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Bayesian population reconstruction of female populations for less developed and more developed countries

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Abstract

We show that Bayesian population reconstruction, a recent method for estimating past populations by age, works in a wide variety of data quality contexts. Bayesian reconstruction simultaneously estimates age-specific population counts, fertility rates, mortality rates and net international migration flows from fragmentary data while formally accounting for measurement error. As inputs, Bayesian reconstruction takes initial bias reduced estimates of standard demographic quantities. We reconstruct the female populations of Laos, a country with little vital registration data where population estimation depends largely on surveys, Sri Lanka, a country with some vital registration data, and New Zealand, a country with a highly developed statistical system and high-quality vital registration data. In addition, we extend the method to countries without censuses at regular intervals. We also use it to assess the consistency between model life tables and available census data, and hence to compare different model life table systems.

Keywords

Bayesian hierarchical model; cohort reconstruction; demographic estimation; fertility; international migration; model life table; mortality; vital registration data

The release of *World Population Prospects 2010* (WPP 2010; United Nations 2011a) coincided with considerable interest in population statistics in both the popular and academic literature (e.g. Alberts 2011; Gillis and Dugger 2011; Reuters 2011; Phillips 2011; Nagarajan 2011; Scherbov et al. 2011). WPP 2010 was released in 2011, and the 31st October of that same year was the day on which the world population was predicted by the United Nations (UN) to reach 7 billion. Despite the precision implied in the declaration of “7 billion day”, there is considerable uncertainty about the sizes of national populations, past and present. In this article, we present evidence that Bayesian reconstruction, developed by Wheldon et al. (2010, 2012, 2013) for estimating past populations and vital rates by age, works in a wide variety of data quality contexts and, therefore, could feasibly be used to reconstruct the female population of any of the countries of the world.

Information about uncertainty can be conveyed by providing interval estimates, rather than simply point estimates as is done for many official statistical releases. Such intervals should have a probabilistic interpretation: they should contain the true value with some specified probability, conditional on the assumed statistical model. Wheldon et al.'s (2010, 2012, 2013) method produces such intervals. It reconstructs population structures of the past by embedding formal demographic relationships in a Bayesian hierarchical model. The outputs are joint probability distributions of demographic rates and population counts by age, from which fully probabilistic interval estimates can be derived in the form of Bayesian confidence intervals (or “credible intervals”). The method has been designed to deal with the lack of reliable data commonly experienced in many less developed countries, but here we show that it also works well for countries with very good data.

In this paper, we show that Bayesian reconstruction works well in a wide range of data quality contexts by reconstructing the populations of countries for which data quality varies from poor to extremely good. Wheldon et al. (2012, 2013) gave only a single example with real data, that of Burkina Faso. This country represents a “medium” case: the available data are fragmentary, but independent estimates of adult and child mortality are available. It is not clear from this single example how Bayesian reconstruction performs in different data quality contexts. In worse cases, for example, there may be insufficient information to yield useful interval estimates. In better cases, there may be little to gain from a probabilistic model. Through a set of case studies chosen specifically to reflect the range of data availability and quality contexts across all countries, we demonstrate that neither of these is true.

In all cases, Bayesian reconstruction indicates when estimates of vital rates are inconsistent with census results. This means that the method can be used to compare competing model life tables. We also extend the method to unevenly spaced censuses.

The remainder of the paper is structured as follows. In the next section we review existing methods of population reconstruction. Following that, we describe the method. Then we apply Bayesian reconstruction to the female populations of three countries: Laos, Sri Lanka and New Zealand. The New Zealand case shows that the model performs sensibly for countries with very good data and the Laos case shows that it works for very fragmentary data. We use the case of Sri Lanka to demonstrate our extension to unevenly spaced censuses. Bayesian reconstruction detected inconsistencies between survey-based estimates of fertility and intercensal population changes, and provided a correction. There is relatively little mortality data for Laos, and we use this case to illustrate how Bayesian reconstruction can be used to choose between competing model life tables. We conclude with a discussion.

Methods of population reconstruction

Many human population reconstructions in the demography literature fall into one of two categories: reconstruction of populations of the distant past using data of the kind commonly found in European parish registers (e.g. Lee 1971, 1974; Wrigley and Schofield 1981; Oeppen 1993a,b; Bertino and Sonnino 2003) and reconstruction of population dynamics after extreme crises such as famine or genocide (e.g. Boyle and Ó Gráda 1986; Daponte et

al. 1997; Heuveline 1998; Merli 1998; Goodkind and West 2001). General methodology has been developed primarily in the former context, the latter being necessarily focused on special cases. In some form or another, the cohort component method of population projection (CCMPP) (Lewis 1942; Leslie 1945, 1948) is central to almost all methods of population reconstruction.

