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An Extended Bayesian-FBP Algorithm

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Abstract

Recently we developed a Bayesian-FBP (Filtered Backprojection) algorithm for CT image reconstruction. This algorithm is also referred to as the FBP-MAP (FBP Maximum *a Posteriori*) algorithm. This non-iterative Bayesian algorithm has been applied to real-time MRI, in which the k-space is under-sampled. This current paper investigates the possibility to extend this Bayesian-FBP algorithm by introducing more controlling parameters. Thus, our original Bayesian-FBP algorithm became a special case of the extended Bayesian-FBP algorithm. A cardiac patient data set is used in this paper to evaluate the extended Bayesian-FBP algorithm, and the result from a well-establish iterative algorithm with L_1 -norms is used as the gold standard. If the parameters are selected properly, the extended Bayesian-FBP algorithm can outperform the original Bayesian-FBP algorithm.

Index Terms

Filtered backprojection; Analytical reconstruction; MAP objective function; Dynamic imaging; MRI; Real time imaging

I. Introduction

Special real-time MRI data acquisition methods are needed to capture moving organs, such as in cardiovascular MRI and in interventional MRI [1–4]. One popular method is to sample the k-space with few (say, 24) uniformly spaced radial lines over the range of 180° [5–8]. Since the k-space data are severely under-sampled, the conventional Fourier transform method is unable to produce satisfactory images. The state-of-the-art reconstruction methods for under-sampled data are the off-line iterative methods to minimize a Bayesian objective function. Many iterative methods use the piecewise-constant constraint [9–14]. Some of the unmeasured k-space data are measured at other time frames. Data at other time frames can assist the image reconstruction at current time frame. This data assisting image reconstruction method has been intensively studied in our group and other groups [15–21]. In our group, iterative spatio-temporal constrained reconstruction (STCR) methods were developed that uses the L_1 norm for the spatial constraint and the L_1 or L_2 norm for the temporal constraint [20].

One drawback of these off-line iterative reconstruction methods is that they are not suitable for real-time applications, for example, real-time MRI temperature monitoring during ultrasound cancer treatment [22], or real-time monitoring the interventional procedure of atrial fibrillation [23].

Recently, we developed an FBP (filtered backprojection) that can be used to minimize a quadratic Bayesian objective function and to reconstruct MRI images with under-sampled data [24–25]. When using radial projection MRI imaging, the mathematics is similar to that in X-ray CT, and the inverse Fourier transform of a measurement is the line-integral of the object at a certain angle. The Bayesian-FBP algorithm developed in [24] will be further generalized in this paper.

The most distinguished advantage of the Bayesian-FBP algorithm over the iterative algorithm is its fast computation time. The clinical X-ray CT systems are able to produce 40 512×512×32 image volumes per second. Thus the FBP algorithm is fast enough for real-time clinical MRI applications.

This current paper has two aims. First, the previously developed Bayesian-FBP algorithm is made more general by introducing more controlling parameters. The original Bayesian-FBP algorithm will become a special case of the extended Bayesian-FBP algorithm. Conceptually, the extended algorithm has a potential to outperform the original algorithm. Second, in the Bayesian term of the objective function, a filtered version of the reference image is compared with a filtered version of the image to be reconstructed. These two filters do not need to be the same. The Bayesian-FBP algorithm uses a quadratic penalty term, while the state-of-the-art iterative algorithms nowadays use non- L_2 norms, for example, the L_1 norm or the total variation norm. We will use the result of an iterative algorithm that uses the L_1 norm in the penalty term as the gold standard [19]. This gold standard will be compared with the results using the new extended Bayesian-FBP algorithm.

