New stability criterion of neural networks with leakage delays and impulses: a piecewise delay method

R. Suresh Kumar¹ · G. Sugumaran² · R. Raja³ · Quanxin Zhu⁴ · U. Karthik Raja⁵

Received: 26 May 2015 / Revised: 6 September 2015 / Accepted: 15 September 2015 / Published online: 29 September 2015
© Springer Science+Business Media Dordrecht 2015

Abstract This paper analyzes the global asymptotic stability of a class of neural networks with time delay in the leakage term and time-varying delays under impulsive perturbations. Here the time-varying delays are assumed to be piecewise. In this method, the interval of the variation is divided into two subintervals by its central point. By developing a new Lyapunov–Krasovskii functional and checking its variation in between the two subintervals, respectively, and then we present some sufficient conditions to guarantee the global asymptotic stability of the equilibrium point for the considered neural network. The proposed results which do not require the boundedness, differentiability and monotonicity of the activation functions, can be easily verified via the linear matrix inequality (LMI) control toolbox in MATLAB. Finally, a numerical example and its simulation are given to show the

conditions obtained are new and less conservative than some existing ones in the literature.

Keywords Asymptotic stability · Time-varying delay · Lyapunov–Krasovskii functional · Leakage delay · Impulse

Introduction

Neural networks have attracted much attention due to their applications in many areas of real world problems such as optimization problems, associative memory, classification of patterns etc., So far, there are various types of neural networks such as cellular neural networks (CNNs), bidirectional associative memory neural networks (BAMNNs), Hopfield neural network (HNNs), Chaotic neural networks and Cohen–Grossberg neural network (CGNNs) which have been studied by many researchers for their enormous applications, see (Cao and Wang 2003; Cho and Park 2007; Haykin 1999; Kosko 1992; Liu 1997; Meng and Wang 2007; Mou et al. 2008; Senan and Arik 2007; Tan et al. 2015; Wang et al. 2008; Yang et al. 2014). These applications heavily depend on the stability of the equilibrium point of neural networks. Therefore, the stability analysis is essential for the design and applications of neural networks. As is well known, time delays is a natural phenomenon frequently encountered in various engineering systems, automatic control systems, population models, inferred grinding models, the AIDS epidemic and so on (Arik 2004; Gopalsamy 1992); Gu et al. 2003. Moreover, the existence of time delays in the network may lead to instability or bad performance of systems. Recently neural networks with various types of delay which have been widely investigated by many authors; see (Cao and Li 2005; Liu et al. 2006; Meng and Wang 2007; Wang et al.

✉ Quanxin Zhu
zqx22@126.com

R. Raja
antony.raja67@yahoo.com

¹ Department of Electrical and Electronic Engineering, Anna University Regional Centre, Coimbatore 641 047, India

² Department of Electrical and Electronic Engineering, Sri Krishna College of Engineering and Technology, Coimbatore 641 008, India

³ Ramanujan Centre for Higher Mathematics, Alagappa University, Karaikudi 630 004, India

⁴ School of Mathematical Sciences and Institute of Finance and Statistics, Nanjing Normal University, Nanjing 210 023, China

⁵ Department of Mathematics, K.S.R College of Arts and Science, Thiruchengodu 637 215, India

2006; Yang and Xu 2005) and references therein. However, so far there has been very little interest in neural networks with time delay in the leakage (or forgetting) term (see Gopalsamy 2007; Peng 2010). Moreover, time delay in the leakage term can also have the ability to crash the activity of dynamic behavior such as instability or poor performance of the given system. Hence it is considered that the leakage delay in dynamical neural networks is an important research topic in the field of stability analysis of neural networks (Li and Huang 2009; Li et al. 2010; Song and Cao 2012).

Furthermore, impulsive effects can be found in a wide range of evolutionary processes, especially in biological systems such as biological neural networks, some bursting rhythm models in pathology, optimal control in economics, frequency-modulated signal processing systems, flying object motions, in which many sudden and sharp changes occur simultaneously, in the form of impulse. In the implementation of neural networks, it has also been shown that the presence of impulsive perturbations is likewise unavoidable (Fu et al. 2005; Ignatyev 2008; Lakshmikantham et al. 1989). So, the combination of impulsive perturbations and time delays in the leakage term can change the dynamic behavior of the neural network. Recently, the survey among the existing results on delayed neural networks and impulsive perturbations can only be regarded as an ideal situation and they contain few errors. Very recently, the authors (Akca et al. 2004; Liu et al. 2005; Xu and Yang 2005), have established some novel methods to reflect such a more realistic dynamics for delayed neural networks in the presence of impulsive perturbations in which the occurring perturbations depend on not only the current state of neurons at impulse times t_k but also the state of neurons in recent history.

