A Geodesics-Based Surface Parameterization to Assess Aneurysm Progression

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Abdominal aortic aneurysm (AAA) intervention and surveillance is currently based on maximum transverse diameter, even though it is recognized that this might not be the best strategy. About 10% of patients with small AAA transverse diameters, for whom

intervention is not considered, still rupture; while patients with large AAA transverse diameters, for whom intervention would have been recommended, have stable aneurysms that do not rupture. While maximum transverse diameter is easy to measure and track in clinical practice, one of its main drawbacks is that it does not represent the whole AAA and rupture seldom occurs in the region of maximum transverse diameter. By following maximum transverse diameter alone clinicians are missing information on the shape change dynamics of the AAA, and clues that could lead to better patient care. We propose here a method to register AAA surfaces that were obtained from the same patient at different time points. Our registration method could be used to track the local changes of the patient-specific AAA. To achieve registration, our procedure uses a consistent parameterization of the AAA surfaces followed by strain relaxation. The main assumption of our procedure is that growth of the AAA occurs in such a way that surface strains are smoothly distributed, while regions of small and large surface growth can be differentiated. The proposed methodology has the potential to unravel different patterns of AAA growth that could be used to stratify patient risks.
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1 Introduction

An aortic aneurysm is a localized dilation of the aorta of more than 1.5 times its normal diameter and occurs due to a local weakness in the arterial wall that is compounded by hemodynamic stresses [1]. AAAs are the most common aortic aneurysm, and while they are usually asymptomatic, if left untreated an AAA may expand and rupture. The AAA rupture is fatal in 90% of cases, thus AAAs that are deemed to have a high risk of rupture are treated either with open surgery or less invasive endovascular procedures [2]. Standard practice calls for repair when the maximum transverse diameter of the AAA either exceeds 5.5 cm (5 cm in women) or grows faster than 1 cm per year [3]. While measuring AAA maximum diameter is relatively simple and thus feasible in clinical practice, maximum transverse diameter alone is imprecise: about 12% of AAAs smaller than 5.5 cm do rupture, putting patients at risk; and about 60% of larger, stable AAAs may receive unnecessary repair [2,4]. Because AAA expansion and rupture depend (at least in part) on mechanical factors [2,5], several groups have been investigating better ways to assess risk based on 3D wall stresses, e.g., see Refs. [6–13]. Further AAA expansion is also presumed to depend on AAA mechanics [14–16].

There are currently no reliable predictors of AAA expansion, and surveillance is recommended. This is in part due to a lack of computational tools to quantify localized expansion of AAAs. Since both wall stress and wall strength properties vary greatly across AAAs [17–20]—spatially as well as demographically—it seems crucial to study the progression of the AAA, that is, how the AAA expands globally and locally. Patient-specific AAA progression could provide insights into patterns that may indicate weakened or susceptible areas of the wall. A key prerequisite for performing such a study, however, is the ability to identify corresponding locations on AAA geometries captured from the same subject at different time points, and quantify the extent of change between them.

We present a methodology to establish point-to-point correspondence between AAA surfaces from the same patient obtained at two (or more) time points as part of surveillance. To the best of our knowledge, the work in this paper is the first attempt to obtain correspondence (also known as shape registration or shape parameterization) between AAA surfaces, and is based on our previous work on embryonic cardiac registration [21,22]. AAA surfaces are characterized by their cylindrical-like shape but generally lack salient features to guide the correspondence. It is then critical to achieve a consistent parameterization between the surfaces to facilitate registration. We propose performing this registration using geodesics-based parameterizations of the AAA surfaces.

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Our methodology assumes that over time the AAA surfaces expand minimizing the deformation (strain) energy between configurations. To exemplify the application of our methodology, we quantified local AAA progression in six patients. Although more studies are needed, our correspondence algorithm may allow early detection of growth-instabilities in the AAA wall.

2 Consistent Parameterization

Our approach is to parameterize the AAA surfaces with a “cylindrical” parameterization, using quasi cross-sectional contours and longitudinal lines. The quasi cross sections are roughly perpendicular to the surface medial axis, and the longitudinal lines follow the surface shape but do not “twist” (see Fig. 1). For computational purposes, we discretize the continuous AAA surfaces and optimize the parameterization using a regular geodesic grid embedded in the surfaces. We present our approach for parameterizing one surface, and for registering two or more AAA surfaces.

