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3D pulse EPR imaging from sparse-view projections via constrained, total variation minimization

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Abstract

Tumors and tumor portions with low oxygen concentrations (pO_2) have been shown to be resistant to radiation therapy. As such, radiation therapy efficacy may be enhanced if delivered radiation dose is tailored based on the spatial distribution of pO_2 within the tumor. A technique for accurate imaging of tumor oxygenation is critically important to guide radiation treatment that accounts for the effects of local pO_2 . Electron paramagnetic resonance imaging (EPRI) has been considered one of the leading methods for quantitatively imaging pO_2 within tumors *in vivo*. However, current EPRI techniques require relatively long imaging times. Reducing the number of projections can considerably reduce the imaging time. Conventional image reconstruction algorithms, such as filtered back projection (FBP), may produce severe artifacts in images reconstructed from sparse-view projections. This can lower the utility of these reconstructed images. In this work, an optimization based image reconstruction algorithm using constrained, total variation (TV) minimization, subject to data consistency, is developed and evaluated. The algorithm was evaluated using simulated phantom, physical phantom and pre-clinical EPRI data. The TV algorithm is compared with FBP using subjective and objective metrics. The results demonstrate the merits of the proposed reconstruction algorithm.

Keywords

Optimization; Image reconstruction; EPR imaging; Compressed sensing; Total variation minimization

1 Introduction

Electron paramagnetic resonance (EPR) imaging (EPRI) has been used to image the oxygen concentration (pO_2) in tumors, which can be an important consideration for treatment with radiation therapy [1][2][3]. Currently, there are two main types of EPRI: pulse EPRI and continuous wave (CW) EPRI. Due to the short magnetic relaxation times of commonly used imaging agents, CW EPRI is more commonly used [4]. The availability of oxygen sensitive

agents with longer relaxation times makes pulse EPRI oximetry possible and, considering higher accuracy and resolution, desirable [1].

For 3D pulse EPRI, Filtered back projection (FBP) is the most common image reconstruction algorithm used because of its straightforward implementation and fast reconstruction speed compared to iterative algorithms [5]. For accurate image reconstruction, FBP requires a complete set of projection data and, therefore, long acquisition times. This is a major limitation in the practical applicability of FBP to magnetic resonance imaging (MRI) and EPRI. Long acquisition times increase the potential for motion artifacts and limit the ability to resolve rapid changes in pO_2 . Several approaches have been considered to reduce the required image acquisition time. Filtering of EPRI data obtained using short acquisition times can help to improve the signal-to-noise (SNR) of reconstructed images. An alternative option is to use partial K-space sampling, which includes half of K-space and circle-K-space located at the center of K-space etc, to reduce acquisition time [6]. These methods, however, risk losing useful information that may not be reconstructed due to the lack of sampling in some portions of K-space. For 3D pulse EPRI, a radial sampling pattern is often used rather than Cartesian sampling, so another possible option for reducing scanning time is to reduce the sampling views (i.e., the projection number) [5] while ensuring that the object is efficiently sampled by the limited views using uniform acquisition strategies [7]. Iterative reconstruction algorithms represent a viable alternative image reconstruction method particularly suited for the reconstruction of high quality images from such rapidly acquired, sparse-view projection data.

The problem of reconstructing an object from its spatial projections is a classical problem in the field of computed tomography (CT), and so the extensive work in CT image reconstruction can be adapted and applied to similar problems in EPRI.

Iterative optimization based image reconstruction is an established method but has often been considered excessively time-consuming [8] causing some to continue to favor FBP instead [9]. With recent data processing acceleration from various hardware developments such as multi-cores, computer clusters and graphics processing units (GPUs), iterative methods have become a more practical and popular option for image reconstruction.

In 2006, a compressed sensing based image reconstruction method was proposed by Candes et al [10]. They reconstructed Shepp-Logan phantom images accurately from sparse-view samples of its discrete Fourier transform (K-space) by solving the convex optimization problem of minimizing the total variation (TV) norm of the image (the ℓ_1 -norm of the discrete gradient transform of the image) subject to data consistency constraints.