Based on the above motivated points, this paper considers a class of neural networks with time delay in the leakage term and impulsive effect to guarantee the global asymptotic stability of the addressed network. By constructing a suitable Lyapunov–Krasovskii functional and LMI technique combined with free weighting matrix method, we obtain new sufficient conditions to ensure the global asymptotic stability of the NN with time delay in the leakage term and impulsive perturbations. Moreover, the limitations on the activation functions like boundedness, monotonicity and differentiability, which are not required for our proposed work and the results obtained can be easily verified using MATLAB LMI Control tool box. In order to show some novelty to this paper, we have assumed that the time-varying delays as piecewise delay and also measured the variation in between the intervals by its central point. Finally, a numerical example is given to demonstrate the effectiveness of the proposed results in this

paper and also we have compared our results with some existing ones in the literature.

Notations: The notations are quite standard throughout this paper; $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; $\mathbb{R}^{n \times m}$ is the set of real $n \times m$ matrices; I is the identity matrix of appropriate dimensions; $\|\cdot\|$ stands for the Euclidean vector norm or spectral norm as appropriate. The notation $X > 0$ (respectively $X < 0$), for $X \in \mathbb{R}^{n \times n}$ means that the matrix X is a real symmetric positive definite (respectively, negative definite). The notation A^T and A^{-1} means the transpose of A and the inverse of the square matrix. For any interval $J \subseteq \mathbb{R}$, set $V \subseteq \mathbb{R}^k$ ($1 \leq k \leq n$), $C(J, V) = \{\phi : J \rightarrow V \text{ is continuous}\}$ and $PC^1(J, V) = \{\phi : J \rightarrow V \text{ is continuously differentiable everywhere except at finite number of point } t \text{ at which } \phi(t^+), \phi(t^-), \dot{\phi}(t^+) \text{ and } \dot{\phi}(t^-) \text{ exist and } \phi(t^+) = \phi(t), \dot{\phi}(t^+) = \dot{\phi}(t), \text{ where } \dot{\phi} \text{ denotes the derivative of } \phi. \text{ The symbol } * \text{ in a matrix is used to denote a term that is induced by symmetry.}\}$

Model description and preliminaries

Consider a continuous-time neural network model with time-delay in the leakage term and impulses as follows:

$$\begin{aligned} \dot{x}(t) &= -Cx(t - \sigma) + Af(x(t)) + Bf(x(t - \tau(t))) \\ &\quad + D \int_{t-\tau(t)}^t f(x(s))ds + I, \quad t \neq t_k, \\ \Delta x(t_k) &= x(t_k^+) - x(t_k^-) = J_k(x(t_k^-), x_{t_k^-}), \quad t = t_k, \\ x(s) &= \phi(s), \quad s \in [-\rho, 0], \end{aligned}$$

where $x(t) = (x_1, x_2, \dots, x_n)^T$ is the neuron state vector of the considered network; $C = \text{diag}(c_1, c_2, \dots, c_n)$ is a diagonal matrix with $c_i > 0$ ($i = 1, 2, \dots, n$); A, B and D are the connection weight matrix, discrete delayed connection weight matrix and distributed delayed connection weight matrix, respectively. I is a constant external input vector; $f(x(\cdot)) = f_1(x_1(\cdot)), f_2(x_2(\cdot)), \dots, f_n(x_n(\cdot))^T$ represents the neuron activation functions; $\tau(t)$ represents the transmission time-varying delay; $\sigma \geq 0$ denote a leakage delay; $\Delta x(t_k)$ describes the evolution process that experiences abrupt change of state at moments t_k , where $J_k(x(t_k^-), x_{t_k^-})$ is the incremental change of state at moments t_k and $J_k(0, 0) = 0$. The fixed moments of time t_k satisfy $t_1 < t_2 < \dots, \lim_{k \rightarrow +\infty} t_k = +\infty$ and $x(t^-) = \lim_{s \rightarrow t^-} x(s)$; $\phi(s)$ is the initial condition for the considered neural network (1), where $\rho = \max\{\sigma, \tau_M\}$, $\phi(\cdot) = (\phi_1, \phi_2, \dots, \phi_n)^T \in PC^1([-\rho, 0], \mathbb{R}^n)$.

In this paper, we make the following assumptions:

Assumption 1 For $i \in 1, 2, \dots, n$, the neuron activation functions $f_i(\cdot)$ is continuous, bounded and satisfies the following condition:

$$\sigma_i^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq \sigma_i^+, \quad \forall s_1 \neq s_2 \in \mathbb{R}, \quad (2)$$

where σ_i^+ and σ_i^- are known constants.

Remark 1 The aforementioned Assumption 1 was proposed by Liu et al. (2006). and Wang et al. (2006), respectively. Here the constants σ_i^- , σ_i^+ in Assumption 1 are allowed to be positive, negative or zero. Hence, the resulting activation functions may be non-monotonic, and the assumption is less conservative than the descriptions on both the sigmoid activation functions and the Lipschitz-type activation functions.

Assumption 2 The time delay $\tau(t)$ is a time-varying differentiable function that satisfies:

$$\tau_m \leq \tau(t) \leq \tau_M, \quad \dot{\tau}(t) \leq \mu < 1, \quad (3)$$

where τ_m , τ_M and μ are known constants.

