Journal of Medical Imaging

MedicalImaging.SPIEDigitalLibrary.org

Improved image reconstruction of low-resolution multichannel phase contrast angiography

Akshara P. Krishnan
Ajin Joy
Joseph Suresh Paul

Improved image reconstruction of low-resolution multichannel phase contrast angiography

Akshara P. Krishnan, Ajin Joy, and Joseph Suresh Paul*

Indian Institute of Information Technology and Management-Kerala, Medical Image Computing and Signal Processing Laboratory, Trivandrum 695581, Kerala, India

Abstract. In low-resolution phase contrast magnetic resonance angiography, the maximum intensity projected channel images will be blurred with consequent loss of vascular details. The channel images are enhanced using a stabilized deblurring filter, applied to each channel prior to combining the individual channel images. The stabilized deblurring is obtained by the addition of a nonlocal regularization term to the reverse heat equation, referred to as nonlocally stabilized reverse diffusion filter. Unlike reverse diffusion filter, which is highly unstable and blows up noise, nonlocal stabilization enhances intensity projected parallel images uniformly. Application to multichannel vessel enhancement is illustrated using both volunteer data and simulated multichannel angiograms. Robustness of the filter applied to volunteer datasets is shown using statistically validated improvement in flow quantification. Improved performance in terms of preserving vascular structures and phased array reconstruction in both simulated and real data is demonstrated using structureness measure and contrast ratio. ©

2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.JMI.3.1.014001](https://doi.org/10.1117/1.JMI.3.1.014001)]

Keywords: nonlocal reverse diffusion; multichannel parallel images; maximum intensity projection; phased array reconstruction; phase contrast magnetic resonance angiography.

Paper 15051RR received Mar. 12, 2015; accepted for publication Dec. 18, 2015; published online Jan. 22, 2016.

1 Introduction

Phase contrast magnetic resonance angiography (PC-MRA) exhibits excellent background signal separation, allowing better visualization of vessels and flow information. Though this is important in the case of slow-flow vessels like those of the venous system, neuroimaging of head and neck calls for larger field-of-view (FOV) scans, typically 30 to 40 cm. Channel images in large FOV acquisition are blurry, and provide only limited details of vessel structures due to lack of high-frequency information.¹ In phase contrast imaging, the contrast between flowing blood and stationary tissues is obtained by application of a velocity encoding gradient. In order to encode the flow in each direction, the flow-encoding gradient is applied on each of the three gradient axes in separate TR intervals. In addition, a fourth nonflow encoded acquisition is also performed in the four-point velocity method.^{2,3} Data from the latter acquisition are subtracted from each of the three flow-encoded acquisitions by taking the complex differences to eliminate phase accumulation from sources other than velocity, such as field inhomogeneities. The final display, often referred to as a “speed image”, is obtained using maximum intensity projection (MIP) applied to the complex difference images.

In MIP, the pixels with maximum intensities are projected along the volume of data, and the resulting projected image will have a thin-structure form.^{4,5} Since intensity projection is performed using component slices with intrinsic blur and noise, the resulting MIP images require preprocessing to mitigate their cumulative effects. Blur is degradation of sharpness and contrast of the image, causing loss of high frequencies. Due to this, vascular edges become obscure on the intensity projected image,

thereby reducing image sharpness and contrast. In each channel, the combined effects of large FOV, and intensity projection can be approximately represented using a linear degradation model

$$\hat{U}_l = g_l \times U_l + \nu, \quad (1)$$

where $\hat{U}_l$ denotes the blurred image, U_l is the undegraded image, g_l is the blurring kernel, and ν is the unknown noise affecting the measurements. The recovery of U_l from the measurements $\hat{U}_l$ is an ill-posed problem.^{6,7} The problem that is addressed in this paper is one of deblurring a coil image $\hat{U}_l$ that has been blurred by a blurring kernel g_l representing some physical process. The problem is modeled by the convolution relation

$$\hat{U}_l(p) = \int U_l(t)g_l(p-t)dt. \quad (2)$$

In general, blind approaches for denoising employ iterative updates computed using diffusion models.^{8–14} Likewise, deblurring is achieved via image updates computed from reverse diffusion models.^{6,15–17} In both cases, the updated image is attempted to fit as closely as possible to the true denoised/deblurred image. At locations where the fitting is not perfect, the updated image will exhibit a speckled appearance. In reverse diffusion, the image is enhanced by reversing time in the diffusion equation. This is numerically achieved by subtracting the Laplacian from the measured image. Due to the ill-posedness in the process of image correction, there have been several attempts to stabilize the solutions. One of the earliest attempts is the shock filter proposed by Rudin and Osher.^{18,19} These filters implement morphological image enhancement based on partial

