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Count-Rate Statistics for Drift Detectors

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Abstract

Synchrotron light sources are low-duty-cycle pulsed X-ray sources, a fact that is often neglected in estimating the count-rate capabilities of photon-counting detectors in synchrotron-based experiments. In this paper, we demonstrate the effect that this has on the pileup statistics of drift detectors. We derive expressions for the cases of continuous and pulsed X-ray sources. We consider a pulsed source with period that is either much less than the shaper support time or much less than the average drift time. We also consider a pulsed source with a period that is long or comparable to both the shaper support and the drift time. These conditions correspond to normal and reduced bunch fill patterns of synchrotrons.

Index Terms

Drift detectors; high count-rate detectors; pile-up statistics

I. Introduction

Solid-state drift detectors have emerged as a promising technology for high-count-rate high-energy-resolution applications, e.g., X-ray spectroscopy and inelastic scattering experiments at synchrotron light sources.

In the solid-state drift detector [1], a relatively large piece of depleted semiconductor is used as the detector medium to affect the conversion of a photon to an ensemble of charge carriers. Instead of immediately inducing a signal on the sensing electrode, however, the charge must first drift through a region that has an electric field defined by field shaping electrodes that largely shield the charge from the sensing electrode. The drift fields are defined such that the charge produced by an event anywhere in the detector will eventually drift toward the sensing electrode.

Since all charges drift to this one location, the sensing electrode can be quite small. This has the advantage of producing a relatively large active area detector with a very small capacitance. And since the detector capacitance is small, very good energy resolutions are achievable with short shaping times [2]. When such detectors can be produced on a

consistent basis, they will perhaps become the standard in high-flux high-resolution experiments.

For drift detectors, the time of detection of an event is not coincident with the time of the event, and the time delay from the occurrence of an event to the detection time is dependent on the position of the event. It is interesting to investigate what happens to the pileup statistics. We consider here the limiting cases: a continuous X-ray source or a pulsed source with period that is either much less than the shaper support time or much less than the average drift time. We also consider a pulsed source where the period is long or comparable to both the shaper support and the drift time.

When a photon enters the detector and generates the drifting charge, an area can be defined around the charge within which no other photons may be permitted to interact with the detector if the event is to be processed without pileup interference or be rejected by a pileup rejection circuit. In the case of a circular drift detector with the sensing electrode at the center, this area would be represented by an annulus with inner and outer radii determined by the location of the charge, the size of the detector, and the shaper left and right support. The annulus is formed at the moment of interaction, shrinks toward the center of the detector as charge drifts toward the sensing electrode, and vanishes completely some time after the charge is collected. The probability that an interfering event occurs during this evolution then depends on the initial radius of the event, the drift velocity profile, and the illumination pattern of the detector. Although the analysis along these lines produces accurate results, it requires knowledge of the illumination pattern and equations of motion for the drifting charges, both of which detract from the generality of the results.

II. Method

A. Continuous X-Ray Sources

We develop another formulation of the problem. In a particular period of time T_x , very long compared to both the maximum drift time T_R and the shaper support time τ , a number of events N occur. If the X-ray flux is constant over time T_x , the arrival time of each of the events is an independent identically distributed (i.i.d.) random variable, uniformly distributed on the interval

$$f_x(t) = \begin{cases} \frac{1}{T_x}, & t \in [0, T_x] \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The detector then adds a delay to each of the event times to produce the detection time of the event. These random variables are also i.i.d. but have a possibly complicated density function that depends on the geometry of the detector, the velocity profile of the drifting charges, and the illumination pattern of the detector. Note, however, that the support of the density function $f_t(t)$ vanishes for t outside the interval $(0, T_R)$

$$f_t(t) = 0 \quad \text{for} \quad t \notin [0, T_R]. \quad (2)$$

Since the detection time is the sum of the arrival time and the transit time, two independent random variables, the density of the detection times is the convolution of the density functions $f_t(t)$ and $f_x(t)$

$$f_d(t) = f_t(t) * f_x(t). \quad (3)$$

Recall that (by supposition) $T_R \ll T_x$. Then there exists a large interval interior to $[0, T_x]$, where the convolution integrand is a constant with respect to t , resulting in a uniform density function

$$f_d(t) = \frac{1}{T_x} \quad \forall t \in [T_R, T_x - T_R] \quad (4)$$

i.e., the pileup statistics for a drift detector used with a continuous (or equivalent) source are computed in precisely the same manner as for other solid-state detectors exposed to the same flux, regardless of the various parameters describing the dynamics of the drift process.

