns, and approximately 10% of those children have hand abnormalities [1,2]. Congenital hand deformities (CHD) are usually occurred between fourth and eighth week after the embryo is formed [4]. Failure of the transformation from arm bud cells to upper limb can result in an abnormal appearing/ functioning upper extremity which is presented at birth. Some causes are linked to genetics while others are affected by the environment - either outside or inside the uterus, and the rest have remained unknown.

CHD are present at birth and may become a challenge for children as they continue to grow and learn to interact with their environment through the use of their hands. The degree of deformity can vary from something minor, such as a digital disproportion, to a more severe deformity, such as total absence of a bone or part of the hand. Children develop prehension with hands as they are, and they usually start being self-conscious of difference when they become socialized in school, as a result, they may intend not to show or use it. The less affected arm and hand are used, the less functional they will be, and the more likely it will affect functionalities of brain. In recent years, a number of researches have been conducted on mechanism, behaviors and psychology for CHD [5–7]. However, the impacts of CHD on brain structure are not well-understood so far.

Most brain MRI scanning protocols acquire volumetric data about the anatomy of the subject. Early researches [30,15] have demonstrated that surface-based brain mapping may offer advantages over volume-based brain mapping work [9] to study structural features of the brain, such as cortical gray matter thickness, complexity, and patterns of brain change over time due to disease or developmental processes. In research studies that analyze brain morphology, many surface-based shape analysis methods have been proposed, such as spherical harmonic analysis (SPHARM) [17,11], minimum description length approaches [13], medial representations (M-reps) [27], cortical gyrification index [31], shape space [25], metamorphosis [33], momentum maps [28] and conformal invariants [34], etc.; these methods may be applied to analyze shape changes or abnormalities in cortical and subcortical brain structures. Even so, a stable method to compute a global intrinsic transformation-invariant shape descriptors would be highly advantageous in this research field.

Here, we propose a novel and intrinsic method to compute the global correlations between various surface region contours in Teichmüller space and apply it to study brain morphology on CHD. The proposed shape signature demonstrates the global geometric features encoded in the interested regions, as a biomarker for measurements of CHD progression and pathology. It is based on the brain surface conformal structure [23,8,18,36] and can be accurately computed using the surface Ricci flow method [35,24].

1.1. Our Approach

For a 3D surface, all the contours represent the 'shape' of the surface. Inspired by the research work of Sharon and Mumford [29] on 2D shape analysis (recently it has been generalized to model multiple 2D contours [26]), we build a Teichmüller space for 3D shapes by using conformal mappings. In this Teichmüller space, every 3D contour (a simple closed curve) is represented by a point in the space; each point denotes a unique equivalence class of diffeomorphisms up to a Möbius transformation. For a 3D surface, the diffeomorphisms of all the contours form a global shape representation of the surface. By using this signature, the similarities of 3D shapes can be quantitatively analyzed, therefore, the classification and recognition of 3D objects can be performed from their observed contours.

We tested our algorithm in segmented regions on a set of brain left cortical surfaces extracted from 3D anatomical brain MRI scans. The proposed method can reliably compute signatures on two cortical functional areas by computing the diffeomorphisms of each observed contour. Using the signature as the statistics, our method achieve about 91% accuracy rate to discriminate a set of CHD subjects from healthy control subjects.

To the best of our knowledge, it is the first work to apply contour diffeomorphism to brain morphometry research. Our experimental results demonstrated that this novel and simple method may be useful to analyze certain functional areas, and it may shed some lights on understanding detecting abnormality regions in brain surface morphometry. Our major **contributions** in this work include:

- i. A new method to compute Teichmüller shape descriptor, in a way that generalized a prior 2D domain conformal mapping work [29].
- ii. The method is theoretically rigorous and general. It presents a stable way to calculate the diffeomorphisms of contours in general 3D surfaces based on Ricci flow.
- iii. It involves solving elliptic partial differential equations (PDEs), so it is numerically efficient and computationally stable.
- iv. The shape descriptors are global and invariant to rigid motion and conformal deformations.