For convenience, we shift the equilibrium point $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ to the origin by the translation $u(t) = x(t) - x^*$, which yields the following system:

$$\begin{aligned} \dot{u}(t) &= -Cu(t - \sigma) + Ag(u(t)) + Bg(u(t - \tau(t))) \\ &\quad + D \int_{t-\tau(t)}^t g(u(s))ds, \quad t \neq t_k, \\ \Delta u(t_k) &= u(t_k^+) - u(t_k^-) = J_k(u(t_k^-), u_{t_k^-}) \\ &= -E_k \left\{ u(t_k^-) - C \int_{t_k-\sigma}^{t_k} u(\theta)d\theta \right\}, \quad t = t_k, \\ u(s) &= \phi(s) - x^*, \quad s \in [-\rho, 0], \end{aligned} \quad (4)$$

where $u(t) = (u_1, u_2, \dots, u_n)^T$, $g(u(t)) = (g_1(u_1(t)), g_2(u_2(t)), \dots, g_n(u_n(t)))^T$, $g(u(t - \tau(t))) = (g_1(u_1(t - \tau(t))), g_2(u_2(t - \tau(t))), \dots, g_n(u_n(t - \tau(t))))^T$, $g_i(u_i(t)) = f_i(x_i(t) + x_i^*) - f_i(x_i^*)$, $E_k (k \in \mathbb{Z}_+)$ are some $n \times n$ real matrices.

By Assumption 1, it can be verified that

$$\sigma_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq \sigma_i^+, \quad \forall s_1 \neq s_2 \in \mathbb{R}, \quad (5)$$

$$g_i(0) = 0, \quad i = 1, 2, \dots, n. \quad (6)$$

In the following, we define two scalars τ_0 and δ related to the variation range of time delay:

$$\tau_0 = \frac{\tau_M + \tau_m}{2}, \quad \delta = \frac{\tau_M - \tau_m}{2}.$$

Remark 2 Here τ_0 is the central point of the interval of the time-varying delay and the method of constructing Lyapunov functional by utilizing the central point was

often named delay center point (DCP) method, which was first proposed in Yue (2004) to study the stabilization for systems with interval time-varying delay. However, in our paper, the DCP method will be improved by introducing a piecewise analysis method in respect to the time delay.

It is easy to see from Assumption 2 that for all $t \in \mathbb{R}^+$, we have $\tau(t) \in [\tau_m, \tau_0]$ or $\tau(t) \in (\tau_0, \tau_M]$. Consequently, we define the following two sets:

$$\begin{aligned} \eta_1 &= \{t | t \in \mathbb{R}^+, \tau(t) \in [\tau_m, \tau_0]\}, \\ \eta_2 &= \{t | t \in \mathbb{R}^+, \tau(t) \in (\tau_0, \tau_M]\}. \end{aligned}$$

Obviously, $\eta_1 \cap \eta_2 = \emptyset$ (an empty set) and $\eta_1 \cup \eta_2 = \mathbb{R}^+$. In the proof of our main results, we will check the variation of derivative of the Lyapunov functional in η_1 and η_2 , respectively.

Now, we need the following lemmas in proving our asymptotic stability results for the addressed network (1).

Lemma 1 Sanchez and Perez (1999) Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ and a scalar $\epsilon > 0$. Then we have:

$$x^T y + y^T x \leq \epsilon x^T x + \epsilon^{-1} y^T y.$$

Lemma 2 Han and Yue (2007) For any symmetric constant matrix $R \in \mathbb{R}^{n \times n}$, $R \geq 0$, scalars τ_m, τ_M with $\tau_m \leq \tau_M$, and a vector valued function $\dot{x}(t) : [-\tau_M, -\tau_m] \rightarrow \mathbb{R}^n$, $t \in \mathbb{R}^+$ such that the following integration is well defined, then

$$\begin{aligned} & -(\tau_M - \tau_m) \int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s) R \dot{x}(s) ds \\ & \leq \begin{bmatrix} x(t - \tau_m) \\ x(t - \tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t - \tau_m) \\ x(t - \tau_M) \end{bmatrix}. \end{aligned}$$

Lemma 3 Zhang et al. (2009) For any constant matrices Ψ_1 and Ψ_2 of appropriate dimensions and a symmetric matrix $\Omega < 0$, scalars $\tau_m \leq \tau_M$ and a function $\tau(t) : \mathbb{R}^+ \rightarrow [\tau_m, \tau_M]$, then

$$(\tau(t) - \tau_m)\Psi_1 + (\tau_M - \tau(t))\Psi_2 + \Omega < 0 \quad (7)$$

holds, if and only if:

$$(\tau_M - \tau_m)\Psi_1 + \Omega < 0, \quad (8)$$

$$(\tau_M - \tau_m)\Psi_2 + \Omega < 0, \quad (9)$$

hold.

Proof (Necessity part:) Let $\tau(t) = \tau_m$ in (7), we can obtain (8) holds. Similarly, $\tau(t) = \tau_M$ in (7) implies (9) holds.