2.1 Surface Parameterization. Let S_t be the input surface at time t , with $t = T_1, T_2, \dots$, and let $u, v \in [0,1] \times [0,1]$ be the parameterization of the surface S_t , with u the circumferential parameter (around the quasi cross-sectional contours) and v the longitudinal parameter (along the tube medial axis). Let $G_t(u, v)$ be the mapping from $[0,1] \times [0,1]$ to the surface S_t . To define this mapping, we use a constructive approach based on intersecting “slicing” planes with S_t .

Let $P(v, \mathbf{n}_v)$, $v \in [0,1]$, be a continuous family of planes, each defined by a position v in the longitudinal direction and a vector normal to the plane, $\mathbf{n}_v$. The first and last planes, $P(0, \mathbf{n}_0)$ and $P(1, \mathbf{n}_1)$, are defined by the top and bottom boundaries of S_t , respectively. The set of planes $P_v = P(v, \mathbf{n}_v)$ sweeps the surface longitudinally without intersecting within the volume of S_t . We select m planes, P_k with $k \in [1..m]$, evenly spaced along the surface medial axis. We then intersect each plane P_k with the surface S_t to create a contour curve, γ_k . We arc-length parameterize γ_k , map it to $[0,1]$, and choose n equally spaced (geodesic) points along each γ_k . Doing this for each plane defines a set of points, p_{ki} , with $k \in [1..m]$ and $i \in [1..n]$, which define the AAA discretized parameterization (see Fig. 1).

The construction process described ensures that: (i) γ_k curves lie in a plane and (ii) the surface sampling is geodesic (equal spacing among points) circumferentially and approximately geodesic longitudinally.

2.2 Grid Parameterization Optimization Algorithm.

Optimization criteria are needed for proper surface parameterization. We optimize the surface grid such that (1) a consistent reference longitudinal line, $u = 0$, is used; (2) longitudinal lines follow the shape of the AAA surface with minimal twisting; (3) circumferential contours lie on planes, which are close to cross-sectional planes; (4) adjacent quasi cross-sectional planes do not intersect within the surface; and (5) difference in distances between neighboring grid points is minimized. This optimization intends to achieve a “minimal-energy” deformation from a canonical cylinder to each surface, and additionally between two (or more) surfaces.

2.2.1 Reference Line. Longitudinal lines, including the reference line, should ideally traverse the AAA from “top” to “bottom” with minimal twisting (see Fig. 1). To this end, we take the shape of the surface into consideration and use the shortest longitudinal line on the AAA surface as an initial reference line. Physical proximity of the AAA to the spine, which restricts bulging in the region, usually generates the shortest longitudinal lines.

Assume that the set of points $p_{k1}, k = 1, \dots, m$ define the reference line. To minimize twisting of this line, we use a reference frame and a local angular parameter. Given the plane P_k , the contour curve γ_k can be represented using a radial coordinate and an angular coordinate, $\theta \in [0, 2\pi)$. The choice of origin of the angular coordinate for each contour, i.e., $\theta = 0$, defines the local angular reference. To ensure a consistent angular reference among planes, we use a reference frame. We apply principal component analysis (PCA) to the coordinates of the surface medial axis, and determine the reference frame (x', y', z') by aligning the z' , x' , and y' axes with the first, second, and third eigenvectors, respectively. Next, we set the origin of the reference frame at the intersection point between the medial axis and each P_k plane. Finally, we project the x' -axis on the P_k plane, and use this projection as the reference angular direction $\theta = 0$ on the plane. Following this procedure, $\theta = \text{constant}$ can be identified as longitudinal trajectories that follow the shape of the surface S_t with minimal twisting. Please note, however, that the relationship might not be unique if the surface folds on itself. To optimize the reference line, we minimize the function

$$R_t = w_1 \left| \frac{\left(\sum_{k=1}^{m-1} l_{k1} \right) - L_{\min}}{L_{\max} - L_{\min}} \right| + \frac{w_2}{m} \sum_{k=1}^m |\theta_k - \langle \theta_k \rangle| \quad (1)$$

