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Spatial Specificity in Spatiotemporal Encoding and Fourier Imaging

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Abstract

Purpose—Ultrafast imaging techniques based on spatiotemporal-encoding (SPEN), such as RASER (rapid acquisition with sequential excitation and refocusing), is a promising new class of sequences since they are largely insensitive to magnetic field variations which cause signal loss and geometric distortion in EPI. So far, attempts to theoretically describe the point-spread-function (PSF) for the original SPEN-imaging techniques have yielded limited success. To fill this gap a novel definition for an apparent PSF is proposed.

Theory—Spatial resolution in SPEN-imaging is determined by the spatial phase dispersion imprinted on the acquired signal by a frequency-swept excitation or refocusing pulse. The resulting signal attenuation increases with larger distance from the vertex of the quadratic phase profile.

Methods—Bloch simulations and experiments were performed to validate theoretical derivations.

Results—The apparent PSF quantifies the fractional contribution of magnetization to a voxel's signal as a function of distance to the voxel. In contrast, the conventional PSF represents the signal intensity at various locations.

Conclusion—The definition of the conventional PSF fails for SPEN-imaging since only the phase of isochromats, but not the amplitude of the signal varies. The concept of the apparent PSF is shown to be generalizable to conventional Fourier- imaging techniques.

Keywords

frequency-swept pulse; magnetic resonance imaging (MRI); RASER; spatiotemporal encoding; ultrahigh magnetic field; point-spread-function

1. Introduction

In standard Fourier imaging, spatial resolution is assumed to be a well-understood property. However, with the development of new imaging sequences based on unconventional spatial

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encoding concepts the true spatial resolution is not self-evident. One of several novel imaging techniques is spatiotemporal encoding (SPEN) [1–5] which will be discussed in detail as part of this work. When performing SPEN with an excitation-pulse, transverse magnetization is created by applying a 90° frequency-swept chirp-pulse in the presence of a magnetic field gradient. Appropriate balancing of the gradient moment during the echo readout results in a quadratic phase profile imprinted on the refocused echo. Ultrafast imaging sequences exploiting SPEN, such as RASER (rapid acquisition with sequential excitation and refocusing) [1, 5], have shown promising results for applications in which significant static magnetic field variations cannot be avoided. In RASER, SPEN replaces phase-encoding in an EPI (echo planar imaging) readout train.

In the SPEN-dimension, spatial localization of the signal is based on dephasing of the transverse magnetization in regions of increasing phase variations moving outward from the vertex of the quadratic phase profile. Previously, spatial resolution in SPEN was derived from the width of the quadratic phase profile [6, 7]. In this paper, it is based on the signal attenuation caused by dephasing. A new definition for an apparent point-spread-function (PSF) will be introduced by quantifying the signal fraction of an isochromat at the distance x' from the vertex contributing to the echo which is comprised of the superimposed phase distribution of the entire excited transverse magnetization.

In Fourier imaging, spatial resolution is determined by modulation and/or truncation of the data which is typically a result of limitations of the acquisition process itself. T_2 - and T_2^* -relaxation are known to attenuate the signal as a function of spatial encoding parameters in ultrafast imaging sequences, such as, EPI and FSE (fast spin echo) resulting in blurring in images. Typically, the PSF describing these effects represents the spatial variations of signal amplitude as signal spreads into neighboring voxels. In this paper, generalizing the definition of the apparent PSF for the SPEN dimension to Fourier imaging is proposed. In Fourier imaging, the apparent PSF represents the fractional contribution of all side lobes of the broadened PSF of neighboring voxels in addition to the part of the signal originating from the within the voxel size to the total signal intensity of the voxel-of-interest. The broadening of the PSF will be simulated by truncating and zero-filling the data acquired with a conventional gradient-echo (GRE) sequence. The resulting Gibbs-ringing described by a conventional PSF based on the sinc-function is often observed in interpolated image data.

To summarize, the true spatial specificity of an imaging sequence will be discussed as a part of the novel concept of the *apparent PSF* quantifying the spatially varying fractional contribution to a voxel's signal. This definition is applicable to line-scan techniques, such as RASER, as well as conventional 2D Fourier imaging methods. Experimental results and simulations are presented to support theoretical derivations of the apparent PSF and its properties.

2. Theory

RASER employs a SPEN scheme [2] to obtain 2D images in a single shot (Fig. 1). To begin, a frequency-modulated chirp-pulse [8, 9] is applied in the presence of a gradient in the

SPEN dimension. Subsequently, two slice-selective 180° pulses are applied to refocus the magnetization whose phase varies non-linearly (quadratically) in the SPEN direction [1–3, 10]. The “vertex” of this phase function is shifted in time using blipped gradient pulses during the sequence block consisting of multiple acquisition periods [1]. Alternatively, a constant gradient can be used to refocus the echoes during the readout train.

In RASER, spatial localization along the SPEN dimension is determined by the quadratic phase profile of the excitation pulse. The signal of each frequency-encoded echo originates from locations along the SPEN dimension where the moving vertex of the quadratic phase function coincides with the center of the respective gradient pulses. For a chirp-pulse, the steepness of the quadratic phase function increases with the time-bandwidth product, R , of the pulse. A more rapid spatial phase variation results in a more efficient signal dephasing which reduces the width of the apparent point-spread-function (PSF). Each of the m acquisition periods in the readout train therefore corresponds to a line directly in image space. RASER is therefore a line-scan technique in which only the frequency-encoded dimension is Fourier-transformed.

If a constant gradient during the RASER echo train is used in RASER to balance gradient moments in the SPEN dimension, the frequency-encoded signal accrues an additional phase, which increases with time. This phase-variation is corrected for as described in reference [1]. All RASER images, which are acquired either with gradient blips or a constant gradient during the echo train readout, are reconstructed with 1D Fourier transformation along the frequency-encoded dimension.

