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Unifying the Gait Cycle in the Control of a Powered Prosthetic Leg

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Abstract

This paper presents a novel control strategy for an above-knee powered prosthetic leg that unifies the entire gait cycle, eliminating the need to switch between controllers during different periods of gait. Current control methods divide the gait cycle into several sequential periods each with independent controllers, resulting in many patient-specific control parameters and switching rules that must be tuned by clinicians. Having a single controller could reduce the number of control parameters to be tuned for each patient, thereby reducing the clinical time and effort involved in fitting a powered prosthesis for a lower-limb amputee. Using the Discrete Fourier Transformation, a single virtual constraint is derived that exactly characterizes the desired actuated joint motion over the entire gait cycle. Because the virtual constraint is defined as a periodic function of a monotonically increasing phase variable, no switching or resetting is necessary within or across gait cycles. The output function is zeroed using feedback linearization to produce a single, unified controller. The method is illustrated with simulations of a powered knee-ankle prosthesis in an amputee biped model and with examples of systematically generated output functions for different walking speeds.

I. INTRODUCTION

To date, there have been several control methods implemented on actuated lower-limb prostheses. Typically, these controllers divide the gait cycle into multiple, sequential periods based on predefined switching criteria. The Vanderbilt leg uses a finite state machine with impedance-based controllers for each discrete period [1]. This requires tuning of the Proportional-Derivative (PD) gains for each period of the gait cycle. Further work has shown that a combination of finite state machines with impedance-based controllers can handle multiple ambulation modes by adjusting the controller parameters [2]. Another impedance-based approach is to encode artificial reflexes from a neuromuscular model in the controller [3]. This method still requires a finite state machine to adjust the control policy or parameters depending on the gait period. Furthermore, multiple controllers may require many control parameters to be specified, potentially requiring hours of tuning to adapt a powered prosthetic leg for just a single lower-limb amputee [4].

The different periods of gait could potentially be unified by virtual kinematic constraints that are enforced using a torque control scheme [5]. Virtual constraints define desired joint trajectories with respect to a phase variable, typically by using polynomial functions. The phase variable is a time-invariant, kinematic quantity that both captures the motion of an unactuated degree of freedom and, during an unperturbed step, monotonically increases. This phase-based control method was originally developed to control underactuated bipedal robots, such as ERNIE [6] and MABEL [7]. These controllers typically divide the stride into stance and swing periods and define separate virtual constraints for each period. The progression through the stride is driven by the phase variable. If the biped is pushed forward (or backward), the phase variable increases (or decreases), which in turn speeds up (or slows down) the joint patterns. The controller is then able to automatically react to disturbances. This would be advantageous for a prosthesis controller because it allows the prosthesis to react to disturbances in a predictable manner that may be similar to the natural human response [8].

Previous work at the Rehabilitation Institute of Chicago developed a virtual constraint controller for a transfemoral (above-knee) powered prosthesis that used the center of pressure (COP) as the phase variable during the stance period [9]–[11]. Because the COP is only defined during stance, the prosthesis switched to a sequential impedance-based controller during swing. Recently, the virtual constraint control method was extended to the swing period of the prosthesis, although separate controllers were still defined for the stance and swing periods [12].

Humans move in a smooth manner over the periodic gait cycle. This smooth periodicity is lost between each of the discrete periods of a finite state machine. We propose a new approach to capture the entire gait cycle with virtual constraints using the Discrete Fourier Transformation (DFT) [13]. The DFT method has been used as a viable approach to predicting accurate joint trajectories for human biomechanics modeling during gait [14]. The DFT converts a signal from the sampled time domain to the frequency domain, which permits examination of the signal's frequency content. The DFT also allows exact recreation of the original signal as a time-domain function. Because the gait cycle is periodic when the entire stride is considered, the number of terms in the DFT representation is finite. This representation also respects the repetitive nature of gait by repeating periodically over the phase variable.

Related work in [15] uses a phase variable to drive a position control loop in a powered prosthetic ankle through-out the gait cycle. Our approach differs by defining a torque control law based on feedback linearization to enforce analytical virtual constraints, which have provable stability properties [12] and are easily generalized to multiple joints and tasks. The powered hip exoskeleton in [16] uses a phase variable to determine when to inject or dissipate energy throughout the gait cycle, which may not be sufficient to replicate joint kinematics in a prosthesis application. In our work, virtual constraints produce the desired kinematics in the absence of biological limb motion.

