Investigation of particle inertial migration in high particle concentration suspension flow by multi-electrodes sensing and Eulerian-Lagrangian simulation in a square microchannel

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The inertial migration of neutrally buoyant spherical particles in high particle concentration ($\alpha_{pi} > 3\%$) suspension flow in a square microchannel was investigated by means of the multi-electrodes sensing method which broke through the limitation of conventional optical measurement techniques in the high particle concentration suspensions due to interference from the large particle numbers. Based on the measured particle concentrations near the wall and at the corner of the square microchannel, particle cross-sectional migration ratios are calculated to quantitatively estimate the migration degree. As a result, particle migration to four stable equilibrium positions near the centre of each face of the square microchannel is found only in the cases of low initial particle concentration up to 5.0 v/v%, while the migration phenomenon becomes partial as the initial particle concentration achieves 10.0 v/v% and disappears in the cases of the initial particle concentration $\alpha_{pi} \geq 15\%$. In order to clarify the influential mechanism of particle-particle interaction on particle migration, an Eulerian-Lagrangian numerical model was proposed by employing the Lennard-Jones potential as the inter-particle potential, while the inertial lift coefficient is calculated by a pre-processed semi-analytical simulation. Moreover, based on the experimental and simulation results, a dimensionless number named migration index was proposed to evaluate the influence of the initial particle concentration on the particle migration phenomenon. The migration index less than 0.1 is found to denote obvious particle inertial migration, while a larger migration index denotes the absence of it. This index is helpful for estimation of the maximum initial particle concentration for the design of inertial microfluidic devices. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4946012>]

I. INTRODUCTION

As a new approach to precisely manipulate particles or fluids, inertial-induced particle migration has demonstrated potentially useful effects for a wide range of applications, including particle transporting, sorting, focusing, and sample mixing due to its advantages such as operation without external fields, low cost, inherent miniaturization, and portability.^{1,2} At small but finite Reynolds number, inertial lift forces induce lateral migration of particles to distinct equilibrium positions in microchannel cross-section, the phenomenon of which is called particle inertial migration.^{3,4}

Many researchers have focused on the particle inertial migration behaviours and succeeded in manipulating different types of particle in microchannel.^{5,6} Kuntaegowdanahalli *et al.*

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separated SH-SY5Y neuroblastoma cells ($\sim 15\ \mu\text{m}$) from C6 rat glioma cells ($\sim 8\ \mu\text{m}$) in a dilute suspension of cells (concentration $\alpha_{pi} = 0.05\ \text{v/v}\%$).⁷ Hur *et al.* sorted red blood cells (RBCs) and white blood cells (WBCs) at certain spanwise positions in the microchannel by using the particle inertial migration. As their sample was sufficiently dilute ($\alpha_{pi} \leq 5.0\ \text{v/v}\%$), migration of different types of cells was visualized by image analysis technique.⁸ Choi *et al.* measured the three-dimensional migration of neutrally buoyant polystyrene microspheres in a dilution of $\alpha_{pi} = 0.05\ \text{v/v}\%$ with water using the digital holographic microscopy technique and reported a transition from the lateral tubular pinch to the cross-lateral focusing with increasing channel Reynolds number.⁹ Chung *et al.* present a novel inertial focusing platform to create a single-stream micro-particle train in a single-focal plane without sheath fluids and external forces and achieved high focusing efficiency in a dilute suspension of $\alpha_{pi} = 0.1\%$ and $0.5\ \text{v/v}\%$.¹⁰

Although these previous studies succeeded in particle manipulation by inertial migration, investigation of particle inertial migration in concentrated particulate solution is a very attractive frontier, especially for further biomedical application.¹¹ For example, in the case of cancer cell isolation from a whole blood sample (concentration of red blood cells $\alpha_{pi} > 40.0\ \text{v/v}\%$), it is preferable to use whole or minimally diluted blood to reduce the total separation time.¹² However, as increase of particle concentration, the increased particle-particle and particle-fluid interaction lead to a significant influence on the particle migration behaviour. Lim *et al.* reported a radical shift in focusing behaviour of PC-3 prostate cancer cells in whole blood.¹³ Tanaka *et al.* investigated migration of cancer cells in high haematocrit (up to $\alpha_{pi} = 40.0\ \text{v/v}\%$) blood flow and reported that cancer cells can migrate towards equilibrium position up to a haematocrit level of $\alpha_{pi} = 10.0\ \text{v/v}\%$.¹⁴ Sollier *et al.* also studied the effect of blood dilution on the collection efficiency of tumour cells; reducing dilution of the blood sample enables faster processing but also results in lower collection efficiency.¹⁵ While these researches reported inhibition of particle inertial migration in high concentration suspensions, the theoretical understanding of the relevance between particle-particle interaction and particle inertial migration in high concentration suspensions is inadequate and need to be figured out. In particular, a quantitative criterion for estimation of the occurrence of particle inertial migration in high concentration suspensions is imperative for improvement of the manipulation efficiency of inertial microfluidic devices.

