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Simple Expressions for the Design of Linear Tapers in Overmoded Corrugated Waveguides

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Abstract

Simple analytical formulae are presented for the design of linear tapers with very low mode conversion loss in overmoded corrugated waveguides. For tapers from waveguide radius a_2 to a_1 , with $a_1 < a_2$, the optimal length of the taper is $3.198a_1a_2/\lambda$. Here, λ is the wavelength of radiation.

The fractional loss of the HE_{11} mode in an optimized taper is $0.0293(a_2 - a_1)^4/a_1^2a_2^2$. These formulae are accurate when $a_2 \lesssim 2a_1$. Slightly more complex formulae, accurate for $a_2 \sim 4a_1$, are also presented in this paper. The loss in an overmoded corrugated linear taper is less than 1 % when $a_2 \sim 2.12a_1$ and less than 0.1 % when $a_2 \sim 1.53a_1$. The present analytic results have been benchmarked against a rigorous mode matching code and have been found to be very accurate. The results for linear tapers are compared with the analogous expressions for parabolic tapers. Parabolic tapers may provide lower loss, but linear tapers with moderate values of a_2/a_1 may be attractive because of their simplicity of fabrication.

Keywords

Corrugated waveguide; Taper; mm wave; THz; Gyrotron

1 Introduction

The applications of high power millimeter wave and terahertz radiation, such as dynamic nuclear polarization nuclear magnetic resonance spectroscopy (DNP NMR) and electron cyclotron heating and electron cyclotron current drive (ECH and ECCD) in fusion plasmas, have pushed up to frequencies of hundreds of gigahertz. Transmission of high-power millimeter wave and terahertz radiation has proven challenging due to the lossy nature of fundamental mode transmission lines at these high frequencies. Overmoded corrugated metallic waveguides have been demonstrated as a viable solution for high frequency applications [1–10]. The applications of circular overmoded corrugated waveguide often require a transition between sections of different radii. Various methods exist for the design of low loss nonlinear tapers [11–14]. In particular, analytical formulae exist for designing very low loss parabolic tapers [15]. Similar work has also been performed for the design of nonlinear, corrugated horns [16, 17]. However, as applications push to higher frequencies,

and waveguide dimensions correspondingly shrink, the simplicity of fabrication afforded by linear tapers becomes attractive. In the past, mode matching techniques using modes of a conical waveguide have been used to calculate mode conversion in smooth-walled linear tapers [18]. In this paper, similar techniques are employed to arrive at a particularly simple solution for corrugated linear tapers.

In Section 2, formulae are derived to approximate the loss in linear tapers for circular overmoded corrugated waveguides. The primary source of loss under consideration is the conversion of the fundamental HE_{11} mode to the HE_{12} mode. In practical application, conversion to modes of higher order than HE_{12} is negligible. This is demonstrated with numerical examples later in the paper. At the end of Section 2, expressions are collected for determining optimum taper length, as well as calculating the expected loss due to mode conversion in an optimized taper.

In Section 3, numerical examples are presented. Additionally, the analytical expressions are benchmarked against a rigorous mode matching code. The mode matching code accounts for conversion between the first 50 HE_{1m} symmetric modes of the waveguide. The mode matching code is also used to identify the parameter space where the approximations made in Section 2 are valid. Section 4 presents a discussion and conclusions.