Fig. 1 AAA surface parameterization strategy. The AAA surface parameterization consists of quasi cross-sectional contour curves and longitudinal lines that are discretized using a grid. The surface medial axis is shown in red and reference curve in orange. (a) Quasi cross-sectional contours were generated by intersection of slicing planes with the surface. Each contour is then discretized to generate n equally spaced sampling points. (b) Sampling points were generated on m contours over the entire surface. The grid consists of p_{ki} points with $i = 1 \dots n$, $k = 1 \dots m$, (c) Connecting the k th sampling points on each contour, the k th longitudinal grid curve is generated. (d) Completed parameterized grid.

where w_1 and w_2 are weighting constants; $L_{\min}$ is the shortest longitudinal geodesic on the surface S_i ; $L_{\max}$ is the largest longitudinal geodesic on the surface S_i (with minimal twisting); l_{k1} is the length of the grid edge connecting the points p_{k1} and $p_{(k+1)1}$ (two adjacent points on the reference line); $\theta_k, k \in [1 \dots m]$, are the angular coordinates of the reference line; and $\langle \theta_k \rangle$ is the mean of θ_k . The first term of Eq. (1) enforces the condition for the reference line to be the shortest line, while the second term enforces minimal twisting.

2.2.2 Set of Quasi Cross-Sectional Planes. To satisfy the conditions that P_k planes do not intersect within the surface volume and are almost perpendicular to the medial axis, we minimize the function

$$Q_t = w_3 \sum_{k=1}^{m-1} |P(v_k, \mathbf{n}_k) \cap P(v_{k+1}, \mathbf{n}_{k+1}) \cap S_t| + \frac{w_4}{m} \sum_{k=1}^m (1 - |\mathbf{n}_k \cdot \mathbf{V}_k|) \quad (2)$$

where w_3 and w_4 are weighting constants; $\mathbf{V}_k$ is the unit tangent vector of the medial axis at point v_k . The first term of Eq. (2) guarantees the validity of the parameterization by forbidding adjacent P_k -planes from intersecting each other inside the volume bounded by the surface, imposing the cardinality of the set of intersection points for a pair of planes and the surface to be zero; the second term ensures quasi cross-sectional planes. We set w_3 to be large, i.e., a penalizing weight, so that the optimization avoids solutions with intersecting planes.

2.2.3 Geodesic Grid. To achieve a geodesic parameterization (evenly spaced sampling points), we include an optimization term that imposes grid edges on each longitudinal curve to be of similar length

$$G_t = \frac{w_5}{n(m-1)} \sum_{i=1}^n \sum_{k=1}^m \left| \frac{l_{ki} - \langle l_i \rangle}{\langle l_i \rangle} \right| \quad (3)$$

where w_5 is a weighting constant; l_{ki} is the length of the grid edge connecting points p_{ki} and $p_{(k+1)i}$; and $\langle l_i \rangle$ is the mean of lengths on the i th longitudinal line.

2.2.4 Objective Function. Combining all the above criteria, the objective function we use to optimize the surface grid is

$$F_t = R_t + Q_t + G_t \quad (4)$$

Minimizing F_t results in an optimized grid for each surface S_t .

2.3 Surface Registration. To achieve registration among the surfaces at times $T_1, T_2, \dots, T_T$, we optimize all surfaces simultaneously. We first align the (two or more) AAA surfaces in 3D (see Sec. 2.4). We then modify the objective function, F , so that the quasi cross-sectional planes and reference lines are optimized for all surfaces simultaneously. To this end, the reference frame that defines the angle θ_k for each plane is computed from the PCA of coordinates of all of the medial axes combined. This gives a reference $\theta = 0$ that is consistent among all surfaces S_t . Further, in the function R_t (see Eq. (1)), the mean angle θ_k now corresponds to the combined mean angles of all surfaces S_t . The objective function to minimize then becomes

$$F = \sum_{t=T_1}^{T_T} F_t = \sum_{t=T_1}^{T_T} (R_t + Q_t + G_t) \quad (5)$$

By minimizing F , we obtain a consistent parameterization for all of the corresponding surfaces S_t .

2.4 MultiSurface Optimization Initialization. We introduce here how to create an initial parameterization (an initial reference

line and set of planes), which is then optimized. We start by aligning the surfaces in space, and define an initial grid.

2.4.1 Surface Alignment. To align the AAA surfaces in space, we use an implementation of the Iterative Closest Point (ICP) method [23]. The ICP is an iterative algorithm that minimizes the sum of square distances (Hausdorff distance) between two surfaces. For more surfaces, we select one as reference and align the others to the reference surface. The ICP algorithm, however, relies on a relatively close initial alignment between the two surfaces. To improve initial alignment, we compute the medial axes of the surfaces [24], and align the points and principal eigenvectors of the medial axes. We use this alignment as an initial guess for the ICP algorithm.