II. Methods

A. The extended algorithm

Let the noisy line-integral measurements be $p(s, \theta)$, where s is position variable and θ is the angular variable. The objective function v considered in [24] can be extended into the following form:

$$v(f) = \|[Rf](s, \theta) - p(s, \theta)\|^2 + \beta \|h_1 * f - h_2 * g\|^2, \quad (1)$$

where Rf means that the Radon transform is applied to the object f , $h_1 * f$ is a filtered version of f by a convolution kernel h_1 whose transfer function is H_1 , $h_2 * g$ is a filtered version of g by a convolution kernel h_2 whose transfer function is H_2 , and g is a reference image. In general, filter H_1 and filter H_2 are two different filters. The purpose of the filters H_1 and H_2 is to extract some desired features from f and g , and those features are in common with f and g . For example, one important feature is the boundaries of the organs. If the reference image g is set to zero, the Bayesian term suppresses the feature $h_1 * f$ in the solution. The objective function (1) is a general expression, and the selection of these two filters is application

dependent. Following the same steps in [24] to minimize the objective function (1), an extended Bayesian-FBP algorithm can be obtained as

$$F = B \left(\frac{|\omega|P}{1+\beta H_1} + \beta \frac{H_2|\omega|P_g}{1+\beta H_1} \right), \quad (2)$$

where B is the backprojection operator, $|\omega|$ is the ramp filter, and P and P_g are the Fourier transform of p and R_g , respectively.

When $\beta = 0$, the Bayesian information is ignored, and (2) reduces to

$$F = B|\omega|P. \quad (2a)$$

which is the conventional FBP algorithm. When $\beta \rightarrow \infty$, the measurements are ignored, and (2) reduces to

$$H_1 F = H_2 B|\omega|P_g = H_2 G, \quad (2b)$$

which is the solution relying only on the Bayesian information.

In this paper, we arbitrarily choose $H_1 = \frac{\beta_1}{\beta} |\omega|^n$ and $H_2 = \frac{1+\beta_1|\omega|^n}{1+\beta_2|\omega|^n} |\omega|^m$ to demonstrate the flexibility of the proposed algorithm. Hence the extended Bayesian-FBP algorithm (2) becomes

$$F = B \left(\frac{|\omega|P}{1+\beta_1|\omega|^n} + \beta_3 \frac{|\omega|^{m+1}P_g}{1+\beta_2|\omega|^n} \right) \quad (3)$$

which is implemented in this paper. When $\beta_1 = \beta_2 = \beta_3 = \beta$ and $m = n = 1$, algorithm (3) reduces to the algorithm developed in [24]. By using $m > 1$ one can extract more boundary information from the reference image g . If m is too large, the high frequency noise will be extracted as the desired features, and this situation should be avoided. The implementation steps for (3) are as follows.

An Extended Bayesian-FBP Algorithm

Step 1: Prepare two sets of projection data: the primary set $p(s, \theta)$ and the secondary set $p_g(s, \theta)$.

Step 2: Apply the modified ramp filter $|\omega|/[1+\beta_1|\omega|^n]$ to $p(s, \theta)$ obtaining $p(\tilde{s}, \theta)$.

Step 3: Apply the modified ramp filter $|\omega|^{m+1}/[1+\beta_2|\omega|^n]$ to $p_g(s, \theta)$ obtaining $p_g(\tilde{s}, \theta)$.

Step 4: Combined the two filtered data sets: $p(\tilde{s}, \theta) + \beta_3 p_g(\tilde{s}, \theta)$.

Step 5: Perform conventional backprojection.

B. Computer simulations

A simple computer generated phantom is also used to test this extended Bayesian-FBP algorithm. The phantom has a static large ellipse, a static small bright disc, and a static small dark disc. The ring in the phantom is dynamic; its size and intensity are time varying. The data size, image size, as well as the data sampling scheme are the same as those in the patient study. Zero-mean Gaussian noise is added to the Fourier domain data so that the resulting k -space signal has a signal-to-noise ratio of 5.8. Figure 9(a) shows the phantom at one time instance.

