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Experimental Study of Acoustic Radiation Force of an Ultrasound Beam on Absorbing and Scattering Objects

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Abstract

Acoustic radiation force is a nonlinear acoustic effect caused by the transfer of wave momentum to absorbing or scattering objects. This phenomenon is exploited in modern ultrasound metrology for measurement of the acoustic power radiated by a source and is used for both therapeutic and diagnostic sources in medical applications. To calculate radiation force an acoustic hologram can be used in conjunction with analytical expressions based on the angular spectrum of the measured field. The results of an experimental investigation of radiation forces in two different cases are presented in this paper. In one case, the radiation force of an obliquely incident ultrasound beam on a large absorber (which completely absorbs the beam) is considered. The second case concerns measurement of the radiation force on a spherical target that is small compared to the beam diameter.

INTRODUCTION

Acoustic radiation force is a nonlinear acoustic effect produced by the transfer of wave momentum to absorbing or scattering objects. This phenomenon is exploited in modern ultrasound metrology for measurement of the acoustic power radiated by a source and is used for both therapeutic and diagnostic sources in medical applications [1].

The relationship between the total acoustic power, W , and the corresponding radiation force acting on an absorbing target that fully intersects the acoustic beam, F , is the basis of the radiation force balance (RFB) method that is widely employed in ultrasound metrology. This relationship is quite simple in the case of plane waves: $W = Fc$, where c is the speed of sound. However, ultrasound beams radiated by real sources used in therapy and imaging can be far from a plane wave, so the relationship between the radiation force and total power is more complex. To relate the power and radiation force for such beams, the method of acoustic holography can be used to represent the acoustic field at any point in space [2–4]. In holography, the true field of an ultrasound beam is recorded by measuring both the

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amplitude and phase of the acoustic wave over a two-dimensional region perpendicular to the axis of beam propagation. Such a record (hologram) may be used to relate total power and radiation force with much better accuracy than other currently available approaches.

For the RFB method, measurements of acoustic radiation force to characterize the field can be realized with not only wide targets, but also with small targets - e.g., a spherical scatterer with diameter much smaller than the beamwidth. In cases with small targets, the radiation force can be accurately predicted theoretically, although the corresponding expression for the force is frequency-dependent and involves more sophisticated theoretical analysis [5]. If the scatterer is smaller than the acoustic field inhomogeneities, the theory can be simplified by using a plane wave approximation, which makes it possible to relate local intensity and radiation force [6].

RADIATION FORCE ON A SCATTERING OR ABSORBING TARGET AND ITS RELATION TO TOTAL ACOUSTIC POWER

Consider a harmonic wave of acoustic pressure $p'(\mathbf{r}, t)$ and particle velocity $\mathbf{v}'(\mathbf{r}, t)$:

$$p' = \frac{P}{2} e^{-i\omega t} + \frac{P^*}{2} e^{i\omega t}, \quad \mathbf{v}' = \frac{\mathbf{V}}{2} e^{-i\omega t} + \frac{\mathbf{V}^*}{2} e^{i\omega t}. \quad (1)$$

Here P and $\mathbf{V}$ are complex amplitudes of velocity and pressure, * indicates complex conjugation, and $f = \omega/(2\pi)$ is the cyclical radiation frequency. The radiation force is proportional to squared values of acoustic perturbations and its definition must take into account the second order quantities that do not vanish after averaging over time. In the quadratic approximation the radiation force on an object surrounded by a closed surface S can be expressed as follows:

$$\mathbf{F} = \left\langle \oint \left[L \mathbf{n} - \rho \mathbf{v}' (\mathbf{v}' \cdot \mathbf{n}) \right] dS \right\rangle, \quad (2)$$

where $L = \rho \mathbf{v}'^2/2 - p'^2/(2\rho c^2)$, $\mathbf{n}$ is the external normal unit vector to surface element dS , $\mathbf{v}'$ and p' are particle velocity and acoustic pressure, the brackets $\langle \dots \rangle$ denote averaging over a wave period, and ρ and c are density and sound speed, respectively.