(Sufficient part:) Define a function as

$$\mathcal{K}(\tau(t)) = (\tau(t) - \tau_m)\Psi_1 + (\tau_M - \tau(t))\Psi_2 + \Omega, \quad (10)$$

which can be further rewritten as

$$\begin{aligned} \mathcal{K}(\tau(t)) &= \frac{\tau(t) - \tau_m}{\tau_M - \tau_m} [(\tau_M - \tau_m)\Psi_1 + \Omega] \\ &+ \frac{\tau_M - \tau(t)}{\tau_M - \tau_m} [(\tau_M - \tau_m)\Psi_2 + \Omega]. \end{aligned} \quad (11)$$

From (7) and (8), we can conclude that $\mathcal{K}(\tau(t)) < 0$ for all $\tau(t) \in [\tau_m, \tau_M]$, that is,

$$(\tau(t) - \tau_m)\Psi_1 + (\tau_M - \tau(t))\Psi_2 + \Omega < 0.$$

Lemma 4 For any constant symmetric positive-definite matrix $J \in \mathbb{R}^{m \times m}$, a scalar $\eta > 0$, and the vector function $v: [0, \eta] \rightarrow \mathbb{R}^m$, the following inequality holds:

$$\eta \int_0^\eta v^T(s) J v(s) ds \geq \left(\int_0^\eta v(s) ds \right)^T J \left(\int_0^\eta v(s) ds \right).$$

Lemma 5 (Schur complement Boyd et al. (1994)). For a given matrix

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} > 0,$$

where $\Sigma_{11} = \Sigma_{11}^T, \Sigma_{22} = \Sigma_{22}^T$, is equivalent to any one of the following conditions:

- (i) $\Sigma_{22} > 0, \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}^T > 0;$
- (ii) $\Sigma_{11} > 0, \Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12} > 0.$

Main results

For our convenience, we use the following notations throughout this paper:

$$\begin{aligned} \Sigma_1 &= \text{diag}\{\sigma_1^+ \sigma_1^+, \sigma_2^+ \sigma_2^+, \dots, \sigma_n^+ \sigma_n^+\}, \\ \Sigma_2 &= \text{diag}\left\{\frac{\sigma_1^- + \sigma_1^+}{2}, \frac{\sigma_2^- + \sigma_2^+}{2}, \dots, \frac{\sigma_n^- + \sigma_n^+}{2}\right\}, \\ N^T &= [N_1^T \ N_2^T \ N_3^T \ N_4^T \ N_5^T \ N_6^T \ N_7^T \ N_8^T \ N_9^T], \\ M^T &= [M_1^T \ M_2^T \ M_3^T \ M_4^T \ M_5^T \ M_6^T \ M_7^T \ M_8^T \ M_9^T], \\ U^T &= [U_1^T \ U_2^T \ U_3^T \ U_4^T \ U_5^T \ U_6^T \ U_7^T \ U_8^T \ U_9^T], \\ V^T &= [V_1^T \ V_2^T \ V_3^T \ V_4^T \ V_5^T \ V_6^T \ V_7^T \ V_8^T \ V_9^T], \\ \xi^T(t) &= [u^T(t) \ u^T(t - \tau_m) \ u^T(t - \tau_M) \ u^T(t - \tau_0) \\ &\quad \times u^T(t - \tau(t)) \left(\int_{t-\sigma}^t u(\theta) d\theta \right)^T \ g^T(u(t)) \\ &\quad \times g^T(u(t - \tau(t))) \left(\int_{t-\tau(t)}^t g(u(s)) ds \right)^T]. \end{aligned}$$

In this section, we study (or investigate) the global asymptotic stability of the continuous-time neural network with time delay in the leakage term and impulsive effects. Based on the Lyapunov–Krasovskii function and piecewise delay method, we obtain the delay-dependent asymptotic stability conditions as follows:

Theorem 1 Suppose that Assumptions 1 and 2 hold. For given scalars $0 \leq \mu < 1$, τ_m and τ_M with $0 \leq \tau_m \leq \tau_M$, the system (4) is globally asymptotically stable if there exist positive symmetric matrices P, Q_i ($i = 1, 2, 3$), R_i ($i = 1, 2, 3$), S , positive diagonal matrices Λ, Γ_i ($i = 1, 2$) and matrices N_l, M_l, U_l and V_l ($l = 1, 2, \dots, 9$) of appropriate dimensions such that for all $i = 1, 2$ and $j = 1, 2$ the following LMIs hold:

$$\begin{bmatrix} \Pi + \Xi_i & \Theta_{ij} \\ * & -R_i \end{bmatrix} < 0 \quad (i, j = 1, 2), \quad (12)$$

$$\begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \geq 0 \quad \text{and} \quad \begin{bmatrix} P & (I - E_k)P \\ * & P \end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+ \quad (13)$$