*Address all correspondence to: Joseph Suresh Paul, E-mail: j.paul@iitmk.ac.in

differential equations. The main drawback of shock filters is the generation of sharp discontinuities at the borders between two regions. Attempt to deblur an image from the temporal evolution of zero crossing of Laplacian was carried out by Humel and Moniot.^{20,21} More recent methods combine anisotropic diffusion filter and shock filter,^{22,23} to selectively deblur the edges. As opposed to the above taxonomy of blind methods, variational^{24–27} approaches for deblurring assume partial knowledge of the blurring kernel, and treat the diffusion process as equivalent to the convolution with a Gaussian kernel. Attempts to regularize reverse diffusion consist of the addition of constraints in the deblurring functional. Using the nonlocal means (NLM) algorithm,²⁸ the constraint is to maintain the degree of similarity throughout the deblurring process.^{29–31} A numerical implementation of this scheme is obtained by smoothing the difference between the image and its Laplacian using a NLM filter.

The application of nonlocal stabilized reverse diffusion (NLSRD) to multichannel PC-MRA magnetic resonance imaging is particularly noteworthy. Combination of channel images in parallel MRI following application of any filtering procedure often requires a prior scaling applied to the individual channel images.³² This scaling factor is determined using calibration procedures, or estimation of the noise levels. In addition to restoring vascular elements, the proposed deblurring filter automatically takes care of this scaling, yielding improved phased array reconstruction with a higher structureness (S) measure and contrast ratio (CR).

The theory and principle of NLM filter, reverse diffusion and statistical model of PC-MRA are discussed in Sec. 2. Section 3 summarizes the experimental results obtained using volunteer data. A quantitative performance overview of the proposed filter using the statistical model of PC-MRA, and comparison of image quality metrics with other state-of-the-art deblurring methods are presented in Sec. 4. A brief discussion and conclusion are included in Secs. 5 and 6, respectively.

2 Material and Methods

2.1 Image Deblurring Using Reverse Diffusion

If the blurring kernel in Eq. (1) is assumed to be a spatially invariant Gaussian function, and the additive noise is zero, the blurring process can be described by the isotropic heat equation. Using this model, the blurring of an image is considered to be proportional to its Laplacian.³² The process of blurring modeled using isotropic heat equation is

$$\frac{\partial U_l}{\partial t} = c \nabla^2 U_l, \quad U_l(p, 0) = \hat{U}_l(p). \quad (3)$$

Here, U_l represents the image being diffused using the heat equation, c is the diffusion coefficient, ∇^2 is the Laplacian of U_l , and $\hat{U}_l(p)$ is the initial degraded channel image. The variance of this kernel is related to the diffusion coefficient c in heat equation via $\sigma^2 = 2tc$. The problem of deblurring is addressed by solving the reverse heat equation, obtained using reversal of time in Eq. (3). This is numerically achieved by subtracting the Laplacian from the observed image. However, solution becomes difficult due to the high-pass nature of the resulting operation and the consequent noise buildup. Even with measures to control the noise buildup, direct application of reverse diffusion can often result in deblurring with structural losses.

Because of the divergence in nature, reverse heat equation remains stable for a short while and degenerates very rapidly, which eventually lead to noise buildup. To make it stable, we can add a regularization term using NLM to the solution of reverse heat equation. A detailed description of the NLM weight calculation and regularization procedure is included in Sec. 2.2.

2.2 Nonlocally Stabilized Reverse Diffusion

2.2.1 Nonlocal means filter

NLM filtering method assumes that the image contains an extensive amount of self-similarity and tries to make use of it. For example, Fig. 1 shows three pixels p , $q1$, and $q2$ and their respective neighborhoods. The neighborhoods of pixels p and $q1$ are similar, but the neighborhoods of pixels p and $q2$ are not similar. Adjacent pixels usually tend to have similar neighborhoods. Additionally, when there is structure in the image, it is observed that even nonadjacent pixels will also have similar neighborhoods. Each pixel p of the NLM filtered image is computed using

$$NLU(p) = \sum_{q \in U} w(p, q) U(q), \quad (4)$$

where weights $w(p, q)$ meet the following conditions $0 \leq w(p, q) \leq 1$, and $\sum_q w(p, q) = 1$. The weights are computed based on the similarity between the neighborhoods of pixels p and q .