Two significant developments were Lee's (1971, 1974) "inverse projection" and Wrigley and Schofield's (1981) "back projection". Inverse projection converts counts of births and deaths into the respective rates. Reconstruction proceeds forwards in time. Counts of baseline population and model age patterns of fertility and mortality are also required. Where at least two independent estimates of population size are available, net migration can also be estimated (Lee 1985). In contrast, back projection takes counts at the terminal year and then moves backward in time, reconstructing population counts and net migration along the way. Several iterations might be required to produce a satisfactory result. There was considerable debate about the efficacy of back projection, centred partly around identifiability issues that arise from trying to "resurrect" members of the open ended age group and simultaneously estimate fertility, mortality and migration rates (Lee 1985, 1993). Further developments are described by Barbi et al. (2004). Oeppen (1993a,b) and Bonneuil and Fursa (2011) frame reconstruction as a high dimensional optimisation problem. All of the above methods are deterministic and produce point estimates only.

Stochastic inverse projection (SIP) was proposed by Bertino and Sonnino (2003). It incorporates a specific kind of stochastic variation into the reconstruction, taking inputs similar to those required by inverse projection. Model age patterns of fertility and mortality are treated as individual-level probabilities of death rather than fixed, population level rates. Like its predecessors, SIP was designed to work with accurate time-series of total births and deaths. The uncertainty in the final estimates comes only from modelling birth and death as stochastic processes at the level of the individual (Lee 1998 called this "branching process uncertainty"). There is no allowance for measurement error in the data, nor is there any stochastic variation in the model fertility and mortality age patterns. For most less-developed countries, information about births and deaths is not highly accurate, and age patterns of births and deaths are known only approximately. In these cases, the uncertainty is due mainly to measurement error. In fact, even for well-measured populations, at the national level where counts are large, Lee (2004) and Cohen (2006) note that uncertainty due to stochastic vital rates is likely to be small relative to uncertainty due to measurement error; see also Pollard (1968).

The aim of Daponte et al. (1997) was to construct a history of the Iraqi Kurdish population from 1977 to 1990, a period during which it was the target of considerable state sponsored violence. A Bayesian approach was taken in which vital rates and population counts were modelled as probability distributions. Prior distributions for fertility and mortality rates were based on survey data and beliefs about the uncertainty founded on studies of the data sources, historical information and knowledge of demographic processes. Conclusions from estimated posterior distributions took the form of fully probabilistic interval estimates. This approach took account of uncertainty due to measurement error and made use of contextual knowledge to make up for fragmentary, unreliable data. However, there were some

restrictions, such as allowing mortality to vary only through the infant mortality rate and specifying fixed age patterns of fertility. Our approach is similar in spirit but more flexible as no model age patterns are assumed to hold throughout the period of reconstruction.

Bryant and Graham (2013) propose a Bayesian method for estimating and forecasting subnational population counts by combining official statistics with administrative data such as tax records, electoral rolls and school rolls. The data required to produce usable subnational estimates are unlikely to be available in many less developed countries, as is access to historical administrative data. Therefore, like the UN, we focus on national level estimates and do not use administrative data.

Method

Our overall goal is to show that Bayesian reconstruction is widely applicable, therefore we give a conceptual overview of the method here and save the mathematical details for the Online Resource. All computation was done using *R*, the freely available statistical software package (R Development Core Team 2012); Bayesian population reconstruction is implemented in the *R* package “popReconstruct”.

Description of the model

Bayesian reconstruction reconciles two different estimates of population counts, those based on adjusted census counts (or similar data) and those derived by projecting initial estimates of the baseline population forward using initial estimates of vital rates. Adjusted census counts are raw counts which have been processed to reduce common biases such as undercount and age heaping. Since projection is done using the CCMPP, the parameters for which we require initial point estimates are the CCMPP inputs, namely population counts for the baseline year, fertility rates, survival proportions and the net number of migrants, all by age group, over the period of reconstruction. Migration is treated in the same way as fertility, mortality and baseline population counts.

Initial judgements of the possible range of the measurement error variance for each parameter are also required for the purpose of defining prior distributions. These can be based on expert judgment or data such as surveys to assess the coverage of vital registration systems. Data and expert knowledge sufficient to generate these inputs are available for most countries from about 1960. The comparison is through a Bayesian hierarchical (or multilevel), statistical model which provides probabilistic posterior distributions of the inputs, as well as population counts at each projection step in the period of reconstruction.

Initial point estimates of the input parameters are derived from data. Baseline population estimates come from adjusted census counts (or similar sources), and fertility and mortality estimates come from surveys such as the Demographic and Health Surveys (DHSs) and from vital registration data. The model defines a joint prior distribution over these parameters which is parameterized by the initial point estimates and standard deviations. Typically, the initial point estimates will serve as the marginal medians of this distribution, but this is not a requirement. The standard deviations represent measurement uncertainty about the point estimates. These distributions induce a probability distribution on the

population counts at the end of each projection step within the period of reconstruction. We impose the necessary condition that the joint prior not lead to projected population counts that are negative. In practice, this truncates the marginal prior for net migration by ruling out extreme negative values.