with

$$\begin{aligned} \Pi &= \begin{bmatrix} \Pi_{11} & R_2 & 0 & 0 & 0 & \Pi_{16} & \Pi_{17} & \Pi_{18} & \Pi_{19} \\ * & \Pi_{22} & 0 & X_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X_3 & -X_2^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & \Pi_{58} & 0 \\ * & * & * & * & * & \Pi_{66} & \Pi_{67} & \Pi_{68} & \Pi_{69} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} & \Pi_{79} \\ * & * & * & * & * & * & * & \Pi_{88} & \Pi_{89} \\ * & * & * & * & * & * & * & * & \Pi_{99} \end{bmatrix}, \\ \Xi_1 &= \begin{bmatrix} 0 & \Xi_{12} & 0 & \Xi_{14} & \Xi_{15} & 0 & 0 & 0 & 0 \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} & \Xi_{27} & \Xi_{28} & \Xi_{29} \\ * & * & -R_3 & \Xi_{34} & \Xi_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & \Xi_{48} & \Xi_{49} \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & \Xi_{58} & \Xi_{59} \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \end{bmatrix}, \\ \Xi_2 &= \begin{bmatrix} 0 & \Xi_{12}^2 & 0 & \Xi_{14}^2 & \Xi_{15}^2 & 0 & 0 & 0 & 0 \\ * & -R_2 & \Xi_{23}^2 & \Xi_{24}^2 & \Xi_{25}^2 & 0 & 0 & 0 & 0 \\ * & * & \Xi_{33}^2 & \Xi_{34}^2 & \Xi_{35}^2 & \Xi_{36}^2 & \Xi_{37}^2 & \Xi_{38}^2 & \Xi_{39}^2 \\ * & * & * & \Xi_{44}^2 & \Xi_{45}^2 & \Xi_{46}^2 & \Xi_{47}^2 & \Xi_{48}^2 & \Xi_{49}^2 \\ * & * & * & * & \Xi_{55}^2 & \Xi_{56}^2 & \Xi_{57}^2 & \Xi_{58}^2 & \Xi_{59}^2 \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \end{bmatrix}, \end{aligned}$$

$$\Theta_{11} = \tau_s N, \Theta_{12} = \tau_s M, \Theta_{21} = \tau_s U, \Theta_{22} = \tau_s V$$

and $\Pi_{11} = -C^T P - PC + Q_1 + R_1 + \tau_s^2 C^T (R_2 + R_3) C + \sigma^2 S + \tau_m^2 (-C^T Q_3 - Q_3 C) - 2\Gamma_1 \Sigma_1 - R_2$, $\Pi_{16} = C^T P C$, $\Pi_{17} = PA - C^T \Lambda - \tau_s^2 C^T (R_2 + R_3) A - \tau_m^2 C^T Q_3 A + \Gamma_1 \Sigma_2$, $\Pi_{18} = PB - \tau_s^2 C^T (R_2 + R_3) B - \tau_m^2 C^T Q_3 B + \Gamma_2 \Sigma_2$, $\Pi_{19} = PD - \tau_s^2 C^T (R_2 + R_3) D - \tau_m^2 C^T Q_3 D$, $\Pi_{22} = -R_1 + X_1 - R_2$, $\Pi_{44} = X_3 - X_1$, $\Pi_{55} = -(1 - \mu)Q_1 - 2\Gamma_2 \Sigma_1$, $\Pi_{58} = \Gamma_2 \Sigma_2$, $\Pi_{66} = -\frac{1}{\sigma}$, $\Pi_{67} = -C^T P A$, $\Pi_{68} = -C^T P B$, $\Pi_{69} = -C^T P D$, $\Pi_{77} = A^T \Lambda + \Lambda A + Q_2 + \tau_s^2 A^T (R_2 + R_3)$

