Analysis of multicrystal pump–probe data sets. II. Scaling of ratio data sets

Bertrand Fournier,* Jesse Sokolow and Philip Coppens*

Chemistry Department, University at Buffalo, State University of New York, Buffalo, NY 14260-3000, USA.

*Correspondence e-mail: betrandf@buffalo.edu, coppens@buffalo.edu

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Two methods for scaling of multicrystal data collected in time-resolved photocrystallography experiments are discussed. The WLS method is based on a weighted least-squares refinement of laser-ON/laser-OFF intensity ratios. The other, previously applied, is based on the average absolute system response to light exposure. A more advanced application of these methods for scaling within a data set, necessary because of frequent anisotropy of light absorption in crystalline samples, is proposed. The methods are applied to recently collected synchrotron data on the tetra-nuclear compound $\text{Ag}_2\text{Cu}_2\text{L}_4$ with $L = 2$ -diphenylphosphino-3-methylindole. A statistical analysis of the weighted least-squares refinement residual terms is performed to test the importance of the scaling procedure.

1. Introduction

The relative scaling of different sets of data has been a subject of discussion since it was treated by Hamilton, Rollet and Sparks in the context of multi-level data collected with the photographic Weissenberg technique (Hamilton *et al.*, 1965) and modified by Fox & Holmes (1966). It recurs in time-resolved pump–probe experiments in which crystals often disintegrate before a full data set can be collected. The cause of the crystal damage is not always obvious, but can often be attributed to the laser exposure, sometimes to the X-ray beam exposure and, for crystals containing solvent molecules, to the loss of solvent. The need for relative data scaling is even more acute in X-ray free-electron laser (XFEL) experiments using a stream of liquid-jet-embedded nanocrystals. Each exposed particle provides a separate diffraction pattern, such that thousands of patterns on the same substance need to be analyzed. All techniques discussed in this publication were developed specifically for pump–probe Laue data using the RATIO method (Coppens *et al.*, 2009; Coppens & Fournier, 2015). This method consists in using the ratios of the intensities with and without light exposure $I^{\text{ON}}/I^{\text{OFF}}$ instead of the absolute intensities I^{ON} , and thus avoids the necessity to identify the wavelength for each of the reflection spots in the Laue pattern as well as differential absorption effects.

In previous papers we discussed specific aspects of the application of the RATIO method to the processing/analysis of the data prior to the refinement based on the observed ratios (Fournier & Coppens, 2014*b*), and defined different models for intensity ratios (Fournier & Coppens, 2014*a*). In the current paper we discuss application of a simple ratio model for analysis of system response anisotropy in a particular data set and scaling of multicrystal data sets.

2. Modeling of the intensity ratios

The ratio $R(\mathbf{H})$ of intensities is obtained by dividing the laser-ON intensity, $I^{\text{ON}}(\mathbf{H})$, by the corresponding laser-OFF one, $I^{\text{OFF}}(\mathbf{H})$,

$$R(\mathbf{H}) = \frac{I^{\text{ON}}(\mathbf{H})}{I^{\text{OFF}}(\mathbf{H})} = 1 + \eta(\mathbf{H}) \quad (1)$$

in which $\eta(\mathbf{H})$ is the relative change of intensity under light exposure.

2.1. General ratio model

The general expression of the modeled ratio of the reflection $\mathbf{H}$ in set i can be written as follows:

$$R_{\text{model}}^i(\mathbf{H}) = K_{\text{model}}^i(\mathbf{H}) S_{\text{model}}^i(\mathbf{H}) T_{\text{model}}^i(\mathbf{H}) \quad (2)$$

where $K_{\text{model}}^i(\mathbf{H})$ is the ratio scale factor, $T_{\text{model}}^i(\mathbf{H})$ is the thermal function representing the effect of the temperature increase and $S_{\text{model}}^i(\mathbf{H})$ is the structure-change function.

In the case of laser-ON and laser-OFF intensities collected separately, the ratio scale factor $K_{\text{model}}^i(\mathbf{H})$ for each observed reflection $\mathbf{H}$ in set i is a function of the absorption correction factors A , Lorentz–polarization factors (L_p) and incident intensity scaling factors K_{incident} of the laser-ON and laser-OFF intensities, respectively. It is defined as

$$K_{\text{model}}^i(\mathbf{H}) = \frac{L_p^{\text{ON}}(\theta) A^{\text{ON}}(\lambda, L) K_{\text{incident}}^{\text{ON}}}{L_p^{\text{OFF}}(\theta) A^{\text{OFF}}(\lambda, L) K_{\text{incident}}^{\text{OFF}}} \quad (3)$$

with θ the scattering angle of reflection $\mathbf{H}$ in set i , L its optical path length inside the sample and λ its scattering wavelength. Appropriate pre-scaling corrections must be performed in this case. However, if at each goniometer position the laser-ON and laser-OFF frames are collected consecutively with the same X-ray incident beam intensity and exposure time, and thus follow the strategy referred to as ON/OFF data collection, $K_{\text{model}}^i(\mathbf{H})$ equals one. Different RATIO models have been introduced by Fournier & Coppens (2014a). In this paper, only the simplest RATIO model is used. Its demonstration is summarized in the following subsections.

2.1.1. The light-induced structure change. In the random distribution (RD) model case assuming a small population P and the cluster formation (CF) model case (Vorontsov & Coppens, 2005), the structure-change function S_{model}^i of the reflection $\mathbf{H}$ in data set i can be modeled as

$$S_{\text{model}}^i(\mathbf{H}) = Q^i \eta_{\text{model}}^{\Delta T=0}(\mathbf{H}) + 1 \quad (4)$$

in which $\eta_{\text{model}}^{\Delta T=0}(\mathbf{H})$ is the averaged η without temperature increase, and Q^i is the relative excited-state (ES) population defined as $Q^i = P^i / \langle P \rangle$ with $\langle P \rangle$ the average ES population $\langle P^i \rangle_{i \in \{\text{sets}\}}$ over all the different ratio data sets.

2.1.2. The light-induced temperature change. The temperature-increase function $T_{\text{model}}^i(\mathbf{H})$ of the reflection $\mathbf{H}$ in data set i can be modeled in two different ways (Fournier & Coppens, 2014a). However, the simplest model, considered here, is obtained by assuming that the laser exposure results in

a global and isotropic increase of the temperature parameters. Thus, the thermal function becomes

$$T_{\text{model}}^i(\mathbf{H}) = \exp[-2\Delta B^i s^2(\mathbf{H})] \quad (5)$$

with $s(\mathbf{H}) = \sin \theta / \lambda$ and ΔB^i the isotropic average atomic B -factor increase in set i .