Pipeline. Figure 1 shows the pipeline for computing the diffeomorphism signature for a surface with 4 closed contours. Here, we use a human brain hemisphere surface whose functional areas are divided and labeled in different color. The contours (simple closed curves) of functional areas can be used to slice the surface open to connected patches. As shown in frames (a,b,g), four contours $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are used to divide the whole brain (a genus zero surface S) to 5 patches S_0, S_1, S_2, S_3, S_4 ; S_0 is a genus zero surface with four boundaries and thus could be conformally mapped to a disk with one exterior circle and three interior circles, as shown in frame (c). In order to study the correlation of left and right functional areas with respect to each half brain, we have cut the whole brain surface into two halves, S_{left} and S_{right} respectively. Each of S_1, S_2, S_3, S_4 is conformally mapped to a circle domain (e.g., disk or annuli), D_1, D_2, D_3, D_4 , in (d). Each of C_1, C_2, C_3, C_4 in frame (e) is

mapped from $(S_{left}-S_1)$, $(S_{left}-S_2)$, $(S_{right}-S_3)$, $(S_{right}-S_4)$, the complement of S_1, S_2, S_3, S_4 with respect to each half brain. One contour γ_i ($i = 1, 2, 3, 4$) is mapped to two unit circles in two mappings, which are boundaries of two topological disk, D_i^{in} and D_i^{out} . Technically, outer topological disks D_i^{out} are mapped from a topological annulus C_p frame (e). The inner boundary of the annulus forms the circle, while the outer boundary represents the connection between left and right half of the brain. The representation of the shape according to each contour is a diffeomorphism of the unit circle to itself, defined as the mapping between periodic polar angles $(Angle_{in}, Angle_{out})$, $Angle_{in}, Angle_{out} \in [0, 2\pi]$, which is determined only by the target functional area and the corresponding half brain surface. The proper normalization is employed to remove Möbius ambiguity. The diffeomorphisms induced by the conformal maps of each curve form a diffeomorphism signature, which is the Teichmüller coordinates in Teichmüller space. As shown in (f,h), the curves demonstrate the diffeomorphisms for two contours; the l_2 norm of the area distance is defined as the metric for shape comparison and classification.

2. Theoretic Background

In this section, we briefly introduce the theoretical foundations necessary for the current work. For more details, we refer readers to the classical books [16,21].

2.1. Surface Uniformization Mapping

Conformal mapping between two surfaces preserves angles. Suppose (S_1, g_1) and (S_2, g_2) are two surfaces embedded in $\mathbb{R}^3$, g_1 and g_2 are the induced Euclidean metrics.

Definition 2.1 (Conformal Mapping)—A mapping $\phi: S_1 \rightarrow S_2$ is called conformal, if the pull back metric of g_2 induced by ϕ on S_1 differs from g_1 by a positive scalar function:

$$\phi^* g_2 = e^{2\lambda} g_1,$$

where $\lambda: S_1 \rightarrow \mathbb{R}$ is a scalar function, called the conformal factor.

For example, all the conformal automorphisms of the unit disk form the *Möbius transformation group* of the disk, each mapping is given by

$$z \rightarrow e^{i\theta} \frac{z - z_0}{1 - \bar{z}_0 z}.$$

All the conformal automorphism group of the extended complex plane $\mathbb{C} \cup \{\infty\}$ is also called Möbius transformation group, each mapping is given by

$$z \rightarrow \frac{az+b}{cz+d}, ad - bc = 1, a, b, c, d \in \mathbb{C}.$$

By stereo-graphic projection, the unit sphere can be conformally mapped to the extended complex plane. Therefore, the Möbius transformation group is also the conformal

automorphism group of the unit sphere. Suppose S is a genus zero closed Riemannian surface, then there exists a conformal mapping $\varphi : S \rightarrow \mathbb{S}^2$, which maps S to the unit sphere. Furthermore, all such conformal mappings differ by a Möbius transformation of the sphere.