C. Patient study

Here we use an example of human cardiac perfusion study to illustrate the how the proposed method can be applied to under-sampled MRI [19]. The MRI data were acquired with a Siemens 3T Trio scanner, using phased array of 12 coils, one of which was chosen to demonstrate the proposed method. In other words, to demonstrate the under-sampled MRI situation, only 1/12 of the measured data are used; measurements from the other 11 coils are discarded. The scanner parameters for the radial acquisition were TR = 2.6 msec, TE = 1.1 msec, flip angle = 12°, Gd dose = 0.03 mmol/kg, and slice thickness = 6 mm. Reconstruction pixel size was $1.8 \times 1.8 \text{ mm}^2$. Each image was acquired in a 62 msec readout. This corresponds to an acceleration factor of ~16 as compared to the Nyquist limit. But the image quality was good with little streaking when 96 projections were used with a standard Inverse Fourier Transform (IFT) reconstruction after gridding. So the current acquisition corresponds to an effective acceleration factor of four. The acquisition matrix size for an image frame was 256×24 , and 60 sequential images were obtained at 60 different times. At each time frame, the k -space is sampled with 24 uniformly spaced radial lines over an angular range of 180°; however, the 24-line sampling patterns of the adjacent time frames are offset by 180°/96. The k -space sampling pattern is shown in Fig. 1 as a scaled-down illustration, where there are 16 (instead of 96) possible radial lines, and at each time frame 4 (instead of 24) lines are measured. The time sequence follows the pattern of A-B-C-D-A-B-C-D-... and so on.

If one sums up the measurements from temporally adjacent 4 time frames, the summed k -space will have a 96-line sampling pattern, uniformly distributed over an angular range of 180°. In our image reconstruction method, each time frame requires current 24-line measurement P and associated time-averaged 96-line measurement $\hat{P}$, which uses the measurements from the current 24-line data, two “immediately after” 24-line measurements, and two “immediately before” 24-line measurements. A symbolic expression for $\hat{P}$ is given as:

$$\hat{P}(t) = P(t-2)/2 + P(t-1) + P(t) + P(t+1) + P(t+2)/2. \quad (4)$$

The fact that one image was acquired with 24 views (i.e., 24 radial lines in the k -space) makes the k -space under-sampled.

The radially sampled k -space MRI data are in the Fourier domain. The k -space data are first converted into the spatial domain by the one-dimensional (1D) Fourier transform. The

spatial-domain data have a real part and an imaginary part. Our strategy is to reconstruct a real-part image, Img_R , using the real-part spatial-domain data and to reconstruct an imaginary-part image, Img_I , using the imaginary-part spatial-domain data, separately. The final image is the norm of the complex image $Img_R + j \text{ } Img_I$, that is, $\sqrt{Img_R^2 + Img_I^2}$. The method of reconstructing Img_R and the method of reconstructing Img_I are identical. In the following, we will assume that the projection data are given in the spatial domain and are real, without specifying whether they are the real-part or the imaginary part. Therefore, the image reconstruction method is the same as that for the Radon transform, which is the line-integral of a two-dimensional (2D) object.

In actual implementation of this Bayesian-FBP algorithm, all variables are discrete. In our MRI data acquisition, each k -space radial readout had 256 samples. In other words, the variable s was sampled at 256 points. After zero-padding, the length N of the array size for each line measurement was chosen as 1024, that is, s now had 1024 samples. The frequency variable ω took 1024 discrete values at $2\pi n/N$, for $n = 0, 1, 2, \dots, 1023$.

The extended Bayesian-FBP algorithm uses a quadratic penalty term, while the state-of-the-art iterative algorithms nowadays use non- L_2 norms, for example, the L_1 norm or the total variation norm. We will use the result of an iterative algorithm that uses the L_1 norm in the penalty term as the gold standard [19]. The main difference between our objective function (1) and the objective function used in the iterative algorithm [19] is that the penalty term is quadratic while the penalty terms in the iterative algorithm use the L_1 norm.

This gold standard [19] will be compared with the results using the new extended Bayesian-FBP algorithm. It is interesting to see how the differences in the objective function can affect the selection of the parameters in the extended algorithm. The use of the L_1 iterative reconstruction as a reference image is not so much to consider the L_1 reconstruction as “best” gold standard, but to demonstrate that the extended Bayesian-FBP algorithm is able to mimic quite well what can only be achieved (with L_1) by means of a slow iterative method.

III. RESULTS

A. The effect of changing β_1 without the secondary data

In Figure 2, the results are obtained by only using the primary data, and the secondary data set is not used. The data under-sampling artifacts are always present. However, as β_1 increases the image becomes smoother; the artifacts and the image contrast are gradually reduced.