2.2. Approximated RATIO model for small geometric and thermal responses

In the case of small conversion, the ΔB -based ratio model can be approximated by a first-order Taylor expansion with respect to ΔB^i and P^i ,

$$R_{\text{model}}^i(\mathbf{H}) \simeq K_{\text{model}}^i(\mathbf{H}) [1 + Q^i \eta_{\text{model}}^{\Delta T=0}(\mathbf{H}) - 2\Delta B^i s^2(\mathbf{H})]. \quad (6)$$

If the thermal-increase factors ΔB and the ES populations P are assumed to be proportional, the constant $A_{\Delta B_p}$ can be defined for any data set i as

$$A_{\Delta B_p} = \frac{\Delta B^i}{P^i}. \quad (7)$$

This assumption is reasonable as a strong correlation between ES populations P and thermal-effect parameters ΔB is expected in the case of data sets collected with the same pump–probe delay.

The calculated η for a unique reflection $\mathbf{H}$ averaged over all sets, $\eta_{\text{model}}(\mathbf{H})$, is defined as

$$\eta_{\text{model}}(\mathbf{H}) = \eta_{\text{model}}^{\Delta T=0}(\mathbf{H}) - 2A_{\Delta B_p} \langle P \rangle s^2(\mathbf{H}) \quad (8)$$

which gives for the first-order Taylor expansion of the modeled ratio [expression (6)] of the reflection $\mathbf{H}$ in set i

$$\begin{aligned} R_{\text{model}}^i(\mathbf{H}) &= K_{\text{model}}^i(\mathbf{H}) \left\{ 1 + Q^i \left[\eta_{\text{model}}^{\Delta T=0}(\mathbf{H}) - 2 \frac{\Delta B^i}{Q^i} s^2(\mathbf{H}) \right] \right\} \\ &= K_{\text{model}}^i(\mathbf{H}) \{ 1 + Q^i [\eta_{\text{model}}^{\Delta T=0}(\mathbf{H}) - 2A_{\Delta B_p} \langle P \rangle s^2(\mathbf{H})] \} \\ &= K_{\text{model}}^i(\mathbf{H}) [1 + Q^i \eta_{\text{model}}(\mathbf{H})] \end{aligned} \quad (9)$$

in which the factor $K_{\text{model}}^i(\mathbf{H})$ is equal to one in the case of ON/OFF data collection.

3. Statistical consideration and data filtering

The data processing of a set of raw frames using the RATIO method produces as output a set of ratios R_{obs} with their corresponding Miller indices. Like any common set of intensity I_{obs} values, a RATIO data set needs to be analyzed and merged with respect to the point group of the studied system. For this purpose, the program *SORTAV* (Blessing, 1997) is used. However, an appropriate preselection is required with the use of observed ratios.

3.1. Ratio distribution

The ON/OFF ratio values are used as estimators of the ratios of expected laser-ON and laser-OFF intensities μ_{ON} and μ_{OFF} . This can be troublesome as the experimental ratio R is a biased estimator of $\mu_{\text{ON}}/\mu_{\text{OFF}}$. Indeed, the bias is due to the expected value μ_R of the variable R , the ratio of two inde-

pendent random variables I^{ON} and I^{OFF} , being different from the ratio of the expected values $\mu_{I^{\text{ON}}}$ and $\mu_{I^{\text{OFF}}}$. The probability density function (p.d.f.) of the ratio of two random normally distributed variables and its properties have been the subject of several publications (Marsaglia, 2006). No analytical formulas of the mean μ_R and the variance σ_R^2 are available. However, an approximation of the mean μ_R can be obtained by second-order Taylor expansion, assuming intensity standard deviations $\sigma_{I^{\text{ON}}}$ and $\sigma_{I^{\text{OFF}}}$ to be small.

For any pair of observed intensities ($I^{\text{ON}}; I^{\text{OFF}}$) with $\delta I^{\text{ON}} = I^{\text{ON}} - \mu_{I^{\text{ON}}}$ and $\delta I^{\text{OFF}} = I^{\text{OFF}} - \mu_{I^{\text{OFF}}}$, the corresponding observed ratio R can be rewritten as follows:

$$\begin{aligned} R &= \frac{I^{\text{ON}}}{I^{\text{OFF}}} \\ &= \frac{\delta I^{\text{ON}} + \mu_{I^{\text{ON}}}}{\delta I^{\text{OFF}} + \mu_{I^{\text{OFF}}}} \\ &= \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \left(\frac{1 + \delta I^{\text{ON}}/\mu_{I^{\text{ON}}}}{1 + \delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}}} \right). \end{aligned} \quad (10)$$

Assuming δI^{ON} and δI^{OFF} are small and share the same order of magnitude,

$$\begin{aligned} R &\simeq \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} (1 + \delta I^{\text{ON}}/\mu_{I^{\text{ON}}}) \\ &\quad \times \left[1 - \delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}} + (\delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}})^2 \right] \\ &\simeq \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \left[1 + \delta I^{\text{ON}}/\mu_{I^{\text{ON}}} - \delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}} \right. \\ &\quad \left. - (\delta I^{\text{ON}}/\mu_{I^{\text{ON}}})(\delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}}) + (\delta I^{\text{OFF}}/\mu_{I^{\text{OFF}}})^2 \right]. \end{aligned} \quad (11)$$

Thus, the expected value of an observed ratio R , μ_R , can be estimated, using $E(\delta I^{\text{ON}}) = 0$, $E(\delta I^{\text{OFF}}) = 0$, $E(\delta I^{\text{ON}}\delta I^{\text{OFF}}) = 0$ and $E(\delta I^{\text{OFF}2}) = \sigma_{I^{\text{OFF}}}^2$, as

$$\mu_R \simeq \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \left(1 + \frac{\sigma_{I^{\text{OFF}}}^2}{\mu_{I^{\text{OFF}}}^2} \right). \quad (12)$$

This expression shows that R tends to overestimate the ratio $\mu_{I^{\text{ON}}}/\mu_{I^{\text{OFF}}}$. The smaller the relative standard deviation $\sigma_{I^{\text{OFF}}}/\mu_{I^{\text{OFF}}}$, the more negligible the bias of the observed ratio becomes.