B. Pulsed X-Ray Sources

When the source of X-rays is pulsed, the situation is different. The density function of the detection times is still of the convolution form above. However, the density of the event times is no longer given by $f_x(t)$. For a synchrotron source, the bunch width is much less than the bunch period T_b , and we may approximate the density function for each of the events that occurs within M bunches as

$$f_b(t) \approx \frac{1}{M} \sum_{k=1}^M \delta(t - kT_b). \quad (5)$$

That is to say, photons arrive in very short, discrete time intervals.

The density of the detection times is then

$$f_d(t) = f_b(t) * f_t(t) = \frac{1}{M} \sum_{k=1}^M f_t(t - kT_b). \quad (6)$$

In Fig. 1 we illustrate a Si drift detector in operation.

A photon incident on the detector will interact somewhere within the volume of the detector. After the interaction, the charge cloud will then drift toward the center (sensing) electrode. The drift velocity is controlled by the presence of field shaping electrodes on the surface of the Si. The electric field of the charge is largely shielded from the sensing electrode until shortly before it is collected; thus, a signal is not induced on the electrode until just before the charge is collected. Note that with a pulsed X-ray source, there is the potential of temporally separating the detection times of events that occurred simultaneously, but at different locations within the detector. If two events are coincident but have sufficiently

different transit times to the sensing electrode, they can be resolved by the system. Note that the density function of the detection times can be made to be uniform on an interval MT_b by proper selection of the transit time density function, i.e., by proper selection of the detector illumination pattern, geometry, and drift field. One such solution, for a uniformly illuminated detector, is to use a strip drift detector where the sensing electrode is at one end of a long narrow strip and a constant drift field such that $T_R = T_b$. In this case, $f_d(t)$ is uniform on $[0, T_b]$, and the resulting density of detection times $f_d(t)$ is also uniform but on the long interval $[0, MT_b]$. In this case, the pileup statistics will again be identical to that of a constant X-ray source with the same total event rate.

We note that performance of sequence estimation [4] for the purpose of resolving pileup pulses may be degraded and/or become much more computationally burdensome when drift detectors are used. Efficient sequence estimation relies on the *a priori* knowledge of event times. Since the drift mechanism destroys that deterministic property, pulse times are not precisely known. If the uncertainty is significant compared to the shaper pulse support, pulse location cannot be assumed.

C. Optimum Drift-Time Density Function

Since drift detectors provide us the opportunity to partially control the arrival time density function through bias conditions and detector geometry, we wish to determine the density function that maximizes the observed event rate for a given average event rate. Consider a pulsed source application and a detector with a pileup rejection system. The event rate can be written with the detection event function

$$\lambda(t) = M f_d(t). \quad (7)$$

We show in the Appendix [(A.15)] that the density function that maximizes the observed average count rate is $\lambda(t) = \lambda_0$, i.e., the uniform distribution, provided that $\lambda_0 \leq 1/\tau_T$. Thus, the case of a continuous source yields an upper bound on achievable count rate. Deviation from the uniform distribution, such as distributions produced by pulsed sources, leads to increased pileup. Experimenters regularly observe this effect when synchrotrons operate in single bunch modes. For similar fluxes, there are more corrupted events than in multibunch modes.

III. Circular Drift Detectors

In this section, we compare the pileup performance of circular drift detectors operated in a pulsed source environment to detectors operated in a continuous source environment (or, equivalently, a linear drift detector with maximum transit time equal to the bunch period).