A *circle domain* on the complex plane is the unit disk with circular holes. A circle domain can be conformally transformed to another circle domain by Möbius transformations,

$$z \rightarrow e^{i\theta} \frac{z - z_0}{1 - \bar{z}_0 z}.$$

All genus zero surfaces with boundaries can be conformally mapped to circle domains:

Theorem 2.2 (Uniformization)—*Suppose S is a genus zero Riemannian surface with boundaries, then S can be conformally mapped onto a circle domain. All such conformal mappings differ by a Möbius transformation on the unit disk.*

2.2. Teichmüller Space

Definition 2.3 (Conformal Equivalence)—*Suppose (S_1, g_1) and (S_2, g_2) are two Riemannian surfaces. We say S_1 and S_2 are conformally equivalent if there is a conformal diffeomorphism between them.*

All Riemannian surfaces can be classified by the conformal equivalence relation. Each conformal equivalence class shares the same *conformal invariants*, the so-called *conformal module*. The conformal module is one of the key component for us to define the unique shape signature.

Definition 1 (Teichmüller Space)—*Fixing the topology of the surfaces, all the conformal equivalence classes form a manifold, which is called the Teichmüller space.*

Suppose a genus zero Riemannian surface S has n boundary components $S = \gamma_1 + \gamma_2 + \cdots + \gamma_n$, $\varphi : S \rightarrow \mathbb{D}$ is the conformal mapping that maps S to a circle domain $\mathbb{D}$ such that (a). $\varphi(\gamma_1)$ is the exterior boundary of the $\mathbb{D}$ (b) $\varphi(\gamma_2)$ centers at the origin; (c) The center of $\varphi(\gamma_3)$ is on the imaginary axis. Then the conformal module of the surface S (also the circle domain $\mathbb{D}$ is given by

$$\text{Mod}(S) = \{(c_i, r_i) | i = 1, 2, \dots, n\}.$$

This shows the Teichmüller space of genus zero surfaces with n boundaries is of $3n-3$ dimensional. The Teichmüller space has a so-called Weil-Peterson metric [29], so it is a Riemannian manifold. Furthermore it is with negative sectional curvature, therefore, the geodesic between arbitrary two points is unique.

2.3. Surface Ricci Flow

Surface Ricci flow is a powerful tool to compute uniformization. *Ricci flow* refers to the process of deforming Riemannian metric g proportional to the curvature, such that the curvature K evolves according to a heat diffusion process, eventually the curvature becomes

constant everywhere. Suppose the metric $\mathbf{g} = (g_{ij})$ in local coordinate. Hamilton [20] introduced the Ricci flow as

$$\frac{dg_{ij}}{dt} = -K g_{ij}.$$

Surface Ricci flow conformally deforms the Riemannian metric, and converges to constant curvature metric [10]. Furthermore, Ricci flow can be used to compute the unique conformal Riemannian metric with the prescribed curvature.

Theorem 2 (Hamilton and Chow [10])—*Suppose S is a closed surface with a Riemannian metric. If the total area is preserved, the surface Ricci flow will converge to a Riemannian metric of constant Gaussian curvature.*

2.4. Conformal Welding Shape Descriptor

Suppose $\Gamma = \{\gamma_0, \gamma_1, \dots, \gamma_n\}$ is a set of non-intersecting smooth closed curves on a genus zero closed surface. Γ segments the surface to a set of connected components $\{\Omega_0, \Omega_1, \dots, \Omega_n\}$, each segment Ω_j is a genus zero surface with boundary components. Construct the uniformization mapping $\phi_k : \Omega_k \rightarrow \mathbb{D}_k$ to map each segment Ω_k to a circle domain $\mathbb{D}_k$, $0 \leq k \leq n$. Assume γ_i is the common boundary between Ω_j and Ω_k , then $\phi_j(\gamma_i)$ is a circular boundary on the circle domain $\mathbb{D}_j$, $\phi_k(\gamma_i)$ is another circle on $\mathbb{D}_k$. Let

$f_i|_{\mathbb{S}^1} := \phi_j \circ \phi_k^{-1}|_{\mathbb{S}^1} : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ be the diffeomorphism from the circle to itself. We called the the diffeomorphism f_i the *signature of γ_i* .

Definition 2.4 (Signature of a Family of Loops)—*The signature of a family non-intersecting closed 3D curves $\Gamma = \{\gamma_0, \gamma_1, \dots, \gamma_k\}$ on a genus zero closed surface is defined as: $S(\Gamma) := \{f_0, f_1, \dots, f_k\} \cup \{Mod(\mathbb{D}_0), Mod(\mathbb{D}_1), \dots, Mod(\mathbb{D}_s)\}$.*

The family of smooth closed curves Γ on a genus zero closed Riemannian surface S is determined by its signature $S(\Gamma)$, unique up to a conformal automorphism of the surface. Furthermore, the signature reflects the intrinsic symmetry of the surface.