Uncertainty about the true population numbers at the time of a census is also modelled by probability distributions. Census counts adjusted for bias are taken as the median of these distributions and measurement uncertainty is represented analogously by standard deviations. There are separate standard deviation parameters for fertility, mortality, migration and population counts. Therefore, uncertainty in each of these parameters remaining after adjustment and bias reduction is explicitly accounted for in the model. We discuss bias and measurement error uncertainty in greater detail below.

Calculation of initial estimates of fertility and survival may require the use of population counts at fixed points in time. It is important, however, that counts (adjusted or otherwise) after the baseline year not be used in any of the following ways. Intercensal survival rates should not be used to estimate mortality. Also, “residual” counts, the difference between census counts and counts based on a projection using fertility and mortality alone, should not be used to estimate migration. These techniques use intercensal changes to estimate vital rates and migration. Bayesian reconstruction uses these changes in the process of updating initial estimates of the vital rates and migration. Doing either of these things would amount to using the intercensal changes twice; once to derive the vital rate initial estimates and again to update them. This could lead to an underestimate of uncertainty, possibly to a large extent.

In standard Bayesian terms, treating the induced distribution of projected counts as a prior distribution and basing the likelihood function on the distribution of measurement errors in the census counts, Bayesian reconstruction yields a posterior distribution of the inputs via Bayesian updating. This distribution can be usefully summarized by marginal Bayesian confidence intervals for each input parameter which express uncertainty probabilistically. Furthermore, confidence intervals for age-summarized parameters such as the total fertility rate (TFR) and life expectancy at birth (e_0) can be obtained. Using simulation, Wheldon et al. (2010, 2012, 2013) found that Bayesian reconstruction produced well-calibrated marginal Bayesian confidence intervals. That is, p -percent Bayesian confidence intervals for each parameter of interest were found to contain the true value p percent of the time.

Often, projected counts based on a sample from the joint prior distribution of the input parameters will not equal the same-year adjusted census counts. This discrepancy is sometimes called an “error of closure” (Preston et al. 2001). The discrepancy could be reduced by manually making appropriate adjustments to any, or all, of the CCMPP input parameters and census counts. Many different combinations of adjustments will have the same effect on the discrepancy; for example, adding a migrant of age x has the same effect on the age- x population count as removing the death of a person of age x . We do not make such manual adjustments but define a posterior distribution over the CCMPP input parameters which assigns higher probability to those combinations leading to larger reductions in the discrepancy. This means that each age-time specific component of the

input parameters is not affected equally, but according to the effect it has on the joint posterior.

Reconstruction can be undertaken for any period at the beginning of which estimates of baseline population are available, and over which estimates of vital rates and international migration are available. However, when the period of reconstruction extends beyond the year of the most recent census, the posterior distribution for the period following the year of the most recent census will be based heavily on the prior distribution of the vital rates over that period. It would essentially be a projection with added uncertainty estimates, based heavily on the prior. Therefore, the information gain from reconstructing beyond the last census in addition to the information contained in the prior is likely to be minimal. For this reason, the periods of reconstruction in our case studies all end at the year of the most recent census.

Bias

Estimates of vital rates and population counts from surveys and censuses are susceptible to bias. For example, fertility rate estimates based on birth histories suffer from omission and misplacement of births due to recall error, and census counts may be biased due to undercount in certain age groups (Zitter and McArthur 1980; Preston et al. 2001). Bayesian reconstruction does not treat bias explicitly because demographic data differ markedly across parameters, time and countries. Many methods for estimating and reducing these biases have been proposed. Methods appropriate for adjusting census data will not, in general, be applicable to vital registration or survey data. Even within these broad categories, there is great variation among countries and time which makes development of a general approach infeasible. Therefore, the analyst applying Bayesian reconstruction will need to select bias reduction methods appropriate to the data being used.

Bias reduction for vital rates is often done using so-called “indirect” methods such as Brass’s (1964, 1996) *P/F* ratio methods, adjustment of siblings and birth histories (e.g., Obermeyer et al. 2010; Schoumaker 2011, 2013), Alkema et al.’s (2012) method for TFR and child mortality (Alkema and New 2014), and others found in standard texts and manuals (e.g., Shryock et al. 1980; United Nations 1983; Preston et al. 2001; Rowland 2003; Moultrie et al. 2013).

Population counts, typically based on censuses, play a central role in the reconstruction. Therefore, efforts should be made to reduce bias using, for example, post-enumeration surveys (PESs) (e.g., United Nations 2008, 2010). These are not available for many countries, however, making bias reduction more difficult. In these cases, the analyst will have to search for well-known examples of miscounting, such as undercounting of young adult males, over counting of age groups at ages of eligibility for state pensions, and heaping at ages ending in 0 and 5. Analysis of counts by single-year of age, calculating ratios of counts in adjacent age groups, and comparison with available data from administrative sources are examples of useful techniques (Moultrie 2013). Intercensal survival ratios can be examined for plausibility but the calculated ratios should not be used to derive initial estimates of survival. Where miscounts are indicated, methods of making defensible adjustments include taking plausible age or survival ratios and age distributions from similar

populations. Age heaping might be reduced by applying statistical smoothing techniques (Booth and Gerland 2015; United States Bureau of the Census 1985; United Nations 1955).