Consider a circular drift detector with constant drifting E-field strength with maximum drift time equal to the bunch period of the source. Since the differential area on the surface of the drift detector increases linearly with the radial distance from the center, it is not difficult to show that the transit time density function for the uniformly illuminated detector is given by

$$f_t(t) = \frac{2}{T^2}t, \quad \text{for } 0 < t < T_R \quad (8)$$

where $T_R = R/v$, R is the radius of the detector, and v is the drift velocity of the charge cloud. Thus for an average incident event rate λ_0 , the instantaneous count rate on one bunch period is

$$\lambda(t) = \lambda_0 \frac{2}{T_R}t, \quad 0 < t < T_R \quad (9)$$

since we have set $T_b = T_R$. The average observed count rate is therefore

$$\begin{aligned} \bar{\lambda}_{ob} &\geq \frac{1}{T_b} \int_0^{T_b} \lambda(t) e^{-\lambda(t)\tau} dt \\ &= \frac{1 - e^{-2\lambda_0\tau}}{2\lambda_0\tau} - e^{-2\lambda_0\tau} = \lambda_c \end{aligned} \quad (10)$$

where τ is the shaper time. Note that the result is independent of the bunch period $T = T_R$. In Fig. 2, we compare the observed count rate derived here against the count rate for a continuous source. For a continuous source, $\lambda u(t) = \lambda u$, and therefore [4]

$$\lambda u = \lambda_0 e^{-\lambda_0\tau}. \quad (11)$$

As expected, the observed count rate for the circular drift detector is bounded from above by detectors with a uniform distribution of arrival times (uniform detectors).

To find the maximum count rates for the two detectors, we take the derivatives of (10) and (11) with respect to λ_0 and set them to zero. For uniform detectors, we have

$$\lambda u_{\max} = \lambda u \left(\frac{1}{\tau} \right) = \left(\frac{1}{\tau} \right) e^{-1}. \quad (12)$$

For the circular detector, the solution is transcendental and is found numerically

$$\lambda_{c_{\max}} = \lambda \left(\frac{0.897}{\tau} \right) = \frac{0.298}{\tau} \quad (13)$$

i.e., the maximum observed count rate for uniform detectors is attained at $\lambda_0 = 1/\tau$ and is equal to e^{-1}/τ . For the circular drift detector, the maximum is attained at $\lambda_0 = 0.897/\tau$ and is equal to $0.298/\tau$. Taking the ratio gives us the pileup performance penalty in maximum count rate for the circular drift detector

$$R_C = \frac{\lambda_{c_{\max}}}{\lambda u_{\max}} = 0.811. \quad (14)$$

Taking the ratio of (10) and (11) gives us the relative efficiency of the circular detector (with respect to a uniform detector). This ratio is plotted in Fig. 3. Notice that the degradation in count-rate performance increases with the event rate.

For example, consider the reasonable detector and synchrotron parameters: 150-nS shaper, 300-nS max drift time circular detector, and 300-nS bunch period (approximate period for two-bunch symmetric mode at the National Synchrotron Light Source (NSLS), Brookhaven National Laboratory). The maximum count rate for such a detector is about 1.98 MHz, while with a continuous source, the same detector could attain a maximum count rate of 2.45 MHz.

IV. Simulation Results

The results of the above analysis were verified with simulations. In brief, continuous and pulsed sources were modeled with appropriate probability densities. For a continuous source, a set of event times was generated by randomly sampling a uniform distribution as many times as appropriate for a given count rate. For a pulsed source, the probability density was modeled as uniform over the width of a single pulse, with a given pulse giving rise to a number of events determined by sampling from a Poisson distribution with mean given by the product of the count rate and pulse period.

To each event time was added a transit time randomly sampled from the density of drift times corresponding to uniform illumination of a given detector geometry. In the case of a constant drift velocity, this density is uniform for a linear detector and a linear ramp for a circular detector [see (8)]. The resulting list of detection times was sorted and analyzed point by point to look for pileup, i.e., interevent intervals shorter than half the shaper support.

The results of two sample simulation runs, which are presented in Fig. 4, agree with the analytically derived results presented in Fig. 2.

V. Conclusion

The analysis of count-rate statistics for drift detectors in terms of the convolution of probability densities for event times and drift times permits an intuitive understanding of the special cases where there is deviation from pixel detector statistics. The first case to summarize is continuous sources: since the event-time probability density is uniform, the convolution with any drift time density is still a uniform density. This is equivalent to saying that a random set of event times yields a random set of detection times even if any independent set of drift times is added to them. Hence, the pileup characteristics are governed entirely by the event rate relative to the shaper support, just as with pixel detectors.