Conventional measurement techniques for studying particle inertial migration—including high-speed bright-field imaging and long exposure fluorescence imaging—are generally limited in complex fluids such as whole blood due to interference from the large numbers of red blood cells. In order to overcome the difficulty of the conventional methods to evaluate the particle inertial migration in the high concentration solution, the multi-electrodes sensing (MES) method had been proposed and been successfully applied in the measurement of cell concentration in concentrated suspensions (up to $\alpha_{pi} = 40.0\ \text{v/v}\%$).¹⁶

In this study, online measurement in a square microchannel under a range of high initial particle concentration (from $\alpha_{pi} = 3.0$ to $40.0\ \text{v/v}\%$) by using MES method is demonstrated to clarify the effect of initial particle concentration on particle inertial migration. In order to quantitatively evaluate the degree of particle inertial migration, cross-sectional particle inertial migration ratios are determined by measured particle concentration near the wall and at the corner of the square microchannel. Moreover, an Eulerian-Lagrangian model is proposed for numerical simulation of particle inertial migration at different initial particle concentrations to clarify the influential mechanism of particle-particle interaction on particle inertial migration.

II. EXPERIMENT

A. Experimental set-up

The experimental set-up used in this work is shown in Figure 1(a). The experiment apparatus primarily consists of a multiplexer, a capacitance acquisition unit, and a personal computer. Microchannel with square cross-section was placed inside a faraday cage to prevent external interference. Particulate suspension was injected using a syringe pump (KD Scientific, USA). Two multi-electrodes sensors are embedded in the square microchannel. Each sensor contains

FIG. 1. Diagrams of the experimental facilities. (a) Experimental set-up; (b) cross-sectional μ CT images of the square microchannel; and (c) multi-electrodes sensors embedded in the square microchannel.

12 electrodes, and the electrode tails are connected to printed circuit board (PCB) through copper terminal connectors. The PCB is connected to multiplexer through pin connectors. The multiplexer switches among all combination pair of electrodes to perform an overall scan of the sensing cross-section of the square microchannel. Simultaneously, the capacitance acquisition device developed based on charge transfer method¹⁷ captures capacitance between electrodes and sent it to the personal computer for data processing.

Schematic representation of the square microchannel is shown in Figures 1(b) and 1(c). The square shape groove cross-sectional microchannel is $H = 450 \mu\text{m}$ per side and $L = 36 \text{ mm}$ in length. The two embedded sensors are 10 mm and 30 mm from the inlet, respectively. Each sensor contains 12 electrodes which is $200 \mu\text{m}$ in width and $10 \mu\text{m}$ in thickness. The square microchannel was fabricated by vapor deposition with a protocol similar to that used in our previous study.¹⁸ First, vapour deposition is carried out to make platinum electrodes on a high transparent quartz substrate by using of a special designed shadow-mask. After precise vapour deposition on substrate, the shadow-mask is removed, and another quartz substrate is compressed to make the electrodes equally embedded in both of these substrates. By repeating these processes, substrate embedded with multi electrode layer can be made. Fabrication of the flow groove is achieved by means of a triangle-shape drilling process. The tomographic images taken by a three-dimensional x-ray microscopic computed tomography scanner (μ CT, Yamato Scientific, Japan) are employed to make sure that the main-flow channel is smoothly grooved.