The goal of this paper is to design a single, unified, periodic controller defined across gait cycles. An additional goal is to systematically design the controller to reduce the effort

necessary to tune a powered prosthetic leg for different ambulation modes or patients. To accomplish these goals, the DFT method was used to construct virtual constraints for a powered prosthetic leg. The method was illustrated using simulations of a transfemoral amputee model.

II. METHODS

A. Model

The planar model of the unilateral, transfemoral amputee consists of seven leg segments plus a point mass at the hip to represent the upper body (Fig. 1, [12]). The full model is divided into a prosthesis subsystem consisting of the prosthetic thigh, shank, and foot, and a human subsystem consisting of the contralateral thigh, shank, and foot, the residual thigh on the amputated side, and the point mass at the hip. The prosthetic thigh and residual stump are rigidly attached. Rather than model all of the degrees of freedom of the foot, the function of the foot and ankle is modeled using a circular foot [17] plus an ankle joint to capture the stance ankle's positive work [18].

For simulation, a stride starts just after the transition from contralateral stance to prosthesis stance and proceeds through the prosthesis stance period, an impact event, the contralateral stance period, and a second impact. The two stance periods are modeled with continuous, second-order differential equations, and the two impact periods are modeled using an algebraic mapping that relates the state of the biped just before impact to the state of the biped just after impact.

The equations of motion during the single-support period for a subsystem can be found using the method of Lagrange [10], [12] and written as

$$M_{ij}\ddot{q}_i + C_{ij}\dot{q}_i + N_{ij} = B_{ij}u_i + J_{ij}^T F, \quad (1)$$

where q_i are the generalized coordinates, M_{ij} is the inertia matrix, C_{ij} is the matrix containing the centripetal and Coriolis terms, N_{ij} is the vector containing the gravity terms, B_{ij} is the matrix relating the input torques to the generalized coordinates, J_{ij} is the Jacobian matrix relating the socket interaction forces to the generalized coordinates, and F is the vector of interaction forces between the human and prosthesis at the socket. The subscript i indicates which subsystem the term is for, with P indicating the prosthesis subsystem and H indicating the human subsystem. The subscript j indicates which leg is in stance, with P indicating that the prosthesis is in stance and C indicating that the contralateral leg is in stance (and that the prosthesis is in swing). Because the biped is assumed to roll without slip, the absolute angle is unactuated. The ideal actuators at the remaining joints generate torque vector u_i . The impacts can be modeled using equations of the form

$$q_i^+ = q_i^-, \quad (2)$$

$$\dot{q}_i^+ = A_{ij}\dot{q}_i^- + \Lambda_{ij}\mathcal{F}, \quad (3)$$

where $\mathcal{F}$ is the socket interaction impulse that depends on the pre-impact state of both subsystems, and A_{ij} and Λ_{ij} are known matrices [12]. The superscripts ‘-’ and ‘+’ refers to the instants before and after impact, respectively.

B. Feedback Linearizing Control

To control both the prosthesis and the human, feedback linearization with virtual constraints is used [10], [12], although the prosthesis controller does not depend on the form of the human controller. Other torque control methods can be used to enforce the virtual constraints such as PD control [10], although it is less precise. To perform feedback linearization, the desired motion of the actuated variables are encoded in output functions to be zeroed [19]. These output functions define the virtual constraints:

$$y_{ij} = h_{ij}(q_i) = H_{0i}q_i - h_{ij}^d(\theta_i(q_i)), \quad (4)$$

where h_{ij} is a vector-valued function to be zeroed, H_{0i} is a matrix that maps the generalized coordinates to the actuated angles, h_{ij}^d is a vector-valued function of the desired joint angles, and θ_i is the phase variable. For the human, a separate output function is defined for the prosthesis single-support period and for the contralateral single-support period, and h_{Hj}^d is encoded using polynomials as in [12]. For the prosthesis, a single, unified output function h_P is defined for the entire stride, i.e., both the stance and swing periods of the prosthesis.