3.2. Data filtering criterion

In practice, the estimated standard deviation of I^{OFF} cannot be used. However, by propagation of errors, the standard deviation of R , σ_R , can also be approximated assuming $\sigma_{I^{\text{ON}}}/\mu_{I^{\text{ON}}} \simeq \sigma_{I^{\text{OFF}}}/\mu_{I^{\text{OFF}}}$,

$$\sigma_R \simeq \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \left(\frac{\sigma_{I^{\text{OFF}}}^2}{\mu_{I^{\text{OFF}}}^2} + \frac{\sigma_{I^{\text{ON}}}^2}{\mu_{I^{\text{ON}}}^2} \right)^{1/2} \simeq 2^{1/2} \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \frac{\sigma_{I^{\text{OFF}}}}{\mu_{I^{\text{OFF}}}}. \quad (13)$$

Therefore the following relation can be deduced from expressions (12) and (13):

$$\mu_R \simeq \frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \left[1 + \frac{\sigma_R^2}{2 \left(\frac{\mu_{I^{\text{ON}}}}{\mu_{I^{\text{OFF}}}} \right)^2} \right]. \quad (14)$$

Thus, selecting the observed ratios with the smallest estimated standard deviations would reduce the risk of biasing the ratios. In our pump-probe experiments, the standard deviation of an observed ratio is not estimated during the data processing. For this reason, in our data collection strategies, each ON/OFF frame pair is collected N times (up to 10). For each reflection $\mathbf{H}$, the statistical analysis of the sample of N observed ratios (up to 10) gives the measured average ratio and also its estimated uncertainty. These average measured ratios do not follow the distribution of a ratio of two independent random normal variables. Actually, according to the central limit theorem, the average measured ratio tends to follow a normal distribution when $N \rightarrow \infty$. However, the bias remains as the expected value $\mu_{(R)}$ of the sample average $\langle R \rangle$ will be equal to the common expected value μ_R of the observed ratios in the sample. Hence, a ratio filtering is still necessary. According to the relation (14), the ratio mean estimator bias is minimal for average measured ratios with small sample standard deviation s_R . At the end of the processing of frame data sets performed with the toolkit *LaueUtil* (Kalinowski *et al.*, 2011, 2012), only average measured ratios with $s_R \leq C$ in which C is a cutoff value, for instance $C = 0.2$, are selected.

4. Scaling methods

The correction for differences in system response between different ratio data sets is an important step in data processing. The reliability of photodifference maps strongly depends on the data set completeness and thus requires combining properly scaled partial ratio data sets. Moreover, performing a joint refinement of the structure model against a large series of small data sets can be technically impossible, making their scaling and merging necessary.

4.1. Average absolute system response (AASR) scaling method

In our previous pump-probe experiment studies (Makal *et al.*, 2011, 2012; Jarzemska *et al.*, 2014), the observed η values of different data sets are scaled using the AASR method.

This method consists of calculating the average absolute observed η value, noted $\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^i$, for each data set i and also the average absolute η value over all data sets, noted $\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^{\text{all}}$,

$$\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^i = \langle |\eta_{\text{obs}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}^i} \quad (15a)$$

$$\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^{\text{all}} = \langle |\eta_{\text{obs}}(\mathbf{H})| \rangle_{i \in \{\text{sets}\} \text{ and } \mathbf{H} \in \{\mathbf{H}\}^i}. \quad (15b)$$

Then, for each data set i , the scaling factor Z^i is given as

$$Z^i = \frac{\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^{\text{all}}}{\langle |\eta_{\text{obs}}| \rangle_{\mathbf{H}}^i}. \quad (16)$$

Thus, the η_{obs} values of each set i are scaled as follows:

$$\eta_{\text{obs}}^i \text{ scaled}(\mathbf{H}) = Z^i \eta_{\text{obs}}^i(\mathbf{H}). \quad (17)$$

The demonstration of this method requires several assumptions.

The ratio model defined by expression (9) is used and so the assumption of proportionality between the ES population P and the temperature-increase parameter ΔB is made. This method is usable either when the laser-ON and laser-OFF intensities are collected on the same sample, which leads to $K_{\text{ratio}}^i(\mathbf{H}) = 1$, or if the $K_{\text{ratio}}^i(\mathbf{H})$ factors have been estimated previously.

These simplifications give the following η model:

$$\begin{aligned} \eta_{\text{model}}^i(\mathbf{H}) &= R_{\text{model}}^i(\mathbf{H}) - 1 \\ &\simeq Q^i \eta_{\text{model}}(\mathbf{H}). \end{aligned} \quad (18)$$

The second assumption implies all data sets share the same set of reflections, $\{\mathbf{H}\}$. This is not a strict limitation in practice, as it would be sufficient if the sets count thousands of observations and share a significant fraction of common reflections.

For each data set i , the average of the $|\eta|$ values becomes proportional to the corresponding relative ES population Q^i ,

$$\langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^i = Q^i \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}}. \quad (19)$$

The second factor on the right-hand side of this expression is common to all data sets. The sum of all $|\eta|$ values from the different data sets, $\langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^{\text{all}}$, can be rewritten as the average value of the $\langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^i$ using the second assumption of a common set of reflections, $\{\mathbf{H}\}$, for all data sets. Thus, using expression (19), its expression can be simplified:

$$\begin{aligned} \langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^{\text{all}} &= \langle \langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^i \rangle_{i \in \{\text{sets}\}} \\ &= \langle Q^i \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}} \rangle_{i \in \{\text{sets}\}} \\ &= \langle Q^i \rangle_{i \in \{\text{sets}\}} \langle \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}} \rangle \\ &= \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}}. \end{aligned} \quad (20)$$

The following relation between the scaling factor Z^i and the relative ES population Q^i is deduced from expressions (19) and (20):

$$\begin{aligned} Z^i &= \frac{\langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^{\text{all}}}{\langle |\eta_{\text{model}}| \rangle_{\mathbf{H}}^i} \\ &= \frac{\langle \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}} \rangle}{Q^i \langle \langle |\eta_{\text{model}}(\mathbf{H})| \rangle_{\mathbf{H} \in \{\mathbf{H}\}} \rangle} \\ &= \frac{1}{Q^i}. \end{aligned} \quad (21)$$

Thus, for each reflection $\mathbf{H}$ of the data set i , the scaling factor corrects the difference of system response in $\eta_{\text{model}}^i(\mathbf{H})$ using expression (18) as follows:

$$\begin{aligned} \eta_{\text{model}}^i \text{ scaled}(\mathbf{H}) &= Z^i \eta_{\text{model}}^i(\mathbf{H}) \\ &= \eta_{\text{model}}^i(\mathbf{H}) / Q^i \\ &= \eta_{\text{model}}(\mathbf{H}). \end{aligned} \quad (22)$$

Hence, the observed η values, η_{obs} , of the reflection $\mathbf{H}$ in the data set i scaled using expression (17) become observations of $\eta_{\text{model}}(\mathbf{H})$.

The AASR method is a simple and fast scaling procedure of the light-induced system response.