For the case of harmonic time dependence as expressed in (1), radiation force can be expressed in terms of complex amplitudes of pressure P and velocity $\mathbf{V} = \nabla P/(i\rho\omega)$ as follows:

$$\mathbf{F} = \oint \left\{ \left(\frac{\rho |\mathbf{V}|^2}{4} - \frac{|P|^2}{4\rho c^2} \right) \mathbf{n} - \frac{\rho}{2} \text{Re} [\mathbf{V}^* (\mathbf{V} \cdot \mathbf{n})] \right\} dS. \quad (3)$$

The acoustic power W enters the volume limited by the surface S is:

$$W = \oint \mathbf{I} \cdot \mathbf{n} dS. \quad (4)$$

Here $\mathbf{I} = \text{Re}(\mathbf{P}\mathbf{V}^*)/2$ is the Umov-Poynting energy flux vector averaged over the wave period.

In the current work, the radiation force was measured for two different targets. In one case, the radiation force of an obliquely incident ultrasound beam on a large absorber (which completely absorbs the beam) is considered. The second case concerns measurement of the radiation force acting on a spherical target that is small compared to the beam diameter. For each experimental arrangement, Eqs. (3) and (4) can be expressed in convenient forms depending on the determination of P and $\mathbf{V}$ and the chosen integration surface – the face of a flat absorber in the case of a large target, and a spherical surface in the case of a small scatterer [6, 7]. Note that the surrounding medium is assumed to be an ideal inviscid fluid.

ACOUSTIC HOLOGRAPHY AND ANGULAR SPECTRUM

The full characterization of an acoustic source – i.e., pressure and velocity at every point in space – can be made with the use of acoustic holography. For example, if the lateral distribution of acoustic pressure magnitude and phase is recorded at some surface ($z = z_0$) in front of the source transducer, the Rayleigh integral can be used to reconstruct the normal velocity at the transducer surface as well as the corresponding acoustic field parameters p and $\mathbf{v}$ at the absorber face [2–4]. Alternatively, the acoustic field can be projected from a hologram recorded at $z = z_0$ using the angular spectrum approach.

The acoustic field can be written in terms of the angular spectrum of the field evaluated at the measurement plane $S(k_x, k_y, z_0)$ as follows:

$$P(x, y, z) = \frac{1}{4\pi^2} \iint_{k_x^2 + k_y^2 \leq k^2} dk_x dk_y S(k_x, k_y, z_0) e^{ik_x x + ik_y y + iz \sqrt{k^2 - k_x^2 - k_y^2}}. \quad (5)$$

Here k_x and k_y are the x and y components of the wave vector, and k is wavenumber. The angular spectrum is calculated based on the complex pressure amplitude distribution in the measurement plane $z = z_0$ obtained from holography measurements:

$$S(k_x, k_y, z_0) = e^{-iz_0 \sqrt{k^2 - k_x^2 - k_y^2}} \int_{-\infty}^{+\infty} \int dx dy P(x, y, z_0) e^{-ik_x x - ik_y y}. \quad (6)$$

Once the angular spectrum is known, it is possible to derive all the components of beam's field [7] and substitute them into Eqs. (3) and (4) to obtain the following expressions for radiation force and total acoustic power:

$$F_z = \frac{1}{8\pi^2 \rho c^2} \iint_{k_x^2 + k_y^2 \leq k^2} \left(1 - \frac{k_x^2 + k_y^2}{k^2}\right) |S(k_x, k_y)|^2 dk_x dk_y \quad (7)$$

$$W = \frac{1}{8\pi^2 \rho c} \iint_{k_x^2 + k_y^2 \leq k^2} \sqrt{1 - \frac{k_x^2 + k_y^2}{k^2}} |S(k_x, k_y)|^2 dk_x dk_y. \quad (8)$$

Here we consider the z -component of the acoustic radiation force to be directed along the beam propagation axis. In general, this component is most easily measured in experimental studies using either a large absorbing target or a small scattering target.