2 Analytical Expressions for Corrugated Linear Taper Design

2.1 Derivation of Azimuthally Symmetric Modes of Conical Overmoded Corrugated Waveguide

In this section, the lens transformation from [19] is used to derive linear modes in a conical overmoded corrugated waveguide. The dimensions of the taper are defined in Fig. 1. The derived modes must satisfy the Helmholtz equation, Eq. 1. The amplitudes of modes are assumed to vary on scales much longer than the guide wavelengths. This assumption allows the use of the paraxial approximation, Eq. 2.

$$\frac{\partial^2 E^c}{\partial z^2} + \frac{\partial^2 E^c}{\partial r^2} + \frac{1}{r} \frac{\partial E^c}{\partial r} + k^2 E^c = 0 \quad (1)$$

$$E^c(r, z) = A^c(r, z) e^{-ikz}, |A^c k| \gg \left| \frac{\partial A^c}{\partial z} \right| \quad (2)$$

The variables r and z are cylindrical coordinates, and k is the wavenumber, $2\pi/\lambda$. Superscript c refers to these modes being evaluated in a conical, as opposed to straight cylindrical, waveguide. Substituting Eq. 2 into Eq. 1 leaves a differential equation for the spatially varying amplitude of the electric fields in the guide. This differential equation has been rewritten in dimensionless form, Eq. 3, using the normalized coordinates, $r_n = r/a_1$ and $z_n = z/(ka_1^2)$. The approximation in Eq. 2 allowed the elimination of the $\partial^2 A^c / \partial z^2$ term in Eq. 3.

$$-2i \frac{\partial A^c}{\partial z_n} + \frac{\partial^2 A^c}{\partial r_n^2} + \frac{1}{r_n} \frac{\partial A^c}{\partial r_n} = 0 \quad (3)$$

The lens transformation was used to rewrite the equation in a solvable form. Equation 3 is transformed, using the transformations of Eq. 4, into Eq. 5. A detailed derivation is presented in [19]. The function $\tilde{A}^c$ is given by Eq. 6. In the transformations, F_n is the normalized focal length of the lens that is used to model the conical waveguide. In a waveguide with taper angle α , $F_n = 1/[ka_1 \tan(\alpha)]$. The corresponding real focal length is $F = a_1/\tan(\alpha)$.

$$\tau = \frac{z_n F_n}{z_n + F_n} \rightarrow z_n = -\frac{F_n \tau}{\tau - F_n} \quad (4a)$$

$$\rho = \frac{r_n F_n}{z_n + F_n} \quad (4b)$$

$$-2i \frac{\partial \tilde{A}^c}{\partial \tau} + \frac{\partial^2 \tilde{A}^c}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \tilde{A}^c}{\partial \rho} = 0 \quad (5)$$

$$\tilde{A}^c(\rho, \tau) = \frac{z_n(\tau)}{\tau} A^c\left(\frac{z_n(\tau)}{\tau} \rho, z_n(\tau)\right) \exp\left(-i \left(1 - \frac{z_n(\tau)}{\tau}\right) \frac{\rho^2}{2\tau}\right) \quad (6)$$

The solution to the differential equation is Eq. 7. The solution in terms of real electric field amplitudes, A^c , is given in Eq. 8.

$$\tilde{A}_m^c(\rho, \tau) = \frac{1}{\sqrt{\pi} |J_1(\nu_{0m})|} J_0(\nu_{0m} \rho) \exp\left(\frac{i}{2} \nu_{0m}^2 \tau\right) \quad (7)$$

$$A_m^c(r_n, z_n) = \frac{F_n}{z_n + F_n} \frac{1}{\sqrt{\pi} |J_1(\nu_{0m})|} J_0\left(\nu_{0m} \frac{F_n r_n}{z_n + F_n}\right) \exp\left(\frac{i}{2} \nu_{0m}^2 \frac{z_n F_n}{z_n + F_n}\right) \exp\left(\frac{-i}{2} \frac{r_n^2}{z_n + F_n}\right) \quad (8)$$

Here, A_m^c is the amplitude of the HE_{1m} mode. J_0 and J_1 are zeroth and first-order Bessel functions of the first kind. The constant ν_{0m} is the m th zero of J_0 . In real coordinates, the HE_{1m} modes of conical overmoded corrugated waveguide are given by Eq. 9.