B. Experimental conditions and method

Polystyrene particles (Duke Scientific Co., USA) were supplied as suspensions in deionized water with trace amount of surfactants to prevent aggregation at defined initial particle concentration ranged from $\alpha_{pi} = 3.0\text{--}40.0 \text{ vol. \%}$. Radius of the polystyrene particle is $10 \mu\text{m}$ with a coefficient of variance (CV) less than 9%. The particle density is $\rho_p = 1.05 \times 10^3 \text{ kg/m}^3$. The density of the deionized water was carefully matched with that of the polystyrene particle by adding glycerine to attain neutral buoyancy. Density and viscosity of the adjusted solution are $\rho_f = 1.05 \times 10^3 \text{ kg/m}^3$ and $\mu = 1.21 \times 10^{-3} \text{ Pa s}$. Relative permittivity of the deionized water and the polystyrene particle is $\epsilon_f = 74.0$ and $\epsilon_p = 2.5$, respectively. In the experiment, polystyrene particle suspensions are load into a glass syringe (Hamilton Co., Switzerland) and flowed through the square microchannel at constant flow rate $Q = 1.0 \times 10^{-7} \text{ m}^3/\text{s}$ so that the experiments show the influence of changing the initial particle concentration. The average flow velocity is $\langle u \rangle = 0.49 \text{ m/s}$ which corresponds to Poiseuille flow at the channel Reynolds number $\text{Re}_c = \langle u \rangle H / \nu = 191$ and particle Reynolds number $\text{Re}_p = 2 \langle u \rangle d^2 / (H \nu) = 0.76$, where d is the particle diameter and ν is the fluid kinematic viscosity. According to the empirical formula of the channel length required for focusing, length required for particles to reach lateral

equilibrium positions is about 18 mm.⁵ Therefore, the microchannel used in the experiments is long enough for particles to reach lateral equilibrium positions.

Since various theoretical models have been proposed to connect the electrical permittivity to the components concentration of the multi-component mixture,¹⁹ the capacitance between two electrodes contains the information of particle concentration nearby. In order to extract the particle concentration from the measured capacitance, the MES approach similar to our previous study is employed.¹⁶ In brief, the capacitance data measured by the capacitance acquisition unit are first normalized by a calibration process to remove the stray capacitance and residual inductances inherent to the cables and electrical leads. The local particle concentration α_p in the square microchannel are then calculated at the two sensing planes by the modified Maxwell model as

$$\alpha_p = \left[1 - \frac{C_n(2+k)}{3+C_n(k-1)} \right] \alpha_{pi} \text{ for } C_n = \frac{C_m - C_{pi}}{C_f - C_{pi}} \text{ and } k = \frac{\epsilon_f}{\epsilon_{pi}}, \quad (1)$$

where C_m , C_f , and C_{pi} present the measured capacitance, the capacitance for the case in which deionized water occupies the entire cross-section, and the capacitance for the case in which particle suspensions with initial particle concentration α_{pi} occupy the entire cross-section. ϵ_{pi} is relative permittivity of the corresponding suspension with initial particle concentration α_{pi} , while

$$\epsilon_{pi} = \epsilon_f \frac{2\epsilon_f + \epsilon_p - 2\alpha_{pi}(\epsilon_f - \epsilon_p)}{2\epsilon_f + \epsilon_p + \alpha_{pi}(\epsilon_f - \epsilon_p)}. \quad (2)$$

According to the geometric position of different electrode pairs, the measurements taken via electrode pairs 1–2, 2–3, 4–5, 5–6, 7–8, 8–9, 10–11, and 11–12 as shown Figures 1(c) and 2(a) are used to determine the particle concentration near the wall, while the measurements taken via electrode pair 1–12, 3–4, 6–7, and 9–10 as shown Figures 1(c) and 2(b) give the particle concentration at corner of the square microchannel. The measured electrode pair is switched by the multiplexer at a time interval of 1 ms, and the capacitance between each electrode pairs is measured by a measurement module in the capacitance acquisition device. The technique used in the measurement module is developed based on the charge transfer method¹⁷ which allows capacitance values down to sub-femto Farads to be resolved.

Considering apply of the electrical field in the measurement process may induce additional electro-kinetic forces and affect the particle inertial migration, amplitude and frequency of the operating electrical source should be carefully determined. In the present work, the potential electro-kinetic forces including dielectrophoresis (DEP) force, thermal buoyancy, electro-thermal

FIG. 2. Measurement approach and electro-kinetic forces evaluation. (a) Electrodes combination for the measurement of particle concentration near the wall; (b) electrodes combination for the measurement of particle concentration near the wall at the corner; and (c) influence of particle and fluid velocities due to various electro-kinetic forces.

force, and electroosmotic force are quantitatively estimated for different electric field intensities based on the theoretical model proposed in the literature.²⁰ Then, a sinusoidal wave with effective peak-to-peak voltage $V_{pp} = 200$ mV and frequency $f = 12.5$ MHz is determined. As shown in Figure 2(c), the particle velocity induced by DEP force u_{DEP} , the fluid velocity induced by electro-thermal force u_{ET} , the fluid velocity induced by thermal buoyancy u_{TB} , and the fluid velocity induced by electroosmotic force u_{EO} in this case are estimated to be only in the magnitude of several microns per second which are fairly small compare to the dimension of square micro-channel and velocity of the main flow.¹⁸ Therefore, their influences on particle motion and fluid flow are negligible. In the experiment, the experiments were operated for 120 s to reach quasi-steady state before capturing the capacitance data for each experimental condition. Furthermore, each experiment was repeated ten times to confirm reliability and reproducibility of the measurement results.