Differentiating Eq. 4 twice and substituting in the equations of motion (Eq. 1) for q_i gives the output dynamics [10]

$$\ddot{y}_{ij} = L_{f_{ij}}^2 h_{ij} + L_{g_{ij}} L_{f_{ij}} h_{ij} \cdot u_i + L_{p_{ij}} L_{f_{ij}} h_{ij} \cdot F, \quad (5)$$

where standard Lie derivative notation [5] has been used, and

$$L_{f_{ij}}^2 h_{ij} = \frac{\partial}{\partial q_i} \left(\frac{\partial h_{ij}}{\partial q_i} \dot{q}_i \right) \dot{q}_i - \frac{\partial h_{ij}}{\partial q_i} M_{ij}^{-1} (C_{ij} \dot{q}_i + N_{ij}), \quad L_{g_{ij}} L_{f_{ij}} h_{ij} = \frac{\partial h_{ij}}{\partial q_i} M_{ij}^{-1} B_{ij}, \quad L_{p_{ij}} L_{f_{ij}} h_{ij} = \frac{\partial h_{ij}}{\partial q_i} M_{ij}^{-1} J_{ij}^T.$$

To cancel the nonlinearities in the output dynamics, set the desired output dynamics to $\ddot{y}_{ij} = v_{ij}$ for some stabilizing controller v_{ij} and solve for the input torques:

$$u_i = \alpha_{ij} + \beta_{ij} \cdot F, \quad (6)$$

where F is known through measurement, and

$$\alpha_{ij} = L_{g_{ij}} L_{f_{ij}} h_{ij}^{-1} (v_{ij} - L_{f_{ij}}^2 h_{ij}), \quad \beta_{ij} = -L_{g_{ij}} L_{f_{ij}} h_{ij}^{-1} \cdot L_{p_{ij}} L_{f_{ij}} h_{ij}.$$

C. Virtual Constraint Design by DFT

The objective is to create virtual constraints that characterize the desired knee and ankle motion over the entire stride for a prosthetic leg. The virtual constraints are time-invariant and depend on a phase variable that is strictly monotonic and unactuated [19]. The

monotonicity of the phase variable over the complete stride allows a single output function to represent one joint's motion over the entire stride. Taking advantage of the periodic kinematics observed in human gait [20], the method of DFT is used to parameterize the virtual constraint for a powered prosthesis controller.

The DFT is a linear transformation of a signal of complex numbers across a spectrum of discrete frequency components. The DFT can be represented as

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, k=0, 1, \dots, K \quad (7)$$

where N is the finite number of samples, K is the highest index for the finite sequence of k up to $N - 1$ samples, and $W_N = e^{-j(2\pi/N)}$ is the complex quantity [13]. The original discrete signal in the sampled time domain is $x[n]$, which is transformed to a discrete sequence in the frequency domain as $X[k]$. Because the signal is periodic, there are a finite number of discrete frequencies. As a result, Eq. 7 can be decomposed using a summation of sinusoids for W_N within the DFT using Euler's relationship $e^{\pm j\Omega} = \cos \Omega \pm j \sin \Omega$, $\Omega \in \mathbb{R}$.

After obtaining the frequency content terms $X[k]$ from a signal, the original signal can be reconstructed using Fourier Interpolation, where the basis function is given by

$$x[n] = \frac{1}{N} \sum_{k=0}^K X[k] W_N^{-kn}, n=0, 1, \dots, N-1. \quad (8)$$

In general, $X[k]$ may be complex. Writing $X[k] = \text{Re}\{X[k]\} + j \text{Im}\{X[k]\}$ and