2.1 The apparent point-spread-function in SPEN imaging

In SPEN a chirp-pulse is typically used as a frequency-swept excitation pulse. This pulse is characterized by a constant amplitude-modulation function

$$B_1(t) = B_1^{\max} \quad (1)$$

and a linear frequency-sweep

$$\Delta\omega(t) = 2A \left[\frac{t}{T_p} - \frac{1}{2} \right] \quad (2)$$

In the presence of a magnetic field gradient, the linear frequency sweep of the chirp-pulse creates a quadratic phase profile along the spatial dimension x :

$$\varphi = \pi R \left(\frac{x - x_0}{\text{FOV}} \right)^2 \quad (3)$$

Considering the vertex of the quadratic phase profile to be located at x_0 at time t_0 , the signal at location x is:

$$M_{xy}(x, t_0) = \rho \exp \left[i\pi R \left(\frac{x - x_0}{\text{FOV}} \right)^2 \right] \quad (4)$$

where ρ is the spin density which is defined as the signal per “volume”. Here the “volume” is determined solely by its extent along the dimension x . Integrating the signal, $M_{xy}(x, t_0)$, of spins along the SPEN dimension, x , produces a localized excitation profile centered at x_0 :

$$\sigma(x, \Delta x) = \int_{x-\Delta x_f/2}^{x+\Delta x_f/2} M_{xy}(\xi, t_0) d\xi = \int_{x-\Delta x_f/2}^{x+\Delta x_f/2} \rho \exp \left[i\pi R \left(\frac{\xi - x_0}{\text{FOV}} \right)^2 \right] d\xi \quad (5)$$

where Δx_f is the “fictitious” spatial resolution, at which the profile is sampled.

To calculate the complex-valued signal intensity of each echo in RASER, the spin density, ρ , is assumed to be spatially invariant,

$$\rho(x) = \rho_0 \quad (6)$$

Defining $y^2 = \pi R \left(\frac{\xi - x_0}{\text{FOV}} \right)^2$ and $y_{\pm}(x) = \frac{\sqrt{\pi R}}{\text{FOV}} \left(x \pm \frac{\Delta x_f}{2} \right)$, the signal intensity at location, x , is given by:

$$s^R(x, \Delta x_f) = \frac{\rho_0 \cdot \text{FOV}}{\sqrt{\pi R}} \int_{y_{-}(x)}^{y_{+}(x)} \exp[iy^2] dy \quad (7)$$

The signal intensity of an echo at time $t_0 = t(x_0)$, where x_0 corresponds to the location of the vertex of the quadratic phase profile, is:

$$S^R_{\text{FOV}} = \rho_0 \int_{-\text{FOV}/2}^{+\text{FOV}/2} \exp \left[i\pi R \left(\frac{\xi - x_0}{\text{FOV}} \right)^2 \right] d\xi \quad (8)$$

Let us assume that $\pi R/4 \gg 2\pi$. In this case, the error in eq. (8) caused by infinite integration limits is small. For this approximation to be valid ρ_0 FOV has to remain constant; that is, the maximal signal is limited by the FOV and spin density of the imaged object. The analytical solution of the integral is:

$$S^R_{\infty} = \rho_0 \int_{-\infty}^{+\infty} \exp \left[i\pi R \left(\frac{\xi}{\text{FOV}} \right)^2 \right] d\xi = \frac{\rho_0 \text{FOV}}{\sqrt{R}} \exp \left(i\frac{\pi}{4} \right) \quad (9)$$

confirming the result in reference [7].

To quantify the signal fraction at the location x that contributes to the echo signal, $s_{R \text{ tot}} = S^R_{\infty} \exp(-i\pi/4)$, the integral in eq. (9) is split into two terms:

$$s_{R \text{ tot}} = \Lambda_R^c(x') + \overline{\Lambda_R^c}(x')$$

$$\Lambda_R^c(x') = \rho_0 \int_{-\infty}^{-x'} \exp \left[i\pi R \left(\frac{\xi}{\text{FOV}} \right)^2 - i\frac{\pi}{4} \right] d\xi + \rho_0 \int_{x'}^{\infty} \exp \left[i\pi R \left(\frac{\xi}{\text{FOV}} \right)^2 - i\frac{\pi}{4} \right] d\xi \quad (10)$$

$$\overline{\Lambda_R^c}(x') = \rho_0 \int_{-x'}^{x'} \exp \left[i\pi R \left(\frac{\xi}{\text{FOV}} \right)^2 - i\frac{\pi}{4} \right] d\xi$$

$\overline{\Lambda_R^c}$ is the fraction of transverse magnetization contributing to the echo signal at distance $x' = x - x_0$ from the phase vertex. Λ_R^c represents the signal attenuation function accounting for the signal fraction of the residual signal arising from outside the fictitious voxel $x_f = x' - (-x')$.

Since the signal $s_{R \text{ tot}}$ is real-valued the imaginary parts of Λ_R^c and $\overline{\Lambda_R^c}$ in eq. (10) cancel.

Hence,

$$s_{R \text{ tot}} = \Lambda_R(x') + \overline{\Lambda_R}(x')$$

$$\Lambda_R(x') = \text{Re} \left[\Lambda_R^c(x') \right] \quad (11)$$

$$\overline{\Lambda_R}(x') = \text{Re} \left[\overline{\Lambda_R^c}(x') \right]$$

Λ_R is referred to as the apparent PSF of RASER.

The *effective resolution* in RASER is defined as the width of the apparent PSF, $x_{R \text{ eff}}$, at which the signal components inside and outside the spatial boundaries $\pm x_f/2$ are equal

$$\Lambda_R(x' = \Delta x_f/2) = \overline{\Lambda_R}(x' = \Delta x_f/2) \quad (12)$$

Numerical integration of eqs. (10) to (11) confirms the relationship

$$\Delta x_{R \text{ eff}} = \frac{\pi^2}{16\sqrt{R}} \text{FOV} \quad (13)$$

$x_{R \text{ eff}}$ coincides with the full-width at half-maximum of the apparent PSF, Λ_R .

The *nominal resolution* $x_m = \text{FOV}/m$ is defined as the FOV divided by the number of acquired echoes of the readout train, m (Fig.1). While in Fourier imaging the width of PSF is inversely proportional to the number of phase-encoding steps, m , the width of the apparent PSF in RASER is independent of the nominal resolution x_m . To maintain consistent edge definition between scans with varying number of the echoes the following relationship between effective and nominal resolution is defined as:

$$\Delta x_m = k \Delta x_{R \text{ eff}} \quad (14)$$

κ can be any constant value. For this work, the relationship

$$\kappa = \sqrt{\frac{32}{\pi^3}} \approx 1. \quad (15)$$

is assumed. In this case, one obtains $R = \frac{2}{\pi} m^2$ from eq. (13).