$$\begin{aligned}
& A + \tau_m^2 A^T Q_3 A - 2\Gamma_1, \quad \Pi_{78} = AB + \tau_s^2 A^T (R_2 + R_3)B + \tau_m^2 A^T Q_3 B, \quad \Pi_{79} = AD + \tau_s^2 A^T (R_2 + R_3)D + \tau_m^2 A^T Q_3 D, \quad \Pi_{88} = \\
& -(1 - \mu)Q_2 + \tau_s^2 B^T (R_2 + R_3)B + \tau_m^2 B^T Q_3 B - 2\Gamma_2, \quad \Pi_{89} = \tau_s^2 B^T (R_2 + R_3)D + \tau_m^2 B^T Q_3 D, \quad \Pi_{99} = \tau_s^2 D^T (R_2 + R_3)D + \\
& \tau_m^2 D^T Q_3 D, \quad \Xi_{12}^1 = \tau_s N_1, \quad \Xi_{14}^1 = -\tau_s M_1, \quad \Xi_{15}^1 = -\tau_s N_1 + \tau_s M_1, \quad \Xi_{22}^1 = \tau_s N_2 + \tau_s N_2^T, \quad \Xi_{23}^1 = \tau_s N_3, \quad \Xi_{24}^1 = \tau_s N_4 - \tau_s M_2, \\
& \Xi_{25}^1 = -\tau_s N_2 + \tau_s M_2 + \tau_s N_5, \quad \Xi_{26}^1 = \tau_s N_6, \quad \Xi_{27}^1 = \tau_s N_7, \quad \Xi_{28}^1 = \tau_s N_8, \quad \Xi_{29}^1 = \tau_s N_9, \quad \Xi_{34}^1 = -\tau_s M_3, \quad \Xi_{35}^1 = -\tau_s N_3 + \tau_s M_3, \\
& \Xi_{44}^1 = -\tau_s M_4 - \tau_s M_4^T, \quad \Xi_{45}^1 = -\tau_s N_4 + \tau_s M_4 - \tau_s M_5, \quad \Xi_{46}^1 = -\tau_s M_6, \quad \Xi_{47}^1 = -\tau_s M_7, \quad \Xi_{48}^1 = -\tau_s M_8, \quad \Xi_{49}^1 = -\tau_s M_9, \\
& \Xi_{55}^1 = -\tau_s N_5 - \tau_s N_5^T + \tau_s M_5 + \tau_s M_5^T, \quad \Xi_{56}^1 = -\tau_s N_6 + \tau_s M_6, \quad \Xi_{57}^1 = -\tau_s N_7 + \tau_s M_7, \quad \Xi_{58}^1 = -\tau_s N_8 + \tau_s M_8, \quad \Xi_{59}^1 = \\
& -\tau_s N_9 + \tau_s M_9, \quad \Xi_{12}^2 = -\tau_s V_1, \quad \Xi_{14}^2 = \tau_s U_1, \quad \Xi_{15}^2 = -\tau_s U_1 + \tau_s V_1, \quad \Xi_{23}^2 = -\tau_s V_2, \quad \Xi_{24}^2 = \tau_s U_2, \quad \Xi_{25}^2 = -\tau_s U_2 + \tau_s V_2, \\
& \Xi_{33}^2 = -\tau_s V_3 - \tau_s V_3^T, \quad \Xi_{34}^2 = \tau_s U_3 - \tau_s V_4^T, \quad \Xi_{35}^2 = -\tau_s U_3 + \tau_s V_3 - \tau_s V_5^T, \quad \Xi_{36}^2 = -\tau_s V_6^T, \quad \Xi_{37}^2 = -\tau_s V_7^T, \quad \Xi_{38}^2 = -\tau_s V_8^T, \\
& \Xi_{39}^2 = -\tau_s V_9^T, \quad \Xi_{44}^2 = \tau_s U_4 + \tau_s U_4^T, \quad \Xi_{45}^2 = -\tau_s U_4 + \tau_s U_5^T + \tau_s V_4, \quad \Xi_{46}^2 = \tau_s U_6^T, \quad \Xi_{47}^2 = \tau_s U_7^T, \quad \Xi_{48}^2 = \tau_s U_8^T, \quad \Xi_{49}^2 = \tau_s U_9^T, \\
& \Xi_{55}^2 = -\tau_s U_5 - \tau_s U_5^T + \tau_s V_5 + \tau_s V_5^T, \quad \Xi_{56}^2 = -\tau_s U_6^T + \tau_s V_6^T, \quad \Xi_{57}^2 = -\tau_s U_7^T + \tau_s V_7^T, \quad \Xi_{58}^2 = -\tau_s U_8^T + \tau_s V_8^T, \quad \Xi_{59}^2 = \\
& -\tau_s U_9^T + \tau_s V_9^T.
\end{aligned}$$

Proof In order to proceed with the stability analysis of system (4), we construct the following Lyapunov function candidate:

$$V(t, u(t)) = \sum_{i=1}^7 V_i(t, u(t)), \quad (14)$$

where

$$\begin{aligned}
V_1(t, u(t)) &= \left[u(t) - C \int_{t-\sigma}^t u(\theta) d\theta \right]^T P \\
&\quad \times \left[u(t) - C \int_{t-\sigma}^t u(\theta) d\theta \right] \\
V_2(t, u(t)) &= 2 \sum_{i=1}^n \lambda_i \int_0^{u_i(t)} g_i(s) ds \\
V_3(t, u(t)) &= \int_{t-\tau(t)}^t u^T(s) Q_1 u(s) ds \\
&\quad + \int_{t-\tau(t)}^t g^T(u(s)) Q_2 g(u(s)) ds \\
V_4(t, u(t)) &= \int_{t-\tau_m}^t u^T(s) R_1 u(s) ds \\
&\quad + \int_{t-\tau_0}^{t-\tau_m} \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix} ds
\end{aligned}$$

$$\begin{aligned}
V_5(t, u(t)) &= \tau_s \int_{t-\tau_0}^{t-\tau_m} \int_s^t \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta ds \\
&\quad + \tau_s \int_{t-\tau_m}^{t-\tau_0} \int_s^t \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta ds \\
V_6(t, u(t)) &= \sigma \int_{-\sigma}^0 \int_{t+s}^t u^T(\theta) S u(\theta) d\theta ds \\
V_7(t, u(t)) &= \tau_m \int_{t-\tau_m}^t \int_s^t \dot{u}_1^T(\theta) Q_3 \dot{u}_1(\theta) d\theta ds. \quad (15)
\end{aligned}$$

Calculating the time derivative of $V(t, u(t))$ along the solution of (4) and in addition to that, from Eqs. (5) and (6), it is easy to obtain that

$$\begin{aligned}
& [g_i(u_i(t)) - \sigma_i^- u_i(t)] [g_i(u_i(t)) - \sigma_i^+ u_i(t)] \leq 0 \\
& [g_i(u_i(t - \tau(t))) - \sigma_i^- u_i(t)] \\
& \quad \times [g_i(u_i(t - \tau(t))) - \sigma_i^+ u_i(t)] \leq 0. \quad (16)
\end{aligned}$$