Since 2006, Xiaochuan Pan and his co-workers systematically designed and evaluated total-variation-minimization-based programs and algorithms motivated by Candes' work [11][12][13][14][15][16][17][18][19]. They used an Adaptive Steepest Descent (ASD) technique to minimize the TV norm and a Projection onto the Convex Sets (POCS) technique to achieve the data-consistency constraint. They have applied and evaluated their ASD-POCS algorithm and its variants to fan-beam, cone-beam, offset-detector cone-beam and micro CT image

reconstruction. Their evaluation results showed that the ASD-POCS algorithm may yield high quality image reconstruction from sparse-view projections.

The TV minimization problem (TV algorithm) relies on the concepts of compressed sensing (CS). The ASD-POCS algorithm is one method for attacking this optimization problem. Other CS problems may rely on different sparsity transforms, such as the wavelet transform, which have been pursued and have yielded satisfactory results. [20] Certainly, ASD is not the only algorithm for minimizing the TV norm. Soft-threshold filtering [21] or shrinkage-threshold filtering [22] can alternatively be used for minimization of the TV norm.

Motivated by the successful application of this TV algorithm in the field of CT image reconstruction, we have formulated an optimization problem and designed a corresponding algorithm for similar application to EPRI. We evaluate these methods using simulated phantom, physical phantom and pre-clinical EPRI data.

In section 2, we describe our TV algorithm for EPRI from sparse-view projections. In section 3, we evaluate the algorithm using simulated, physical and pre-clinical data. In section 4, we give a brief conclusion.

2 Method

In this section, we describe the model of the imaging system, design the reconstruction program, implement the reconstruction algorithm, and establish metrics used to assess image quality.

2.1 System model

Generally speaking, analytic algorithms (e.g., FBP) are based on continuous-to-continuous (C2C) models, while iterative algorithms are based on discrete-to-discrete (D2D) models. For a C2C model, the imaged object is a continuous object and the projections are continuous as well. There are corresponding analytic reconstruction formulae comprising the continuous system model, which provide a unique solution to the reconstruction problem. For a D2D model, the imaged object is considered to be a discretized object and the projections are discrete as well. In this case, a linear system of equations can be used to express the imaging problem, where the linear system represents the discrete system model. Depending on the number of equations and number of unknowns, this system of equations may have many possible solutions.

2.1.1 C2C model for 3D pulse EPRI—For 3D pulse EPRI, a demodulated EPR signal acquired with a linear magnetic field gradient applied along a specific direction for spatial encoding is the Fourier transform of the corresponding spatial projection at a specific view-angle [23]. Spatial projections can therefore be obtained by performing the inverse Fourier transform (IFT) for each demodulated EPR signal. Each measured data point in an acquired projection of the object at a given view-angle (ϕ, θ) can be mathematically represented as the area-integral of the imaged object at that point on the projection axis (t) in the plane perpendicular to the projection axis (see Fig. 1). This is the basis for defining the set of 3D spatial projections for the C2C model using the Radon transform of the imaged object [24]:

$$p(t, \phi, \theta) = \iiint_{-\infty}^{+\infty} f(x, y, z) \delta(x \cos \phi \sin \theta + y \sin \phi \sin \theta + z \cos \theta - t) dx dy dz \quad (1)$$

Here, $p(t, \phi, \theta)$ is the spatial projection at the angle (ϕ, θ) , $f(x, y, z)$ is the imaged object and δ is the standard Dirac delta function.

2.1.2 D2D model for 3D pulse EPRI—Iterative image reconstruction algorithms rely on D2D models. For the 2D case, the imaged object is discretized onto a square grid (pixels). For the 3D case, the imaged object is discretized onto a cubic grid (voxels). If the 3D object has A rows, B columns and C layers, every voxel of the object can be expressed as $f_{a,b,c}$ with $a = 1, 2, \dots, A$; $b = 1, 2, \dots, B$; $c = 1, 2, \dots, C$. If the voxels are rearranged into a single vector, f , with $J = A \times B \times C$ entries, (arranged row-by-row, column-by-column, and then layer-by-layer) the relation between the vector index and the original 3D array indices is