$$A_m^c(r, z) = \frac{a_1}{a_1 + z \tan(\alpha)} \frac{1}{\sqrt{\pi} |J_1(\nu_{0m})| a_1} J_0\left(\nu_{0m} \frac{r}{a_1 + z \tan(\alpha)}\right) \exp\left(\frac{i}{2} \nu_{0m}^2 \frac{1}{ka_1} \frac{z}{a_1 + z \tan(\alpha)}\right) \exp\left(-\frac{i}{2} \frac{kr^2 \tan(\alpha)}{a_1 + z \tan(\alpha)}\right) \quad (9)$$

2.2 Mode Matching

Incident waves at $z = 0$ are composed of a pure HE_{11} mode, with electric field amplitude given in Eq. 10. The amplitude of the incident wave is normalized such that the incident power is equal to 1. In the taper, as well as the outgoing waveguide, the first two modes, HE_{11} and HE_{12} , are accounted for in the total amplitude of the fields, Eqs. 11 and 12. In an optimized taper, conversion to higher order modes (HE_{1m} , $m > 2$) is assumed to be of negligible amplitude. The system is illustrated in Fig. 1. The geometry is divided into three regions, in which different propagating modes are evaluated. A_{inc} is the electric field amplitude of the incoming HE_{11} mode. C_1 and C_2 are the mode amplitudes of the HE_{11} and HE_{12} modes of the conical waveguide. A_{tr} is the electric field amplitude of the transmitted wave in the guide of radius a_2 . T_1 and T_2 are the mode amplitudes of the HE_{11} and HE_{12} modes of straight waveguide transmitted through the taper.

$$A_{inc}(r) = \frac{1}{\sqrt{\pi} |J_1(\nu_{01})| a_1} J_0\left(\nu_{01} \frac{r}{a_1}\right) \quad (10)$$

$$A^c(r, z) = \sum_{m=1}^2 C_m \frac{a_1}{a_1 + z \tan(\alpha)} \frac{1}{\sqrt{\pi} |J_1(\nu_{0m})| a_1} J_0\left(\nu_{0m} \frac{r}{a_1 + z \tan(\alpha)}\right) \cdot \exp\left(\frac{i}{2k} \nu_{0m}^2 \frac{z}{a_1 (a_1 + z \tan(\alpha))}\right) \exp\left(\frac{-i}{2} \frac{kr^2 \tan(\alpha)}{a_1 + z \tan(\alpha)}\right) \quad (11)$$

$$A_{tr}(r) = \sum_{m=1}^2 T_m \frac{1}{\sqrt{\pi} |J_1(\nu_{0m})| a_2} J_0\left(\nu_{0m} \frac{r}{a_2}\right) \quad (12)$$

Standard mode matching techniques, found in [20, 21], are employed to determine the amplitude of individual modes in each section of waveguide. Amplitudes of the modes in the conical section are given by Eqs. 13 and 14.

$$\begin{aligned} C_1 &= \frac{2}{|J_1(\nu_{01})|^2 a_1^2} \int_0^{a_1} J_0^2\left(\nu_{01} \frac{r}{a_1}\right) \exp\left(\frac{i}{2a_1} kr^2 \tan(\alpha)\right) r \, dr \\ &\approx 1 + i \frac{k \tan(\alpha)}{a_1^3 |J_1(\nu_{01})|^2} \int_0^{a_1} J_0^2\left(\nu_{01} \frac{r}{a_1}\right) r^3 \, dr \\ &= 1 + i \frac{k \tan(\alpha)}{2} \frac{\nu_{01}^2 - 2}{3\nu_{01}^2} a_1 \end{aligned} \quad (13)$$