Measurement error uncertainty

Bias reduced initial point estimates of the CCMPP input parameters and adjusted population counts are still subject to measurement error; that is, variation that is non-systematic and cannot realistically be eliminated or otherwise modelled. In Bayesian reconstruction, measurement error is represented by the prior standard deviations of the initial estimates. In many cases there is not much data with which to estimate these parameters, but there is often relevant expert knowledge. This can be included by giving the variances themselves prior distributions and using the expert knowledge to set the fixed hyperparameters of these distributions. This defines a hierarchical model. To do this, we require a value for p in statements of the form “there is a 90 percent probability that the true fertility rates are within plus-or-minus p percent of the initial point estimates”, and similarly for survival proportions, migration proportions and population counts. We asked United Nations Population Division (UNPD) analysts to provide p , which we refer to as the “elicited relative error”.

Case studies

Our principal aim in this paper is to show that Bayesian reconstruction works across the range of data quality found in practice. To this end, we selected three countries based on the quality of their mortality rate data: 1) New Zealand, with complete vital rate data based on vital registration; 2) Sri Lanka with reasonably good vital rate data requiring only small adjustments; and 3) Laos with only limited under-five mortality estimates available and fertility data from a few demographic surveys. Thus we analyzed New Zealand with excellent data, Sri Lanka with intermediate data, and Laos with poor data. Wheldon et al. (2012, 2013) analyzed Burkina Faso which, in terms of data availability and quality, sits between Laos and Sri Lanka, having survey data on both adult and under-five mortality, but no vital registration data.

Each case is discussed separately below. We briefly describe the original data sources and the processes used to derive the initial estimates. We study the results for a number of age-summarized demographic parameters such as TFR, net number of migrants, e_0 and under-five mortality. We give the limits of 95 percent Bayesian confidence intervals of our initial estimates and the posterior distributions of selected parameters using the notation: “(lower, upper)”. We compare our results for fertility and mortality to those published in WPP 2010 for years with comparable estimates. WPP 2010 was based on a different procedure but the same data, so the comparison is useful.

We cover the highlights here. More detailed descriptions of the data sources, initial estimates and results are in the Online Resource, which also contains all results in comma-delimited files.

Laos, 1985–2005

Data and initial estimates—National censuses were conducted in 1985, 1995 and 2005. These data allowed us to reconstruct the female population between 1985 and 2005. We

used the census year counts in WPP 2010; there were no PESs, but the census counts were adjusted to compensate for undercount in certain age groups. Further details are in the Online Resource.

Initial estimates of age-specific fertility rates were based on direct and indirect estimates from the available surveys. Age-specific initial estimates were obtained by multiplying smoothed estimates of TFR by smoothed estimates of the age pattern of fertility. Due to the small number of data points, smoothing was done by taking medians across data source for each age-time period combination.

The only available mortality data were for infant and under-five mortality. Therefore our initial estimates came from Coale et al.'s (1983) West (CD West) model life tables with values of ${}_1q_0$ and ${}_5q_0$ close to those estimated from available data.

Elicited relative errors for population counts, fertility and mortality were set to 10 percent on the population count scale, the fertility rate scale and the survival proportion scale, respectively.

There was not much information about migration. To model this, we set initial point estimates to zero for all ages and time periods, but used an elicited relative error of 20 percent in terms of net number of migrants. This means that the *a priori* central 90 percent probability interval for net international migration ranges from -20 percent to 20 percent of the population for each age group.

Results—Figure 1 shows our prior and posterior distributions for TFR and e_0 , together with WPP 2010 estimates. The Bayesian reconstruction estimate of TFR differs from the initial estimates in the five-year periods beginning 1985, 1990 and 2000. While both imply consistent decreases in fertility, the initial estimates appear to be too high in all but the third five-year period. Our posterior intervals suggest a level of fertility more similar to WPP 2010, except that our estimates suggest that the acceleration in the decline begins one five-year period later.

Figure 1a shows that the posterior intervals are not constrained to lie inside the prior intervals. Moreover, the posterior intervals can be wider than the prior intervals. This is the case for age-specific fertility rates for Laos.

The prior distributions of TFR and e_0 are asymmetric because the age-specific parameters from which they were calculated, namely age-specific fertility rates and survival proportions, were modelled on different scales. Age-specific fertility rates were modelled on the log scale and age-specific survival proportions were modelled on the log-odds (logit) scale. Further details are in the Online Resource.