$$j = (c - 1)AB + (b - 1)A + a \quad (2)$$

Similarly, the measured projection data can be rearranged into a single vector, g , with I entries, where I is equal to the product of the number of measurements in each projection and the total number of projections or views. Then, a system matrix, M , is constructed, with dimensions $I \times J$. Each individual element of the system matrix, M_{ij} , is the contribution of the voxel f_j to the measured data. This contribution is equivalent to the area of intersection between the voxel f_j and the plane corresponding to the measured data. From the definition of the 3D radon transform, we can model the D2D forward projection problem using:

$$g = Mf \quad (3)$$

The above expression represents a system of linear equations. As such, the image reconstruction problem has been reformulated to be a solution of this system of linear equations. If the coefficient matrix (system matrix) and its augmented matrix have the same rank and if the rank is equal to the number of unknown variables (number of voxels), there is a unique solution for the linear system. However, for image reconstruction applications, the system of linear equations is usually underdetermined and ill-posed (many more unknown variables or object voxels compared to equations or projection measurements). Additionally, the system matrix is often very large. Classical linear algebra methods therefore tend to be insufficient for solving such problems.

In order to determine an approximate solution, one must employ optimization techniques. The algebraic reconstruction technique (ART) minimizes the sum-of-squares of the object-voxel values and the expectation maximization (EM) algorithm minimizes the Kullback-Liebler distance between measured projection data and calculated projection data of the estimated object [11]. Seeking a solution that minimizes the total variation (TV) of the imaged object can potentially further assist in producing solutions that yield high-quality images from sparse-view projections. This constrained optimization problem will be referred to as the TV algorithm. In this work, we investigate and evaluate the TV algorithm as a method for reconstruction of EPRI images from sparse-view projections.

2.2 Reconstruction program

As discussed in the previous section, to solve the underdetermined system of linear equations with many possible solutions, optimization techniques must be used to determine an appropriate solution. Here, we develop a TV-based reconstruction program as follows.

$$\min \|f\|_{TV} \quad s.t. \quad g = Mf \quad (4)$$

This is a constrained optimization problem. The objective function is $\|f\|_{TV}$ and the optimization process is constrained by data consistency, $g = Mf$, which ensures consistency between the projections and the imaged object.

The TV norm of the object is the ℓ_1 -norm of the gradient transform (GT) of the object.

The GT of a 3D object $f_{a,b,c}$ is:

$$GT(f_{a,b,c}) = \sqrt{(f_{a,b,c} - f_{a-1,b,c})^2 + (f_{a,b,c} - f_{a,b-1,c})^2 + (f_{a,b,c} - f_{a,b,c-1})^2} \quad (5)$$

Thus, the TV norm is:

$$\|f\|_{TV} = \|GT(f_{a,b,c})\|_1 = \sum_{a,b,c} GT(f_{a,b,c}) \quad (6)$$

2.3 Reconstruction algorithm

We use the ASD-POCS (adaptive steepest descent-projection on convex sets) algorithm proposed by the Pan group to solve the optimization program above [11] (see Section 2.3.2).

2.3.1 TV norm gradient vector—The ASD method is used to achieve minimization of the TV norm. For an n dimensional function, the corresponding gradient function is a vector function with n dimensional elements. In other words, at a particular point in the n dimensional gradient function, the gradient value at that point is a vector, which points along the direction in which the value of the function increases at the maximum rate, also known as the slope direction. The inverse is true and it can be assumed that the value of the function will decrease at a maximum rate along the opposite direction to the gradient direction [12].

The TV norm is a J dimensional function, and our goal is to determine the direction of steepest descent in order to achieve minimization of the TV norm. The gradient vector consists of the partial derivatives, so the problem now becomes a matter of how to calculate these partial derivatives for every variable.