$$\begin{aligned} C_2 &= \frac{2}{|J_1(\nu_{01}) J_1(\nu_{02})| a_1^2} \int_0^{a_1} J_0\left(\nu_{01} \frac{r}{a_1}\right) J_0\left(\nu_{02} \frac{r}{a_1}\right) \exp\left(\frac{i}{2a_1} kr^2 \tan(\alpha)\right) r \, dr \\ &\approx i \frac{k \tan(\alpha)}{2a_1} \frac{2}{|J_1(\nu_{01}) J_1(\nu_{02})| a_1^2} \int_0^{a_1} J_0\left(\nu_{01} \frac{r}{a_1}\right) J_0\left(\nu_{02} \frac{r}{a_1}\right) r^3 \, dr \\ &= i \frac{k \tan(\alpha)}{2} \frac{(-8)\nu_{01}\nu_{02}}{(\nu_{01}^2 - \nu_{02}^2)^2} a_1 \end{aligned} \quad (14)$$

The transmitted amplitude of the HE_{12} mode, T_2 , is given in Eq. 15. The loss in the taper is very nearly equal to the amount of power converted to the HE_{12} mode, $|T_2|^2$, Eq. 16. Equations 15 and 16 are written in terms of $L = (a_2 - a_1)/\tan(\alpha)$, the length of the taper.

$$T_2 = \sum_{m=1}^2 C_m \frac{1}{a_2^2} \frac{2}{|J_1(\nu_{0m}) J_1(\nu_{02})|} \int_0^{a_2} J_0\left(\nu_{0m} \frac{r}{a_2}\right) J_0\left(\nu_{02} \frac{r}{a_2}\right) \cdot \exp\left(\frac{i}{2k} \nu_{0m}^2 \frac{L}{a_1 a_2}\right) \exp\left(-i \frac{k r^2 (a_2 - a_1)}{2 a_2 L}\right) r dr$$

$$\approx \frac{8i \nu_{01} \nu_{02}}{(\nu_{01}^2 - \nu_{02}^2)^2} \frac{k(a_2 - a_1)}{2L} \left[a_2 \exp\left(i \frac{\nu_{01}^2}{2k} \frac{L}{a_1 a_2}\right) - a_1 \exp\left(i \frac{\nu_{02}^2}{2k} \frac{L}{a_1 a_2}\right) \right] \quad (15)$$

$$|T_2|^2 = \left(\frac{k a_1 (a_2 - a_1)}{2L} \right)^2 \left(\frac{8 \nu_{01} \nu_{02}}{(\nu_{01}^2 - \nu_{02}^2)^2} \right)^2 \left| 1 - \frac{a_2}{a_1} \exp\left(i \frac{\nu_{01}^2 - \nu_{02}^2}{2k} \frac{L}{a_1 a_2}\right) \right|^2 \quad (16)$$

It should be noted that Eq. 16 has a similar form to Eq. 153 in [22] for mode conversion in a conical horn. Rather than use modes of a conical waveguide, Dragone integrated the amplitudes of the modes of a straight waveguide over a slowly changing radius to produce an expression for mode conversion. However, the a_2/a_1 factor in front of the exponential in Eq. 16 does not appear in Dragone's expression and will result in disagreement for tapers with larger ratios of output and input radii.

2.3 Optimization

To find the optimal taper length, the derivative of Eq. 16 is set equal to zero. For compactness, the term $a \equiv a_2 - a_1$ is defined. The oscillatory portion of Eq. 16 is expanded in Eq. 17. The derivative of loss with respect to taper length, L , is given in Eq. 18.

$$|T_2|^2 = \left(\frac{a_1 \Delta a}{\lambda L} \right)^2 \left(\frac{8 \pi \nu_{01} \nu_{02}}{(\nu_{01}^2 - \nu_{02}^2)^2} \right)^2 \left(1 - \frac{2a_2}{a_1} \cos\left(\frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda L}{a_1 a_2}\right) + \frac{a_2^2}{a_1^2} \right) \quad (17)$$

$$\frac{\partial |T_2|^2}{\partial L} = 2 \left(\frac{a_1 \Delta a}{\lambda} \frac{8 \pi \nu_{01} \nu_{02}}{(\nu_{01}^2 - \nu_{02}^2)^2} \right)^2 \left[\frac{-1}{L^3} \left(1 - \frac{2a_2}{a_1} \cos\left(\frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda L}{a_1 a_2}\right) + \frac{a_2^2}{a_1^2} \right) + \frac{2a_2}{a_1} \frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda}{a_1 a_2} \sin\left(\frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda L}{a_1 a_2}\right) \right] = 0 \quad (18)$$