$W_N^{-kn} = \text{Re}\{W_N^{-kn}\} + j \text{Im}\{W_N^{-kn}\}$, $x[n]$ can be separated into its real and imaginary parts:

$$\begin{aligned} x[n] &= \frac{1}{N} \sum_{k=0}^K (\text{Re}\{X[k]\} + j \text{Im}\{X[k]\}) \\ &\quad \cdot (\text{Re}\{W_N^{-kn}\} + j \text{Im}\{W_N^{-kn}\}) \\ &= \frac{1}{N} \sum_{k=0}^K (\text{Re}\{X[k]\} \text{Re}\{W_N^{-kn}\} \\ &\quad - \text{Im}\{X[k]\} \text{Im}\{W_N^{-kn}\}) \\ &\quad + j(\text{Re}\{X[k]\} \text{Im}\{W_N^{-kn}\} \\ &\quad + \text{Im}\{X[k]\} \text{Re}\{W_N^{-kn}\}). \end{aligned} \quad (9)$$

It is obvious that the joint kinematic signals are real numbers, thus the only portion necessary for $x[n]$ is the real part. It can be validated using the calculated DFT signal that $X^*[k] = X[-k]$ and $X^*[N-k] = X[k]$ holds, which means that the imaginary components of Eq. 9 are zero. The real, conjugate symmetric properties means the magnitude of $X[k]$ for the k^{th} index is the same as the $(N-k)^{\text{th}}$ index [13]. Therefore, the joint kinematics over a stride (Eq. 9) simplify to

$$x[n] = \frac{1}{N} \sum_{k=0}^K \text{Re}\{X[k]\} \text{Re}\{W_N^{-kn}\} - \text{Im}\{X[k]\} \text{Im}\{W_N^{-kn}\}, \quad (10)$$

where the summation terms are strictly real.

Eq. 10 can be represented by a function with a set of frequency terms from 0 up to $N/2$, the Nyquist sampling frequency, which is half the number of samples in the discrete, equally spaced signal. Recall that the magnitudes of $X[k]$ and $X[N-k]$ are equal because the signal is a real-valued sequence, so the interpolation only needs to be computed for half of its frequency range, as beyond $N/2+1$ the samples are duplicate values for the DFT. This results in the following exact representation of the original signal

$$x[n] = \frac{1}{2} \alpha_0 + \sum_{k=1}^{\frac{N}{2}-1} \left[\alpha_k \text{Re}\{W_N^{-kn}\} - \beta_k \text{Im}\{W_N^{-kn}\} \right] + \frac{1}{2} \alpha_{\frac{N}{2}} \text{Re}\{W_N^{-\frac{N}{2}n}\}, n=0, 1, \dots, N-1 \quad (11)$$

where $\alpha_k = 2 \text{Re}\{X[k]\} \in \mathbb{R}$ and $\beta_k = 2 \text{Im}\{X[k]\} \in \mathbb{R}$ are the scalar coefficients. Note that Eq. 11 is a lower order functional representation of Eq. 10.

The Fourier Interpolation presented in Eq. 11 is used to create the unified prosthesis virtual constraint, h_p^d , for the entire gait cycle that contains both stance and swing periods. To find the virtual constraint, we sample the desired angular trajectories of the knee and ankle over the entire gait cycle at n discrete, equally-spaced values of the phase variable. For this work, the phase variable was chosen as the hip x -position q_x , measured relative to a coordinate frame created at the transition from the contralateral stance to the prostheses stance period, although other options may exist [21]. The phase variable was normalized between 0 and 1 using

$$s_p(\theta_p(q_p)) = \frac{\theta_p^- - \theta_p^+}{\theta_p^- - \theta_p^+}, \quad (12)$$

where the '+' signifies the start of the stance period for the prosthetic leg and the '-' indicates the end of its swing period. Using Eq. 7 to solve for $X[k]$, the real and imaginary parts of $X[k]$ are used to compute the coefficients of Eq. 11. Eq. 11 is converted into an output function (Eq. 4) to generate the prosthesis virtual constraints:

$$h_p(q_p) = H_{0p} \cdot q_p - \left(\frac{1}{2} \alpha_0 + \frac{1}{2} \alpha_{\frac{N}{2}} \cos(\pi N s_p) + \sum_{k=1}^{\frac{N}{2}-1} [\alpha_k \cos(2\pi k s_p) - \beta_k \sin(2\pi k s_p)] \right). \quad (13)$$

III. SIMULATION RESULTS

A. Virtual Constraint Design for Simulated Prosthesis

Applying the method from Section II-C, we now design a unified output function for a powered prosthetic leg, consisting of a knee and ankle joint (Fig. 1). To begin, a desired

trajectory that generates a stable gait is needed. For a prosthesis controller, it is generally desirable to mimic human able-bodied motion, so the desired trajectory can be chosen from one of many gait studies [20]. For this simulation work, the trajectory was chosen based on a model designed to predict healthy human walking [12].