4.2. Weighted least-squares (WLS) scaling method

4.2.1. WLS error function minimization. The method developed by Fox & Holmes (1966) for the scaling of unmerged observed intensity data sets is a good departure point to develop a more advanced RATIO scaling method. Let us assume N_{sets} ratio data sets collected on crystals of the same polymorph. This ratio scaling method minimizes the error function $\varepsilon_{\text{min}}^R$. It depends on the variable vector $\mathbf{x}$ consisting of the different RATIO model variables,

$$\begin{aligned} \varepsilon_{\text{min}}^R(\mathbf{x}) &= \sum_{i \in \{\text{sets}\}} \left(\sum_{\mathbf{H} \in \{\mathbf{H}\}_{\text{unique}}} \left\{ \sum_{j \in \{R_{\text{obs}}^i\}_{\mathbf{H}}} w_{\text{obs}}^{(i,j)}(\mathbf{H}) \right. \right. \\ &\quad \left. \left. \times \left[R_{\text{obs}}^{(i,j)}(\mathbf{H}) - R_{\text{model}}^i(\mathbf{H}) \right]^2 \right\} \right) \end{aligned} \quad (23)$$

where $\{\mathbf{H}\}_{\text{unique}}^i$ is the set of unique reflections of data set i , $\{R_{\text{obs}}^i\}_{\mathbf{H}}$ is the set of observed ratios of the unique reflection $\mathbf{H}$ in the data set i , $R_{\text{obs}}^{(i,j)}(\mathbf{H})$ the j th of these observed ratios, and $w_{\text{obs}}^{(i,j)}(\mathbf{H})$ the corresponding weight [by default $w_{\text{obs}}^{(i,j)}(\mathbf{H}) = 1/s_{R_{\text{obs}}^{(i,j)}(\mathbf{H})}^2$ with $s_{R_{\text{obs}}^{(i,j)}(\mathbf{H})}$ the estimated standard deviation of $R_{\text{obs}}^{(i,j)}(\mathbf{H})$].

The minimization of the function $\varepsilon_{\text{min}}^R$ must be done under constraints to avoid over-parametrization and to preserve the properties of the variables.

4.2.2. Constraints using the approximated ratio model. The constraint $m_1 = 0$ is applied to the relative populations Q^i to satisfy the property that their average value must be equal to 1.0:

$$m_1(\mathbf{x}) = \langle Q^i \rangle_{i \in \{\text{sets}\}} - 1. \quad (24)$$

If the global ratio scale factors K_{ratio} are refined, an extra constraint $m_2 = 0$ is necessary to properly scale the observed ratios. This constraint forces the intercept of the linear regression on the set of variables $\eta_{\text{model}}(\mathbf{H})$ to equal 1.0. For more details, see the supporting information.

4.3. Intra-dataset scaling

A scaling procedure can be applied to analyze and correct the orientation dependence of the light absorption and excitation by dividing a data set into several subsets, one subset per goniometer orientation (in our case, per φ -angle value). In the case of the WLS method, this 'intra-dataset' scaling can be done only if each goniometer orientation subset shares enough unique reflections with the others. With pink-Laue X-ray diffraction, the use of a polychromatic beam increases the effective mosaicity and causes a reflection to be recorded at several consecutive goniometer settings when the rotation angle step of a data collection run is small enough (less than 2°). To avoid an overcorrection of the system response anisotropy, a restraint can be added. Assuming the relative ES population Q profile as a function of φ to be continuous, the restraint on the variables Q can be defined as

Table 1

Description of the $\text{Ag}_2\text{Cu}_2\text{L}_4$ data sets.Set #1 collected with a pair redundancy $\langle N \rangle = 10$ and a φ -angle step $\Delta\varphi = 1^\circ$; all others collected with $\langle N \rangle = 5$ and $\Delta\varphi = 2^\circ$.

Set	Experiment			Statistics				
	Power (mJ mm ⁻²)	Strategy angle range	Usable angle range	No. observed ratios	No. unique <i>hkl</i>	Completeness (%)	Redundancy	Max. resolution
1	0.50	0–90	0–18; 20–69	9403	3158	36.8	3.0	0.526
2	0.40	0–90	all	5243	2887	24.2	1.8	0.587
3	0.25	0–180	all	12708	5295	39.2	2.4	0.611
4	0.25	0–90	all	6744	3621	29.8	1.9	0.591

$$r^Q(\mathbf{x}) = \frac{\sum_{i=1}^{N_{\text{sets}}-1} (Q_{i+1} - Q_i)^2}{N_{\text{sets}} - 1}. \quad (25)$$

The term $r^Q(\mathbf{x})$ is added to the minimization function $\Phi_{\min}^R$ and multiplied by a weight factor w^Q :

$$\Phi_{\min}^R(\mathbf{x}) = \varepsilon_{\min}^R(\mathbf{x}) + w^Q r^Q(\mathbf{x}). \quad (26)$$

The larger the restraining weight, the smaller the short-range fluctuations of the Q profile will be.

4.4. Implementation of the WLS scaling technique

The WLS scaling technique has been implemented as a FORTRAN90 program named *RSCALING*. The minimization of the WLS function $\Phi_{\min}^R$ under constraints is performed using the augmented Lagrangian method developed and implemented in the library *ALGENCAN* by Andreani *et al.* (2007) under the GNU General Public License.

At convergence, the standard deviations of relative populations Q and, if refined, of global ratio scale factors K_{ratio} are estimated at the minimum $\tilde{\mathbf{x}}$ of $\Phi_{\min}^R$ by inversion of its bordered Hessian matrix. For more details, see the supporting information.

After the data scaling, each ratio data set is analyzed for outlier detection and merged over equivalent observations using the program *SORTAV* (Blessing, 1997).

Figure 1
Number of unique reflections per (φ angle) goniometer setting (blue solid line) and number of reflections collected at least at two goniometer settings and so useful for the WLS scaling method (red solid line).

5. Applications

The tetra-nuclear complex $\text{Ag}_2\text{Cu}_2\text{L}_4$ with $L = 2$ -diphenylphosphino-3-methylindole (Jarzemska *et al.*, 2014) studied recently in our group is selected as an example. The determination of its structure under light excitation shows a remarkable structural change due to ligand–metal charge transfer (LMCT) with a shortening of the argentophilic contact $\text{Ag}_1 \cdots \text{Ag}_2$ of 0.38 (3) Å.

5.1. Data sets collected by time-resolved photocrystallography experiments

Time-resolved single-crystal Laue X-ray data were collected at the 14-ID BioCARS beamline of the Advanced Photon Source (Chicago). For the current study, each raw frame data set has been reprocessed using the latest version of the toolkit *LaueUtil* (Kalinowski *et al.*, 2011, 2012; Coppens & Fournier, 2015). The laser-ON structure model defined in the original paper, which assumes a random distribution (RD) of the excited molecules (Vorontsov & Coppens, 2005), is used. The characteristics for each data set are summarized in Table 1. All data sets have been collected with a frame-pair redundancy per goniometer setting (N) of 5 and a φ goniometer angle step $\Delta\varphi$ of 2° except set #1 for which $\langle N \rangle = 10$ and $\Delta\varphi = 1^\circ$.