Sri Lanka, 1951–2001

Data and initial estimates—Censuses were conducted in Sri Lanka in 1953, 1963, 1971, 1981 and 2001 and so we reconstructed the female population between 1953 and 2001. We took population counts from WPP 2010 which were adjusted to account for under enumeration. Initial estimates of age-specific fertility rates were derived in a manner similar

to that used for Laos, although at the level of TFR we used loess (Cleveland 1979; Cleveland et al. 1992) to smooth multiple data points across time period. Initial estimates of age-specific survival proportions were based on abridged national life tables calculated from death registration and available surveys. Elicited relative errors for all of these parameters were set at 10 percent on the same scales as described above for Laos.

We used the same default initial estimate of international migration as for Laos. Luther et al. (1987) provide age-specific estimates for the periods 1971–1975 and 1976–1980 using census data as well as information about vital rates. Their results are not suitable as a basis for initial estimates because they were partly derived from intercensal changes in population counts; we used them for comparison instead. These results are in the Online Resources.

Interpolation to handle irregular census intervals—Wheldon et al. (2010, 2012, 2013) assumed that censuses were taken at regular intervals but there is an irregular gap between the 1963 and 1971 censuses. Therefore, we propose interpolating the CCMPP outputs on the growth rate scale such that they coincide with the census years. We explain by way of an example.

Consider the number in the population aged $[x, x + 5]$ for which we have a census based estimate at 1963 and another census-based estimate at 1971. Initial estimates for vital rates are available at 1963, 1968, 1973, and at subsequent five-year increments. The CCMPP can be used with these data to derive projected counts for this age group in 1968 and 1973. To compare the CCMPP output with the census counts at 1971, we assume that the growth rate for this age group, $r_{x,1968}$, was constant between 1968 and 1973, and estimate it from the projected counts. The estimate is then used to interpolate the CCMPP output to 1971. Using a “hat” () to denote “estimate”, this is compactly expressed as:

$$\hat{r}_{x,1968} = \frac{1}{5} \log \left(\frac{n_{x,1973}}{n_{x,1968}} \right)$$

$$\hat{n}_{x,1971} = (n_{x,1968}) e^{3\hat{r}_{x,1968}}$$

We use a similar method to extrapolate the population counts from the 1953 census back to 1951 using the 1953–1963 growth rate. Interpolating in this manner is adequate for periods of length less than five years.

Results—Posterior distributions for the mortality and migration parameters are summarized in the Online Resource. Our posterior estimates of mortality and migration agree closely with those in WPP 2010 and Luther et al. (1987).

Applying Bayesian reconstruction suggests, however, that the sources upon which the initial estimates of fertility rates were based are inconsistent with intercensal changes in the number of births (Figure 2). The posterior estimates of TFR from Bayesian reconstruction differ noticeably from the initial estimates in the periods 1951–1956 and 1956–1961 (posterior intervals (5.09, 5.69) and (5.21, 5.93); initial estimates 5.01 and 5.03 children per woman, respectively). Our method has automatically provided a correction which, in this case, yields results similar to the WPP 2010 estimates.

New Zealand, 1961–2006

Data and initial estimates—Census counts came from national censuses conducted every five years between 1961 and 2006. Initial estimates of fertility rates were calculated from published age-specific fertility rates (Statistics New Zealand 2011a) and numbers of births (Statistics New Zealand 2012) by age group of mother by year. Initial estimates for survival proportions were calculated from New Zealand life tables (Statistics New Zealand 2011b).

Information about the measurement errors of these parameters was available in the form of PESs and estimates of the coverage achieved by the birth and death registration systems. Elicited relative errors were based on this information and were set to 2.5 percent, one percent, and one percent for population counts, fertility and mortality, respectively. Elicitation was done on the same scales as those used for Laos.

Information about international migration is more reliable than in most other countries because New Zealand is a small island nation with a well-resourced official statistics system. The basis of our initial estimates of international migration were counts of permanent and long-term migrants (PLT) migrants taken from arrivals and departures cards (Statistics New Zealand 2010b). The largest source of error in these data as estimates of international migration is the discrepancy between the stated intentions and actual behaviour of travellers. To reflect this, we set the elicited relative error of the migration input parameter to five percent.

Results—The posterior distributions for TFR, e_0 and under-five mortality are summarized in the Online Resource. Our posterior estimates of mortality and fertility follow the initial estimates closely. This is not unexpected; the initial estimates were based on data of high quality and coverage. The least reliable data, *a priori*, were those for migration. Our posterior intervals suggest corrections in some time periods (Figure 3). The initial estimates for periods between 1961 and 1974 appear to be too high while those for periods between 1976 and 1989 are too low.

Choosing between alternative initial estimates of mortality

In our application to Laos we derived initial estimates of over-five mortality from the CD West model life table. This choice was made by UNPD analysts who drew on previous studies (Hartman 1996a,b; United Nations 2011b). However, other approaches are possible. Therefore, we compared the results above with those given by an alternative set of initial estimates of survival based on a different model life table, and used them to explain why the CD West model should be preferred. To do this, we looked at the age specific mortality rates, rather than e_0 .