The NLM filtered value of a given pixel (p) is a weighted average of all pixels in the image as in Eq. (4). The magnitude of weight assigned to a pixel will depend on the similarity between the neighborhood of that pixel and the neighborhood of p . For example, in Fig. 1, the weight $w(p, q1)$ is much greater than $w(p, q2)$ because pixels p and $q1$ have similar neighborhoods and pixels p and $q2$ do not have similar neighborhoods. In order to compute the similarity, a neighborhood is first defined.

Let N_i be the square neighborhood centered about pixel i . The similarity between two regions is computed using

$$d(p, q) = \|\bar{\mathbf{u}}_p - \bar{\mathbf{u}}_q\|_2^2, \quad (5)$$

Fig. 1 Example of self-similarity in an image. Pixels p and $q1$ have similar neighborhoods, but pixels p and $q2$ do not have similar neighborhoods. Because of this, pixel $q1$ will have a stronger influence on the filtered value of p than $q2$.

where $\bar{\mathbf{u}}_i$ denotes a vector whose elements consist of intensities $U(i)$ such that $i \in N_i$. The weights can then be computed using the following formula:

$$w(p, q) = \frac{1}{Z(p)} e^{-\frac{d(p, q)}{h^2}}, \quad (6)$$

where $Z(p)$ is the normalizing constant defined as

$$Z(p) = \sum_q e^{-\frac{d(p, q)}{h^2}}, \quad (7)$$

and h is the weight-decay control parameter. A criterion for choosing the value of h for the current application is presented in Sec. 3.3.2.

2.2.2 Nonlocal means regularization

The NLM based regularization term has the potential to suppress noise while preserving edges and other details more effectively than conventional regularization techniques. A common image restoration approach is to minimize a cost function such as

$$E(U_I) = \|\hat{U}_I - HU_I\|^2 + \lambda \gamma(U_I). \quad (8)$$

The first term in Eq. (8) ensures fidelity of the estimate from the data $\hat{U}_I$. The second term is a regularizing or stabilizing constraint $\gamma(U_I)$, which may exploit prior knowledge of the underlying image and can be used to avoid noise amplification during deblurring. A regularization parameter λ controls the trade-off between the two terms. The set of weights used in NLM filter is computed from the observed image $\hat{U}_I$. The deblurred image in each iteration can be used to reestimate a better set of weights to refine, in a second pass, the deblurred result. Since pixels belonging to the whole image are used for the estimation of each new pixel, this regularization prior is described as nonlocal. In order to reduce the computation time, the search region for calculation of weights in Eq. (4) is limited to a search window around the pixel to be estimated. The NL stabilized reverse equation is given by

$$\frac{\partial U_I}{\partial t} = -\nabla^2 U_I + \lambda \text{NLU}_I(p). \quad (9)$$

2.3 Statistical Analysis of Phase Contrast Magnetic Resonance Angiography

In this article, we present the filtered form of complex difference (CD) speed images in PC-MRA. A phase contrast complex difference angiogram is an image showing the magnitude of the CD vector as pixel intensity.³³ This kind of angiogram will ideally have no signal intensity in pixels with stationary spins. Signal intensity in pixels with flowing spins will vary with flow velocity. Hence, the intended task of this particular form of PC-MRA display is to quantify the flow velocity.

Since noise contained in the complex difference of the two oppositely encoded images is known to possess a zero-mean Gaussian distribution, the CD speed image obtained by the addition of squared modulus of the complex difference images encoded along x , y , and z directions will exhibit a Maxwell distribution.³⁴