With a pulsed source, the convolution of densities necessarily deviates from a uniform distribution whenever the maximum transit time is shorter than the pulse period. As pointed out above, a linear drift detector can be made to have uniformly distributed detection times by matching its maximum transit time to the pulse period. In this condition, the drift detector will have fewer corrupted events than a pixel detector that is exposed to the same flux and has the same shaper support, e.g., a small pixel detector in a bright source. We also show

that the uniform distribution of event times maximizes the count rate for a given event rate less than the inverse of the shaping time.

When the maximum drift times and the shaper support are less than one pulse period, the drift detector can have different count-rate statistics than a pixel detector and will depend on the exact relationships between these parameters. If the shaper support is just long enough to permit the most delayed events from one pulse to interfere with events in the subsequent pulse, then there will be more corrupted events in the drift detector. If the shaper support is much shorter than the maximum drift time, on the other hand, then the drift detector could temporally separate events generated within the same pulse and have a smaller pileup fraction.

At current synchrotron light sources, the pulse periods during normal operating modes are quite short (on the order of 20 ns) and event-time densities are generally indistinguishable from continuous sources. However, during single bunch operations, as are often used for kinetic studies, the pulse period will be comparable to reasonable shaper times and the dynamics of the drift process will affect the count-rate statistics. Indeed, the dramatic increase in the number of pileup events observed by experimenters working with pixel detectors in single bunch mode could be avoided with drift detectors with suitable drift-time densities, even if the shaper support could not be reduced.

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Appendix

If we assume that the shaper support time τ is small compared to the bunch period, then taking the time average over a bunch period gives the approximation of the average observed count rate

$$\bar{\lambda}_{ob} = \frac{1}{T_b} \int_0^{T_b} \zeta(t) e^{-\zeta(t)\tau} dt \approx \frac{1}{T_b} \int_0^{T_b} \lambda(t) e^{-\lambda(t)\tau} dt$$

where

$$\zeta(t) = \frac{1}{\tau} \int_t^{t+\tau} \lambda(\beta) d\beta \rightarrow \begin{cases} \lambda(t), & \text{for small } \tau \\ \lambda_0, & \text{for large } \tau. \end{cases} \quad (\text{A.1})$$

Note that $\zeta(t)$ is a running average of $\lambda(t)$ and thus will be smoothed toward a constant λ_0 , the time average of $\lambda(t)$. From the result of this section, we conjecture that this implies that the approximation in (A.1) is a lower bound on the observed count rate.

We wish to maximize (A.1) subject to the constraint that the average incident event rate is held constant

$$\frac{1}{T_b} \int_0^{T_b} \lambda(t) dt = \lambda_0. \quad (\text{A.2})$$

Let $\lambda(t)$ be Riemann integrable on $[0, T_b]$. We can then write $\lambda(t)$ as

$$\lambda(t) = \lambda_0 + \lambda_1(t) \quad \text{where} \quad \int_0^{T_b} \lambda_1(t) dt = 0. \quad (\text{A.3})$$

Define the functions

$$\begin{aligned} \lambda^+(t) &= \begin{cases} \lambda_1(t), & \lambda_1(t) > 0 \\ 0, & \text{otherwise} \end{cases} \\ \lambda^-(t) &= \begin{cases} \lambda_1(t), & \lambda_1(t) < 0 \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (\text{A.4})$$

Let $\mathbf{P}$ be a partition of $[0, T_b]$ such that $\forall t \in I_i, \lambda^+(t) > 0$ or $\lambda^+(t) = 0$. For all such partitions $\mathbf{P}$, $\exists$ a partition $\mathbf{Q}$ of $[0, T_b]$ such that

$$\begin{aligned} \forall i \int_{t \in I_i} \lambda^+(t) dt &= - \int_{t \in J_i} \lambda^-(t) dt \\ \text{mesh}(\mathbf{P}) \rightarrow 0 &\Rightarrow \text{mesh}(\mathbf{Q}) \rightarrow 0 \end{aligned} \quad (\text{A.5})$$

where I_i is an interval of $\mathbf{P}$ and J_i is an interval of $\mathbf{Q}$. Letting $\text{mesh}(\mathbf{P})$ go to zero

$$\begin{aligned} \lambda^+(t_{I_i}) I_i &\rightarrow \int_{I_i} \lambda^+(t) dt, \quad \text{and} \quad \lambda^-(t_{J_i}) J_i \rightarrow \int_{J_i} \lambda^-(t) dt \\ \Rightarrow \frac{\lambda^+(t_{I_i}) I_i}{\lambda^-(t_{J_i}) J_i} &\rightarrow -1 \quad \forall i \text{ where the ratio exists.} \end{aligned} \quad (\text{A.6})$$