B. The effect of changing β_2 and the number of filtering m (with fixed $n = 1$)

In this part, the primary data set is not used; only the secondary data are used. The parameter β_2 is changed, and the parameter m is varied. The reconstructed images are shown in Figure 3. The first row of Figure 3 is for $m = 0$, that is, the ramp filter is NOT applied to the projections; the second row is for $m = 1$ (applying the ramp filter once); the third row is for $m = 2$ (applying the ramp filter twice); the forth row is for $m = 3$ (applying the ramp filter

thrice). Increasing m brings up more detailed structures in the image; at the same time, the noise is increased. Increasing β_2 is equivalent to application of a lowpass filter with a narrower bandwidth, which removes high frequency components and reduces the contrast.

C. Comparisons with the iterative reconstruction (with $n=1$)

In this section, the iterative algorithm reconstruction (using 120 iterations) with temporal and spatial constraints, details described in [19], is used as the gold standard to compare with the results using the Bayesian-FBP algorithm. The iterative algorithm reconstruction for the time frame #50 is shown in Figure 4(Left). The reconstruction at the same time frame with $\beta = 0.07$ is shown in Figure 4(Right), and its mean-square-error (MSE) is 0.0703.

Four sets of clinical comparison studies are presented in Figures 6~9 and Tables 1~4, respectively. The tables show the mean-square-errors (MSE) with respect to the iterative reconstruction gold standard. The first set contains the cases of $m = 0$. The second set is for the cases of $m = 1$, and the third set is for $m = 2$, and the forth set is for $m = 3$.

The computer simulation results are shown in Figure 9 and Table V.

IV. Discussion and Conclusions

Among our limited cases in the Results section, the minimum MSE is reached at $m = 2$, $n = 1$, $\beta_1 = 10$, $\beta_2 = 5$, and $\beta_3 = 100$. However, for the computer simulation, the minimum MSE is reached at $m = 1$, $n = 1$, $\beta_1 = 10$, $\beta_2 = 5$, and $\beta_3 = 100$. These limited cases cannot be used to suggest any optimal way to select the parameters.

When $\beta_3 = 0$, the extended algorithm is the conventional FBP algorithm and the modulation transfer function (MTF) is identically equal to 1. When $\beta_3 \neq 0$, there are two inputs to the algorithm (i.e., the “system”). The algorithm becomes a multiple-input single-output “system.” If we ignore the under-sampling issue, the MTF of the “sub-system” with the sinogram input is still 1. For the “reference image” g input, the MTF of the “sub-system” is never be 1 unless g is the same as the true image f that we are looking for. Some features of the reference image are desired. Some features of the reference image are not desired. A filtered version of f should be close to a filtered version of g . The filter for f and the filter for g , in general, are different. Parameters β_1 , β_2 , m , and n are not the only way to define filters; the filters H_1 and H_2 can be defined using other functional forms. Parameter β_3 controls the influence of the Bayesian information for the final solution f . The motivation of this paper is to show the flexibility of a general Bayesian-FBP algorithm. The point is not to tell readers what parameters to use, because they are application dependent.

To see the effect of the parameter n , let us consider the objective function (1) and the special case that $\beta = \beta_1 = \beta_2$. We have $H_1 = |\omega|^n$ and $H_2 = |\omega|^m$. Both H_1 and H_2 are high-pass filters. If f and g are almost the same, we can use $m = n$. If g is more blurry than f , we can use $m > n$. The purpose of g is to supply some assistance to f . If g can assist f for the DC of very low frequency contents, n can be chosen as 0. If g is to assist f for its boundaries, n can be chosen as 1. In most of our examples, we set $n = 1$. Two examples of $n = 2$ are also shown. However, a larger n does not seem to be helpful in our examples.

The paper has no intension to find the optimal parameters. We are unable to conclude a general rule to select the beta values. Parameter selection is a universal issue in all Bayesian algorithms. Proper parameters can give much better results than those using randomly chosen or fixed parameters. Our message to the reader is that the extended Bayesian-FBP algorithm has more flexibility in controlling image quality, and allows the final image to reach a smaller MSE than the original Bayesian-FBP algorithm. This extended Bayesian-FBP algorithm is not restricted to MRI applications. In fact, the significance of introducing the filters H_1 and H_2 makes it possible that the reference image g and the desired image f can be loosely related and can come from difference imaging modalities. The reference image g can even be zero, e.g., for the purpose of high-frequency noise control when a high-pass filter H_1 is used.