In modeling the intensity characteristics of vessel voxels, a laminar flow pattern is generally assumed. It has been established that the intensity distribution of vascular components in such cases can mostly be treated to have a uniform distribution.^{35–37} A deviation from the stated model occurs when the individual distribution in a given direction does not conform to a perfect Gaussian shape. In such cases, it is often customary to consider the background in the combined CD signal to be modeled as a mixture of a Maxwell and a Gaussian distribution. Using the mixture model and expectation maximization algorithm,³⁸ we construct a statistical model for a given voxel (i) of the form

$$f(i) = C_M f_M(i) + C_G f_G(i) + C_U f_U(i), \quad (10)$$

where C 's denote the mixture weights, f 's the underlying distributions, and suffixes M , G , and U represent the Maxwell, Gaussian, and uniform distributions, respectively. From this model, we derive a threshold using

$$C_M f_M(I_t) + C_G f_G(I_t) = C_U f_U(I_t). \quad (11)$$

In the presence of blur inherent in low-resolution PC-MRA, statistical modeling becomes less efficient due to increase in the number of misclassified voxels. When a significant number of voxels are blurred, this misclassification leads to an increased peak frequency and mean value of Maxwell component representative of background. This leads to a high-threshold value computed using Eq. (11). This implies that any approach to reduce the effect of blur, should also minimize the resulting misclassification. Deblurring achieved during each iteration of NLSRD filter results in more number of voxels representing flow. This increase in the number of voxels, if reliable, should result in a reduced threshold computed using statistical approach. Therefore, we make use of the thresholds from statistical mixture model to validate the filter performance. A quantitative measure indicative of improvement due to filtering is shown using reduction of the resultant intensity threshold with each iteration in Sec. 4.1.

2.4 Structureness Measure Applied to Maximum Intensity Projection Images

Intensity projected coil images contain both structural and smooth regions. Degradation in the form of blur reduces the structural information and results in poor visual quality. The structural information is measured using structureness (S) measure, computed using second-order derivatives at each voxel

$$S = \sqrt{U_{xx}^2 + U_{yy}^2}, \quad (12)$$

where $U_{xx} = \partial^2 U / \partial x^2$ and $U_{yy} = \partial^2 U / \partial y^2$.

3 Experimental Results

3.1 Simulated Data

Radio frequency reception sensitivity is a function of coil geometry, and its position relative to the imaging plane. It can be calculated using Biot–Savart law in combination with the principle of reciprocity.³⁹ Simulated multichannel data are obtained by multiplying a MIP image with coil sensitivities computed for circular coils of a given radius. Figure 2 shows the individual

Fig. 2 (a) Channel images prior to filtering; (b) channel images after filtering.

Fig. 3 Plots of iterations versus structureness measure for simulated data with coil radius = 10.

Fig. 4 L-curve obtained using simulated data.

Fig. 5 Complex Difference Method for multichannel phase contrast angiogram.

Fig. 6 (a) MIP images prior to filtering for first dataset; (b) MIP images after filtering for first dataset; (c) MIP images prior to filtering for second dataset; and (d) MIP images after filtering for second dataset.

Fig. 7 (a) Channel images of first dataset prior to filtering; (b) channel images of first dataset after filtering; (c) channel images of second dataset prior to filtering; and (d) channel images of second dataset after filtering.

channel images prior to and following the application of NLSRD filter.

Structureness measure computed for filtered images in each iteration is shown in Fig. 3. It is seen that S increases with iterations and levels off after a few steps. Maximum enhancement is achieved when the iterations are continued till this saturation point.

3.1.1 Choice of regularization parameter

In the generalized form, the deblurred image is constructed with a constraint to have similarity to vascular structures of the blurred version. This is achieved using a nonlocal deblurring functional

$$E_{\text{NLU}_l^{(k)}} = \|U_l^{(k)} - \hat{U}_l\|^2 + \lambda \|U_l^{(k)} - \text{NLU}_l^{(k)}\|^2, \quad (13)$$

where λ represents the Tikhonov regularization parameter and k is the iteration number. The first term can be considered as a fidelity term and second as the regularization term. With increase in regularization, the regularization error increases $\|U_l^{(k)} - \text{NLU}_l^{(k)}\|^2$ and perturbation error $\|U_l^{(k)} - \hat{U}_l\|^2$ decreases.

If too much regularization is performed on the solution, then it will not fit the data properly, and the regularization error will be too large. On the other hand, if the regularization performed on the residual is too little, then fitting of data will be perfect but the solution will be dominated by the contributions from data errors, and hence perturbation error will be too large. The major advantage of regularizing the solution is that the contributions from both data and rounding errors can be damped and keeps $\|U_l^{(k)} - \hat{U}_l\|^2$ within limits. The plot of these two norm quantities versus each other, parameterized by λ gives the L-curve.