2.2 The apparent point-spread-function in Fourier imaging

In Fourier imaging, the conventional PSF is determined by the modulation of the signal in k-space. In this paper, the ideal case of no relaxation-weighting of k-space is discussed. Hence, only truncation of k-space contributes to the broadening of the PSF. This effect is mathematically described by a hat-function Π multiplied with the idealized k-space representation $s_{\text{k-space}}^{\text{F}\infty}$ of the imaged spin density distribution ρ assuming infinite integration limits:

$$S_{\text{k-space}}^{\text{F}}(\mu) = s_{\text{k-space}}^{\text{F}\infty}(\mu) \cdot \Pi(\mu)$$

$$s_{\text{k-space}}^{\text{F}\infty}(\mu) = \int_{-\infty}^{+\infty} \rho \exp[i\mu \Delta k_x x'] dx' \quad (16)$$

$$\Pi(k_x) = \begin{cases} 1, & -m_c \leq \mu \leq (m - m_c - 1) \\ 0, & \text{otherwise} \end{cases}$$

where m is the total number of spatial phase-encoding steps, m_c represents the center of k-space and $k_x = \gamma G_p \tau$ is the k-space increment. In a gradient-echo (GRE) acquisition, τ is the duration and G_p the increment of the amplitude of the phase-encoding gradient pulse.

The Fourier transform of $S_{\text{k-space}}^{\text{F}}$ provides the image $S_{\text{img}}^{\text{F}}$ represented by the convolution

$$S_{\text{img}}^{\text{F}}(\nu) = s_{\text{img}}^{\text{F}} \otimes s_{\text{PSF}}^{\text{F}} = \sum_{\varsigma=-m_c}^{m-m_c-1} S_{\text{img}}^{\text{F}}(\varsigma) S_{\text{PSF}}^{\text{F}}(\nu - \varsigma) \quad (17)$$

where $s_{\text{img}}^{\text{F}}(\nu) = \sum_{\mu=-m_c}^{m-m_c-1} s_{\text{k-space}}^{\text{F}\infty}(\mu) \exp[-i\mu \Delta k_x \nu \Delta x_m]$ is the image of the spin density

distribution ρ [eq. (A.2)]. The conventional PSF $s_{\text{PSF}}^{\text{F}}(\nu) = \Delta x_m \sum_{\mu=-m_c}^{m-m_c-1} \exp[-i\mu \Delta k_x \nu \Delta x_m]$ is the discrete Fourier transform of the hat function [eq. (A.4)]. According to eq. (17), blurring as a result of a broadened PSF causes signal, which originates from location ς , to leak into the signal of the voxel at $x = \nu x_m$. Hence, the total signal of the ν -th voxel is the superposition of the signal from the location νx_m and the side lobes of the conventional PSF from neighboring voxels.

Let us assume that the spin density in eq. (16) is constant: $\rho(x) = \rho_0$. Then, the image is

$s_{\text{img}}^{\text{F}}(\nu) = \rho_0$. Using the identity $\sum_{\varsigma=-m_c}^{m-m_c-1} s_{\text{img}}^{\text{F}}(\varsigma) s_{\text{PSF}}^{\text{F}}(\nu - \varsigma) = \sum_{\varsigma=-m_c}^{m-m_c-1} s_{\text{img}}^{\text{F}}(\nu - \varsigma) s_{\text{PSF}}^{\text{F}}(\varsigma)$ and choosing the voxel-of-interest to be $\nu = 0$, the convolution in eq. (17) is simplified to

$$S^{\text{F}} = \rho_0 \sum_{\varsigma=-m_c}^{m-m_c-1} s_{\text{PSF}}^{\text{F}}(\varsigma) \quad (18)$$

Applying zero-filling to k-space, the signal $S_{\text{img}}^{\text{F}}$ can be approximated by the continuous conventional PSF [eqs. (A.5) and (A.9)]

$$s_{\text{PSF}}^{\text{F}}(x) = \Delta x_{\text{m}} m \exp\left(i\pi \frac{x}{\text{FOV}}\right) \text{sinc}\left(\frac{\pi}{m} - \pi \frac{x}{\Delta x_{\text{m}}}\right) \quad (19)$$

As a result, eq. (18) is restated

$$S^{\text{F}} = \rho_0 \int_{-\text{FOV}/2}^{+\text{FOV}/2} s_{\text{PSF}}^{\text{F}}(\varsigma) d\varsigma \quad (20)$$

Based on eq. (20), an apparent PSF Λ_{F} and its complimentary function $\overline{\Lambda_{\text{F}}}$ can be defined in Fourier imaging in an analogous manner to SPEN [eqs. (10) and (11)]

$$S^{\text{F}\infty} = \rho_0 \int_{-\infty}^{+\infty} s_{\text{PSF}}^{\text{F}}(\varsigma) d\varsigma = \Lambda_{\text{F}}(x') + \overline{\Lambda_{\text{F}}}(x')$$

$$\Lambda_{\text{F}}(x') = \text{Re} \left[\rho_0 \int_{-\infty}^{-x'} s_{\text{PSF}}^{\text{F}}(\varsigma) d\varsigma + \rho_0 \int_{x'}^{\infty} s_{\text{PSF}}^{\text{F}}(\varsigma) d\varsigma \right] \quad (21)$$

$$\overline{\Lambda_{\text{F}}}(x') = \text{Re} \left[\rho_0 \int_{-x'}^{x'} s_{\text{PSF}}^{\text{F}}(\varsigma) d\varsigma \right]$$

where the total signal intensity is

$$S^{\text{F}\infty} = \rho_0 m \Delta x_{\text{m}} \quad (22)$$

Using an relationship equivalent to eq. (12) $\Lambda_{\text{F}}(x' = \Delta x_{\text{f}}/2) = \overline{\Lambda_{\text{F}}}(x' = \Delta x_{\text{f}}/2)$ the full-width at half-maximum of the apparent PSF in Fourier imaging $x_{\text{F eff}}$ is

$$\Delta x_{\text{F eff}} = \frac{1}{2}. \quad (23)$$

2.3 The experimental point-spread-function

Experimentally the PSF can be determined by mapping the profile perpendicular to the edge of a block suppressing the signal in half of the FOV [11, 12]. This phantom is described mathematically by a spin density distribution represented by a step-function

$$\rho_{\text{edge}}(x) = \begin{cases} \rho_0, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (24)$$