Then from the above equation it can be deduced that there exist positive diagonal matrices $\Gamma_1 = \text{diag}\{\gamma_{1,1}, \gamma_{1,2}, \dots, \gamma_{1,n}\} > 0$, $\Gamma_2 = \text{diag}\{\gamma_{2,1}, \gamma_{2,2}, \dots, \gamma_{2,n}\} > 0$, such that

$$\begin{aligned}
& 2 \sum_{i=1}^n \gamma_{1,i} [g_i(u_i(t)) - \sigma_i^- u_i(t)] [g_i(u_i(t)) - \sigma_i^+ u_i(t)] \\
&= \sum_{i=1}^n \gamma_{1,i} \begin{bmatrix} u(t) \\ g(u(t)) \end{bmatrix}^T \begin{bmatrix} 2\sigma_i^- \sigma_i^+ e_i e_i^T & -(\sigma_i^- + \sigma_i^+) e_i^T e_i \\ * & 2e_i e_i^T \end{bmatrix} \\
&\quad \times \begin{bmatrix} u(t) \\ g(u(t)) \end{bmatrix} \\
&= \begin{bmatrix} u(t) \\ g(u(t)) \end{bmatrix}^T \begin{bmatrix} 2\Gamma_1 \Sigma_1 & -\Gamma_1 \Sigma_2 \\ * & 2\Gamma_1 \end{bmatrix} \begin{bmatrix} u(t) \\ g(u(t)) \end{bmatrix} \leq 0, \quad (17)
\end{aligned}$$

where e_i denotes a column vector having 1 element on its i th row and zeros elsewhere. Similarly,

$$\begin{aligned}
& 2 \sum_{i=1}^n \gamma_{2,i} [g_i(u_i(t - \tau(t))) - \sigma_i^- u_i(t - \tau(t))] \\
&\quad \times [g_i(u_i(t - \tau(t))) - \sigma_i^+ u_i(t - \tau(t))] \\
&= \sum_{i=1}^n \gamma_{2,i} \begin{bmatrix} u(t - \tau(t)) \\ g(u(t - \tau(t))) \end{bmatrix}^T \\
&\quad \times \begin{bmatrix} 2\sigma_i^- \sigma_i^+ e_i e_i^T & -(\sigma_i^- + \sigma_i^+) e_i^T e_i \\ * & 2e_i e_i^T \end{bmatrix} \begin{bmatrix} u(t - \tau(t)) \\ g(u(t - \tau(t))) \end{bmatrix} \\
&= \begin{bmatrix} u(t - \tau(t)) \\ g(u(t - \tau(t))) \end{bmatrix}^T \begin{bmatrix} 2\Gamma_2 \Sigma_1 & -\Gamma_2 \Sigma_2 \\ * & 2\Gamma_2 \end{bmatrix} \\
&\quad \times \begin{bmatrix} u(t - \tau(t)) \\ g(u(t - \tau(t))) \end{bmatrix} \leq 0. \quad (18)
\end{aligned}$$