Figure 4 shows L-curve obtained using simulated data with λ varying from 0.01 to 10. The optimal λ is chosen corresponding to the knee point of the L-curve. For simulated data, the optimal λ was obtained as 1.97. For experiments with simulated channel images, each one of them is blurred with a Gaussian kernel g . The plot of regularization error versus perturbation error is influenced by variance of g . It is seen that increase in variance results in reduction of both regularization and perturbation errors, with a slight increase in optimized regularization parameter.

Characteristics of the error component influence the shape of the L-curve. Experiments with the added Gaussian noise to the channel image show that higher the noise variance, less

sharp the L-curve appears. Convergence is obtained when $\|U_l^{(k+1)} - U_l^{(k)}\|^2 \leq \delta$, for a preset tolerance δ . When applied to coil images with large extent of blur, the convergence slows down.

3.2 Volunteer Data

Two volunteer PC-MRA data were acquired on a Siemens 1.5T machine with six head coils. The imaging parameters used were echo time, TE = 9 ms and repetition time, TR = 56.70 ms. The flip angle used was 15 deg. The phase encoding direction was anterior–posterior with velocity encoding, 10 cm/s.

The data acquired for each coil consisted of 256 slices. These were separated into four partitions S11, S12, S13, and S14, each consisting of 64 slices, corresponding to the nonflow encoded and flow-encoded acquisitions along x , y , and z directions, respectively. The process was repeated for all the six channels. CD method was applied to the partitions in each channel using steps outlined in Fig. 5 to obtain structural and flow information.

Using complex difference, magnitude flow information is obtained as maximum intensity projections along x , y , and z direction for each channel.^{16,17} NLSRD filtering was individually applied to the directionally combined images per channel. Due to sensitivity encoding in multichannel imaging, the intensity projections per channel exhibit a spatially modulated thin-structure form. We see that a direct application of NLSRD filter to each channel compensates for this spatial variation, and provides an improved reconstruction. Block diagram of the filtering steps applied to phased array reconstruction of multichannel PC-MRA is shown in Fig. 5.

Figure 6 shows results of PC-MRA acquired using six head coils (channels) for the two volunteer datasets. Panels (a) and (b) show the MIP images prior to and after filtering. Individual channel images are shown in Fig. 7.

3.3 Filter Parameters

3.3.1 Window size

In neuro-vascular structures, the maximum possible width of an artery is 10 mm. In order to limit computation time and also

Fig. 8 Weight histograms computed for different h values. The bins represent weights subtracted from unity. h values correspond to images with normalized maximum gray scale of unity.

incorporate maximum possible similar patches within a search window, a neighborhood of 121 pixels (approximately equal to 16 mm $\times$ 16 mm with 1.5 mm in-plane resolution) is used. This clearly indicates that an increase in window size beyond 11 $\times$ 11 will not result in any further improvement.

3.3.2 Weight decay control

The weight-decay control parameter h influences the distribution of weights in the image. High values of h introduces structural losses in the filtered image. Decreasing the value of h leads to weight values at pixels having maximum similarity to deviate from unity. A balance between these two cases is desired. An optimum value of h would result in weights close to unity for the vascular content, and sharper transition in the weight distribution. Figure 8 shows NLM weight histograms corresponding to different h values. The bins shown correspond to weight values subtracted from unity. The h values shown are for images normalized to have maximum gray scale of unity.

Of the three cases shown ($h = 0.01$, $h = 0.05$, and $h = 0.4$), first corresponds to the situation where the maximum weight deviates away from unity. As compared to higher values ($h = 0.4$), choice of $h = 0.05$ results in histogram with a sharper transition from the high-to-low weight regions. The latter choice is more ideal for better separation of vascular and background regions. With increase in h , halo effects begin to appear around the vessel edges. Further increase can lead to loss of structural information.

Fig. 9 (a) Maxwell–Gaussian–Uniform fitted model of unfiltered speed image and (b) Maxwell–Gaussian–Uniform fitted model of filtered speed image.

Fig. 10 Left panels: Segmented MIP images using mixture model with background represented by only the Maxwell distribution ($CG = 0$). Right panels: Segmented MIP images using mixture model with background represented by combination of Maxwell and Gaussian distributions. Insets show threshold value (I_t) in each iteration. The thresholds in the top panels correspond to the unfiltered image.

4 Filter Performance

We utilize the relationship between the physiological nature of origin of PC-MRA, and statistical properties of its maximum intensity projected speed image to demonstrate the quantitative improvement using NLSRD filter. Using statistical mixture modeling discussed in Sec. 2.3, the filtered image is analyzed at successive iterations. In addition to this, visual quality of the restored image is compared based on image quality metrics such as CR and contrast per pixel measures.