5.1.1. Intra-set scaling of data set #3. The intra-set scaling procedure is here illustrated in detail for data set #3 which has the highest maximal resolution and the largest completeness,

Figure 2
Maximal resolutions per (φ angle) goniometer setting considering all observations (blue solid line) and only the reflections appearing at least at two goniometer settings, and thus useful for the WLS scaling method (red solid line).

but also the largest φ -angle range with 180° . Fig. 1 shows (blue solid line) the number of reflections collected at each goniometer setting (φ angle) in data set #3. The number of observations per goniometer setting varies between 110 and 175 with an average of about 140. The WLS scaling method is performed considering only reflections collected at multiple goniometer settings as the others with a unitary redundancy do not provide any information. This does not decrease significantly the number of reflections as illustrated by a red line in Fig. 1. For this reason, the maximal resolution per subset is not affected significantly by selecting exclusively redundant observations with an average decrease of around $0.01\text{--}0.02\text{ \AA}$ as shown in Fig. 2.

The crystals of $\text{Ag}_2\text{Cu}_2\text{L}_4$ have a low symmetry (space group $P\bar{1}$), which limits the redundancy of the collected data sets (Table 1). However, the intra-set scaling procedure can still be performed. This is illustrated by the square map (Fig. 3) in which each pixel element of coordinates $(\varphi_i; \varphi_j)$ is colored with respect to the number of common unique reflections $N_{i,j}$ between the two subsets i and j collected, respectively, at the φ angles φ_i and φ_j . Diagonal elements illustrate numbers of unique reflections in the different subsets. The elements close to the diagonal appear in orange ($N_{i,j}$ around 40), clearly indicating largest numbers of common observations. It means each goniometer setting subset i shares its unique reflections mainly with the previous and next setting subsets $i-1$ and $i+1$ which is expected when using the pink-Laue technique.

Both scaling methods introduced in §4 have been applied. The series of relative ES populations Q are plotted in Fig. 4 and are characterized by a long-range sine-like profile which reveals a system response anisotropy with respect to the crystal orientation.

Figure 3
Representation in color of the number $N_{i,j}$ of common unique reflections for each pair $(i; j)$ of subsets collected at φ angles $(\varphi_i; \varphi_j)$. The diagonal elements correspond to the numbers of unique reflections in the different subsets. The color bar is defined with a quasi-logarithmic scale $[\log(N_{i,j} + 1)]$.

The Q series obtained by applying the AASR scaling (black line) agrees qualitatively with the three other series obtained using the WLS method, although the amplitude of the AASR Q series is narrower with extremum values around 0.57 and 1.64. The WLS Q series have been obtained for different restraining weights: 0, 5000 and 25 000. The restraint is applied on the relative populations Q to smooth their Q profile (§4.3). With a weight of 25 000, only the long-range changes are observed with extremum values around 0.22 and 1.58. As the system response anisotropy is expected to be related to physical properties, Q series are assumed to be a continuous function of the goniometer setting and thus the one obtained with the largest restraining weight is selected to avoid any overcorrections during scaling.

5.1.2. Intra-set scaling of other $\text{Ag}_2\text{Cu}_2\text{L}_4$ data sets. The procedure described in the case of set #3 has been applied for the other data sets listed in Table 1.

During application of the WLS scaling procedure for each data set independently, the restraining weight has been optimized empirically to preserve only the long-range fluctuations of their Q series.

The effect of the intra-set scaling on data merging can be globally estimated using merging R -based indicators of quality. Two unbiased indicators $w\mathcal{R}_1$ and $w\mathcal{R}_2$ are used and defined as follows:

$$w\mathcal{R}_1(R) = \frac{\sum_{\mathbf{H} \in \{\mathbf{H}\}} \left(\frac{N_{\mathbf{H}}}{N_{\mathbf{H}} - 1} \right)^{1/2} \left\{ \sum_{j \in \{R_{\text{obs}}\}_{\mathbf{H}}} [w_{R_{\text{obs}}}^j(\mathbf{H})]^{1/2} |R_{\text{obs}}^j(\mathbf{H}) - \langle R_{\text{obs}} \rangle(\mathbf{H})| \right\}}{\sum_{\mathbf{H} \in \{\mathbf{H}\}} \left[\sum_{j \in \{R_{\text{obs}}\}_{\mathbf{H}}} \sum_{\mathbf{H}} [w_{R_{\text{obs}}}^j(\mathbf{H})]^{1/2} |R_{\text{obs}}^j(\mathbf{H})| \right]} \quad (27)$$

and

$$w\mathcal{R}_2(R) = \left\{ \frac{\sum_{\mathbf{H} \in \{\mathbf{H}\}} \frac{N_{\mathbf{H}}}{N_{\mathbf{H}} - 1} \left[\sum_{j \in \{R_{\text{obs}}\}_{\mathbf{H}}} w_{R_{\text{obs}}}^j(\mathbf{H}) |R_{\text{obs}}^j(\mathbf{H}) - \langle R_{\text{obs}} \rangle(\mathbf{H})|^2 \right] \right\}^{1/2} \frac{1}{\sum_{\mathbf{H} \in \{\mathbf{H}\}} \left[\sum_{j \in \{R_{\text{obs}}\}_{\mathbf{H}}} \sum_{\mathbf{H}} w_{R_{\text{obs}}}^j(\mathbf{H}) R_{\text{obs}}^j{}^2(\mathbf{H}) \right]} \quad (28)$$

Generally speaking, the differences in $\mathcal{R}$ -factor values between the different cases are quite small. This can be explained by the limited range of the anisotropic system response and the low redundancy of the $\text{Ag}_2\text{Cu}_2\text{L}_4$ data sets (Table 1). The intra-set WLS scaling method without restraint gives the smallest $\mathcal{R}$ -factor values, which is expected as the WLS method explicitly reduces discrepancies between equivalent observations. As explained previously (§4.3), it is recommended to perform the WLS intra-set scaling with restraint to reduce short-range system response fluctuation and thereby limit the risk of overcorrection. The intra-set scaling procedure using AASR or restrained WLS reduces the $\mathcal{R}$ -factor values for all data sets except the $w\mathcal{R}_2$ $\mathcal{R}$ factor of data set #1 (Tables 2 and 3). The comparison of $\mathcal{R}$ factors shows that the restrained WLS scaling technique does not usually lead to smallest values. This is attributed to the AASR

Table 2
Indicators of quality $w\mathcal{R}_1$ per data set without and with intra-set scaling performed using AASR or WLS.

For each data set, we give in bold the smallest $w\mathcal{R}_1$ among all excluding the one obtained using WLS without restraint (given in *italics*).

Set	No intra scaling	AASR unrestrained	WLS unrestrained	WLS restrained
1	1.466%	1.459%	<i>1.390%</i>	1.502%
2	1.464%	1.445%	<i>1.385%</i>	1.456%
3	2.384%	2.247%	<i>2.121%</i>	2.197%
4	1.793%	1.785%	<i>1.736%</i>	1.786%

Table 3
Indicators of quality $w\mathcal{R}_2$ per data set without and with intra-set scaling performed using AASR or WLS.

For each data set, we give in bold the smallest $w\mathcal{R}_2$ among all excluding the one obtained using WLS without restraint (given in *italics*).