III. NUMERICAL SIMULATION

A. Governing equations

The Eulerian–Lagrangian approach on a microscopic level devoted to the physical interactions between particles is applied to simulate the particle migration behaviours. The motion of particles was modelled by Newton’s laws of motion on an individual particle scale, whereas the fluid flow was treated as a continuum phase, described by the mass balance and momentum balance equations. In order to accurately predict the movement of particle, the exact forces acting on the particle must be determined. Strictly speaking, the most accurate method of determining the forces is to integrate the hydrodynamic stress tensor and the particle-particle interactions over the particle. Many researchers have spent a lot of efforts to develop complicated numerical models that account for the microscale hydrodynamic fields.²¹ Based on these researches, an Eulerian–Lagrangian model is presented here to verify the validity and to provide a physical interpretation of the experimental data.

Due to the almost same density of the particle and fluid, buoyancy and gravity force act on particles cancel each other out. In order to evaluate the diffusion effect due to Brownian motion in the present study, the diffusion coefficient D is calculated by the Stokes-Einstein relation $D = k_B T / (3\pi\mu d) = 1.77 \times 10^{-14}$ m²/s, where k_B is the Boltzmann constant and T is the temperature in Kelvin $T = 293$ K.²² This means that the Brownian displacement of particles in 1 s here is about $0.19 \mu\text{m}$ and corresponds to the Péclet number $\text{Pe} = \langle u \rangle H / D = 1.25 \times 10^{10}$. Therefore, the diffusion effect due to the Brownian motion here is negligible. Moreover, particle-particle interaction which is a function of the distance between particles must be introduced to account for the increase of particle concentration.

Figure 3 shows diagrammatic sketch of the considered forces acting on particles in this work. The differential equation that governs the motion of particle i is expressed by

$$m_p \frac{d\mathbf{v}_p^i}{dt} = \mathbf{F}_{vm}^i + \mathbf{F}_d^i + \mathbf{F}_{pp}^i + \mathbf{F}_{IL}^i, \quad (3)$$

where m_p , $\mathbf{v}_p^i$, $\mathbf{F}_{vm}^i$, $\mathbf{F}_d^i$, $\mathbf{F}_{pp}^i$, and $\mathbf{F}_{IL}^i$ represent the particle mass, the particle velocity vector, the virtual mass force, the fluid-particle drag force, the particle-particle interaction force, and the inertial lift force, respectively. The virtual mass force $\mathbf{F}_{vm}^i$, which is the force required to accelerate the fluid surrounding the particle, is written as

$$\mathbf{F}_{vm}^i = \frac{1}{2} m_p \left(\frac{D\mathbf{u}}{Dt} - \frac{d\mathbf{v}_p^i}{dt} \right), \quad (4)$$

where D/Dt is the material derivative and $\mathbf{u}$ is fluid velocity around particle. The fluid-particle drag force $\mathbf{F}_d^i$ is given by

FIG. 3. Diagrammatic sketches of the considered forces acting on particles.

$$\mathbf{F}_d^i = \frac{\pi}{8} C_D \rho_f d^2 |\mathbf{v}_p^i - \mathbf{u}| (\mathbf{u} - \mathbf{v}_p^i), \quad (5)$$

where C_D is fluid-particle drag coefficient. In the present study, C_D is determined by the Gidaspow model which is a combination of the Ergun's equation and Wen and Yu's equation.²³ Considering the discontinuous in particle concentration of this transition proposed by Gidaspow, a switch function is introduced to give a smooth transition from the dilute regime to the dense regime to avoid discontinuity.²⁴