To measure the apparent PSF, the RASER sequence is modified to acquire a one-dimensional profile along the SPEN-dimension without the alternating readout-gradient pulses for frequency-encoding (Fig. 3a). The constant gradient during signal acquisition causes the transverse magnetization to be refocused exactly at $t_{\text{acq}} = t(x_0)$ when the vertex of the quadratic phase profile is located at x_0 . Hence, the vertex of the quadratic phase profile shifts along the SPEN-dimension as a function of the acquisition time $x_0 = x_0(t_{\text{acq}})$. The acrylic glass block suppresses a part of the quadratic phase profile depending on the location of the vertex x_0 . Accordingly, eq. (8) can be modified to account for the non-constant spin-density distribution in eq. (24)

$$S_{\text{edge}}^{\text{R FOV}}(x_0) = \int_{-\text{FOV}/2}^{+\text{FOV}/2} \rho_{\text{edge}}(\xi - x_0) \exp \left[i\pi R \left(\frac{\xi - x_0}{\text{FOV}} \right)^2 \right] d\xi \quad (25)$$

Substituting $\eta = \xi - x_0$ and $x = x_0$, one obtains

$$S_{\text{edge}}^{\text{R FOV}}(x) = \rho_0 \begin{cases} \int_{-\text{FOV}/2}^{+\text{FOV}/2-x} \rho_{\text{edge}}(\eta) \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x \geq 0 \\ \int_{-\text{FOV}/2-x}^{+\text{FOV}/2} \rho_{\text{edge}}(\eta) \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x < 0 \end{cases} \quad (26)$$

taking into consideration that magnetization is not excited outside the FOV. Eq. (26) can be restated using eq. (24),

$$\begin{aligned} S_{\text{edge}}^{\text{R FOV}}(x) &= \rho_0 \begin{cases} \int_0^{+\text{FOV}/2-x} \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x \geq 0 \\ \int_{-x}^{+\text{FOV}/2} \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x < 0 \end{cases} \\ &= \rho_0 \begin{cases} \int_0^{+\text{FOV}/2} \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta - \int_x^{+\text{FOV}/2} \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x \geq 0 \\ \int_{-x}^{+\text{FOV}/2} \exp \left[i\pi R \left(\frac{\eta}{\text{FOV}} \right)^2 \right] d\eta, & x < 0 \end{cases} \end{aligned} \quad (27)$$

The vertex of the quadratic phase profile coincides with the edge of the block when $x = 0$. For large FOV and after phasing the profiles, eq. (27) is approximated by the apparent PSF $\Lambda_R(x)$ and its complementary function $\overline{\Lambda_R}(x)$ [eq. (10)]

$$s_{\text{edge}}^{\text{R FOV}}(x) \approx \frac{1}{2} \begin{cases} s_{\text{R tot}} - \Lambda_{\text{R}}(x), & x \geq 0 \\ \Lambda_{\text{R}}(x), & x < 0 \end{cases} = \frac{1}{2} \begin{cases} \overline{\Lambda_{\text{R}}}(x), & x \geq 0 \\ \Lambda_{\text{R}}(x), & x < 0 \end{cases} \quad (28)$$

In Fourier imaging, the signal in image space is the convolution of the high-resolution quasi-continuous profile of the edge $s_{\text{edge}}^{\text{F}}(x) = \rho_{\text{edge}}(x)$ [eqs. (A.2), (16) and (24)] and the conventional PSF $s_{\text{PSF}}^{\text{F}}$ [eqs. (17) and (19)]

$$S_{\text{edge}}^{\text{F FOV}}(x) = s_{\text{edge}}^{\text{F}} \otimes s_{\text{PSF}}^{\text{F}} = \int_{-\text{FOV}/2}^{+\text{FOV}/2} \rho_{\text{edge}}(\xi) s_{\text{PSF}}^{\text{F}}(x - \xi) d\xi = \rho_0 \int_0^{+\text{FOV}/2} s_{\text{PSF}}^{\text{F}}(x - \xi) d\xi \quad (29)$$

Substituting $\eta = x - \xi$, one obtains

$$S_{\text{edge}}^{\text{F FOV}}(x) = \rho_0 \int_x^{x-\text{FOV}/2} s_{\text{PSF}}^{\text{F}}(\eta) d\eta = -\rho_0 \int_{x-\text{FOV}/2}^0 s_{\text{PSF}}^{\text{F}}(\eta) d\eta - \rho_0 \begin{cases} \int_0^x s_{\text{PSF}}^{\text{F}}(\eta) d\eta, & x \geq 0 \\ \int_x^0 s_{\text{PSF}}^{\text{F}}(\eta) d\eta, & x < 0 \end{cases} \quad (30)$$

In analogy to SPEN-imaging [eq. (28)], eq. (30) can be approximated by the corresponding apparent PSF $\Lambda_{\text{F}}(x)$ and its complimentary function $\overline{\Lambda_{\text{F}}}(x)$ defined in eq. (21)

$$s_{\text{edge}}^{\text{F}}(x) \approx -\frac{1}{2} \begin{cases} [s_{\text{F tot}} + \overline{\Lambda_{\text{F}}}(x)], & x \geq 0 \\ [s_{\text{F tot}} - \overline{\Lambda_{\text{F}}}(x)], & x < 0 \end{cases} = -\frac{1}{2} \begin{cases} [s_{\text{F tot}} + \overline{\Lambda_{\text{F}}}(x)], & x \geq 0 \\ \Lambda_{\text{F}}(x), & x < 0 \end{cases} \quad (31)$$

In addition, the conventional PSF $s_{\text{PSF}}^{\text{F}}$ can be obtained from the measured edge profile $s_{\text{edge}}^{\text{F}}(x)$ according to

$$s_{\text{PSF}}^{\text{F}}(x) = \frac{1}{\rho_0} \frac{d}{dx} S_{\text{edge}}^{\text{F}}(x) \quad (32)$$

This relationship is derived from eq. (29) using the identity

$$\int_{-\text{FOV}/2}^{+\text{FOV}/2} s_{\text{edge}}^{\text{F}}(\xi) s_{\text{PSF}}^{\text{F}}(x - \xi) d\xi = \int_{-\text{FOV}/2}^{+\text{FOV}/2} s_{\text{edge}}^{\text{F}}(x - \xi) s_{\text{PSF}}^{\text{F}}(\xi) d\xi \quad \text{and} \quad \frac{d}{dx} S_{\text{edge}}^{\text{F}}(x) = \rho_0 \delta(x) \quad \text{where} \\ \delta \text{ is the Dirac delta-function.}$$

2.4 Simulations

The apparent PSFs in SPEN-imaging were calculated using a Bloch simulator [13] written in MATLAB (The MathWorks Inc., Natick, Massachusetts, USA) and the GPU of the Tesla C870 video card (NVIDIA, Santa Clara, California, USA). Bloch simulations were performed by sequentially applying incremental rotations of the magnetization depending on the B_1 - and B_0 -fields occurring during the simulated sequences with quasi-continuous spatial and temporal resolution.