Mode conversion loss in the taper varies much more rapidly due to beating between the HE_{11} and HE_{12} modes than the slow L^{-2} dependence. Because of this, the argument of the oscillatory sin and cos terms is nearly equal to $2m\pi$. Choosing $n = 1$ provides us with the shortest optimized taper length, labeled L_1 . The sin and cos expansions about this point are Eq. 19. Substituting these expansions into Eq. 18 yields an algebraically solvable expression for the optimized taper length, Eq. 20.

$$\cos\left(\frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda}{a_1 a_2} L_1\right) \approx 1$$

$$\sin\left(\frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda}{a_1 a_2} L_1\right) \approx \frac{\nu_{02}^2 - \nu_{01}^2}{4\pi} \frac{\lambda}{a_1 a_2} L_1 - 2\pi \quad (19)$$

$$\left. \frac{\partial |T_2|^2}{\partial L} \right|_{L_1} = 8 \left(\frac{\Delta a}{a_2} \frac{\nu_{01} \nu_{02}}{(\nu_{02}^2 - \nu_{01}^2)} \right)^2 \left[- \left(\frac{4\pi a_1 a_2}{(\nu_{02}^2 - \nu_{01}^2) \lambda} \right)^2 \frac{\Delta a^2}{a_1^2 L_1^3} + \frac{a_2}{a_1 L_1} \left(1 - \frac{8\pi^2 a_1 a_2}{(\nu_{02}^2 - \nu_{01}^2) \lambda L_1} \right) \right] = 0 \quad (20)$$

2.4 Results

The shortest taper length at which there is a minimum in HE₁₂ mode content is given by Eq. 21. The optimal length scales linearly with frequency. The predicted loss at this optimal length is given by Eq. 22. This predicted minimum loss depends only upon the ratio of the output and input radii of the taper. A plot of HE₁₁ loss versus this ratio, a_2/a_1 , is shown in Fig. 2a. The loss is less than 1 % when $a_2/a_1 \geq 2.12$ and less than 0.1 % when $a_2/a_1 \geq 1.53$. Figure 2b shows the nearly linear behavior of the normalized optimal length, $L_1 \lambda / a_1^2$, as a_2/a_1 is varied.

$$L_1 = \frac{4\pi k a_1 a_2}{\nu_{02}^2 - \nu_{01}^2} \left(1 + \frac{\Delta a^2}{4\pi^2 a_1 a_2} \right) \approx 3.198 \frac{a_1 a_2}{\lambda} \left(1 + \frac{\Delta a^2}{4\pi^2 a_1 a_2} \right) \quad (21)$$

$$\begin{aligned} |T_2|^2 &= \left(\frac{1}{\pi} \frac{\Delta a^2}{a_1 a_2} \frac{\nu_{01} \nu_{02}}{\nu_{02}^2 - \nu_{01}^2} \right)^2 \left(1 - \frac{\Delta a^2}{2\pi^2 a_1 a_2} \right) \\ &\approx 0.0293 \frac{\Delta a^4}{(a_1 a_2)^2} \left(1 - \frac{\Delta a^2}{2\pi^2 a_1 a_2} \right) \end{aligned} \quad (22)$$

For small changes in waveguide radius, $\Delta a \lesssim 2a_1$, Eqs. 21 and 22 can be simplified to Eqs. 23 and 24.