Set	No intra scaling	AASR unrestrained	WLS unrestrained	WLS restrained
1	1.659%	1.672%	<i>1.459%</i>	1.698%
2	1.479%	1.440%	<i>1.257%</i>	1.469%
3	2.425%	2.259%	<i>1.931%</i>	2.115%
4	1.781%	1.763%	<i>1.653%</i>	1.758%

scaling method being applied without restraint on the resulting Q series. For set #3 presenting the largest system response fluctuations, the intra-set scaling quite noticeably decreases $w\mathcal{R}_2$ from 2.425% to 2.259% with the AASR method and to 2.115% with the WLS scaling (Table 3).

5.1.3. Inter-dataset scaling. As mentioned in the methodology section (§4), these methods have been initially developed to scale data sets collected on different crystal samples and/or with different experimental settings before data merging. The inter-set scaling can be performed with prior intra-set scaling applied to each data set independently as illustrated in Fig. 5.

The relative populations Q obtained by inter-set scaling after intra-set scaling or not are compared in Table 4. A first look at the Q_{inter} values shows the WLS method leads to

Figure 4
Series of relative ES populations Q , as a function of the goniometer setting subsets. The Q -factor series obtained using the AASR scaling technique is plotted as a black solid line. The three dot-line combinations represent series of Q factors obtained using the WLS scaling method with different restraining weights: 0 in blue, 5000 in red and 25 000 in green.

Table 4
Relative ES population Q obtained by inter-set scaling with and without intra-set scaling using AASR or restrained WLS.

Set	AASR inter	AASR intra/inter	WLS inter	WLS intra/inter
1	0.665	0.668	0.492 (9)	0.56 (1)
2	0.984	0.987	1.09 (1)	1.12 (1)
3	1.278	1.267	1.270 (9)	1.209 (9)
4	1.074	1.079	1.14 (1)	1.10 (1)

somewhat larger ranges of Q values. The most significant difference between the Q_{inter} values obtained without intra-set scaling is for set #1 with a decrease of its Q value using the WLS method. Using the AASR method requires in theory data sets sharing the same set of reflections (§4.1). Even though this condition is not so strict in practice, set #1 is distinguishable by its smaller maximal resolution of only 0.526 Å (Table 1) which could explain this result. Application of the intra-set scaling does not change the Q_{inter} values in the case of the AASR method. In contrast, the intra-set scaling decreases the Q_{inter} -value range using the WLS method, with minimal and maximal Q_{inter} values of 0.56 (1) and 1.209 (9), respectively, after intra-set scaling, compared with 0.429 (9) and 1.270 (9) without such scaling.

It is of interest to compare the relative populations obtained for the different data sets with or without intra-set scaling. In the case of intra-set scaling followed by inter-set scaling, for each data set i the effective relative population $Q_{\text{eff}}^{(i,j)}$ of the subset j , collected at a specific φ angle φ_j , is defined as $Q_{\text{eff}}^{(i,j)} = Q_{\text{inter}}^i Q_{\text{intra}}^j$, the product of the inter-set scaling relative population Q_{inter}^i and its subset j intra-set population Q_{intra}^j . The Q_{inter} values and Q_{eff} series obtained, respectively, by inter-set scaling and consecutive intra- and inter-set scalings are illustrated in Fig. 6.

The four graphs in Fig. 6 are plotted sharing the same Y axis. As for data set #3 (§5.1.1), the WLS method applied on the other sets gives Q_{eff} series with larger fluctuations. Set #1,

Figure 5
Illustration of the intra + inter scaling procedure applied to several data sets.

regardless of its short φ -angle range of 65° , shows a large Q_{eff} -series increase from 0.5 to 1.27. Both data sets #2 and #4 have Q_{eff} series decreasing according to the WLS scaling method while the AASR method does not show clear evidence of Q -profile long-range variations. The changes in the intra/inter scaling effective Q series are large compared with the range of the inter-set scaling Q variations. This indicates the system response anisotropy within a data set is not negligible and hence the importance of a careful data analysis.

5.2. Light-induced structure model refinement

Several joint refinements have been performed using the program *LASER2010* (Vorontsov *et al.*, 2010) against the four data sets merged independently after various types of scaling.

5.2.1. Structure changes. The structure model refined against data sets without intra-set scaling discussed here differs from the model reported by Jarzemska *et al.* (2014), which is due to the raw data processing being redone with different filtering criteria, altering the WLS error function minimum. Even though their differences are within the estimated uncertainties, general tendencies can be observed. In the newly refined model, the ES populations are somewhat smaller (Table 5) while the Ag and Cu atomic shifts become

slightly larger (Table 6). The comparison of the newly reported structure models refined jointly against the merged data sets with or without intra-set scaling shows that they are equivalent. The only remarkable differences are the small increase of the ES populations P and thermal factors k_B for data sets #2 and #3, and the increase of the atom Cu₂ shift when using intra-set scaling.

The shortenings of interatomic distances $\text{Ag}_1 \cdots \text{Cu}_2$ and $\text{Ag}_2 \cdots \text{Cu}_1$ under light exposure are slightly more pronounced when the intra-set scaling is performed, particularly using the WLS method (Table 7).

The application of intra-set scaling does not significantly alter the structure model which is expected as the $\text{Ag}_2\text{Cu}_2\text{L}_4$ compound presents a remarkable structural change under light exposure with a considerable shortening of the contact $\text{Ag}_1 \cdots \text{Ag}_2$ [-0.38 (3) Å as recalled in Table 7]. Moreover, set #3 is the only data set which has large fluctuations of system response with respect to the goniometer setting as explained in §5.1.3. Still, the intra-set scaling has an influence on the WLS refinement validation as discussed in the following section.

5.2.2. Validation by statistical analysis of residuals. Each structure model is refined by minimizing the error function $\varepsilon_{\text{min}}^R$,

Figure 6

Series of effective relative ES populations Q obtained by inter-set or intra/inter scaling using both methods AASR and WLS. For each data set graph, the horizontal lines represent its unique Q populations obtained by inter scaling, and the dot-lines the effective Q series deduced from intra/inter scaling. The Q populations obtained using the methods AASR and WLS are, respectively, in black and red. Note: the inter scaling horizontal lines superimpose for set #3.

$$\varepsilon_{\min}^R(\mathbf{X}) = \sum_{i \in \{\text{sets}\}} \left\{ \sum_{\mathbf{H} \in \{\mathbf{H}\}_{\text{unique}}^i} [r_w^i(\mathbf{H}, \mathbf{X})]^2 \right\} \quad (29)$$

with $\mathbf{X}$ the vector of model variables, and for each unique reflection $\mathbf{H}$ in set i , the weighted residual term $r_w^i(\mathbf{H}, \mathbf{X})$ defined as

$$r_w^i(\mathbf{H}, \mathbf{X}) = \frac{[R_{\text{obs}}^i(\mathbf{H}) - R_{\text{calc}}^i(\mathbf{H}, \mathbf{X})]}{s_{R_{\text{obs}}^i(\mathbf{H})}} \quad (30)$$

where $R_{\text{obs}}^i(\mathbf{H})$ and $R_{\text{calc}}^i(\mathbf{H}, \mathbf{X})$ are, respectively, the observed and calculated ratio values of the unique reflection $\mathbf{H}$ in set i , and $s_{R_{\text{obs}}^i(\mathbf{H})}$ the estimated standard deviation of the observed ratio.