The apparent PSF in the SPEN dimension was simulated using the RASER sequence without frequency-encoding (Fig. 3a). The “imaged” object was a plastic block ($\rho = 0$) immersed in water ($\rho = 1$) represented by the spin density distribution:

$$\rho(x) = \begin{cases} 1, & 0 \leq x \leq +\text{FOV}/2 \\ 0, & -\text{FOV}/2 \leq x < 0 \end{cases} \quad (33)$$

Simulations were performed in 1D with 19,200 spins at a temporal resolution of 1 μs . Imaging parameters were: $T_p = 57.6$ ms, flip angle 90° , FOV = 288 mm and echo time $T_E = 128.78$ ms. R -values of the excitation pulse ranged from 900 to 5400 in increments of 900. The resulting curves were binned (bin size = 42) for smoothing.

In Fourier imaging, the conventional PSF was simulated by assuming a point source of the spin density distribution

$$\rho(x) = \begin{cases} 1, & x=0 \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

The k-space data were generated by applying phase-encoding numerically according to eq. (A.1) with $M = 20$ phase-encoding steps. The k-space data were zero-filled by a factor of 501. Numerical fast Fourier-transformation of the zero-filled k-space data produced the interpolated conventional PSF.

3. Methods

All imaging experiments were carried out on a 7 T/90 cm bore whole-body magnet (MagneX Scientific, Oxford, UK) equipped with a Siemens console with body gradients. A 8-channel transmit/32-channel receive head coil which was driven with 8 individual 1 kW amplifiers (CPC Hauppauge, NY) was used. In all experiments, B_1 -shimming [14] was performed and the RF-power calibrated using the double-flip angle method based on a GRE sequence. All image reconstruction and data analysis were performed in MATLAB (The MathWorks Inc., Natick, Massachusetts, USA).

The PSFs in SPEN- and Fourier-imaging were experimentally determined by imaging the edge of an acrylic glass block which was immersed in saline (100 mM NaCl, 0.5 mM CuSO₄ in de-ionized water). Data were acquired using a modified RASER sequence without readout gradient (Fig. 3a). The constant gradient during the acquisition refocuses continuously the transverse magnetization along the SPEN dimension (see Fig. 3b). The edge of the block was oriented perpendicular to the SPEN dimension and centered in the FOV matching eq. (33). The duration of the acquisition period was equal to the pulse duration $T_p = 19.2$ ms. The dwell time was 1 μs . The first refocusing pulse is slice-selective in the G_s -direction (8 mm), and the second 180° -pulse refocuses signal within a slab (16 mm) along the G_f -dimension. The FOV in the SPEN dimension is 96 mm. The R -value of the chirp-pulse varied from 100 to 600 in increments of 100. The resulting curves are binned (bin size = 52) for smoothing.

Fourier-encoded data of the saline phantom with the acrylic glass block were acquired with a standard gradient-echo sequence. The phase-encoded dimension was oriented perpendicular to the edge of the block (Fig. 4a). The imaging parameters were: FOV 35.2 cm $\times$ 17.6 cm, slice-thickness 8 mm, in-plane resolution 0.25 mm, acquired matrix 1408 $\times$ 704, acquisition bandwidth 418 Hz/voxel, echo time 4.5 ms and repetition time 4 s. Images were reconstructed using 2D-fast Fourier transform. To remove signal variation originating from the profiles of receiver coil elements of the RF head coil sensitivity maps were calculated from the acquired GRE data by smoothing the images with a kernel of 5 mm $\times$ 0.25 mm (frequency-encoded $\times$ phase-encoded dimension). The sharpness of the edge was preserved by eliminating voxels from the smoothing kernel which had a signal intensity which differed by more than 98% from the central voxel of the kernel. To flatten the images and remove phase-variations related to the receiver coil profiles the complex-valued image of the receiver channels were divided by the corresponding complex sensitivity maps. The original noise in the flattened images was retained using a mask based on the signal distribution of the sensitivity maps. A profile was generated by averaging the flattened complex-valued image data along the frequency-encoded dimension within a 12-mm wide region-of-interest (red ROI in Fig. 4a) which was centered relative to the edge of the acrylic glass block. The resulting curve was smoothed with a bin size of four providing a final resolution of 1 mm (Fig. 4b). To simulate blurring, the inverse Fourier transform of the average profile was calculated. The obtained k-space data were truncated providing $M_1 = 40$ and $M_2 = 96$ phase-encoding steps. The truncated k-space data were zero-filled to the original number of 400 points and Fourier-transformed to generate profiles exhibiting blurring.

4. Results

Blurring in images is typically described by the conventional PSF which characterizes the signal intensity at varying locations. The apparent PSF - as defined in this paper - represents the fractional signal contribution to a voxel's total signal as a function of distance from the respective voxel. The proposed concept of the apparent PSF can be applied to different types of imaging sequences. Fig. 2 illustrates the derivation for a line-scan technique based on SPEN (RASER) and for conventional Fourier-imaging (GRE). The theoretical results are confirmed by experimental results shown in Figs. 3 and 4.

RASER (Fig. 1) relies on the quadratic phase variation imprinted on the transverse magnetization by the frequency-swept excitation pulse [3]. Fig. 2a shows the real and imaginary part of the transverse magnetization along the SPEN dimensions for an echo whose quadratic phase vertex is centered at $x' = 0$. Localization in RASER is a result of signal attenuation caused by dephasing of signal on either side of the vertex of the quadratic phase profile. The signal attenuation could be calculated by simply binning the complex-valued transverse magnetization with a spatial resolution Δx . However, this approach produces different signal attenuation functions for the same quadratic phase profile depending on the voxel size (Fig. 2b).