Noting that variable $f_{a,b,c}$ only appears in the expressions of $GT(f_{a,b,c})$, $GT(f_{a+1,b,c})$, $GT(f_{a,b+1,c})$ and $GT(f_{a,b,c+1})$, we get the expression of partial derivative:

$$\begin{aligned}
v_{a,b,c} = \frac{\partial \|f\|_{TV}}{\partial f_{a,b,c}} \approx & \frac{(f_{a,b,c} - f_{a-1,b,c}) + (f_{a,b,c} - f_{a,b-1,c}) + (f_{a,b,c} - f_{a,b,c-1})}{\sqrt{\varepsilon + (f_{a,b,c} - f_{a-1,b,c})^2 + (f_{a,b,c} - f_{a,b-1,c})^2 + (f_{a,b,c} - f_{a,b,c-1})^2}} \\
& + \frac{-(f_{a+1,b,c} - f_{a,b,c})}{\sqrt{\varepsilon + (f_{a+1,b,c} - f_{a,b,c})^2 + (f_{a+1,b,c} - f_{a+1,b-1,c})^2 + (f_{a+1,b,c} - f_{a+1,b,c-1})^2}} \\
& + \frac{-(f_{a,b+1,c} - f_{a,b,c})}{\sqrt{\varepsilon + (f_{a,b+1,c} - f_{a-1,b+1,c})^2 + (f_{a,b+1,c} - f_{a,b,c})^2 + (f_{a,b+1,c} - f_{a,b+1,c-1})^2}} \\
& + \frac{-(f_{a,b,c+1} - f_{a,b,c})}{\sqrt{\varepsilon + (f_{a,b,c+1} - f_{a-1,b,c+1})^2 + (f_{a,b,c+1} - f_{a,b-1,c+1})^2 + (f_{a,b,c+1} - f_{a,b,c})^2}}
\end{aligned}$$

where, ε is a small positive number (e.g., $\varepsilon = 10^{-8}$) that is used to avoid issues with singular values arising from division by zero. From Eq. (7), it can be seen that one may consider the gradient vector, $v_{a,b,c}$, to be a 3D ‘object’ vector (similar to $f_{a,b,c}$), with $a = 1, 2, \dots, A$; $b = 1, 2, \dots, B$; $c = 1, 2, \dots, C$. In the reconstruction algorithm below, we employ the normalized TV gradient vector $\hat{v}$ which can be obtained by dividing vector v by its vector norm.

2.3.2 ASD-POCS algorithm—To solve the optimization program, an ASD-POCS algorithm is developed. We use ART to implement the constraint $g = Mf$, i.e. ART is used to implement POCS. We use ASD to minimize the TV norm. The steps are as follows.

Step 1: Initialization

Let $f = 0$

Step 2: ART process

$$for i=1: I do: f = f + \frac{M_i}{(M_i, M_i)} (g_i - M_i f)$$

Step 3: TV

$$d_{ART} = |f_{ART-END} - f_{ART-BEGIN}|$$

Loop N_{TV} times :

$$\{\text{Calculate according to Eq. (7)} \quad \hat{v} = \frac{v}{|v|} \quad f = f - w * d_{ART} * \hat{v}\}$$

Step 4: Iteration control

If the stopping criteria are achieved, the reconstruction is stopped.

Otherwise, return to step 2.

In Step 1, the initial image is set to be a zero image for generality, but other choices are also acceptable. The choice of initial image can impact the convergence speed.

In Step 2, a standard ART methodology is used to ensure data consistency for the POCS implementation. The expression (M_i, M_j) denotes the inner product of the vector M_i , or the i th row vector of the matrix M , with itself.

In Step 3, $f_{ART-END}$ is the estimated image following completion of the ART process and $f_{ART-BEGIN}$ is the estimated image prior to the ART process. Therefore, d_{ART} is the image-distance resulting from the ART process. The parameter w is a weighting factor for selecting a proper step-size, $w \times d_{ART}$, to take along the steepest descending direction during each iteration. In this work, this parameter is set to be $w = 0.5$ and the number of iterations for the TV process is set to $N_{TV} = 5$.

In Step 4, some stopping criteria are specified. Potential stopping criteria include image change, data consistency change or iteration number. For instance, if the change in the image following an iteration is found to be smaller than some predetermined threshold, no more iterations are necessary. Alternatively, if the estimated projections obtained by forward projection of the estimated image are found to closely approximate the measured projections to within predefined limits, the solution may be acceptable without further iterations. Also, one can use the iteration number itself as a stopping criterion based on experience and/or reconstruction speed requirements.

2.4 Image quality assessment

In this work, to evaluate image reconstruction quality, an error criterion and a spatial resolution criterion are used.