The particle-particle interaction force $\mathbf{F}_{pp}^i$ is estimated by using Lennard-Jones potential (L-J potential) as the inter-particle potential.²⁵ For instance, the particle-particle interaction force acting on particle i is modelled by

$$\mathbf{F}_{pp}^i = \sum_{j=1, j \neq i}^{N_p} 4\delta \left(12 \frac{d^{12}}{r_j^{13}} - 6 \frac{d^6}{r_j^7} \right) \hat{\mathbf{r}}_j^i, \quad (6)$$

where N_p is the number of surrounding particles, r_j^i is the distance between particle i and j , δ is the depth of the potential well which is calculated to be 1.0×10^{-14} [J] according to the particle interaction force scale and its effective interaction length in the present study, and $\hat{\mathbf{r}}_j^i$ is the unit transformation operator for unification of the Eulerian and Lagrangian coordinate. In Eq. (6), the first term is repulsive term that describes particle repulsion and the second term is attractive term that describes attraction at long ranges such as van der Waals force.

To account for the lift forces (the inertial and wall-induced), an expression derived by Asmolov²⁶ is employed to calculate the lift force acting on particle i located at (x^i, y^i, z^i) as

$$\mathbf{F}_{IL}^i = C_L(r^i) \frac{\rho_f u_{\max}^2 d^4}{H^2} \hat{\mathbf{r}}^i, \quad (7)$$

where $u_{\max}$ is the maximum fluid velocity approximated as $2\langle u \rangle$, $C_L(r^i)$ is the lift coefficient which is a function of the distance r^i from particle i to the centreline of the square microchannel, and $\hat{\mathbf{r}}^i$ is the unit vector point to particle i corresponds to $(x^i/r^i, y^i/r^i, 0)$. In the present study, the position dependent function of lift coefficient $C_L(r^i)$ is calculated by a semi-analytical approach proposed by Hood *et al.*²⁷ In brief, in this semi-analytical approach, numerical simulation of a Poiseuille flow through an infinitely long square channel disturbed by a rigid spherical particle is proposed. The particle is only allowed to translate in the axial direction (z) and rotate. The particle velocity in the flow direction and in particular, the particle angular velocity of rotation are varied until it is drag-free and torque-free. Then, the transverse lift force on the drag-free and torque-free particle is calculated by Finite Element Method (FEM) Lagrange multipliers.

The governing equations for the fluid domain are written as

$$\frac{\partial \alpha_f}{\partial t} + \nabla \cdot (\alpha_f \mathbf{u}) = 0, \quad (8)$$

$$\rho_f \left[\frac{\partial (\alpha_f \mathbf{u})}{\partial t} + (\mathbf{u} \cdot \nabla)(\alpha_f \mathbf{u}) \right] = -\alpha_f \nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{f}_{fp}, \quad (9)$$

where α_f is the fluid volume fraction, p is the pressure, and $\mathbf{f}_{fp}$ is the fluid-particle interaction term given by

$$\mathbf{f}_{fp} = -\frac{1}{V} \sum_i^N (\mathbf{F}_d^i + \mathbf{F}_{IL}^i), \quad (10)$$

where V is the volume of the discrete computational cell and N is the particle number inside the cell.

B. Simulation methodology and conditions

The majority of the simulations were on a square column with a side length of $450 \mu\text{m}$ and a length of 36 mm which is the same size as the square microchannel used in the experiments. As addressed above, the simulation domain is discrete into approximately 0.7×10^6 hexahedron cells of which the average side length is about three times of the particle diameter except near wall and at corner of the square column. Each cell consists of fluid phase contacting with particles, and the particle concentration can be defined by the number of particle existing in the cell. A preliminary grid convergence study was carried out in order to verify that the solution is grid independent. Fluid flow rates of $Q = 1.0 \times 10^{-7} \text{ m}^3/\text{s}$ was specified by setting inlet or outlet boundary conditions to fully developed Poiseuille inflow or outflow. Hard spheres with different initial particle concentrations ranged from $\alpha_{pi} = 3.0$ to $40.0 \text{ v/v\%}$ were evenly dispersed into the simulation domain. The corresponding physical properties of the sphere are set exactly same as the polystyrene particles used in the experiment. Considering the high flowrate and the pre-acceleration of particles in tube between the syringe and the inlet of microchannel as shown in Figure 1(a), particles are assumed to be dynamic equilibrium at the inlet of the microchannel. Therefore, the particles were forced to flow at the same velocity as the fluid in the simulation by assigning a negligible velocity difference at the inlet. Nonslip boundary conditions were applied at channel walls. Mixture of deionized water and glycerine was used as the operating fluid by setting the corresponding density and viscosity.