4.1 Performance Analysis Using Statistical Model

With reference to the description in Sec. 2.3, the observed histograms and fitted models for unfiltered and filtered MIP images of volunteer dataset-2 are as shown in Fig. 9. Using either the Maxwell–Gaussian–uniform or Maxwell–uniform mixture model, threshold is computed using Eq. (11). A binary segmentation is performed by labeling voxels with intensities above the threshold as vessels, and the rest as background. It is seen that as

Table 1 Performance comparison of state-of-the-art methods against proposed method.

Method	C	CR
Shock filter	83	0.0849
Iterative Wiener filter	85	0.0746
NLSRD	120	0.7553

the threshold reduces with each iteration of the filtering process, the segmentation yields better results with inclusion of the Gaussian component in the mixture model.

It is also seen that the peak frequency and mean value of the Maxwell component reduces for the first few iterations, and then remains steady for the remaining iterations. This evidently means that for the first few iterations, many voxels represented initially as background, get reclassified as vascular elements due to the filtering process. The initial misclassification of these elements can be attributed to the inherent blur.

Left panels of Fig. 10 show images segmented using mixture model containing only Maxwell distribution ($C_G = 0$) representing the background. Segmented images in right panels are obtained using a mixture of Maxwell and Gaussian distributions. In both cases, voxels having flow information is represented using uniform distribution. Beginning from second row, the left and right panels show images segmented during 1st, 6th, and 12th iteration of the filtering process.

4.2 Visual Quality

As a measure of the visual quality, we compare the shock filter, iterative Wiener filter, and nonlocally stabilized reverse diffusion based on CR and contrast per pixel computation. The CR is computed as

$$CR = \frac{\max[U] - \min[U]}{\max[U] + \min[U]}, \quad (14)$$

where U is an image of size $M \times N$. The contrast per pixel (C) for U is defined as the average difference in gray level between adjacent pixels and is given by

$$C = \frac{\sum_{i=1}^N \sum_{j=1}^M (\sum_{(m,n) \in R(i,j)} |U(i,j) - U(m,n)|)}{M \times N}, \quad (15)$$

where R represents a local neighborhood of (i,j) th pixel. Increase in C indicates average increase in contrast difference for each pixel compared to the neighboring pixels. The comparisons of results for various filters are summarized in Table 1. Major drawbacks of the iterative Wiener filter are the failure to detect less intense vascular structures and background suppression. Shock filter increases background noise and the edge curvatures get lost. From Table 1, it is clear that NLSRD outperforms the existing methods in terms of CR and contrast per pixel.

Fig. 11 (a) Combined MIP prior to filtering and (b) combined MIP images after filtering.

5 Discussion

Application of stabilized reverse diffusion to low-resolution PC-MRA is able to capture signals of fine details such as small vessels, when acquisition is performed in a larger FOV. A major drawback of this method is the inability to enhance narrow vessels masked by intravoxel dephasing effects. For a given extent of degradation, the method exhibits faster convergence than the Laplacian form of reverse diffusion. With increase in blur, the convergence slows down. Using simulated data, the evolution of structureness measures in successive iterations conforms to the convergence behavior of the deconvolution process. Further, the robustness of our filter applied to real datasets is demonstrated using statistical models elucidating the improvement in flow quantification. As applicable to validation of deconvolution methods using standard forms of image quality metrics, iteratively filtered MIP images show improvement in terms of contrast per pixel and CR as compared with other state-of-the-art methods.

Simulated results show that the filter works equally well with all coil sizes, and its performance is not particularly affected under nonstationary conditions introduced by increased coil-to-coil coupling at larger radii. These characteristics indicate the fact that unknown blurring kernel applies to the sensitivity encoded channel images. The results are particularly promising for the case of images reconstructed using intensity projections. Figure 11 shows that the amount of information encoded in combined multichannel MIP images increases with coil radius. Columns (a)–(b) shows channel images for different coil radii combined using sum-of-squares method, without and with application of the NLSRD filter.

6 Conclusion

We demonstrated the application of nonlocally stabilized form of reverse diffusion to deblur intensity projected channel images prior to sum-of-squares reconstruction in multichannel PC-MRA. The blur introduced during acquisition reduces the contrast of flow images reconstructed using complex difference method. Application of stabilized reverse diffusion to flow channel images is found to yield high-quality images after sum-of-square combination. Best performance of the filter requires an *a priori* adjustment of the regularization parameter. The extension to motion blur kernels could be an interesting generalization, as well as the extension for application to three-dimensional partial Fourier reconstruction.