The posterior distribution of e_0 in Figure 1b was computed from the posterior distribution of the age specific survival proportions, ${}_5S_x[t, t + 5]$, which are an output of Bayesian reconstruction. These were converted into age-specific annual mortality rates using the separation factors implicit in the CD West life table. Medians and limits of 95 percent Bayesian confidence intervals for the marginal posterior distributions of these parameters for

the period 1985–1989 are shown in Figure 4 on the logarithmic scale. Posterior uncertainty about these quantities is very low in this and all subsequent reconstructed periods (see the Online Resource); the mean half-widths of the 95 percent Bayesian confidence intervals of mortality rate over age, within year, are all less than 0.002 (raw scale). Mean half-widths of confidence intervals were derived by taking half the difference between the upper and lower limits of the intervals for each age group and time period, and then averaging over age groups and time periods.

An alternative set of initial estimates for the ${}_5S_x[t, t+5]$ was generated from the same data on under-five mortality, but adult mortality was estimated using the Brass two parameter relational logit model with the United Nations South Asian (UNSA) model life table, $e_0 = 57.5$ years (United Nations 1982). Figure 5 gives the initial estimates and marginal posteriors of the log mortality rates using these alternative survival estimates, but keeping the initial estimates of all other parameters the same. The posterior intervals of mortality rate are much wider under this set of initial estimates; the mean half-widths over age, within year, are between 0.015 and 0.025 (raw scale), a large increase.

The wider intervals show that using the alternative initial estimates greatly increases posterior uncertainty. In addition, for many of the older age groups, the posterior medians are actually closer to the CD West initial point estimates than those used to fit the model. While not a formal statistical test, this does suggest that the initial estimates based on the CD West life tables are more consistent with the intercensal changes in population counts, given the initial estimates for the other parameters, and that they are preferable to the UNSA-derived initial estimates.

Looking at e_0 in Figure 6 leads to the same conclusion. Again, uncertainty is much greater under the alternative set of initial estimates (cf. Figure 1b). The posterior distribution has shifted away from the initial estimates used to fit the model toward those derived from the CD West model life table. In fact, all the CD West initial point estimates are contained within the 95 percent posterior interval based on the alternative estimates, while this is not the case for the initial estimates used to fit the model.

We emphasize that our preferred set of initial estimates are those generated using the CD West standard. Our purpose here is not to advocate for the UNSA standard, or the Brass two-parameter logit model, but to present an alternative, plausible set of initial estimates which we can use to generate an alternative set of posterior estimates for use in a comparative analysis.

Discussion

We have shown that Bayesian reconstruction (Wheldon et al. 2010, 2012, 2013) works well when applied to different countries spanning a very wide range of data quality characteristics. For Laos, all mortality data are for ages five and below and come from surveys. New Zealand has complete period life tables based on vital registration. Burkina Faso (analyzed in Wheldon et al. 2012, 2013) and Sri Lanka lie between these extremes. Furthermore, the magnitude of the estimated uncertainty reflected the quality of the

available data. For instance, posterior intervals for New Zealand are much narrower than those for Laos, reflecting the greater accuracy and coverage of the New Zealand data.

From the single example presented by Wheldon et al. (2012, 2013), it was not clear that Bayesian reconstruction would produce useful estimates in cases of very fragmentary data such as Laos. Our results show that, even in a case where data on only infant and under-five mortality are available, the posterior uncertainty is small enough to make the results useful. New Zealand, in contrast, has very good data, but even here there is uncertainty that Bayesian reconstruction is able to estimate. That the method performed well in such a diverse range of cases suggests that it could feasibly be used to reconstruct the female population of any country. Bayesian reconstruction is likely to be of greatest value for those countries which lack comprehensive, reliable vital registration data, and where the uncertainty is therefore greater. Roughly half of all the countries and areas included in the WPP fall into this category (United Nations 2011a). Where data are even more fragmentary than for Laos, the uncertainty could be greater than that shown in our examples. Ultimately, of course, the quality of the outputs will be dependent on the quality of the inputs, including the bias reduction process and elicited relative errors.

Bayesian reconstruction embeds the standard CCMPP in a hierarchical statistical model which takes initial estimates of vital rates and population counts as inputs, together with expert opinion about their relative error (informed by data if available), and yields fully probabilistic interval estimates for all of the inputs. International migration is handled in the same way as the other inputs. The approach is Bayesian as the initial estimates serve as the medians of informative priors for CCMPP input parameters, hence population counts, which are then updated using population counts over the period of reconstruction. These latter population counts, typically derived from available census data, play a central role in the reconstruction, but they are not assumed to be known without error. Rather, measurement error uncertainty is accounted for through variance parameters in the same way as for the vital rates.

The age patterns in the initial estimates will be plausible *a priori* by construction, or possibly flat in the case of net migration, but there are no further impositions on age patterns during the modelling process. The posterior estimates will depart from the initial estimates only if there is strong evidence for this in the data. We did not observe this in our case studies, but we prefer not to exclude this possibility completely. In general, posterior distributions for age-specific parameters can be inspected for plausibility. We give these for our case studies in the Online Resource.