Figure 7
Average of weighted residual terms calculated at each φ angle of set #3 considering the collected set of unique reflections. The series of average residuals are deduced from the model of each *LASER2010* refinement without intra-set scaling in blue dots (a), with intra-set scaling using the AASR method in red (b) and using WLS in green (c). $\pm 3\sigma$ error bars are added.

Table 5
ES populations P and thermal factors k_B obtained by joint model refinements done with the program *LASER2010* (Vorontsov *et al.*, 2010) against independently merged data sets with and without intra-set scaling performed using AASR or restrained WLS.

The P and k_B reported in the original paper (Jarzembska *et al.*, 2014) are given.

Set	Reported in paper	No scaling	AASR method	WLS method
P^1	0.49 (4)	0.46 (4)	0.46 (4)	0.46 (4)
P^2	1.07 (6)	0.97 (5)	0.97 (4)	1.02 (5)
P^3	0.73 (4)	0.66 (4)	0.72 (4)	0.75 (4)
P^4	1.00 (5)	0.94 (5)	0.94 (4)	0.94 (4)
k_B^1	1.035 (1)	1.035 (1)	1.035 (1)	1.036 (1)
k_B^2	1.054 (2)	1.054 (2)	1.055 (2)	1.054 (2)
k_B^3	1.064 (1)	1.065 (2)	1.067 (1)	1.068 (1)
k_B^4	1.047 (2)	1.049 (2)	1.049 (2)	1.049 (2)

Table 6
Atomic shifts in the refined structure models obtained by joint model refinements performed with the program *LASER2010* (Vorontsov *et al.*, 2010) against independently merged data sets with and without intra-set scaling using AASR or restrained WLS.

The atomic shifts reported in the original paper (Jarzembska *et al.*, 2014) are given.

Set	Reported in paper	No scaling	AASR method	WLS method
Ag_1	0.30 (2)	0.31 (2)	0.31 (2)	0.32 (2)
Ag_2	0.27 (1)	0.27 (2)	0.27 (2)	0.27 (2)
Cu_1	0.09 (2)	0.13 (3)	0.12 (3)	0.12 (2)
Cu_2	0.33 (2)	0.34 (3)	0.35 (3)	0.36 (2)

From a mathematical point of view, a refinement by least-squares fitting is valid if at convergence its weighted residual terms $r_w^i(\mathbf{H}, \mathbf{X})$ [expression (30)] are random and share the same probability distribution, presumably Gaussian. Their independence is expected when the number of observations N_{obs} is significantly larger than the number of variables N_X ($N_{\text{obs}} > 10N_X$). Recent studies by Henn & Meindl (2014*a,b*, 2015) emphasize the relevance of statistical analysis of weighted residual terms to identify systematic errors in data sets or model refinement bias.

As discussed in §5.1.1 and §5.1.3 and illustrated by Figs. 4 and 6, the pre-analysis of the observed ratio data sets before data reduction reveals a dependence of the system response on the goniometer setting (in this case, the φ angle). For each data set, a first step to evaluate the impact of this anisotropy on the residual term distribution is calculating the average weighted residual value per φ angle. For a given set, at each φ angle, the average weighted residual term is calculated over the corresponding subset of unique reflections collected. The plots obtained for set #3 are shown in Fig. 7. Following Henn & Meindl (2015), for each subset the average residual uncertainty is estimated as three times the sample mean standard deviation (s^2/N)^{1/2} with s^2 the sample unbiased variance and N the number of elements in the subset.

It is represented by error bars in Fig. 7. Each average residual is expected to be equal to zero with error fluctuation. This is not the case in the plot (Fig. 7*a*) in which the average residuals obtained without intra-set scaling show a sine-like

Table 7

Interatomic distances in the ground-state (GS) and excited-state (ES) conformations and elongation/shortening under light exposure of the different refined structure models.

The joint refinements have been performed with the program *LASER2010* (Vorontsov *et al.*, 2010) against independently merged data sets with and without intra-set scaling using AASR or restrained WLS. The atomic shifts reported in the original paper (Jarzembska *et al.*, 2014) are given.

	$\text{Ag}_1 \cdots \text{Ag}_2$			$\text{Ag}_1 \cdots \text{Cu}_1$			$\text{Ag}_1 \cdots \text{Cu}_2$		
	GS	ES	Δ	GS	ES	Δ	GS	ES	Δ
Reported	3.0345 (2)	2.66 (3)	−0.38 (3)	2.7640 (2)	2.82 (3)	0.06 (3)	3.4655 (2)	2.88 (3)	−0.59 (3)
None	3.0345 (2)	2.63 (4)	−0.40 (4)	2.7640 (2)	2.82 (3)	0.05 (3)	3.4655 (2)	2.86 (3)	−0.61 (3)
AASR	3.0345 (2)	2.62 (3)	−0.41 (3)	2.7640 (2)	2.80 (3)	0.04 (3)	3.4655 (2)	2.85 (3)	−0.62 (3)
WLS	3.0345 (2)	2.62 (3)	−0.41 (3)	2.7640 (2)	2.81 (3)	0.04 (3)	3.4655 (2)	2.82 (3)	−0.65 (3)

	$\text{Ag}_2 \cdots \text{Cu}_1$			$\text{Ag}_2 \cdots \text{Cu}_2$		
	GS	ES	Δ	GS	ES	Δ
Reported	3.2110 (2)	2.93 (3)	−0.29 (3)	2.8043 (2)	2.70 (3)	−0.10 (3)
None	3.2110 (2)	2.91 (3)	−0.30 (4)	2.8043 (2)	2.71 (3)	−0.09 (3)
AASR	3.2110 (2)	2.90 (3)	−0.31 (3)	2.8043 (2)	2.72 (3)	−0.09 (3)
WLS	3.2110 (2)	2.89 (3)	−0.32 (3)	2.8043 (2)	2.71 (3)	−0.09 (3)

profile mirroring the Q profile (Fig. 4) with respect to the φ angle. It can be easily explained by the tendency of observed R values to decrease with the increase of the system response. Moreover, the merging of equivalent observed ratios prior to model refinement did not alter the sine-like profile thanks to equivalent observations being usually collected at consecutive φ angles as explained in §5.1.1 and illustrated in Fig. 3. Thus, if at a given φ angle the system response is relatively stronger, the observed ratios will be globally underestimated and so will the residuals. Performing intra-set scaling minimized the φ -angle dependence of the average weighted residuals (Fig. 7b). The WLS scaling method is efficient with the correction of the φ dependence in the φ range from 100° to 180° and performs an improved reduction of the positive deviation from 30° to 100°. We note that the φ -dependence plot is not based on a

discrete subdivision of the set of residual terms as a unique reflection can have been collected at different φ angles.