After alignment, the quasi cross-sectional planes at the two ends of the AAA surfaces (P_l and P_m) are determined. We impose the condition that P_l and, separately, P_m are parallel among the surfaces (although P_l and P_m planes do not need to be parallel to each other). To this end, we “clip” the original surfaces using an average plane, while minimizing the amount of surface being clipped off. After this, P_l and P_m remain fixed during the optimization of the parameterization.

2.4.2 Initial Grid. We next select an initial guess for the reference line $u = 0$ and the set of quasi cross-sectional planes P_k . We initially set $u = 0$ as the shortest longitudinal geodesic (for each surface) [25]. Then, to generate an initial set of P_k -planes, we cut open the surface along the shortest longitudinal geodesic and map it to a unit square in 2D, essentially mapping the local surface coordinates u and v to $[0,1] \times [0,1]$. We then discretize the unit square in the v direction with m equally spaced lines, and in the u direction with n equally spaced lines. Lines in the u direction, however, will not necessarily form a plane when mapped back to the 3D surface. To define the initial set of planes P_k we chose, for each line, three points along the u direction: $u = 0, 1/3$, and $2/3$, which define a plane P_k with $k = 2, 3, \dots, m-1$. Once the initial set of P_k planes is selected, each plane is intersected with the AAA surface, forming a contour, γ_k , and each contour then discretized into n geodesic (evenly spaced) points. This discretization defines the initial guess points p_{ki} , with $k = 1, 2, \dots, m$ and $i = 1, 2, \dots, n$ that form, for each surface, the AAA surface grid to be optimized.

3 Methods

3.1 Data Sets. To exemplify the use of our parameterization methodology, we registered pairs of AAA surface meshes from the same patient. The surface meshes corresponded to the AAA outer wall surface of six individuals, extracted from patient-specific CT

Table 1 AAA surface mesh data sets from the six patients used in this study. For each patient, AAA dimensions at the two time points considered (T_1 and T_2) are listed, together with the time interval (ΔT) between the two CT scans and the maximum transverse diameter expansion rate. To compute major and minor axis data, an ellipse was fitted to the AAA contour, γ_k , corresponding to the maximum transverse diameter. Bold numbers indicate that the patient satisfies clinical criteria for repair.

Patient	Plane of maximum transverse diameter		ΔT $T_2 - T_1$ (years)	Diameter expansion (mm/year)
	Major axis, minor axis (mm) time T_1	Major axis, minor axis (mm) Time T_2		
1	51.8, 48.1	70.7 , 69.1	4.2	4.5
2	30.9, 22.8	39.3, 33.4	4.3	2.0
3	34.2, 33.3	44.7, 42.8	4.0	2.6
4	46.3, 46.1	50.5, 50.2	5.7	0.7
5	45.2, 41.7	69.2 , 51.6	2.6	9.2
6	55.2 , 44.1	58.9 , 53.1	2.0	1.8

Table 2 Weight values used to optimize the parameterization of AAA corresponding surfaces

Function	Equation	Weight	Weight value
R_i	Equation (1)	w_1	5
		w_2	10
Q_i	Equation (2)	w_3	1000
		w_4	0.1
G_i	Equation (3)	w_5	5

scans taken as part of routine patient care at two different time points: T_1 and T_2 . De-identified input AAA surface meshes were provided by Vorp. No demographics of the patients were provided. Construction of the AAA surface meshes from CT scan images started just below the renal arteries and ended prior to the iliac

bifurcation for all patients and time points, so that the upper and lower ends of the surfaces were approximately corresponding. Table 1 summarizes some characteristics of the AAA surface data set.

3.2 AAA Parameterization and Grid Optimization.

Surface parameterization and grid optimization were performed as described in Sec. 2. We used a sampling grid of 80×80 points, which was sufficient to capture the geometry of the surfaces. Once an initial grid was generated, we optimized the location of grid points to obtain a geodesics-based correspondence for each pair of AAA surfaces (Sec. 2.3). Table 2 shows the values of the weights used to solve the grid optimization problem (Eq. (5)). These values were empirically found to give the best results for the AAA surfaces. We set w_3 to a prohibitively large value (in effect a penalizing factor) to ensure the validity of the P_k planes.