Acknowledgments

The authors wish to thank the Kerala State Council for Science, Technology and Environment (No. 199/2014/KSCSTE under ETP scheme) for scholarship and financial support to carry out this work.

References

1. Y. Pang, B. Yu, and X. Zhang, "Enhancement of the low resolution image quality using randomly sampled data for multi-slice MR imaging," *Quant. Imaging Med. Surg.* **4**(2), 136–144 (2014).
2. N. J. Pelc et al., "Encoding strategies for three-direction phase-contrast MR imaging of flow," *J. Magn. Reson. Imaging* **1**, 405–413 (1991).
3. M. A. Bernstein et al., "Reconstructions of phase contrast, phased array multicore data," *Magn. Reson. Med.* **32**(3), 330–334 (1994).
4. M. Lustig, D. L. Donoho, and J. M. Pauly, "Sparse MRI: the application of compressed sensing for rapid MR imaging," *Magn. Reson. Med.* **58**(6), 1182–1195 (2007).
5. M. Lustig, D. D. J. Santos, and J. Pauly, "Compressed sensing MRI," *IEEE Signal Process. Mag.* **25**(2), 72–82 (2008).
6. A. Buades, B. Coll, and J. M. Morel, "Image enhancement by non-local reverse heat equation," Technical Report 22, CMLA (2006).
7. S. Lefkimmatis, A. Bourquard, and M. Unser, "Hessian-based norm regularization for image restoration with biomedical applications," *IEEE Trans. Image Process.* **21**(3), 983 (2012).
8. P. Perona and J. Malik, "Scale space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Anal. Mach. Intell.* **12**(7), 629–639 (1990).
9. A. P. Witkin, "Scale-space filtering," in *Proc. Eighth Int. Joint Conf. on Artificial Intelligence (IJCAI, 83)*, Aug. 8–12, 1983, Vol. 2, pp. 1019–1022, Karlsruhe (1983).
10. J. Du et al., "Noise reduction in MR angiography with nonlinear anisotropic filtering," *J. Magn. Reson. Imaging* **19**, 632–639 (2004).
11. P. Thirion, "Image matching as a diffusion process: an analogy with Maxwell's demon," *Med. Image Anal.* **2**(3), 243–260 (1998).
12. J. George and S. P. Indu, "Fast adaptive anisotropic filtering for medical image enhancement," *IEEE Int. Symp. Signal Process. Inf. Technol.* 227–232 (2008).
13. Y. L. You et al., "Behavioral analysis of anisotropic diffusion in image processing," *IEEE Trans. Image Process.* **5**(11), 1539–1553 (1996).
14. M. J. Black et al., "Robust anisotropic diffusion," *IEEE Trans. Image Process.* **7**(3), 421–432 (1998).
15. V. P. Namboodiri and S. Chaudhuri, "Image restoration using geometrically stabilized reverse heat equation," in *Int. Conf. on Image Processing (ICIP '07)*, Vol. 4, pp. 413–416 (2007).
16. S. Kindermann, S. Osher, and P. W. Jones, "Deblurring and denoising of images by nonlocal functionals," *Multiscale Model. Simul.* **4**, 1091–1115 (2005).
17. I. Pollak, A. S. Willsky, and J. Krim, "Image segmentation and edge enhancement with stabilized inverse diffusion equations," *IEEE Trans. Image Process.* **9**(2), 256–266 (2000).
18. L. I. Rudin, "Images, numerical analysis of singularities and shock filters," PhD thesis, California Institute of Technology, Pasadena, CA, (1987).
19. S. Osher and L. I. Rudin, "Feature-oriented image enhancement using shock filters," *SIAM J. Numer. Anal.* **27**(4), 919–940 (1990).
20. R. Hummel and R. Moniot, "Reconstructions from zero-crossings in scale space," *IEEE Trans. Acoust. Speech Signal Process.* **37**, 2111–2130 (1989).
21. R. A. Hummel, B. Kimia, and S. W. Zucker, "Deblurring Gaussian blur," *Comput. Vision Graphics Image Process.* **38**(1), 66–80 (1987).
22. G. Gilboa, N. Sochen, and Y. Y. Zeevi, "Regularized shock filters and complex diffusion," in *Proc. European Conf. Computer Vision (ECCV '02)*, pp. 399–313 (2002).
23. L. Alvarez and L. Mazorra, "Signal and image restoration using shock filters and anisotropic diffusion," *SIAM J. Numer. Anal.* **31**(2), 590–605 (1994).
24. L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Phys. D* **60**(1–4), 259–268 (1992).
25. M. Grasmair and F. Lenzen, "Anisotropic total variation filtering," *Appl. Math. Optim.* **62**(3), 323–339 (2010).
26. M. Jung and L. A. Vese, "Nonlocal variational image deblurring models in the presence of Gaussian or impulse noise," *Lect. Notes Comput. Sci.* **5567**, 401–412 (2009).
27. X. Liu and L. Huangb, "A new nonlocal total variation regularization algorithm for image denoising," *Math. Comput. Simul.* **97**, 224–233 (2014).
28. A. Buades, B. Coll, and M. J. Morel, "A non-local algorithm for image de-noising," in *IEEE Computer Society Conf. on Comp. Vision and Patt. Recog.*, Vol. 2, pp. 60–65, (2005).
29. A. Buades, B. Coll, and J. M. Morel, "A review of image denoising methods, with a new one," *Multiscale Model. Simul.* **4**(2), 490–530 (2005).
30. Y. Lou et al., "Image recovery via nonlocal operators," *J. Sci. Comput.* **42**, 185–197 (2010).
31. G. Gilboa and S. Osher, "Nonlocal operators with applications to image processing," *Multiscale Model. Simul.* **7**, 1005–1028 (2008).
32. F. Guichard and J. M. Morel, "A note on two classical enhancement filters and their associated PDE's," *Int. J. Comput. Vision* **52**(2/3), 153–160 (2003).