Theorem 2.5—*Suppose $(S, \mathbf{g})$ is a genus zero Riemannian surface, $\eta : S \rightarrow S$ is an isometry, γ_1 and γ_2 are two closed loops on S with signatures f_1 and f_2 respectively, such that*

$$\gamma_2 = \eta(\gamma_1).$$

Then the signatures f_1 and f_2 are equal after Möbius normalization.

By applying this theorem, we can detect the intrinsic symmetry of the shapes.

3. Algorithm

In this section, we explain each step of the pipeline in Figure 1 in details.

3.1. Circular Uniformization Mapping

The surface is represented as a triangular mesh $\Sigma(V, E, F)$. We apply discrete Ricci flow method [24] and discrete Yamabe flow method.

Definition 3.1 (Discrete Conformal Factor)—The discrete conformal factor function is defined on the vertex set $u: V \rightarrow \mathbb{R}$, such that for each edge $[v_i, v_j]$, the edge length

$$l_{ij} = e^{u_i} \beta_{ij} e^{u_j},$$

where β_{ij} is the initial edge length.

The discrete Gaussian curvature on each vertex v_i is defined as angle deficit

$$K_i = \begin{cases} 2\pi - \sum_{ij} \theta_i^{jk} & v_i \notin \partial\Sigma \\ \pi - \sum_{ij} \theta_i^{jk} & v_i \in \partial\Sigma \end{cases}$$

where θ_i^{jk} is the corner angle at v_i in the face $[v_i, v_j, v_k]$. Then the discrete Gauss-Bonnet theorem holds

$$\sum_{v_i \in \Sigma} K_i = 2\pi\chi(\Sigma).$$

Definition 3.2 (Discrete Ricci Energy)—The discrete Ricci energy is given by

$$E(\mathbf{u}) = \int \sum_i^{\mathbf{u}} K_i du_i$$

where $\mathbf{u}$ is the vector of conformal factors $(u_1, u_2, \dots, u_n)$. The discrete Ricci energy is convex on the space

$$\sum_i u_i = 0.$$

The Hessian matrix of the Ricci energy is given by

$$\frac{\partial^2 E}{\partial u_i \partial u_j} = \frac{\partial K_i}{\partial u_j}$$

which has explicit geometric interpretation. Suppose $[v_i, v_j]$ is an interior edge on Σ , which is adjacent to two faces $[v_i, v_j, v_k]$ and $[v_j, v_i, v_l]$,

$$\frac{\partial K_i}{\partial u_j} = -(\cot \theta_k^{ij} + \cot \theta_l^{ji}),$$

if $[v_i, v_j]$ is an boundary edge adjacent to $[v_i, v_j, v_k]$, then

$$\frac{\partial K_i}{\partial u_j} = -\cot \theta_k^{ij}.$$

Furthermore,

$$\frac{\partial K_i}{\partial u_i} = -\sum_{[v_i, v_j] \in \Sigma} \frac{\partial K_i}{\partial u_j}.$$

Definition 3.3 (Delaunay Triangulation)—A closed triangle mesh is Delaunay, if for each edge $[v_i, v_j]$ adjacent to faces $[v_i, v_j, v_k]$ and $[v_j, v_i, v_l]$,

$$\theta_k^{ij} + \theta_l^{ji} \leq \pi.$$

Given target curvature $\bar{K}: V \rightarrow \mathbb{R}$, satisfying Gauss-Bonnet theorem, the discrete Ricci flow is given by

$$\frac{du_i}{dt} = \bar{K}_i - K_i,$$

which is the gradient flow of the following energy

$$F(\mathbf{u}) = \int \sum_i (\bar{K}_i - K_i) du_i,$$

this energy is strictly concave in the space $\sum_i u_i = 0$, and can be optimized directly using Newton's method,

$$\nabla F = (\bar{K}_1 - K_1, \bar{K}_2 - K_2, \dots, \bar{K}_n - K_n)^T,$$

the Hessian matrix is the negative of that of $E(\mathbf{u})$. Furthermore, during the flow, we preserve the mesh to be Delaunay all the time. This guarantees the existence of the solution.