As explained in §4, the data sets collected in time-resolved experiments have partial completeness and an uneven distribution in reciprocal space. Moreover, the system response under light exposure depends on the crystal sample orientation. For these reasons, checking the angular dependence in the reciprocal space of weighted residual terms is a second relevant way to evaluate the effect of the intra-set scaling. A fuzzy subdivision is possible by using a solid angle with an appropriate opening δ to probe the angular space along several directions d . The method consists of calculating the average values of the residual terms within a solid angle of 15° centered on each vertex, and subsequently estimating the tendency of the residuals within the triangle formed by the

three vertices by interpolation between the values obtained. For more details, see Appendix A. The angular-dependence average weighted residual plots have been done for set #3 for each model refinement using a solid-angle probe with an opening of 15° and can be put in parallel with the φ -dependence plots.

In the case of no intra-set scaling, the φ range from 30° to 100° in which the average residuals tend to be positive (Fig. 7) clearly appears on the first view of the reciprocal-space angular-dependence plot (Fig. 8) as a large blue patch. In the same way, the φ range from 100° to 180° with negative average residuals corresponds to red domains on the second view. Both intra-set scaling methods minimize the deviations from zero of the average residuals which can be observed with an attenuation of the

Figure 8

Angular-dependence plots of the average weighted residuals of set #3 for each model refinement without intra-set scaling ('None'), with intra-set scaling using the AASR method and using WLS. Two different views are proposed with respect to the orthonormal axis system of the reciprocal space. The solid-angle probe used has an opening angle of 15°.

red and blue domain colors. The WLS method qualitatively gives better results which is in agreement with the φ -dependence plots.

Improvements of the angular-dependence plots can be observed also for the other data sets (not reported here), although they are less significant due to the smaller system response fluctuations observed (Fig. 4).

6. Conclusion

Scaling of multocrystal data sets is a crucial component of time-resolved pump–probe crystallography at fast and ultra-fast radiation sources. In general, they include laser-ON/laser-OFF scaling, inter-dataset scaling as well as intra-dataset scaling. Analysis of a synchrotron data set on the organo-metallic complex discussed shows the importance of the last two scaling methods, the need for laser-ON/laser-OFF scaling being avoided by the ON/OFF data collection technique. In the particular example studied the impact of intra-set scaling is substantial, but varies among the four data sets considered. The relevance of the more advanced WLS scaling method is confirmed by the statistical analysis of the residuals at convergence of the structure model refinement. The consequences of the partial completeness of RATIO data sets and of the heterogeneous angular coverage of their reciprocal subspace on refined models require additional studies.

APPENDIX A

Residual vector analysis

Analyzing the angular dependence of residual terms is not trivial as the reciprocal-space angular part is a two-dimensional non-Euclidean subspace. A possible way to visualize this dependence is proposed and consists of three steps. A sample of directions in the angular subspace is first defined. To work with a quasi-regular sample, the vertices of a 20×64 -triangle icosahedron-based geodesic sphere mesh are used as shown in Fig. 9(a). Then, a solid angle with an opening δ of 15° is applied as a probe. The average value of the weighted residuals within this solid-angle probe centered on each geodesic mesh vertex direction of the subspace is calculated (Fig. 9b). As the reciprocal space is a discrete space, any quantity g function of the reflections and calculated within the solid-angle probe, and hence noted angular, will be a discontinuous function of the solid-angle probe direction d . However, for a simple representation of the angular dependence of the average residuals, each triangle element of the geodesic mesh is colored by interpolation of the angular average values calculated along the directions of its three vertices (Fig. 9c).

Figure 9 Angular-dependence visualization method. (a) Step 1: 20×64 -face icosahedron-based geodesic mesh with, in white dots, its vertices used as an angular-space sample. (b) Step 2: along each vertex direction d , calculation of the residual term average value of the reflections included in a solid angle of opening δ . (c) Step 3: coloring of each mesh triangle by interpolation according to the average values of its three vertices.

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References

- Andreani, R., Birgin, E. G., Martínez, J. M. & Schuverdt, M. L. (2007). *SIAM J. Optim.* **18**, 1286–1309.
- Blessing, R. H. (1997). *J. Appl. Cryst.* **30**, 421–426.
- Coppens, P. & Fournier, B. (2015). *J. Synchrotron Rad.* **22**, 280–287.
- Coppens, P., Pitak, M., Gembicki, M., Messerschmidt, M., Scheins, S., Benedict, J., Adachi, S., Sato, T., Nozawa, S., Ichiiyanagi, K., Chollet, M. & Koshihara, S. (2009). *J. Synchrotron Rad.* **16**, 226–230.
- Fournier, B. & Coppens, P. (2014a). *Acta Cryst.* **A70**, 514–517.
- Fournier, B. & Coppens, P. (2014b). *Acta Cryst.* **A70**, 291–299.
- Fox, G. C. & Holmes, K. C. (1966). *Acta Cryst.* **20**, 886–891.
- Hamilton, W. C., Rollett, J. S. & Sparks, R. A. (1965). *Acta Cryst.* **18**, 129–130.
- Henn, J. & Meindl, K. (2014a). *Acta Cryst.* **A70**, 248–256.
- Henn, J. & Meindl, K. (2014b). *Acta Cryst.* **A70**, 499–513.
- Henn, J. & Meindl, K. (2015). *Acta Cryst.* **A71**, 203–211.
- Jarzembska, K., Kamiński, R., Fournier, B., Trzop, E., Sokolow, J., Henning, R., Chen, Y. & Coppens, P. (2014). *Inorg. Chem.* **53**, 10594–10601.
- Kalinowski, J. A., Fournier, B., Makal, A. & Coppens, P. (2012). *J. Synchrotron Rad.* **19**, 637–646.
- Kalinowski, J. A., Makal, A. & Coppens, P. (2011). *J. Appl. Cryst.* **44**, 1182–1189.
- Makal, A., Benedict, J., Trzop, E., Sokolow, J., Fournier, B., Chen, Y., Kalinowski, J. A., Graber, T., Henning, R. & Coppens, P. (2012). *J. Phys. Chem. A*, **116**, 3359–3365.
- Makal, A., Trzop, E., Sokolow, J., Kalinowski, J., Benedict, J. & Coppens, P. (2011). *Acta Cryst.* **A67**, 319–326.
- Marsaglia, G. (2006). *J. Stat. Softw.* **16**, 1–10.
- Vorontsov, I. I. & Coppens, P. (2005). *J. Synchrotron Rad.* **12**, 488–493.
- Vorontsov, I., Pillet, S., Kamiński, R., Schmökel, M. S. & Coppens, P. (2010). *J. Appl. Cryst.* **43**, 1129–1130.