Fig. 2 Correspondence on the AAA surfaces between times T_1 and T_2 . (a) Sagittal view of patient AAAs. (b) Frontal view of patient AAAs. The correspondence is obtained after parameterization and optimization of the parameterization. The plot shows correspondence of lines, with the reference line for each patient marked with an arrow. Note that longitudinal grid lines follow the global shape of the surfaces, and the reference line is the shortest longitudinal line for each patient.

In contrast, w_4 is very small, indicating that enforcing planes to be strictly cross-sectional planes is the least important criteria in our optimization. Our results are most sensitive to the remaining three terms, which optimize the reference line (w_1 and w_2) and the geodesic distance between neighboring nodes (w_5).

While optimizing the parameterization, we calculate grid points from the medial axis v_k , a set of planes P_k (defined by a point on the medial axis and a normal vector $\mathbf{n}_k$) and the local angular coordinate θ_k of the reference line. To ensure that v_k , $\mathbf{n}_k$, and θ_k change smoothly along the tube, we used splines which reduce the number of variables we optimize to two 1D and one 3D splines. The number of control points we use for each spline was set by how “curvy” the surface was, with a minimum of three and a maximum of six control points for each spline. This resulted in 30–40 variables to optimize for the surfaces. At each iteration of the optimization, we adjust the control points of the splines, recalculate the parameterization, and evaluate the optimization equation F (Eq. (5)) at the resulting $(2 \times 80 \times 80)$ 12,800 grid points. Optimization iterations (performed using a generic simplex algorithm) end when the difference in F between successive iterations is less than 10^{-6} .

3.3 Correspondence Refinement. After parameterization and grid optimization, we use a strain minimization algorithm to further refine the correspondence between AAA surfaces from the same patient [26]. In our implementation, strain serves as a measure of the deformation of the surface from time T_1 (used here as a reference or “undeformed” configuration) to time T_2 . Our algorithm assumes that the AAA progresses in such a way that gradients of strain are minimized. Since our strain optimization algorithm is local, we still expect to find regions of large deformation featuring relatively high strains.

3.4 AAA Progression Metrics. We applied different metrics to assess the localized deformation of the AAA and thus the AAA progression from time T_1 to T_2 . Stretches provide a point-by-point local indication of the AAA change between times T_1 and T_2 for each patient. Stretches are computed at time T_2 with respect to the (reference) configuration at time T_1 . Another way of analyzing AAA progression is to quantify local changes in surface area and their rates of change. This can be easily done using the principal

stretches. The local area at time T_2 , A_{T2} , with respect to the reference area at time T_1 , A_{T1} , is

$$\frac{A}{A_0} = \frac{A_{T2}}{A_{T1}} = \lambda_1 \lambda_2 \quad (6)$$

This surface expansion ratio, A/A_0 , can therefore be used to determine how much the AAA surface expanded locally. To get a meaningful map of the local rate of surface expansion, we can simply calculate the change in area assuming a constant rate

$$\frac{d}{dt} \left(\frac{A}{A_0} \right) \cong \frac{1}{\Delta T} \left(\frac{A_{T2} - A_{T1}}{A_{T1}} \right) = \frac{1}{\Delta T} (\lambda_1 \lambda_2 - 1) \quad (7)$$

where $\Delta T = T_2 - T_1$. Plotting local surface expansion rates on the patient surface gives an estimation of regions that expanded/changed faster from time T_1 to time T_2 , and allow for a better comparison of patient data, especially when the time span between images varies.

4 Results and Discussion

We applied our parameterization and strain relaxation procedures to data from six patients. Analyses of results show widespread differences in rates of expansion among patients. Some patients exhibited large expansion rates, suggesting an unstable, rapidly growing AAA, and some patients exhibited almost stable AAAs with negligible rates of AAA expansion.