3.1.1. Topological Annulus: Suppose Σ is a topological annulus, $\Sigma = \gamma_1 - \gamma_2$. We set the target curvature to be zero everywhere, then use discrete Yamabe flow to get a flat metric. Then we find a shortest path τ between γ_1 and γ_2 . By slice Σ along τ to get a mesh topological disk $\bar{\Sigma}$. Then we isometrically embed $\bar{\Sigma}$ face by face, this embedding is denoted

as $\phi : \Sigma \rightarrow \mathbb{C}$. By translation, rotation and scaling, we maps γ_1 to the imaginary axis, and the length is 2π . Then the composition of the complex exponential map and the embedding

$$\exp \circ \phi : \Sigma \rightarrow \mathbb{D},$$

maps the mesh to the canonical annulus with the unit exterior radius.

3.1.2. Topological Disk: Suppose Σ is a topological disk, then by puncture a small hole in the center, we convert it to a topological annulus. After mapping the topological annulus to the canonical annulus, we can fill the center hole. This gives the Riemann mapping from the original topological disk to the unit disk.

3.1.3. Multiply Connected Domains: Suppose Σ is a genus 0 surface with multiple boundaries, $\Sigma = \gamma_1 + \gamma_2 \cdots \gamma_n$, we apply the following Koebe's iteration method. We fill $n-1$ holes bounded by γ_i , $i > 1$ to convert the mesh to a topological disk, map it to the planar unit disk, γ_1 is mapped to the unit circle. Then we remove the area bounded by γ_2 , choose a small circle inside γ_2 , by reflecting the whole plane with respect to the small circle, we make γ_2 to be the exterior boundary, γ_1 the interior boundary. Then we fill γ_1 , compute a Riemann mapping to map the whole mesh to the unit disk, γ_2 to the unit circle. Then we repeat the whole process, each time, remove an area bounded by γ_k , and fill all other holes, and map the mesh to the unit disk, make γ_k to the unit circle. The whole iteration procedure will converge, and eventually, all boundary curves γ_k 's become rounder and rounder, and the mesh is mapped to a circle domain.

3.2. Computing Shape Descriptor

After the computation of the conformal mapping, each connected components is mapped to a circle domain. We define an order for all the loops on the surface, this induces an order for all the boundary components on each segment. Then by the definition for the conformal module of a circle domain, we normalize each circle domain using a Möbius transformation, then compute the conformal modules directly. For those segments, which are simply connected and mapped to the unit disk, we compute its mass center, and use a Möbius transformation to map the center to the origin.

Each loop on the surface becomes the boundary components on two segments, both boundary components are mapped to a circle under the uniformization mapping. Then we compute the signature directly. Suppose $\gamma \subset S_1 \cap S_2$, where S_1 and S_2 are two segments. $\phi_1 : S_1 \rightarrow \mathbb{D}_1$ and $\phi_2 : S_2 \rightarrow \mathbb{D}_2$ are two conformal mappings, then the signature of γ is given by

$$f_\gamma := \phi_2|_\gamma \circ \phi_1^{-1}|_\gamma.$$

4. Experimental Results

We demonstrate the efficiency and efficacy of our method by analyzing the human brain cortexes of CHD subjects and healthy controls. The brain surfaces are represented as

triangular meshes; a brain surface with 300K triangles. We implement the algorithm using generic C++ on windows 7 platform, with Intel Core i7-2630QM CPU 2.00 GHz, 6.00 GB RAM. The numerical systems are solved using C++ libraries, graphs are plotted with C++ and Matlab libraries. In general, the signature calculation on each half brain surface with 2 contours on each half takes less than 20 minutes to compute, even on complicated domains.

We applied our method on both pre-central gyrus and post-central gyrus, which are known to be the main sensory receptive area for the sense of touch/planing and executing movements.

4.1. Data acquisition

The experimental data include patients missing the distal part of the upper limb, and matched healthy controls. The segmentation algorithm is implemented in C++ and the user interface is developed using FLTK (<http://www.fltk.org/>). The data set consists of brain MRI scans of patients and matched controls obtained using 3-Tesla Siemens Trio Scanner with a standard 8-channel head coil in the Department of Psychological Sciences at the University of Missouri (TR = 1920 ms, TE = 3.75 ms, flip angle = 8, in-plane resolution = 1×1 mm, slice thickness = 1 mm, number of images = 160, matrix = 256×256).