The posterior distributions will be somewhat sensitive to the elicited relative errors because the initial estimates themselves do not contain much information about measurement error. However, in cases like New Zealand, these are likely to be heavily based on data from register coverage studies and post-censal surveys. In countries without such studies, expert opinion and knowledge about the situation in neighbouring countries under similar conditions provide an important source of information. Elicited relative errors specified for each component can vary depending on the reliability of the information available. In both

cases, it is appropriate that these sources of information have some bearing on the resulting estimates.

In our case studies, the periods of reconstruction were delimited by the earliest and most recent censuses. Reconstruction can be done beyond the year of the most recent census if initial estimates of vital rates and international migration are available, but the posterior distribution for this latter period will not be updated and will depend heavily on the prior. In practice, therefore, a minimum of two sets of population counts are required for a useful reconstruction.

We have presented 95 percent Bayesian confidence intervals for the marginal distributions of TFR, total net number of migrants, e_0 and under-five mortality. Ninety-five percent intervals cover the range of most likely values. Results for TFR and age-specific fertility for Laos show that the posterior intervals are not constrained to lie inside prior intervals, nor are they necessarily narrower than prior intervals. Our posterior estimates of TFR for Laos and Sri Lanka suggest that, in some years, the initial estimates based mainly on surveys are inconsistent with intercensal changes in the number of births and Bayesian reconstruction is able to provide an appropriate correction.

In our studies of Laos and Sri Lanka the *a priori* central 90 percent probability interval for net international migration ranged from -20 percent to 20 percent of the population for each age group. This is very spread out and reduces the influence of the migration prior on the posterior relative to the priors for the other vital rate parameters and the population counts. Posterior Bayesian confidence intervals are much narrower, however, and posterior medians display a noticeable age-pattern despite one being absent from the prior. Further discussion, including age-specific posterior distributions for migration, is in the Online Resources.

The method as described in Wheldon et al. (2010, 2012, 2013) was limited by the fact that it required census data at regular intervals. Here, we relaxed this requirement by showing that linearly interpolating census counts on the growth rate scale produces good results.

We have also shown how Bayesian reconstruction might be used to help choose between two sets of initial mortality estimates. We compared the posterior distributions of age-specific mortality rates for Laos derived from initial estimates based on the CD West model life table and the Brass two-parameter relational logit with the UNSA model life table. In the latter case, the interval widths are much greater. This implies that the CD West based initial estimates agree more closely with the data on fertility, mortality and population counts and that they should be preferred.

Model life tables are a useful source of initial estimates of mortality in cases where data are fragmentary, but they do not need to be used in isolation. Where the analyst has additional information about mortality patterns, independent of intercensal population changes, this can be used in the reconstruction by incorporating it into the initial estimates. For example, knowledge about how wars and epidemics affect mortality age patterns could be used in cases where such events are known to have occurred. In countries experiencing substantial HIV/AIDS mortality, mortality rates from all causes of death based on vital registration or retrospective information from surveys can be used.

Bias and measurement error variance are handled separately under Bayesian reconstruction. Existing demographic techniques, such as indirect estimation via P/F ratios and model life tables, and adjustments using PESs, are used to reduce bias in initial point estimates and population counts based on raw data collected from surveys, vital registration and censuses. The nature of bias varies greatly across parameters, time and country, so we do not propose a general purpose method to replace the many existing techniques. Instead, the analyst is able to select the most appropriate technique for the data at hand. In the case of population counts, this may be difficult to achieve if PESs are not available and the analyst will have to search for well-known examples of miscounting, such as undercounting of young adult males, over-counting of age groups at ages of eligibility for state pensions, and heaping at ages ending in 0 and 5.

Measurement error variance is accounted for through the standard deviations of the initial point estimates. Expert opinion is used *a priori* to set reasonable ranges for measurement error uncertainty. Separate ranges for fertility, mortality, migration, and population counts are used to account for *a priori* uncertainty in initial estimates and population counts that remains after adjustment and bias reduction. These are then updated using all the available initial estimates and census counts.

Cross-sectional population counts over the period of reconstruction may be needed to construct initial estimates of vital rates and migration, but intercensal change quantities should not be used for this purpose. Therefore, forward or reverse survival methods, methods of estimating migration using “residuals” after projecting using fertility and mortality alone, and other techniques that use intercensal changes to estimate vital rates and migration are not compatible with Bayesian reconstruction. Using any of these could lead to an underestimate of uncertainty. If no reliable migration data are available, the default initial point estimates should be centred at zero for all ages and time periods with a large elicited relative error. If no reliable mortality data are available, a model life table must be chosen from a family of standards or derived using data from another country. In both cases, the resulting increase in uncertainty can be fed into the model via the elicited relative error. Since a key aim of Bayesian reconstruction is to quantify the uncertainty in population reconstructions, the usefulness of the method, *per se*, will not be compromised because posterior uncertainty will still be estimable, and this, itself, is important information.