2.4.1 Error criterion: normalized mean squared error—The normalized mean square error (*NMSE*) is an error metric, which is applied on a voxel-by-voxel basis as shown in Eq. (8).

$$NMSE = \left[\frac{\sum_{m=1}^M (f(m) - r(m))^2}{\sum_{m=1}^M (f(m) - \bar{f})^2} \right]^{\frac{1}{2}} \quad (8)$$

Here, $\bar{f}$ is the average value found for all of the voxels of the ideal object, f , and r is the reconstructed object.

2.4.2 Spatial resolution criterion: edge spread function method—The edge spread function (ESF) is used here to measure the spatial resolution [25]. First, a set of 1D profiles, orthogonal to an edge in the image, are selected from the reconstructed image. These discrete profiles are then fit to a series of Gaussian error functions. A set of FWHM (full width at half maximum) values are obtained according to the parameters of the fitted error functions. The spatial resolution is obtained by averaging these FWHM values to approximately determine the point spread function.

2.4.3 Visual assessment—This work is comprised of three studies: a simulation study, a physical phantom study and a tumor imaging study. For the simulation study, the two

assessment methods above may be used. However, for the non-simulated studies there is no a priori knowledge of the ‘true’ object, so a visual assessment method is used to evaluate the reconstructed images. For each reconstructed image, we evaluate the image quality via qualitative examination of 2D slices of the reconstructed object.

3 Results and discussion

The main advantage of the TV algorithm is its ability to accurately reconstruct an image of the object using sparse-view projections. We are, therefore, interested in investigating the reconstruction quality as a function of view number. We also compare TV and FBP to demonstrate the advantages of TV versus other commonly used EPRI image reconstruction techniques. Previous work has demonstrated that TV outperforms the ART or EM algorithms for similar applications, so we do not repeat this evaluation by comparing TV with ART or EM.

3.1 Inverse-crime study

Inverse-crime studies permit the characterization of reconstruction programs and algorithms in the absence of data inconsistency (aside from computer precision errors) [26][27][28]. In the case of an inverse-crime study, if the input data are sufficiently large, the object can theoretically be reconstructed exactly. The value of an inverse-crime study is that it can ensure that the design and implementation of a reconstruction algorithm is correct. Generally speaking, it is impossible for an algorithm to obtain acceptable results with real-data if it cannot obtain acceptable results in an inverse-crime study.

The simulated phantom used in this study consists of six spheres. Five non-overlapping small spheres are embedded into a larger sphere. Each sphere has a density in the range of [0, 1]. The parameters used to simulate this phantom are shown in Table 1. The simulated phantom contains both high and low contrast objects, making it suitable for both qualitative and quantitative evaluation of image precision and spatial resolution. The object is discretized, consisting of $64 \times 64 \times 64$ voxels, whose center is (33, 33, 33). The physical size of the object is $10 \times 10 \times 10 \text{ cm}^3$, so each voxel is $10/64 \times 10/64 \times 10/64 \text{ cm}^3$. The length of the virtual detector is 10cm and the number of points in a projection is 64, therefore the sampling interval for the projection signal is (10/64) cm. Images are reconstructed from sets of 26, 52, 104, and 208 projections, distributed using an equal-solid-angle sampling pattern providing approximate uniformity [29].

The images reconstructed using TV and FBP are shown in Fig. 2. The error comparison is shown in Fig. 3 and the spatial resolution comparison in Fig. 4. From Fig. 2, it can be seen that the FBP results are contaminated by noise and streak artifacts, whereas the TV algorithm yields visually superior images and more accurate representation of finer structural detail (e.g., object edges). The dominant streak artifacts in the FBP reconstructed images are associated with angular under sampling in the sparse-view data. TV suppresses these artifacts by iteratively reducing the TV norm of the reconstructed object. As the number of projections used for image reconstruction increases, the appearance of both FBP- and TV-reconstructed images improves.

For the case in which 208 projections are used, there are $208 \times 64 = 13312$ measurements, i.e. 13312 equations that form a system of linear equations in which the number of unknown variables (number of voxels) is $64 \times 64 \times 64 = 262144$. In general, when solving a linear system, there is a unique solution only if the number of equations and number of unknowns are the same. However, even for the case of 208 projections, the sparseness ratio (ratio of the number of equations to the number of unknowns) is much less than one $\left(\frac{13312}{262144} \approx 5\%\right)$, resulting in a highly underdetermined system. From Fig. 2 it can be seen that TV can reconstruct relatively high quality images from only 208 projections, demonstrating that the TV algorithm performs sparse reconstruction well.