In contrast, the apparent PSF is defined as the superposition of all isochromats outside a fictitious voxel Δx_f which varies in size from infinity to zero. This function is equivalent to

the contribution of a spin's signal to the signal intensity of a RASER echo at a distance $x' = \pm x_f/2$ from the vertex of the quadratic phase profile at $x' = 0$ [eqs. (8) to (11)] as it experiences dephasing in the presence of all other spins. Fig. 3c shows the real parts of two signal components representing the average signal outside and within the fictitious voxel corresponding to the apparent PSF $\Lambda_R(x')$ and its complementary function $\overline{\Lambda_R}(x')$. This definition for the apparent PSF depends solely on the width of the quadratic phase profile and is independent of the nominal voxel size $x_m = \text{FOV}/m$ where m is the number of echoes. As a consequence, the width of Λ_R provides an unambiguous definition for the effective spatial resolution $x_{R\text{ eff}}$ [eq. (13)]. In Fig. 2c, $x_{R\text{ eff}}$ is marked by dotted vertical lines.

Equivalent relationships for the apparent PSF in Fourier imaging are derived in the theory section [eqs. (16) to (23)]. Blurring in Fourier imaging is caused by additional attenuation of k-space besides the modulation produced by spatial encoding and is conventionally described by an amplitude-modulated PSF in image space referred to as conventional PSF in this paper. Eq. (19) represents the conventional PSF s_{PSF}^F for k-space which is truncated to m phase-encoding steps and zero-filled to its original number of k-space points. This definition describes the signal intensity variations as a function of the spatial dimension x' for a voxel-of-interest at $x' = 0$. The simulated conventional PSF based on eq. (34) is displayed in Fig. 2d. The corresponding apparent PSF Λ_F representing the fractional signal contribution from all spatial locations to the signal intensity of the voxel-of-interest at $x' = 0$ and its complementary function $\overline{\Lambda_F}$ [eq. (21)] are shown in Figs. 2e. The effective resolution $x_{F\text{ eff}}$ is marked by vertical dotted lines.

In Fig. 2f, the apparent PSFs of RASER and Fourier imaging are directly compared. The width of Λ_R depends on the R -value of the frequency-swept excitation pulse. For the chosen pulse parameters [eqs. (13), (14) and (15)], the width of Λ_R , as given by $x_{R\text{ eff}}$ (dotted red lines), is similar to the full-width-at-half-maximum of the apparent PSF $x_{F\text{ eff}}$ in Fourier imaging (dotted blue lines). At this effective resolution of RASER, 69% of the echo signal originates from spins within a voxel of size x_m . In Fourier imaging, spins located within x_m contribute to 87% of a voxel's signal.

To confirm the analytically derived equation for the apparent PSF of RASER [eq. (11)] experiments and Bloch simulations were performed. They are based on the modified RASER sequence in Fig. 3a for which the readout gradient in the echo train is removed. The constant gradient during acquisition refocuses continuously the transverse magnetization as a function of time. The temporal coordinate is proportional to the SPEN spatial dimension x . In order to measure the effective spatial resolution in the SPEN dimension spatial signal variation perpendicular to the edge of acrylic glass block immersed in saline is mapped (Fig. 3b). Since the imaged block suppresses the signal of half of the FOV along the SPEN dimension the coherences, which would contribute to the signal component corresponding to the apparent PSF Λ_R , are not excited. As a result, the recorded signal corresponds to the complementary function of the apparent PSF, $\overline{\Lambda_R}$. The analytical solution of the spatial signal variation produced with this phantom is given by eq. (28).

In Figs. 3d and e, the Bloch simulation and the experimental results using the modified RASER sequence (Fig. 3a) are displayed. The gray shaded areas represent the saline solution which extends up to the acrylic glass block covering the range $0 < x \leq \text{FOV}/2$. Alleviating truncation artifacts from the edges of the excitation bandwidth the FOV in the simulations is three fold larger than the one in the experiments. In order to ensure the same effective spatial resolution $\Delta x_{\text{R eff}}$ [eq. (13)], the R -values in the Bloch simulations have to be 9 times higher than in the experiments (see insets). The Bloch simulations reproduce the oscillations observed in the experimental results for $x \geq 0$.

The Bloch simulations and experiments (Figs. 3d and e) reproduce only the right half ($x \geq 0$) of the analytical result for the complementary function of the apparent PSF [eqs. (11) and (28)] shown in Fig. 3c. The differences for the left side of the phantom ($x < 0$) arise from the fact that the analytical derivation assumes that the water compartment extends to minus infinity. Without signal excitation at $x = \text{FOV}/2$ in the Bloch simulations and experiments, dephasing of transverse magnetization is imperfect and leads to incomplete signal attenuation at $x = 0$. This error is larger for smaller R -values producing broader quadratic phase profiles.

The same phantom was used to determine the PSF in Fourier imaging using a standard GRE sequence (Fig. 4a). The phase-encoded dimension was oriented perpendicular to the edge of the plastic block. The image was flattened using the sensitivity maps of the receive elements of the RF coil. The signal within the red ROI of the flattened image was averaged to obtain a profile of the spatially varying signal intensity perpendicular to the edge of the plastic block (Fig. 4b). The inverse Fourier transform of this profile produces the signal distribution in k -space as shown in Fig. 4c. To simulate broadening of the conventional PSF, the k -space signal is truncated at the levels indicated by the red and blue dashed lines and zero-filled to the original number of k -space points. The forward Fourier-transform generates the profiles in Fig. 4d which now show the typical oscillations for truncation also referred to as Gibbs-ringing. The red and blue color-code correspond to the level of truncation in Fig. 4c. The spatial coordinate is normalized to the nominal resolution $x_{m1} = \text{FOV}/m_1$ and $x_{m2} = \text{FOV}/m_2$ determined by the truncation limiting the number of phase-encoding steps to m_1 (red) and m_2 (blue). Zero-filling of the truncated k -space results in interpolation of the profiles. The oscillations observed in the experimental profiles (Fig. 4d) qualitatively reflect the analytical solution for this phantom [eq. (31)].

Fig. 4e shows the derivative of the profiles in Fig. 4d which accurately reproduce the conventional PSF $s_{\text{PSF}}^{\text{F}}$ representing a sinc-function. For qualitative comparison the theoretically derived PSF [eq. (32)] from Fig. 2 is shown in Fig. 4f. The corresponding apparent PSFs Λ_{F} are obtained by numerically integrating the curves in Figs. 4e and f according to eq. (21). The experimental apparent PSF displayed in Fig. 4g reproduces the overall shape of the analytical solution (Figs. 2e and 4h). As expected the amplitude of the side lobes of the PSFs depend on the degree of truncation, that is, the smaller the number of phase-encoding steps the more signal from regions outside the nominal voxel size contribute to the total signal of the voxel-of-interest.