Remaining bias in cross-sectional population counts used to construct initial estimates of vital rates and migration may be propagated to initial estimates themselves. For example, if death and population counts are used to produce initial estimates of survival, and population is under-counted but deaths are not, the resulting survival proportions will be too low. However, Bayesian reconstruction is done simultaneously across all age groups and time periods and population undercount tends to be limited to specific age ranges. In this case, the cohort will eventually age into an age group where bias is not significant and there will be an error of closure due to a discrepancy between the population count at this point and the counts implied by the projection of the under counted earlier population using under estimated survival. Bayesian reconstruction will respond by exploring all possible combinations of changes in fertility, survival, and migration across the age-time range of the reconstruction, assigning higher posterior probability to those combinations leading to larger

reductions in the discrepancy. Changes in the survival proportions for the period of population undercount will be included in this exploration and adjustments made according to the impact on the joint-posterior.

Bayesian reconstruction was developed and demonstrated for female-only populations. In Wheldon et al. (2015) we have extended the method to two-sex populations. A further potential refinement is to use single-year age groups and time periods.

A great deal of attention has already been directed at the estimation of uncertainty in demographic forecasts, as opposed to estimates about the past which we focus upon here. The study of stochastic models for forecasting dates back at least to Pollard (1966) and Sykes (1969). Further developments are reviewed by Booth (2006) with more recent additions in Hyndman and Booth (2008), Scherbov et al. (2011) and Alkema et al. (2011). One component of error in forecasts of population size is the error in estimates of population size and the vital rates prevailing at the baseline time. While the ergodic theorems of demography (Lotka and Sharpe 1911; Lopez 1961) imply that these become irrelevant if one forecasts far enough into the future, short term forecasts can be significantly affected (e.g., Keilman 1998; National Research Council 2000). It is possible, then, that Bayesian reconstructions could contribute to improved forecasting methods by providing important information about the uncertainty in estimates of baseline populations.

The fact that official statistical estimates are not perfect is undisputed. The UNPD acknowledges this both explicitly (United Nations 2011a) and implicitly in the fact that the WPP are revised biannually as new sources of data become available and methods are improved. Therefore, augmenting point estimates with quantitative estimates of their uncertainty is an important contribution. For many countries, the available data are fragmentary and subject to bias and measurement error, and so the expert opinions of demographers are valuable. A Bayesian approach is especially appropriate since this knowledge can be used in conjunction with the available data in a statistically coherent manner.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.

Prior and posterior medians and 95 percent Bayesian confidence intervals of selected parameters for the reconstructed female population of Laos, 1985–2005. Prior medians correspond to initial estimates. (a) Total fertility rate. (b) Female life expectancy at birth.

Source: Coale et al. (1983), 1986–1988 Multi-Round Survey, 1993 Laos Social Indicator Survey, 1994 Fertility and Birth Spacing Survey, 1995 and 2005 censuses, the 2000 and 2005 Lao Reproductive Health Surveys, United Nations (2011a), Bureau of East Asian and Pacific Affairs (2011), 2006 MICS3 survey.

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Figure 2.

Prior and posterior medians and 95 percent Bayesian confidence intervals and WPP 2010 estimates of total fertility rate for the reconstructed female population of Sri Lanka, 1950–2000. Prior medians correspond to initial estimates.

Source: 1953–2001 censuses, 1972–2006 vital registration, 1975 Sri Lanka WFS, 1987, 1993, 2000, 2006–07 Sri Lanka DHS, United Nations (2011a).

Figure 3.

Prior and posterior medians and 95 percent Bayesian confidence intervals of the total net number of female migrants (average annual) for the reconstructed female population of New Zealand, 1961–2006. Prior medians correspond to initial estimates.

Source: 1961–2006 censuses, 1961–2005 vital registration and international arrivals and departures, 1996, 2001, 2006 PESs, Statistics New Zealand (1998; 2010).

Figure 4.

Prior and posterior medians and 95 percent Bayesian confidence intervals of the age-specific log mortality rates for the reconstructed female population of Laos, 1985–1990. Prior medians correspond to initial estimates which were calculated using the CD West model life table. Note that the posterior Bayesian confidence intervals are the darker, inner bands in this plot.

Source: As for Figure 1a.

Figure 5.

Prior and posterior medians and 95 percent Bayesian confidence intervals of age specific log mortality rates for the reconstructed female population of Laos, 1985–2005. Prior medians correspond to initial estimates. Initial estimates and posterior distributions were calculated using the Brass two-parameter relational logit model with the United Nations South Asian (UNSA) model life table. Note that the posterior Bayesian confidence intervals are the lighter, wider bands in this plot.

Source: As for Figure 1a, United Nations (1982).

Figure 6.

Initial and posterior estimates of e_0 for Laos females, 1985–2005, using Brass two-parameter logit model and the United Nations South Asian (UNSA) model life table. This figure summarizes the results shown in Figure 5. Note that the posterior Bayesian confidence intervals are the darker bands in this plot.

Source: As for Figure 5.