The position dependent function of the inertial lift force was added using a user defined function. Considering the interactions between particles far away with each other are very weak, it is too costly and unnecessary to conduct a particle-particle interaction force evaluation that involves all the particles. To address this issue, the particle-particle interaction evaluation is only conducted for particles included in the neighbouring hexahedron cells because particles more remote are clearly out of reach as shown in Figure 3. Moreover, the computational cost of tracking all of the particles individually is also prohibitive due to the huge particle numbers in the case of high particle concentration. Therefore, in the present study, parcels, which denote a proxy for an agglomeration of eight real particles, are employed as the basic Lagrangian cells and released to the calculation domain.

To solve the differential equations of particle motion as shown in Eq. (3), a solver to switch automatically between the implicit Euler integration scheme and the high order Runge-Kutta scheme is employed. Basically, in the case that the particle is far from dynamic equilibrium, high order Runge-Kutta scheme is activated to achieve an accurate solution quickly, while the solver is switched to implicit Euler integration scheme to facilitate larger integration step in the case that the particle reaches dynamic equilibrium. The continuity and momentum conservation equations as shown in Eqs. (8) and (9) for flow fields were solved using a second order

upwind scheme, along with the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) algorithm for pressure-velocity coupling.²⁸

IV. RESULTS AND DISCUSSION

A. Measured particle concentration by MES

In order to estimate measurement uncertainty of the experiment, standard deviations of measured capacitances are calculated by means of the root-sum-square (RSS) method for each experimental condition. As a result, the standard deviation of the measured capacitances from ten repeated experiments is quite constant with a maximum of 7.0%. Therefore, it is reasonable that the stochastic external interference is restricted to a low level, and the measured capacitances are mostly affected by the permittivity distribution nearby the electrodes which relates to the particle concentration.

Averaged capacitances from the ten repeated experiments are used to calculate the particle concentration near the wall and at the corner of the square microchannel, and the results are shown in Figure 4. In Figure 4(a), the particle concentration near the wall at sensing plane I $\alpha_{p,I}^{wall}$ which is located at the square microchannel upstream presents a linear relationship with the initial particle concentration α_{pi} . However, the particle concentration near the wall at sensing plane II $\alpha_{p,II}^{wall}$ which is located at the square microchannel downstream shows different features. In the case of low initial particle concentration from $\alpha_{pi} = 3.0$ to 5.0 v/v%, $\alpha_{p,II}^{wall}$ is obviously larger than $\alpha_{p,I}^{wall}$ which indicates strong particle inertial migration to the wall. In the case of intermediate initial particle concentration $\alpha_{pi} = 10.0$ v/v%, $\alpha_{p,II}^{wall}$ is still larger than $\alpha_{p,I}^{wall}$ even though $\alpha_{p,I}^{wall}$ is gradually decreased, which indicates a weakened particle inertial migration to the wall. As well, in the case of the sufficiently high initial particle concentration from $\alpha_{pi} = 15.0$ to 40.0 v/v%, $\alpha_{p,II}^{wall}$ is increased with the same linear rate as increase of $\alpha_{p,I}^{wall}$, while the difference between them are also quite small. This indicates an absence of the particle inertial migration to the wall.

Figure 4(b) also shows particle concentration at the corner of square microchannel for different initial particle concentrations α_{pi} . From Figure 4(b), the particle concentrations at the corner of sensing plane I $\alpha_{p,I}^{corner}$ and sensing plane II $\alpha_{p,II}^{corner}$ which are, respectively, located at upstream and downstream of the square microchannel are proportional to α_{pi} with the similar linear rate. It is difficult to recognize the consistent difference between $\alpha_{p,I}^{corner}$ and $\alpha_{p,II}^{corner}$ in the cases of high $\alpha_{pi} = 15.0$ to 40.0 v/v%, while a decrease between $\alpha_{p,I}^{corner}$ and $\alpha_{p,II}^{corner}$ is observed in the cases of lower $\alpha_{pi} = 3.0$ to 10.0 v/v%. Moreover, the particle concentration at the corner is always smaller than the particle concentration near the wall. This phenomenon can be attributed to the complex and unstable flow field caused by the asymmetric interaction between the suspension flow and walls at the corner.