33. A. B. Matt et al., "Reconstructions of phase contrast, phased array multicoil data," *Magn. Reson. Med.* **32**, 330–334 (1994).
34. C. S. C. Albert, J. A. Noble, and P. Summers, "Vascular segmentation of phase contrast magnetic resonance angiograms based on statistical mixture modeling and local phase coherence," *IEEE Trans. Med. Imaging* **23**(12), 1490–1507 (2004).
35. A. H. Andersen and J. E. Kirsch, "Analysis of noise in phase contrast MR imaging," *Med. Phys.* **23**, 857–869 (1996).
36. A. C. S. Chung, *Vessel and Aneurysm Reconstruction Using Speed and Flow Coherence Information in Phase Contrast Magnetic Resonance Angiograms*, Department of Engineering Science, University of Oxford, Trinity, Oxford, United Kingdom (2001).
37. M. Griswold, R. Heidemann, and P. M. Jakob, "The GRAPPA operator," in *Proc. 11th Annual Meeting ISMRM*, Toronto Canada, pp. 2348 (2003).
38. J. A. Bilmes, *A Gentle Tutorial of the EM Algorithm and its application to Parameter Estimation for Gaussian Mixture and Hidden Markov Models*, Dept. of Electrical Engineering and Computer Science, Berkeley (1998).
39. M. Sabry et al., *Statistical Cerebrovascular Segmentation for Phase Contrast MRA Data*, pp. 32–37, University of Louisville, Louisville ACBME (2002).

Akshara P. Krishnan recently completed her MPhil thesis on restoration of intensity projected angiograms. She is currently working as a project fellow at the Medical Image Computing and Signal Processing Laboratory, Indian Institute of Information Technology, India. Ajin Joy holds a postgraduate degree in engineering and currently is pursuing a PhD at the Medical Image Computing and Signal Processing Laboratory, Indian Institute of Information Technology, Kerala, India. His research interests include application of compressed sensing and image reconstruction to magnetic resonance imaging.

Ajin Joy holds a postgraduate degree in engineering and currently pursuing a PhD at the Medical Image Computing and Signal Processing Laboratory, Indian Institute of Information Technology, Kerala, India. His research interests include application of compressed sensing and image reconstruction to magnetic resonance imaging.

Joseph Suresh Paul is currently serving as associate professor in the Medical Image Computing and Signal Processing Lab at the Indian Institute of Information Technology, Kerala, India. He holds a PhD degree from Indian Institute of Technology, Madras, in the area of signal processing. His current research focuses on parallel magnetic resonance image reconstruction and compressed sensing.