$$L_1 \approx 3.198 \frac{a_1 a_2}{\lambda} \quad (23)$$

$$|T_2|^2 \approx 0.0293 \frac{(\Delta a)^4}{(a_1 a_2)^2} \quad (24)$$

3 Numerical Examples and Benchmarking

The present derivation of loss in a linear overmoded corrugated taper relies upon the assumption that there is no significant mode conversion of HE₁₁ to modes of higher order than HE₁₂. This assumption is verified in Fig. 3. Mode content at the taper output was calculated using a rigorous mode matching code for overmoded corrugated waveguide developed in [23]. This mode matching code was developed to calculate scattering matrices of the radially symmetric HE_{1n} modes of corrugated waveguide at junctions between waveguide sections of different radii. The tapers examined in this paper were modeled as a series of many steps. The number of steps in each taper was increased until the results converged. Fifty modes were used in all calculations presented here, to account for all propagating modes, and many evanescent modes in each waveguide. In addition to mode conversion, the mode matching code includes the effects of reflections at each waveguide

junction. More details about the code and its performance for modeling tapers can be found [23].

At taper lengths of interest, at the optimized length and longer, loss is dominated by mode conversion to HE_{12} . For these examples, large ratios of output to input radii were chosen, so that loss is great enough to make an easy comparison. In Fig. 3a, mode content was calculated at the output of a taper between 10 and 20 mm radii operating at 170 GHz. In Fig. 3b, mode content was calculated at the output of a taper between 4 and 10 mm radii operating at 330 GHz. In both cases presented in Fig. 3, the content of the lowest neglected mode, HE_{13} , is less than 0.1 % at the optimized length. The agreement between analytical and numerical results also justifies the neglect of reflections, which are omitted in the derivation of Eq. 16 but included in the mode matching code.

The parameter space in which the assumptions made in Section 2 are valid is investigated in Fig. 4. Our rigorous mode matching code was used to calculate loss in a linear taper as the ratio a_2/a_1 is varied between 1.5 and 5. The resulting loss is plotted versus a normalized taper length, $L\lambda/a_1^2$. In these units, the optimal taper length from Eq. 21, L_1 , is independent of frequency and the physical radii of the taper, depending only on the ratio a_2/a_1 . Equations 21 and 22 are shown to accurately predict the properties of an optimized linear taper when $1 < a_2/a_1 < 4$. The numerically calculated loss shown in Fig. 4 was calculated for a taper with an input radius of $a_1 = 10$ mm operating at 170 GHz. The agreement between Eqs. 21 and 22 and the numerical code is preserved as frequency and input radius are varied.

4 Discussion and Conclusions

Analytical formulae for the design of linear, corrugated tapers of overmoded waveguides have been derived. These formulae have been benchmarked against a rigorous mode matching code in Section 3 and found to be in excellent agreement. The approximations used in the derivation of Eqs. 21 and 22 produce accurate results when $a_2/a_1 < 4$.

The loss in an overmoded corrugated linear taper is less than 1 % when $a_2 < 2.12a_1$ and less than 0.1 % when $a_2 < 1.53a_1$. For more radical changes in waveguide radius, it may be preferable to use a series of two tapers, rather than a single taper. For example, a linear taper with a ratio of $a_2/a_1 = 4$ has 13 % mode conversion loss. Using two tapers, each with $a_2/a_1 = 2$, would result in a combined loss of 1.4 %. The combined system of two tapers would, however, be 2.4 times as long as the single taper.