5. Discussion

Spatial specificity of an imaging sequence is an essential criterion for all MRI techniques, but has not been addressed thoroughly for many existing imaging methods. For example, in the early papers on SPEN imaging, the spatial resolution was estimated from the width of the quadratic phase profile instead of the spatially varying signal attenuation caused by dephasing [6, 7]. In publications by Ben-Eliezer *et al.*, the signal attenuation was determined by binning the signal with a spatial resolution of $\Delta x_m = \text{FOV}/m$ determined by the FOV and the number of acquired echoes m [4, 15, 16]. This approach is appropriate for superresolution-reconstruction. However, the width of this type of signal attenuation function depends on the voxel size Δx_m . In more recent works, Fourier-transforming the SPEN-dimension was used to determine the “PSF” in the Fourier-domain [17]. In contrast, the apparent PSF discussed in this work quantifies spatial specificity of the original SPEN imaging techniques without reconstructing the SPEN-dimension as shown by the example RASER.

The approach by Li *et al.* [17] of determining the PSF in the Fourier-domain is analogous to the typical method of characterizing blurring in the conventional Fourier-imaging. The conventional PSF describes the amplitude of the signal intensity of a voxel “leaking” into neighboring voxels. While in SPEN this leakage is determined by the width of the quadratic phase profile, in Fourier imaging blurring, that is, broadening of the PSF in image-space, is a result of a modulation of the k-space signal in addition to the attenuation caused by spatial encoding. In ultrafast imaging techniques, such as FSE and EPI, T_2 - or T_2^* -relaxation can generate significant blurring and, hence, reduce the effective spatial resolution.

Previous definitions of the PSF in SPEN- and Fourier-imaging are primarily based on describing the spatially varying signal amplitude at and adjacent to a voxel-of-interest. In contrast, the spatial specificity of an imaging technique as proposed in this paper relies on the novel definition of the *apparent PSF* which quantifies the fractional contribution of isochromats originating from the location of the voxel as well as its surroundings to the total signal of the respective voxel. Based on this definition, the effective spatial resolution or spatial specificity of SPEN imaging solely depends on the degree of dephasing caused by the varying steepness of quadratic the phase profile. The apparent PSF in the SPEN dimension is therefore purely a function of the bandwidth time product and is independent of the resolution at which the signal phase is sampled. In Fourier imaging, the apparent PSF represents the superposition of the signal from within the voxel and the signal spreading into the voxel-of-interest from neighboring voxels as result of blurring.

In summary, the novel concept of the apparent PSF proposed in this paper has been shown by theoretical derivations, simulations and experiments to be an adequate description for the true spatial specificity for SPEN- and Fourier-imaging techniques. While this new definition is not strictly necessary to describe blurring in Fourier-imaging, it is the only approach so far to unambiguously quantify spatial specificity in SPEN-imaging. Future work will extend this concept to non-cartesian techniques, such as, radial and spiral imaging.

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Abbreviations

SPEN	spatiotemporal encoding, spatiotemporal encoded
RASER	rapid acquisition with sequential excitation and refocusing
PSF	point-spread-function

Appendix

A. Conventional PSF in Fourier imaging

In Fourier imaging, the k-space variable $k_x = \mu \cdot k_x$ is produced by μ integer increments of the k-space interval k_x . The k-space signal encoded with gradient pulse of duration τ and amplitude G_p incremented in steps G_p for spatial phase-encoding in a standard GRE-sequence ($k_x = \gamma \cdot G_p \tau$) is

$$s_{\text{k-space}}^F(\mu) = \int_{-\text{FOV}/2}^{+\text{FOV}/2} \rho \exp[i\mu \Delta k_x x'] dx' \quad (\text{A.1})$$

To encode the full-FOV, the equation $2\pi = m \cdot k_x \cdot x_m$ has to be fulfilled where m is the total number of phase-encoding steps. The image is the Fourier transform of eq. (A.1):

$$s_{\text{img}}^F(\nu) = \sum_{\mu=-m_c}^{m-m_c-1} s_{\text{k-space}}^F(\mu) \exp[-i \mu \Delta k_x \nu \Delta x_m] \quad (\text{A.2})$$

m_c corresponds to the center of k-space which is typically $m_c = m/2 + 1$ for even m . The total signal intensity is identical before and after Fourier transform.

To calculate the PSF for Fourier imaging, a spin density distribution with spins confined to a point in the image center is used:

$$\rho_F(x) = \Delta x_m \delta(x) \quad (\text{A.3})$$

where $\delta(x)$ is the Dirac delta function. The amplitude of the point source $\delta(x)$ is scaled to match the signal intensity originating from a voxel of size. Based on eqs. (A.1) to (A.3) the PSF in Fourier imaging is defined as

$$s_{\text{PSF}}^F(\nu) = \Delta x_m T(\nu)$$

$$T(\nu) = \sum_{\mu=-M_c}^{M-M_c-1} \exp[-i \mu \Delta k_x \nu \Delta x_m] \quad (\text{A.4})$$

To generate a continuous PSF

$$s_{\text{PSF}}^{\text{F}}(x) = \Delta x_m T_z(x) \quad (\text{A.5})$$

k-space in eqs. (A.1) and (A.2) is zero-filled and the discrete argument ν is replaced with the continuous variable x

$$T_z(x) = \sum_{\mu=-\infty}^{\infty} \Pi_z(-m_c, m - m_c - 1) \exp(-i \mu \Delta k_x x) \quad (\text{A.6})$$

where Π represents the hat function

$$\Pi_z(a, b) = \begin{cases} 1, & a \leq \mu \leq b \\ 0, & \text{otherwise} \end{cases} \quad (\text{A.7})$$

The asymmetry of the hat function limits relative to $\mu = 0$ in eq. (A.6) results in a linear phase variation along the spatial dimension. Symmetrizing the hat function and using the relationships $2\pi = m \Delta k_x \Delta x_m$ and $\text{FOV} = m \Delta x_m$ yields

$$T_z(x) = \exp\left(i\pi \frac{x}{\text{FOV}}\right) \sum_{\mu=-\infty}^{\infty} \Pi_z(-m/2, +m/2) \exp(-i \mu \Delta k_x x). \quad (\text{A.8})$$

Employing the Nyquist-Shannon sampling theorem, the sum can be represented by a sinc-

function $\text{sinc}(x) = \frac{\text{sinc}(x)}{x}$

$$T_z(x) = m \exp\left(i\pi \frac{x}{\text{FOV}}\right) \text{sinc}\left(\frac{\pi}{m} - \pi \frac{x}{\Delta x_m}\right) \quad (\text{A.9})$$

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Figure 1.