EXPERIMENTAL STUDY OF THE RADIATION FORCE OF AN ULTRASOUND BEAM ON ABSORBING AND SCATTERING OBJECTS

Experimental measurements of acoustic radiation force were carried out in two cases. In the first series of measurements, an elastic scatterer with linear dimensions comparable with the beamwidth was considered. In the second experiment a large totally absorbing object was used as a target. In both cases, the source was an ultrasound transducer with a diameter of 100 mm, it was operated in continuous-wave mode at a frequency of $f=1.119$ MHz. Methods of measuring radiation force were different for the two types of targets.

Radiation Force Acting on a Small Spherical Object

A spherical stainless steel scatterer with radius $a=1.8$ mm was used as a target. An ultrasound source and the spherical target were placed in water. Using two different methods for suspending the steel target, radiation force was measured by detecting target displacement from equilibrium.

Two approaches of such measurements were considered. The first one is based on the suspension of the scatterer on thin wires (Fig. 1a) folded in the form of the letter “V,” with the lower part of the thread glued to the spherical target. The plane of the “V” was perpendicular to the beam axis. Displacement of the target due to radiation force in this case was determined by measuring the displacement of a laser beam, which was focused onto the edge of the sphere before and after the force was applied. The magnitude of the force is expressed by the following equation:

$$F_{rad} = \frac{4}{3} \pi a^3 (\rho_s - \rho) g \frac{d}{l}. \quad (9)$$

Here a is the radius of the scatterer, ρ_s is the density of the scatterer, ρ is the density of the fluid, d is the displacement from the equilibrium position, and l is the vertical projection of the wires' length.

In the second approach, the spherical target was held in place by four elastic rubber threads that were glued to four sites on the scatterer (Fig. 1b). The other end of each thread was attached to a rigid circular frame with a diameter larger than the beamwidth (to avoid reflections); in this way, the target was elastically held at the center of the frame. The frame was attached to a positioning system that could translate the target with 0.01 mm accuracy. Displacements caused by radiation force were measured using the ultrasound echo-location principle with a use of an additional diagnostic transducer. Ultrasound echo pulses were detected and their shift in time was used to calculate the target displacement. With this approach, the radiation force was determined from Hooke's law: $F_{rad} = -\kappa d$, where κ is the effective stiffness of the threads and d is the measured displacement. Displacement measurements based on the echo-location principle were more accurate than those based on

the laser. Uncertainties in echo-location timing and knowledge of κ yielded radiation force measurements with errors on the order of 5–7%, which corresponds to about 5 μN of force inaccuracy.

Radiation Force Acting on a Large, Totally Absorbing Target

The acoustic beam radiated by the piezoelectric source of 100 mm diameter was directed to an absorber (rubber puck of 200 mm diameter). Force on the absorber was measured by precision scales (Model Precise 310C, accuracy 0,01 g). The ultrasound source and absorber were placed in a water-filled tank with the scales outside the tank.

The measured dependence of acoustic radiation force on transducer electric power was linear, as it should have been according to theory. To test the effect of the beam structure on radiation force, measurements were performed for different angles of oblique incidence of the beam onto the absorber. The reduction in measured force was approximately parabolic with increasing beam inclination from normal incidence (when the angle equals zero). An angle of 28° resulted in force reduction by 10%. The results of experimental measurements coincide well with those derived from holography.

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FIGURE 1.

Radiation force measurements on a stainless steel spherical scatterer suspended on thin wires (a) and elastic threads (b)

FIGURE 2.

The acoustic radiation force at different angles of incidence of the ultrasonic beam. Solid thin lines are the linear approximation of experimental data. Error bars for measurements represent $\pm$ one standard deviation of xx measurements. Data obtained on the basis of holographic measurements at an angle of 28° (dashed line) match experimental results within the measurement error.