Figure 3 shows the reconstruction error as a function of the number of views for the two reconstruction algorithms. It can be seen that the error is always lower using TV when compared to FBP and that the error decreases with increasing number of views for both algorithms. It should be noted that FBP is more sensitive to the number of views than TV, as demonstrated by the fact that the error change when going from 52 views to 208 views for TV is smaller than that for FBP.

The dependence of spatial resolution (FWHM in mm) on the number of views is plotted for both TV and FBP in Fig. 4. The spatial resolution of images reconstructed using the TV algorithm improves with increasing number of views and, beyond approximately 60 views, surpasses that of images reconstructed using FBP.

The TV algorithm thus yields reasonable quality reconstructed images with relatively sparse projection views.

3.2 Physical data evaluation

It is, of course, more challenging to accurately reconstruct an image from experimental projection data, mainly due to noise obscuring the signal.

A bottle phantom filled with a uniform solution containing a 1mM concentration of triarylmethyl, the EPRI spin probe imaging agent [30], was imaged (Fig. 5). We consider our image or reconstructed object to be a discrete object consisting of $64 \times 64 \times 64$ voxels, whose center is (33, 33, 33). The physical size of the object is $3\sqrt{2} \times 3\sqrt{2} \times 3\sqrt{2} \text{ cm}^3$, so the size of each individual voxel is $3\sqrt{2}/64 \times 3\sqrt{2}/64 \times 3\sqrt{2}/64 \text{ cm}^3$. The length of the virtual detector is $3\sqrt{2} \text{ cm}$ and the number of points in a projection is 64, resulting in a sampling interval for the projection signal is $\left(3\sqrt{2}/64\right) \text{ cm}$. Images are reconstructed from 26, 52, 104 and 208 projections, distributed using an equal-solid-angle sampling pattern providing approximate uniformity [29].

The reconstructed images using TV and FBP are shown in Fig. 6. It can be seen that the images reconstructed using TV are of higher quality than those reconstructed using FBP, regardless of the number of projections used. The FBP images all have obvious streak artifacts and are relatively noisy. However, the TV method recovers the high frequency image content (e.g., the sharp bottle edge) while simultaneously suppressing unwanted noise

and streak artifacts. From Fig. 6 it can be seen that noise and artifacts decrease with increasing number of views when using the FBP algorithm. It should be noted that there is a fog artifact in the TV images reconstructed from 26 projections. The fog is surrounding the bottle, especially at the axial bottle direction. As the number of views increases, the fog artifact becomes less appreciable. When 208 views are used for the TV reconstruction, a high-quality image is produced. This demonstrates that the TV method can provide high-quality images reconstructed from sparse-view projections (sparseness ratio of 5%).

3.3 Pre-clinical data evaluation

The utility of TV reconstruction was evaluated using *in vivo* animal experimental data. The experimental conditions are the same as those detailed in Section 3.2, except that the imaged object is a tumor bearing leg of a mouse and the sampling view-numbers used were 104, 130, 160, 208 and 828. For this evaluation, we investigate how many projections are required for the TV method to obtain image quality similar to that using FBP with a larger number of projections (828 projections).

The reconstruction results using either FBP or TV with different view-numbers are shown in Fig. 7. Again, it can be seen that the main disadvantage of FBP are the streak artifacts that result from sparsely sampled projections and that TV suppresses these artifacts by iteratively reducing the total variation of the reconstructed image. Also, the fog artifact in the FBP images, which surrounds the tumor, is more pronounced than that in the TV images. As expected, noise and artifacts are reduced in FBP images and spatial resolution is improved in TV images with increasing number of projection views (Fig. 7).

For a quantitative image quality evaluation, the NMSE is plotted in Fig. 8 and 9, with the FBP and TV images reconstructed using 828 projections being the reference images, respectively. These plots show that the precision of TV images always exceeds that of the FBP images for all of the sparse-view cases tested here. This is due to the fact that FBP is not suited for image reconstruction from sparsely sampled projections and tends to produce images degraded by noise and undersampling-induced streak artifacts. On the other hand, TV suppresses these unwanted effects during the iterative optimization.