RASER pulse sequence. AM and FM: amplitude and frequency modulation function of the frequency-swept RF-pulses. G_f : frequency-encoding gradient for spatial encoding similar to an EPI-echo train. G_{SPEN} : SPEN gradients. The gradient moment is balanced using blipped gradients or a constant gradient during the echo train. G_s : slice-selective gradients. A double-spin echo is refocused using two 180° hyperbolic secant (HS1) pulses. The blue dashed lines mark the times when the magnetization, which is excited early during the chirp-pulse, is refocused and the red dashed lines when magnetization, which is excited at the end of the pulse, is in phase.

Figure 2.

The spatial modulation of the transverse magnetization and the apparent PSF in RASER and in Fourier-imaging. The displayed example is computed with an R -value of 255. The spatial dimension is scaled relative to the nominal spatial resolution $x_m = \text{FOV}/m$ ($m = 20$: number of echoes) in all plots. **(a)** Quadratic phase modulation of the transverse magnetization in RASER [eq. (4)]. **(b)** Examples of signal attenuations function of the quadratic phase profile in **(a)** obtained by averaging of isochromats of three different voxel sizes x (dark blue: $x = 0.203 x_m$, red: $x = 0.414 x_m$, light blue: $x = 0.625 x_m$). **(c)** Numerical integration of eq. (10) representing the apparent PSF Λ_R (red solid line) and its complement $\bar{\Lambda}_R$ (blue solid line). The vertical dotted lines mark the crossing of Λ_R and $\bar{\Lambda}_R$ which defines the effective resolution $x_{R \text{ eff}}$ [eq. (13)]. x' is defined as the modified spatial variable $x' = x_f/2$ where x_f is the fictitious voxel size over which the isochromats are averaged [eq. (4)]. **(d)** Simulated conventional complex-valued PSF in Fourier imaging using the single-point spin-density distribution in eq. (34). **(e)** The apparent PSF Λ_F (red solid line) and its complement $\bar{\Lambda}_F$ (blue solid line) in Fourier imaging obtained by numerical integration [eq. (21)] of the conventional PSF in **(d)**. The dashed vertical lines mark the effective resolution $x_{F \text{ eff}}$ [eq. (23)]. 20 spatial encoding steps are used. The spatial variable is defined as $x' = x_f/2$. **(f)** Comparison of the normalized apparent PSFs of RASER (red) and Fourier imaging (blue). The vertical dotted lines mark the effective resolution color-coded according to the corresponding apparent PSFs. All plots are obtained using the integration limits $\pm \text{FOV}/2$.

Figure 3.

Experimental results and Bloch simulations of the apparent PSF in RASER. **(a)** The modified RASER sequence which is used to simulate and to measure the complementary function of the apparent PSF of RASER $\overline{\Lambda_R}$. The readout gradient is removed and the second 180°-pulse is slice-selective in the former frequency-encoded dimension. The blipped gradients for SPEN during the readout train in Fig. 1 are replaced by a constant gradient to refocus magnetization continuously during the acquisition period ACQ. **(b)** Saline phantom with acrylic glass block. The SPEN-dimension of the acquired FOV

(yellow) is oriented perpendicular to the edge of the block. The red ROI marks the data used for the binned profile shown in (d) and (e). **(c)** Schematic of the complementary function of the apparent PSF $\overline{\Lambda_R}$ from Fig. 2c for qualitative comparison with results displayed in (d) and (e). **(d)** Bloch simulation of a plastic block immersed in saline (33) showing the signal intensity as a function of the SPEN dimension x . The extent of the aqueous solution is marked by the gray shaded area. The Bloch simulation used the sequence in (a). x is proportional to the time during acquisition. The nominal resolution x_m is defined in eqs. (14) and (15). Curves are simulated for various R -values listed in the inset. **(e)** Experimental results corresponding to the Bloch simulation in (d). The R -values used in the experiment (see inset for color-coding of the curves) are nine times smaller than for the Bloch simulation to account for a three times smaller FOV maintaining the same effective spatial resolution [eq. (13)]. The gray-shaded area marks the region covered by the aqueous solution.

Figure 4.

(a) Saline phantom with acrylic glass block as in Fig. 3b. A 2D GRE-image was acquired covering the whole phantom to avoid aliasing (yellow region). The phase-encoded dimension was oriented perpendicular to the edge of the block. (b) The profile was obtained by averaging the image data along the frequency-encoded dimension and binning in the phase-encoded dimension within the ROI marked in red in (a). The gray-shaded area marks the region covered by the aqueous solution. The labels 'Re' and 'Im' refer to real and imaginary part of the profile in the image domain. (c) Magnitude of the inverse Fourier

transform of the averaged profile in (b). The dashed lines represent two different levels $m_1 = 40$ (red) and $m_2 = 96$ (blue) of simulated truncation of the ‘acquired’ k-space data. The truncated data are zero-filled to the original number of points. **(d)** Profiles obtained by forward Fourier-transformation of the truncated k-space data in (c). The extent of the aqueous solution is marked by the gray shaded area. The solid and dashed lines represent the real and imaginary part of the profiles. The red and blue curves correspond to the truncation level of the same color-coding used in (c). The spatial dimension x is normalized with the nominal spatial resolution defined as $x_{m1} = \text{FOV}/m_1$ and $x_{m2} = \text{FOV}/m_2$. **(e)** Conventional PSF obtained by calculating numerically the derivative [eq. (32)] of the complex-valued profiles in (d). The solid and dashed lines represent the real and imaginary part of the conventional PSF. The same color-coding for the corresponding truncation levels are used as in (c). **(f)** Schematic of the theoretically derived conventional PSF from Fig. 2d displayed for qualitative comparison with the experimental results in (e). **(g)** Apparent PSFs Λ_F using the same color-coding for the truncation levels in (c). The curves were obtained by numerically integrating the experimental results in (e) according to eq. (21). **(h)** Schematic of the theoretically derived apparent PSF from Fig. 2e displayed for qualitative comparison with the experimental results in (g).