4.1 Limitations. Our procedure has some limitations that need consideration. First, our grid-based parameterization is limited to surfaces for which there exists a valid set of quasi cross-sectional planes and with a cylindrical topology (see Sec. 2). Thus, the procedure can be applied to fusiform aneurysms, which are the majority of AAAs. Saccular aneurysms, especially those with a small diameter neck, cannot be parameterized by the proposed procedure. Second, we cannot guarantee that the correspondence obtained is a true correspondence, as it is impossible to “mark” points in the patient AAA—and even validation using animal models could be flawed because the mechanisms of

Fig. 3 Principal stretch components λ_1 and λ_2 on the AAA surfaces at T_2 (with respect to T_1) for the six patients. The gray scale reflects the minimum and maximum of λ_1 and λ_2 for each patient.

expansion can be different in animal models and humans. With these limitations in mind, the assumption that the tissue stretches almost evenly in all directions, minimizing differences in growth stretches (and strains), provides a reasonable map of local expansion rates in a patient AAA. More studies are needed, however, to show the applicability of such map.

4.2 Surface Parameterization. The parameterization of the surfaces resulted in a seemingly closed correspondence (see Fig. 2). Applying our strain relaxation methodology [26] resulted in only minor changes in the final strain distribution (not shown),

indicating that our initial correspondence was already a good initial guess, with evenly distributed strains.

4.3 AAA Surface Deformation. To analyze AAA progression from time T_1 to T_2 for each patient, we applied different metrics. Diverse stretch patterns were found in the patients analyzed (see Fig. 3). By definition, $\lambda_1 > \lambda_2$, and the principal stretches therefore show maximum and minimum stretch on the surface. Note that a value of $\lambda_2 < 1$ does not directly imply shrinkage of the surface, but only a local dimensional shrinkage that could still result in an increase in local surface area. We also determined

Fig. 4 Relative surface area expansion (λ_1, λ_2) and surface expansion rates shown on AAA surfaces at time T_2 . Gray scale reflects the minimum and maximum relative areas, first on an individual basis (individual scale) and then for all patients (common scale).

local changes in surface area and their rates (see Eqs. (6) and (7) and Fig. 4). For the untreated AAAs considered here, we expected that either the surface is stable and does not change over time ($\lambda_1 \lambda_2 \simeq 1$), or that the AAA expands over time ($\lambda_1 \lambda_2 > 1$). In practice, errors in image segmentation, due to the segmentation procedure or image quality, and initial surface alignment could result in surface shrinkage ($\lambda_1 \lambda_2 < 1$). Surface shrinkage, however, can be used as an indication of the accuracy of the surface extraction procedure and initial surface alignment. In the patients analyzed, surface shrinkage was generally restricted to very small regions, and associated with segmentation and clipping uncertainties. Patient 4, however, showed extensive overall shrinkage. Two possible explanations for this shrinkage are: (1) Poor selection of endpoints when segmenting the AAA surfaces. The total length of patient 4 AAA is smaller at time T_2 than at time T_1 , and while λ_1 is generally greater than 1 (see Fig. 4), surface shrinkage prevails. (2) Actual circumferential expansion of the AAA at the expense of AAA length. Cases like this require closer analysis and comparison of segmentations to CT scan images. Unfortunately for this study, we do not have access to the original CT scan data. We decided to include these results since they illustrate how our strategy can point to uncertainties in patient data that can be later verified.

Each patient considered not only showed different patterns of AAA progression but also they had “hot spots,” which were regions where surface expansion or surface expansion rate were more elevated (Fig. 4). Hot-spot regions, and the specific AAA geometry, were unique to each patient, pointing to differences that are not captured by maximum diameter alone. When analyzing surface expansion rates (see Eq. (7) and Fig. 4(b)) patients 1, 5, and 6 showed the fastest expansions, which were also localized to distinct regions of the AAA for each patient. In general, rates of surface expansion could be used to assess response to treatments, such as after repair or use of a drug, pointing to potential uses in clinical practice. In the future, surface expansion rates could also be used to predict AAA enlargement and risk rupture.

4.4 Concluding Remarks. Individual AAA patients showed markedly different AAA shapes and progression patterns. With advances in technology, a more thorough evaluation of the AAA is now becoming feasible and available for clinicians, and this availability could change AAA evaluation, significantly reducing risks for patients. While more extensive studies are clearly needed, our results demonstrate that by following maximum transverse diameter alone, clinicians are missing a great deal of the shape change dynamics on patient-specific AAAs. This becomes especially important for patients that are below the threshold for repair (AAA diameter < 5.5 cm) and yet may be at risk of rupture, and for those patients who have larger AAAs but have other complications that make repair riskier and could perhaps wait if their AAAs are stable. Local changes in AAA shape could be the key in predicting an unstable versus a stable AAA, and thus could be a more accurate tool than transverse diameter to stratify patients.

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