B. Cross-sectional migration ratio

In order to quantitatively evaluate particle inertial migration behaviour, cross-sectional migration ratio is introduced based on the measured particle concentration in the experiments as

FIG. 4. Measured particle concentration at two sensing planes. (a) Near the wall; and (b) at the corner.

$$\psi^{wall} = \frac{\alpha_{p,II}^{wall} - \alpha_{p,I}^{wall}}{\alpha_{p,I}^{wall}} \quad \text{and} \quad \psi^{corner} = \frac{\alpha_{p,II}^{corner} - \alpha_{p,I}^{corner}}{\alpha_{p,I}^{corner}}. \quad (11)$$

The value of cross-sectional migration ratio denotes intensity of particle inertial migration, whereas the positive-negative sign presents migration direction. For example, positive ψ^{wall} shows particle inertial migration towards the wall, and negative ψ^{corner} shows particle inertial migration apart from the corner. Figure 5 shows results of the cross-sectional migration ratio near the wall and at the corner of the square microchannel in the cases of different initial particle concentrations α_{pi} . As shown in Figure 5, ψ^{wall} in the case of low $\alpha_{pi}=3.0$ to 5.0 v/v% are considerably high which means a large number of particles migrate to the wall. As α_{pi} is increased to $\alpha_{pi}=10.0$ v/v%, ψ^{wall} is still noticeable but decreases to a lower value of $\psi^{wall}=0.53$. When α_{pi} reaches or exceeds $\alpha_{pi}=15.0$ v/v%, ψ^{wall} dramatically reduces to nearly zero and presents little particle inertial migration. Similarly, the particle inertial migration apart from the corner is barely observed only in the case of low $\alpha_{pi}=3.0$ – 10 v/v% by noticing that ψ^{corner} is around -0.5 .

These phenomena are verified by the numerical simulation results shown in Figure 6. As shown in Figures 6(a) and 6(b), in the case of low $\alpha_{pi}=3.0$ and 5.0 v/v%, particles are accumulated at four equilibrium positions near centre of each face of the square channel rather than the so called Segre–Silberberg annulus in an axisymmetric Poiseuille pipe flow at $z=30$ mm, which agrees well with the previous research.³ When particle transports through the square microchannel, the inertial lift force that made particles migrate radially is actually the combinations of two major lateral forces. One is the wall induced repulsion force due to the particle-wall interaction, and another is the shear-induced lift force that originates from the shear-gradient of the flow. The wall induced repulsion force pushes particles away from the wall and the shear-induced lift force draws the particle toward the wall. In particular, these two oppositely directed forces are in equilibrium only at eight symmetrically placed points around the square microchannel and induces eight possible equilibrium positions—four near the centre of each face of the square microchannel and four at the corner. However, only the equilibrium positions near the wall are stable, while the rest of them at the corner are unstable²⁷ which well explained the simulation results in the present study. As α_{pi} increases, the equilibrium positions become blurred in the case of $\alpha_{pi}=10.0$ v/v% as shown in Figure 6(c) and even disappeared in the case of $\alpha_{pi}=15.0$ v/v% as shown in Figure 6(d) which implies the weakened migration effect.

Figure 6(e) shows the calculated radial particle concentration profiles. The square microchannel width and height were normalized by the hydraulic radius $h=H/2$. As shown in Figure 6(e), in the case of low $\alpha_{pi}=3.0$ and 5.0 v/v%, the axial particle concentration profiles clearly show the formation of a peak position where the particles are accumulated. The peak positions are found to be around $0.7h$ away from the square microchannel centre which shifts slightly towards the wall comparing with the previous studies at a lower Reynolds number.^{29,30} This can be explained by the more significant influence of the Reynolds number on the shear-induced lift

FIG. 5. Cross-sectional migration ratios at the wall and corner of the square microchannel in the experiment.

FIG. 6. Simulation results of spatial particle distribution and radial particle concentration profiles at $z = 30$ mm. (a) $\alpha_{pi} = 3.0$ v/v%; (b) $\alpha_{pi} = 5.0$ v/v%; (c) $\alpha_{pi} = 10.0$ v/v%; (d) $\alpha_{pi} = 15.0$ v/v%; and (e) radial particle concentration profiles.

force than that on the wall-induced lift force. That is to say, higher Reynolds number causes the shear gradient lift to become more dominant and consequently, the particle equilibrium positions shift closer to the wall. With the increase of α_{pi} , the peak positions become difficult to be recognized. The reason lies in that particles are forced to stay in close proximity to each other and cause a large increase in particle–particle interactions in the case of high α_{pi} . As a result, a portion of the particles are pushed out of the equilibrium positions and make the peak positions become ambiguous.