4.2. Quantitative analysis

Figure 2 shows an example of diffeomorphism signatures for a brain cortical surface. We selected two contours: pre-central and post-central gyrus marked by experts on the left and the right half brain cortical surfaces, which correspond to the main sensory receptive area for the sense of touch/planing and executing movements. More results are demonstrated in figure 3 and figure 4. Early researches [22,19] have indicated that these two areas may have significant atrophy in congenital anomalies.

The diffeomorphism signature for each contour is plotted as a monotonic curve $Angle_{out} = f(Angle_{in})$ within the square $[0, 2\pi] \times [0, 2\pi]$. The area difference between the plotted curves, $d = \int_0^{2\pi} |Angle_{out}^{left} - Angle_{out}^{right}|^2 dAngle_{in}$, is used as the metric to represent the global shape of both contours. As a result, the signature of the whole brain surface is represented as a pair (d_{pre}, d_{post}) for combining the pre-central and post-central brain shape signatures. The method was tested on CHD subjects and matched healthy controls, with average signatures L1-norm on CHD (1.5790, 5.1543) and healthy (2.3496, 2.9778); L2-norm on CHD (1.7605, 8.6638) and healthy (4.4331, 4.6525), respectively. We have applied paired t-test to the two group. The p-value of the methods are 0.0168 and 0.0512, which are strong presumptions against neutral hypothesis.

4.3. Conjectures from preliminary result

From the statistics, we may give the conjecture that (a) Compared with patients with CHD, healthy controls yield relatively stable contours for both pre-central and post-central gyrus of the brain. (b) CHD may provide a greater impact on post-central gyrus than pre-central gyrus for patients brains, however, we cannot rule out the possibility that both CHD and variations of post-central gyrus development result from genetic causes.

5. Conclusion

In this paper, we propose a novel method that computes the global shape signatures on specified functional areas on brain cortical surfaces based on Teichmüller space theory. Conformal welding signature reflects the geometric relations among different functional areas on the cortex surface, which is intrinsic to the Riemannian metric, invariant under conformal deformation, and encodes complete information of the functional area boundaries. The computational algorithm is based on discrete surface Ricci flow, which has theoretic guarantees for the existence and uniqueness of the solutions. Discrete Ricci flow is equivalent to a convex optimization problem, therefore has high numerical stability. We applied the method to study the shape difference of cortical surfaces between Congenital Hand Deformities (CHD) and healthy control groups. The method is lossless, robust, and effective; it also has great potential to be employed to general brain morphometry study. Currently, the method is implemented to limited sets of data from recruited patients and healthy controls but it has the potential to extend to a greater number of subjects. The statistical comparison from the preliminary result indicates the impacts of CHD on brain structure. In the future, we will further explore and validate numerous applications of this global correlation shape signature in neuroimaging and shape analysis research.

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Fig. 1.

Diffeomorphism signature via uniformization mapping for genus zero surfaces(left and right half of the brain) with 4 simple closed contours $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ in (a), which correspond to the boundaries c_1, c_2, c_3, c_4 of the circle domains D_1, D_2, D_3, D_4 in (c), respectively. These four contours are also mapped to the boundaries of the base circle domain D_0 in (a).

Fig. 2.

Diffeomorphism signatures of a subject with CHD and a healthy control brain cortex. Each (*left* and *right*) half brain is a genus zero surface with 2 contours. Each (pre-central and post-central) functional area is a surface patch with one contour. Frame (a,d) shows the difference between 2 halves (*left* and *right*) on pre-central gyrus, while frame (c,f) corresponds to post-central gyrus. The figure illustrates the different impact of CHD to different functional areas. In this group, subject with CHD makes greater signature difference on post-central gyrus.

Fig. 3. Diffeomorphism signatures of a subject with CHD and a healthy control brain cortex. In this group, subject with CHD gives greater signature difference on post-central gyrus.

Fig. 4. Diffeomorphism signatures of two CHD subjects brain cortex. The figure illustrates the unstableness of CHD subject signatures for both(pre-central and post-central) functional areas.