By definition of the Riemann integral

$$\begin{aligned} \lim_{\text{mesh}(\mathbf{P}) \rightarrow 0} \sum_i \left[\lambda^+(t_{I_i}) I_i + \lambda^-(t_{J_i}) J_i \right] \\ = \int_0^{T_b} \lambda^+(t) dt + \int_0^{T_b} \lambda^-(t) dt = \int_0^{T_b} \lambda_1(t) dt. \end{aligned} \quad (\text{A.7})$$

We may then write the average observation rate in terms of the same partitions $\mathbf{P}$ and $\mathbf{Q}$

$$\begin{aligned} \frac{1}{T_b} \int_0^{T_b} \lambda_{ob}(t) dt &= \frac{1}{T_b} \sum_{i \in \Psi} \left[\int_{I_i} \lambda(t) e^{-\lambda(t)\tau} dt + \int_{J_i} \lambda(t) e^{-\lambda(t)\tau} dt \right] \\ &= \lim_{mesh(P) \rightarrow 0} \sum_{i \in \Psi} \left[\lambda(t_{I_i}) e^{-\lambda(t_{I_i})\tau} I_i + \lambda(t_{J_i}) e^{-\lambda(t_{J_i})\tau} J_i \right] \end{aligned} \quad (\text{A.8})$$

where Ψ is the set of indexes such that

$$\lambda^+(t), \lambda^-(t) \neq 0, \quad \text{for } t \in I_i, J_i, \text{ respectively.}$$

Note that for each i in (A.8), we may write, with some algebraic manipulation, the incremental change in the observed count rate

$$\begin{aligned} \Delta \lambda_{ob_i} &= I_i \left[\left(\lambda_0 + \lambda^+(t_{I_i}) \right) e^{-(\lambda_0 + \lambda^+(t_{I_i}))\tau} - \lambda_0 e^{-\lambda_0 \tau} \right] - I_i \frac{\lambda^+(t_{I_i})}{\lambda^-(t_{J_i})} \left[\left(\lambda_0 + \lambda^-(t_{J_i}) \right) e^{-(\lambda_0 + \lambda^-(t_{J_i}))\tau} - \lambda_0 e^{-\lambda_0 \tau} \right] \\ &= \Delta_i^+ + \Delta_i^- \end{aligned} \quad (\text{A.9})$$

where we have made use of the fact that $I_i \lambda^+(t_{I_i}) = -J_i \lambda^-(t_{J_i})$. However, incremental change in observed count rate is nonpositive if either $\lambda_0 \in (0, 1/\tau)$ or $\lambda^+(t_{I_i}) \in (0, 2/\tau)$.

Proof

Take the first and second derivatives of the observed count-rate function

$$\frac{d}{d\lambda} \lambda_{ob}(\lambda) = (1 - \lambda\tau) e^{-\lambda\tau} \quad (\text{A.10})$$

$$\frac{d^2}{d\lambda^2} \lambda_{ob}(\lambda) = (\lambda\tau - 2)\tau e^{-\lambda\tau}. \quad (\text{A.11})$$

Notice that the second derivative is negative for $\lambda \in (0, 2/\tau)$, and therefore λ_{ob} is convex on that interval. Define the linear function tangent to the observed count-rate function at $\lambda = \lambda_0$ using (A.10)

$$g(\lambda, \lambda_0) = K + \left[(1 - \lambda_0\tau) e^{-\lambda_0\tau} \right] \lambda$$

where

$$K = \lambda_0^2 \tau e^{-\lambda_0\tau}. \quad (\text{A.12})$$

Since λ_{ob} is convex on the indicated interval and has a single maximum at $\lambda = 1/\tau$, $g(\lambda, \lambda_0)$ $\lambda_{ob}(\lambda) \forall \lambda_0 \in (0, 1/\tau) \vee \lambda \in (0, 2/\tau)$ with equality iff $\lambda = \lambda_0$. Therefore

$$\begin{aligned}
\Delta_i^+ &\leq \left[g \left(\lambda^+ (t_{I_i}), \lambda_0 \right) - K \right] I_i \\
&= \left[(1 - \lambda_0 \tau) e^{-\lambda_0 \tau} \right] \lambda^+ (t_{I_i}) I_i \\
\Delta_i^- &\leq \frac{-\lambda^+ (t_{I_i})}{\lambda^- (t_{J_i})} \left[g \left(\lambda^- (t_{J_i}), \lambda_0 \right) - K \right] I_i \\
&= - \left[(1 - \lambda_0 \tau) e^{-\lambda_0 \tau} \right] \lambda^+ (t_{I_i}) I_i.
\end{aligned} \tag{A.13}$$