C. Migration index

In order to quantitatively explain the effect of initial particle concentration α_{pi} on the particle inertial migration shown in our experiment and numerical simulation results, the characteristic length between the adjacent particles in the Poiseuille flow through the square microchannel need to be determined first due to its direct contact with the particle–particle interaction. Assuming that particles are uniformly distributed in the suspension, the characteristic length between the adjacent particles is defined by the inter-particle distance D_{pp} as

$$D_{pp} = d \left(\frac{0.76}{\alpha_{pi}} \right)^{1/3}, \quad (12)$$

where 0.76 is the assumed maximum value of particle volume fraction which is corresponding to the close-packing state of spheres. Then, a dimensionless parameter named migration index M which determines the particle inertial migration possibility in the high particle concentration suspensions is defined by the ratio of the particle–particle interaction force and the inertial lift force as

$$M = \frac{4\epsilon h^2}{\langle C_L \rangle \rho_f u_{\max}^2} \left(12 \frac{d^8}{D_{pp}^{13}} - 6 \frac{d^2}{D_{pp}^7} \right), \quad (13)$$

where $\langle C_L \rangle$ is the averaged lift coefficient calculated by the semi-analytical approach proposed by Hood *et al.*²⁷ Figure 7 shows the calculated migration index for different α_{pi} together with its corresponding inter-particle distance. From Figure 7, the inter-particle distance falls quickly with the increase of α_{pi} . Simultaneously, the migration index rises sharply which denotes that the particle–particle interaction becomes dominant and thus limits the inertial migration effect. The migration results in our experiment and numerical simulation under different α_{pi} are summarized in Table I. As shown in Table I, in the case of low $\alpha_{pi} = 3.0$ and 5.0 v/v%, the inter-particle distance is sufficiently large, and the particle–particle interaction forces are lower than

FIG. 7. Relationship among migration index, inter-particle distance, and initial particle concentration.

the inertial lift force by two orders of magnitude. Thus, the particles are successfully focused to the four stable equilibrium positions near the centre of each face of the square microchannel. As α_{pi} increases to $\alpha_{pi} = 10.0$ v/v%, the inter-particle distance decreases dramatically, and the particle-particle interaction forces are increased to a level of only one order of magnitude lower than the inertial lift force. In this case, only partial particle migration occurs without clear stable equilibrium position. Moreover, further increase of α_{pi} causes sustained decrease of the inter-particle distance, and the particle-particle interaction forces become more and more dominant than the inertial lift force. Consequently, particle migration does not occur when α_{pi} exceeds $\alpha_{pi} = 15.0$ v/v%.

V. CONCLUSIONS

The present study reveals an applicability of the multi-electrodes sensing method to determine the particle inertial migration behaviour in the high particle concentration suspension. The online measurements of particle concentration near the wall and at the corner of the square microchannel are achieved for the experimental estimation of the particle cross-sectional migration ratio. In order to evaluate the experiment results, numerical simulations of the particle migration behaviours are also presented by an Eulerian–Lagrangian model. A dimensionless parameter named migration index is defined to provide a quantitative indicator for the migration phenomena in the high particle concentration suspension. The results can be summarized as follows:

- (1) The particle inertial migration in the case of initial particle concentration up to $\alpha_{pi} = 5.0$ v/v% is obvious. Same as the previous experimental works in the literatures, four stable equilibrium

TABLE I. Migration results under different initial particle concentrations.

Initial particle concentration α_{pi} (v/v%)	Inter-particle distance D_{pp} (μm)	Migration index M [-]	Migration results
3	35.0	0.0077	Migration
5	29.5	0.0253	
10	23.4	0.126	Partial migration
15	20.4	0.31	
20	18.6	0.59	No migration
25	17.2	0.95	
30	16.2	1.36	
35	15.4	1.77	
40	14.8	2.16	

positions appear near the centre of each face of the square channel except a slight shift towards the wall is observed.

- (2) Only partial particle migration occurs as the initial particle concentration increases to $\alpha_{pi} = 10.0$ v/v%, while the migration phenomena completely disappear in the case of larger initial particle concentration. The reason can be lie in that as increase of the initial particle concentration, instead of behaving as independent individual, particles behave as aggregate group with high interaction and thus weaken the effect of inertial migration.
- (3) Influence of the initial particle concentration on the particle inertial migration can be attributed to the dimensionless migration index which describes the inter-relation between the particle interaction and the inertial lift. Migration index below one order of magnitude less than unit denotes obvious particle inertial migration phenomena, while the larger migration index indicates the considerable particle interaction that prevents particle migration.

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