In order to reconstruct images in a $64 \times 64 \times 64$ matrix using FBP, we use 828 projections so as to avoid the prominent undersampling artifacts. From Fig. 7 and 8, it can be seen that the TV result using 208 projections approaches similar image quality compared to the FBP result using 828 projections. It is therefore not unreasonable to conclude that TV can achieve accurate sparse reconstruction when applied in practical situations. If we consider 828 to be

the ‘full’ sampling number, the practical relative sparseness ratio is $\frac{208}{828} \approx \frac{1}{4}$.

Sparse-view image reconstruction offers several important practical advantages. It can reduce scanning time for *in vivo* imaging. It can also improve the SNR for images reconstructed from the same quality EPRI projection data. Reducing scanning time is very important for *in vivo* imaging. Shorter scanning times reduce patient-motion artifacts and may allow imaging of clinically relevant high-frequency changes in physiology.

3.4 Why can TV achieve superior sparse reconstruction?

3.4.1 Iterative methods are well suited to sparse data.—FBP is a standard analytic reconstruction algorithm. It requires a sufficient number of projections to achieve accurate image reconstruction. The FBP algorithm can be thought of as a single-pass algorithm, in that there is no feedback process for adjusting the distance between the measured projections and the approximated projections obtained by forward projecting the reconstructed image. When the projections are sampled sparsely, FBP will produce less accurate images due to the presence of serious streak artifacts. It can be stated that, for sparsely sampled projection data, FBP does not make adequate use of the available information. More subtle information buried within the data may be ignored by the simple but coarse single-pass reconstruction process. Iterative reconstruction algorithms, such as TV, are more delicate and utilize a feedback process to iteratively update the image to better match the acquired data on a more subtle level, and thus tend to squeeze all available information from the projection data.

The iterative methods model the reconstruction problem as a linear inverse problem, which requires solving a system of linear equations. Projections of the same object from various views will inevitably tend to have certain differences but they will also have strong correlations and interdependencies. By formulating the problem using a large system matrix representing the many linear equations to be simultaneously solved, the iterative methods are more robust with respect to how they make use of these similarities and important consistencies between projection views. Iterative methods provide potentially more accurate image reconstruction by modelling all of the projections together and by iteratively adjusting the image so as to achieve optimal consistency between the measured data and the guessed data.

3.4.2 TV minimization using a priori information—When solving the system of linear equations, the solution space can be narrowed if prior information is used. Prior information can include object-support information, object-value-range information or other unique information. For the TV reconstruction algorithm presented here, of all potential prior information, the greatest weight is given to the constraint that the TV norm (L_1 norm of the gradient image) should be minimal. This assumption is based on the notion that, for most applications, the majority of imaged objects will be relatively smooth, resulting in sparse gradient images, the TV norm of which will be small. When the prior information that the TV norm should be minimized is incorporated, the solution space becomes considerably more narrow, aiding in the determination of an accurate solution.

3.4.3 TV embodies the idea of compressed sensing—The theory of compressed sensing has recently garnered a great deal of attention. There are, however, some misunderstandings regarding its application in the field of image reconstruction. The biggest of them is that CS can somehow overcome the limitations dictated by the Nyquist sampling theorem. In fact, the Nyquist sampling theorem describes how one must sample a continuous signal when dealing with a continuous-to-discrete (C2D) problem. However, CS theory is only applicable to discrete data, i.e. CS must be applied to a D2D model. A simulation experiment has been performed to illustrate that CS cannot accurately recover a continuous image [9].

Accurate TV reconstruction of sparse projection data presumably is related to the incorporation of CS, wherein a signal can be sparsely represented (i.e. represented by small amounts of data) in some linear transform domain [31][32][33]. Accurate reconstruction of a sparse-view projection set can then be achieved by adding the sparsity constraint to the iteration process.