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Figure 1.

Illustration of k -space sampling pattern using an example with 16 possible radial lines [24]. At each time frame, 4 lines are sampled. The sampling pattern follows the sequence of A-B-C-D-A-B-C-D- ... and so on.

Figure 2.

Images reconstructed with only the primary data. The effect of increasing β_1 : a larger β_1 causes a smoother image.

Figure 3.

Images reconstructed using the secondary data only with $n=1$. As β_2 increasing the image becomes smoother. The first row is for $m = 0$, the second row is for $m = 1$, the third for $m = 2$, and forth for $m = 3$.

Figure 4.

(Left) Iterative reconstruction at the time frame #50. This image is used as the gold standard.

(Right) Bayesian-FBP reconstruction using $\beta_1 = \beta_2 = \beta_3 = 0.07$ at the time frame #50. Its MSE is 0.0703 with respect to the gold standard.

Figure 5.

Four cases for $m = 0$ and $n = 1$. The corresponding mean-square-errors are displayed in Table I.

Figure 6.

Four cases for $m = 1$ and $n = 1$. The corresponding mean-square-errors are displayed in Table II.

Figure 7.

Four cases for $m = 2$ and $n = 1$. The corresponding mean-square-errors are displayed in Table III.

Figure 8.

Four cases for $m = 3$ and $n = 1$. The corresponding mean-square-errors are displayed in Table IV.

Figure 9.

Computer simulation results for one time instance using a dynamic phantom. (a) The true image. (b) The image reconstructed with the current 24-line k-space data. (c) Bayesian reconstruction using $m = 1$, $n = 1$, $\beta_1 = 10$, $\beta_2 = 5$, and $\beta_3 = 100$. (d) Bayesian reconstruction using $m = 2$, $n = 2$, $\beta_1 = 200$, $\beta_2 = 55$, and $\beta_3 = 0.5$.

Table I

MSE Bayesian-FBP reconstruction with respect to the iterative reconstruction ($m = 0$ and $n = 1$)

β_1	β_2	β_3	MSE
12	3	30	0.2230
10	3	30	0.2230
10	3	50	0.2230
3	3	50	0.2215

Table II

MSE for Bayesian-FBP reconstruction with respect to the iterative reconstruction ($m = 1$ and $n = 1$)

β_1	β_2	β_3	MSE
0.07	0.07	0.07	0.0763
0.15	0.25	0.50	0.0726
0.12	0.30	0.12	0.0755
0.12	0.22	0.22	0.0740
12	3	30	0.0859
10	3	30	0.0583
12	1	50	0.0755
3	3	50	0.0858

Table III

MSE for Bayesian-FBP reconstruction with respect to the iterative reconstruction ($m = 2$ and $n = 1$)

β_1	β_2	β_3	MSE
0.07	0.07	0.07	0.0796
0.09	0.12	0.20	0.0777
0.07	0.15	0.10	0.0783
0.12	0.30	0.12	0.0781
0.12	0.22	0.22	0.0773
10	10	20	0.0671
12	10	22	0.0663
12	10	50	0.0603
10	5	100	0.0478

Table IV

MSE for Bayesian-FBP reconstruction with respect to the iterative reconstruction ($m = 3$ and $n = 1$)

β_1	β_2	β_3	MSE
12	3	30	0.2230
10	3	30	0.2230
10	3	50	0.2230
3	3	50	0.2215

Table V

MSE for Bayesian-FBP computer simulation with respect to the true image.

m	n	β_1	β_2	β_3	MSE
1	1	50	10	25	0.0048
2	2	200	55	0.5	0.0053
1	1	10	10	10	0.0049
2	1	20	50	1.0	0.0062
1	1	25	2.5	0.15	0.0053
1	1	0.07	0.07	0.07	0.0418
2	2	25	2.5	0.15	0.0098
0	1	200	55	0.5	0.2103