Using MATLAB to determine the DFT of the desired gait trajectory, the real and imaginary parts of $X[k]$ and W_N^{-kn} can be computed using Eq. 7 to create the Fourier Interpolation in Eq. 11. The DFT spectrum (Fig. 2) for the knee and ankle joint trajectories show that between the 10th frequency and the Nyquist sampling frequency of $N/2$, the magnitude is approximately zero and thus does not have a significant impact on recreating the joint trajectory for h_p^d . As a result, the series can be truncated to $k \in \{1, \dots, 10\}$, which reduces the number of coefficients for the virtual constraint and in turn lessens the computational complexity of the feedback linearization for y_{ij} .

To verify that the first 10 indices are needed to accurately capture the desired trajectory, virtual constraints with $K = 5, 10$, and $N/2$ were determined for each joint. As expected, the virtual constraints for $K = 10$ and $N/2$ are similarly accurate and are more accurate than $K = 5$ (Fig. 3, Table I). A virtual constraint with index $K = 5$ is an adequate representation of the original joint trajectory for both the ankle and knee with a coefficient of determination, $r^2 = 0.995$ and $r^2 = 0.999$, respectively. The remaining virtual constraints ($K = 10$ and $N/2$) both result in a coefficient of determination of $r^2 = 1.000$. In all cases, the root mean square error (RMSE) is less than $2.25\text{e-}04$ rad.

B. Simulated Walking of Amputee Model

Two different sets of virtual constraints ($K = 5$ and 10) were used to control the prosthetic leg in the amputee biped simulation. For each set of virtual constraints, the entire human plus prosthetic leg was simulated until steady-state was reached (Fig. 4, Video 1). Not surprisingly, the two simulations produce two different periodic orbits due to the differences in their virtual constraints. The steady-state prosthesis joint angles and velocities for $K = 10$ are shown in Fig. 5. Due to the lower RMSE (Table I) with a virtual constraint at $K = 10$ as compared with $K = 5$, the $K = 10$ virtual constraint is preferred.

IV. EXTENSIONS OF DFT DESIGN

This control design strategy can be applied to other desired gaits or ambulation modes. For example, virtual constraints can be designed for multiple walking speeds that an amputee may execute on a daily basis. Following the DFT design methodology, virtual constraints for slow, normal and fast walking were created to match able-bodied data reported in [20]. Fig. 6 shows that DFT virtual constraints with $K = 10$ can accurately match different walking speeds.

For comparison, we also designed virtual constraints for the normal speed using the popular formulation of Bézier polynomials [7], [19] by least squares polynomial fitting. For each method, tenth-order virtual constraint functions were found from able-bodied walking data [20]. Table II shows goodness-of-fit statistics for DFT ($K = 10$) and for Bézier ($p = 10$). The DFT virtual constraint matches the desired trajectory better than the Bézier polynomial

virtual constraint (Fig. 7) despite the fact that both methods use a tenth-order function. To obtain comparable results, the Bézier polynomial needed to be 20th order, which is twice the order required for the DFT method. However, from a practical standpoint, both 10th order functions are acceptably accurate.

The DFT method also unifies control across the gait cycles of an ambulation mode, which can be observed through the behavior of the virtual constraint outside the design region of its phase variable ($0 \leq s_p \leq 1$), as shown in Fig. 7. For comparison, the Bézier polynomials immediately diverge to undesirable values outside the design range of the phase variable. As a result, the phase variable must be reset back to zero between gait cycles to avoid exceeding its design range ($0 \leq s_p \leq 1$). Practical implementations typically saturate the phase variable at the design limits to prevent undesirable commanded angles [6]. In contrast, the DFT virtual constraint is periodic over the phase variable, so this formulation transitions seamlessly to the next gait cycle outside of the design bounds. Thus, the phase variable does not have to be reset as the amputee transitions from the end of the swing period to the beginning of the next stance period. This also eliminates the need for saturating the phase variable, which may simplify the controller implementation. In summary, the DFT virtual constraints unify not only a single stride but also periodic steady-state locomotion since the same controller can be used without any resetting for an arbitrary number of strides.