Using this in (A.9), we have

$$\begin{aligned}
\Delta \lambda_{ob_i} &= \Delta_i^+ + \Delta_i^- \\
&\leq \left[(1 - \lambda_0 \tau) e^{-\lambda_0 \tau} \right] \lambda^+ (t_{I_i}) I_i - \left[(1 - \lambda_0 \tau) e^{-\lambda_0 \tau} \right] \lambda^+ (t_{I_i}) I_i = 0.
\end{aligned} \tag{A.14}$$

Furthermore, $\lambda^-(t_{J_i}) \rightarrow 0 \Rightarrow \lambda^+(t_{J_i}) \rightarrow 0$. Taking this limit in (A.9) gives

$$\lim_{\lambda^+(t_{I_i}) \rightarrow 0^+} \lim_{\lambda^-(t_{J_i}) \rightarrow 0^-} \Delta \lambda_{ob_i} = 0 \quad \forall i. \tag{A.15}$$

Therefore, (A.8) is maximized for $\lambda^-(t_{J_i}) = 0, \lambda^+(t_{J_i}) = 0 \forall i$, i.e., when $\lambda(t) = \lambda_0$.

This implies simply that the event detection distribution function is just the uniform distribution and does not depend on the average event rate provided that the condition $\lambda_0 \in (0, 1/\tau)$ or $\lambda^+(t_{J_i}) \in (0, 2/\tau)$ is preserved, i.e.,

$$\lambda_1(t) = \arg \left[\max_{\lambda_1(t)} \int_0^{T_b} \lambda(t) e^{-\lambda(t)\tau} dt \right] = 0. \tag{A.16}$$

This says that for a given average event rate $\lambda_0 = 1/\tau$, the uniform distribution of events maximizes the observed count rate. [We work with the constraint $\lambda_0 = 1/\tau$, since it is easier to predict and because the other constraint, $\lambda^+(t) < 2/\tau$, is less likely to hold for a synchrotron radiation source at high count rates. Satisfaction of either constraint is sufficient. Note that for shaping times nonnegligible compared to the bunch period, $\zeta(t)$ is “closer” to a uniform distribution than $\lambda(t)$, justifying the conjecture that

$$\lambda_0 e^{-\lambda_0 \tau} \geq \bar{\lambda}_{ob} \geq \frac{1}{T_b} \int_0^{T_b} \lambda(t) e^{-\lambda(t)\tau} dt. \tag{A.17}$$

Fig. 1. Circular drift detector operated in a pulsed source environment. Spatially separated, coincident events may be seen as temporally separated events at the output since they may experience different drift times.

Fig. 2.

Observed count rates for $\lambda_c(\lambda_0)$: uniformly illuminated circular drift detector in a pulsed source environment with bunch period T_b equal to the maximum drift time T_R ; and $\lambda_u(\lambda_0)$: an arbitrary geometry detector with a continuous source, or, equivalently, a uniformly illuminated linear drift detector with $T_b = T_R$. Both axes are normalized the shaper support time τ .

Fig. 3.

The ratio of the observed count rates for the circular detector and a uniform detector. The performance of the circular detector tends toward that of the uniform detector at low count rates and degrades as the event rate is increased.

Fig. 4.

Simulated count rates for $\lambda_c(\lambda_0)$: uniformly illuminated circular drift detector in a pulsed source environment with bunch period T_b equal to the maximum drift time T_R ; and $\lambda_u(\lambda_0)$: an arbitrary geometry detector with a continuous source, or, equivalently, a uniformly illuminated linear drift detector with $T_b = T_R$. The shaper support time is set to 5% of the pulse period. Both axes are normalized the shaper support time τ .