3.5 On the applicability of TV and TpV minimization

The first issue that should be discussed is whether a piece-wise constant function is a necessary condition for the TV algorithm. In some early papers on TV reconstruction, the explanation for why TV is able to achieve sparse reconstruction was based on the fact that a majority of medical and industrial images are piece-wise constant which can lead to sparse gradient images [11]. However, upon further investigation, it was found that piece-wise constancy is *not* a necessary condition [12]. This is because the TV algorithm simply seeks a solution with minimal TV that falls within the solution space dictated by the system of linear equations. So, even if an image is not piece-wise constant, there still must exist a solution with minimal TV relative to other solutions. Furthermore, in the real world there essentially are no truly piece-wise constant objects. So, the idea that piece-wise constancy is a necessary condition for TV reconstruction should be abandoned.

TV is the ℓ_1 norm of the gradient transform of an image. The reason why ℓ_1 norm is used to define TV is that ℓ_1 norm minimization is a classical convex optimization problem whose corresponding algorithm is considered to be relatively simple. In fact, the ℓ_0 norm should be used to truly enforce the sparsity of the gradient image, considering the fact that sparsity means a majority of the pixels of the gradient image are zero or near zero and that the ℓ_0 norm counts the number of non-zero pixels. However, use of the ℓ_0 norm results in a non-convex optimization problem, which is not as straightforward to solve as a convex optimization problem, hence the prevalent use of the ℓ_1 norm. A variety of TV algorithms can be formulated based on different definitions of 'total'. For example, instead of the ℓ_1 norm, one could make use of the ℓ_0 norm, $\ell_{1/4}$ norm, $\ell_{1/3}$ norm, $\ell_{1/2}$ norm, $\ell_{2/3}$ norm and ℓ_1 norm, etc. These alternatives are referred to as TpV minimization. There have already been some preliminary explorations into the potential advantages of using these different TpV minimization algorithms [34]. Future studies are required to investigate the potential of using TpV minimization in EPRI image reconstruction.

4 Conclusions

In this work, a TV-minimization-based EPRI image reconstruction program and its corresponding ASD-POCS algorithm were described. A mathematical phantom, a physical phantom and a mouse tumor were used to evaluate the designed program and algorithm. All of the experimental results had shown that TV minimization can provide accurate image reconstruction using sparse-view projections. Additional studies are necessary to evaluate the TV image reconstruction algorithm on more complicated physical phantoms and to establish better ways to tune algorithm parameters for achieving optimal image reconstruction.

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Fig.1.
The spatial projection in 3D FBP algorithm (Inverse Radon transform)

Fig 2.

The reconstruction results (the central horizontal slices) of FBP and TV for the simulated model using different number of views. The display window is $[0, 1]$. The upper numbers are numbers of projection views used. The first row images are from FBP and the second ones are from TV.

Fig. 3. NMSE as a function of the number of views for comparing the precision of FBP and TV in reconstructing the simulated model.

Fig. 4. Spatial resolution (in mm) as a function of the number of views for comparing FBP and TV in reconstructing the simulated model.

Fig.5.
The scanned physical bottle

Fig. 6.

Images of the bottle phantom (horizontal slices) reconstructed using TV and FBP methods from 26, 52, 104 and 208 projections. The display window is $[0, 0.007]$. The meaning of the upper numbers is the same with that of Fig. 2.

Fig.7.

The reconstructed images of a mouse-leg-tumor (horizontal slices) using the TV and FBP methods from 104, 130, 160, 208 and 828 projections respectively. The display window is $[-0.006, 0.00015]$. The meaning of the upper numbers is the same with that of Fig. 2.

Fig. 8. NMSE as a function of the number of views for FBP and TV with the FBP image reconstructed from 828 projections considered to be the reference image.

Fig. 9. NMSE as a function of the number of views for FBP and TV with the TV image reconstructed from 828 projections considered to be the reference image.

Table 1

The parameters of the simulated model

	Sphere 1	Sphere 2	Sphere 3	Sphere 4	Sphere 5	Sphere 6
Density	0.5	0.6	0.7	0.8	0.9	1
Coordinate of center of Sphere	[0,0,0]	[-2, 2,0]	[2,2,0]	[2,-2,0]	[-2,-2,0]	[0, 0,0]
Radius	4cm	1cm	1cm	1cm	1cm	1cm