V. CONCLUSIONS

We developed a single, unified prosthesis controller that captures the entire gait cycle by using the DFT to design virtual constraints for a feedback-linearizing controller. The unified controller eliminates the need to divide the gait into different periods with independent controllers. Since the DFT virtual constraint is periodic, the controller does not need to be reset at the start of each stride. The feasibility of the controller was demonstrated using simulations of a walking gait, but the DFT approach can be applied equally well to other locomotion activities with well-characterized joint kinematics from able-bodied data [20] or model-based optimizations [18]. For instance, we plan to apply our control strategy to different tasks such as upward/downward slopes. This DFT approach could also be used to characterize wholebody joint motion for the control of autonomous bipedal robots.

Future work will involve hardware experimentation, where this control strategy will be implemented on a powered knee-ankle prosthesis built at the University of Texas at Dallas. A dSPACE real-time interface will be used to implement our control algorithms onto the powered prosthesis for experiments with transfemoral amputee subjects. One of the challenges for hardware implementation will be measuring the phase variable (i.e., the hip position). Real-time estimation methods using inertial measurement units (e.g., [22]) could be used to estimate the hip position. Alternatively, other phase variables could be measured from other sensors on-board the prosthetic leg [21].

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Fig. 1.

Schematic of the unilateral, transfemoral amputee model. The prosthesis is shown in black and the human is shown in gray. The generalized coordinates used in the model are indicated with q terms. Angle q_1 is unactuated and angles q_{2-6} have ideal actuators.

Fig. 2.

The DFT spectrum for the knee (left) and ankle (right) joint trajectories over the entire gait cycle. The magnitude is the power spectrum of the signal for the respective k indices. Note that the frequency content appears to end after the 10th index.

Fig. 3.

The error between the desired trajectory and the virtual constraint for the knee (left) and ankle (right) vs. phase variable. $K = 10$ has minimal error compared to $K = 5$ and is more aligned to the lowest error of $K = N/2$.

Fig. 4.

The angular phase portrait for the prosthetic knee (left) and ankle (right) with $K = 5$ and 10 for the virtual constraints. The simulation shows convergence to a periodic orbit in both cases, resulting in steady-state stable gaits for the amputee biped.

Fig. 5.

The simulated joint angles (left) and velocities (right) of the prosthetic knee and ankle for the $K = 10$ virtual constraints, plotted against the normalized phase variable.

Fig. 6.

The desired joint trajectory and the virtual constraint by DFT at $K = 10$ for the knee joint (left) and the ankle joint (right) across different walking speeds for an entire gait cycle. Note, SW = Slow Walking, NW = Normal Walking, and FW = Fast Walking, the experimental data is indicated with solid lines and the virtual constraint is indicated with broken lines.

Fig. 7.

The virtual constraint by DFT and Bézier polynomial for normal walking [20] at the knee (left) and ankle (right) both designed for a normalized phase variable s_P from 0 to 1. The DFT method continues to the next periodic gait cycle for phase variable values $s_P > 1$ and $s_P < 0$, i.e., the ranges of $-0.5 \leq s_P < 0$ and $0.5 \leq s_P < 1$ are identical. In contrast, the Bézier polynomial virtual constraints diverge to undesirable trajectories on both sides of the design region.

TABLE I

Statistical Results with DFT Design

	Knee		Ankle	
<i>K</i> value	r^2	RMSE (rad)	r^2	RMSE (rad)
5	0.999	2.23e-04	0.995	1.33e-04
10	1.000	6.14e-05	1.000	3.14e-05
$N/2$	1.000	4.50e-05	1.000	1.17e-05

TABLE II

Statistical Comparison between Design Methods

Virtual Constraint	Knee		Ankle	
	r^2	RMSE (rad)	r^2	RMSE (rad)
DFT $K = 10$	1.000	2.27e-05	1.000	2.44e-05
Bézier $p = 10$	0.999	2.02e-04	0.985	2.98e-04