The taper designs in this paper should be compared with the parameters of the most likely alternative, a parabolic taper designed using the expressions in [15]. The optimized length of a parabolic taper in circular overmoded corrugated waveguide is given by Eq. 25. In the limit $a_2 \rightarrow a_1$, the optimized length of a parabolic taper and linear taper both converge to Eq. 23. At a larger ratio, such as $a_2/a_1 = 3$, the optimal length of the parabolic taper is 13 % greater than that of a linear taper. In an ideal parabolic taper, there is no conversion from HE_{11} to the HE_{12} mode. The main mechanism of loss is conversion to the HE_{13} mode. In Section 3, it is shown that, in a linear taper, very little power is converted into this higher order mode. The same is true in a parabolic taper. Less than 0.5 % (−0.022 dB) mode

conversion loss is expected in a parabolic taper in most practical applications. The linear taper is, however, simpler to fabricate than the parabolic taper and would be useable if the mode conversion loss predicted in Eq. 22 is acceptable.

$$L_{para} = 1.599 \frac{a_2^2 - a_1^2}{\lambda \ln(a_2/a_1)} \left(1 - 0.117 \left(\ln \frac{a_2}{a_1} \right)^2 \right)^{1/2} \quad (25)$$

The formulae presented in this paper are accurate for overmoded waveguide with corrugations of approximately $\lambda/4$ depth. This is necessary for the approximations of the hybrid HE modes of the waveguide to remain valid.

In cases where the waveguide radius is very large, $a_1/\lambda \gg 1$, and the change in radius is very small, $\Delta a/a_1 \ll 1$, the taper length predicted by Eq. 21 may be impractically large. In such a case, it may be worthwhile to consider a taper with a very short length, just a few centimeters. Our simulations show that the loss in a very short length taper is comparable to the loss in the limiting case of a taper of zero length, that is, an abrupt discontinuity between waveguides differing in radius by Δa . At such a radial discontinuity, loss due to mode conversion of the HE_{11} mode to the HE_{1n} modes is given by Eq. 26 from [1]. For very small steps, the loss is well approximated by the special case of mode conversion to only the HE_{12} mode, Eq. 27, which is the first term of Eq. 26 and also has been previously presented in [2]. More terms from Eq. 26 can be kept to achieve greater accuracy for estimating the loss in an abrupt radial discontinuity for moderate values of $\Delta a/a_1$.

$$\frac{P_{HE_{1n}}}{P_{in}} = \left(\frac{2\nu_{0n}\nu_{01}}{\nu_{0n}^2 - \nu_{01}^2} \right)^2 \left(\frac{\Delta a}{a_1} \right)^2 \quad (26)$$

$$\frac{P_{HE_{12}}}{P_{in}} = 1.2 \left(\frac{\Delta a}{a_1} \right)^2 \quad (27)$$

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Fig. 1.

Definition of coordinates and variables used in derivation of HE_{11} loss in linear overmoded corrugated tapers. All inner surfaces are assumed to be corrugated. A_{inc} is the amplitude of the incoming HE_{11} mode. C_1 and C_2 are the amplitudes of the first two modes of the conical waveguide. T_1 and T_2 are the amplitudes of the HE_{11} and HE_{12} modes transmitted through the taper

Fig. 2.

Properties of an optimized linear taper versus the ratio of output to input radii, a_2/a_1 . **a**

Percent loss of the HE_{11} mode. **b** The normalized taper length, $L_1\lambda/a_1^2$, only slightly deviates from a linear relationship with a_2/a_1

Fig. 3.

Mode content of taper output as calculated by a rigorous mode matching code versus loss predicted by Eq. 16. An x marks the parameters of an optimized taper predicted by Eqs. 21 and 22. **a** Taper between 10 and 20 mm radii operating at 170 GHz. **b**) Taper between 4 and 10 mm radii operating at 330 GHz. Both examples assume a pure HE_{11} mode input

Fig. 4.

Accuracy of analytical optimization formulae versus the ratio a_2/a_1 . Each line represents loss in a linear taper as calculated by a rigorous mode matching code. Near each line, an x marks the optimized length and loss predicted by Eqs. 21 and 22. The ratio a_2/a_1 is varied between each line. A normalized length, $L\lambda/a_1^2$, is displayed as the x -axis. In these units, L_1 is solely a function of the ratio a_2/a_1 . The analytical formulae are seen to be accurate at ratios of a_2/a_1

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