$$\begin{aligned}
\dot{V}(t, u(t)) = & u^T(t) [-C^T P - PC + Q_1 + R_1 + \tau_s^2 C^T \\
& \times (R_2 + R_3)C + \sigma^2 S + \tau_m^2 (-C^T Q_3 \\
& - Q_3 C)] u(t) + 2u^T(t) C^T P C \\
& \times \left(\int_{t-\sigma}^t u(\theta) d\theta \right) + 2u^T(t) [PA - C^T A \\
& - AC - \tau_s^2 C^T (R_2 + R_3)A - \tau_m^2 C^T Q_3 A] \\
& \times g(u(t)) + 2u^T(t) [PB - \tau_s^2 C^T \\
& \times (R_2 + R_3)B - \tau_m^2 C^T Q_3 B] g(u(t - \tau(t))) \\
& + 2u^T(t) [PD - \tau_s^2 C^T (R_2 + R_3)D - \tau_m^2 \\
& \times C^T Q_3 D] \left(\int_{t-\tau(t)}^t g(u(s)) ds \right) \\
& + u^T(t - \tau_m) [X_1 - R_1] u(t - \tau_m) \\
& + u^T(t - \tau_m) X_2 u(t - \tau_0) - u^T(t - \tau_m) X_3 \\
& \times u(t - \tau_m) - u^T(t - \tau_m) X_2^T u(t - \tau_0) \\
& + u^T(t - \tau_0) [X_3 - X_1] u(t - \tau_0) \\
& + u^T(t - \tau(t)) [-(1 - \mu)Q_1] u(t - \tau(t)) \\
& + g^T(u(t)) [A^T A + \Lambda A + Q_2 + \tau_s^2 \\
& \times A^T (R_2 + R_3)A + \tau_m^2 A^T Q_3 A] g(u(t)) \\
& - 2 \left(\int_{t-\sigma}^t u(\theta) d\theta \right)^T C^T P A g(u(t)) \\
& - 2 \left(\int_{t-\sigma}^t u(\theta) d\theta \right)^T C^T P B g(u(t - \tau(t))) \\
& 2 \left(\int_{t-\sigma}^t u(\theta) d\theta \right)^T C^T P D \left(\int_{t-\tau(t)}^t g(u(s)) ds \right) \\
& + 2g^T(u(t)) [AB + \tau_s^2 A^T (R_2 + R_3)B \\
& + \tau_m^2 A^T Q_3 B] g(u(t - \tau(t))) + 2g^T(u(t)) [\Lambda D \\
& + \tau_s^2 A^T (R_2 + R_3)D + \tau_m^2 A^T Q_3 D] \\
& \times \left(\int_{t-\tau(t)}^t g(u(s)) ds \right) + g^T(u(t - \tau(t))) \\
& \times [-(1 - \mu)Q_2 + \tau_s^2 B^T (R_2 + R_3)B \\
& + \tau_m^2 B^T Q_3 B] g(u(t - \tau(t))) + 2g^T(u(t - \tau(t))) \\
& \times [\tau_s^2 B^T (R_2 + R_3)D + \tau_m^2 B^T Q_3 D] \\
& \times \left(\int_{t-\tau(t)}^t g(u(s)) ds \right) + \left(\int_{t-\tau(t)}^t g(u(s)) ds \right)^T \\
& \times [\tau_s^2 D^T (R_2 + R_3)D + \tau_m^2 D^T Q_3 D] \\
& \times \left(\int_{t-\tau(t)}^t g(u(s)) ds \right) - \sigma^2 \int_{t-\sigma}^t u^T(\theta) S u(\theta) d\theta \\
& - \tau_s \int_{t-\tau_0}^{t-\tau_m} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta - \tau_s \int_{t-\tau_M}^{t-\tau_0} \dot{u}_1^T(\theta) R_3 \\
& \times \dot{u}_1(\theta) d\theta - \tau_m^2 \int_{t-\tau_m}^t \dot{u}_1^T(\theta) Q_3 \dot{u}_1(\theta) d\theta.
\end{aligned} \tag{19}$$

By Lemma 2.4 and 2.6, it can be seen that

$$\sigma^2 \int_{t-\sigma}^t u^T(\theta) S u(\theta) d\theta \leq -\frac{1}{\sigma} \left(\int_{t-\sigma}^t u(\theta) d\theta \right)^T S \left(\int_{t-\sigma}^t u(\theta) d\theta \right) \tag{20}$$

$$\begin{aligned}
-\tau_m^2 \int_{t-\tau_m}^t \dot{u}_1^T(\theta) Q_3 \dot{u}_1(\theta) d\theta \leq & \begin{bmatrix} u(t) \\ u(t - \tau_m) \end{bmatrix} \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \\
& \times \begin{bmatrix} u(t) \\ u(t - \tau_m) \end{bmatrix}.
\end{aligned} \tag{21}$$

Combining (17)–(21), we get

$$\begin{aligned}
\dot{V}(t, u(t)) \leq & \xi^T(t) \Pi \xi(t) - \tau_s \int_{t-\tau_0}^{t-\tau_m} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta \\
& - \tau_s \int_{t-\tau_M}^{t-\tau_0} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta.
\end{aligned} \tag{22}$$

Next, we will discuss the variation of derivatives of $V(t, u(t))$ under two cases, i.e., $t \in \eta_1$ and $t \in \eta_2$, respectively.

Case I. For $t \in \eta_1$, i.e., $\tau(t) \in [\tau_m, \tau_0]$. It can be deduced from Lemma 2.4 that

$$\begin{aligned}
-\tau_s \int_{t-\tau_M}^{t-\tau_0} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta \leq & \begin{bmatrix} u(t - \tau_0) \\ u(t - \tau_M) \end{bmatrix} \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \\
& \times \begin{bmatrix} u(t - \tau_0) \\ u(t - \tau_M) \end{bmatrix}.
\end{aligned} \tag{23}$$

We further introduce two variable matrices N and M of appropriate dimensions. By combining (22) and (23) using Leibniz formula, we get

$$\begin{aligned}
\dot{V}(t, u(t)) \leq & \xi^T(t) \Pi \xi(t) - \tau_s \int_{t-\tau_0}^{t-\tau_m} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta \\
& + \begin{bmatrix} u(t - \tau_0) \\ u(t - \tau_M) \end{bmatrix} \begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3 \end{bmatrix} \begin{bmatrix} u(t - \tau_0) \\ u(t - \tau_M) \end{bmatrix} \\
& + 2\tau_s \xi^T(t) N [u(t - \tau_m) - u(t - \tau(t))] \\
& - \int_{t-\tau(t)}^{t-\tau_m} \dot{u}_1(\theta) d\theta + 2\tau_s \xi^T(t) M [u(t - \tau(t)) \\
& - u(t - \tau_0) - \int_{t-\tau_0}^{t-\tau(t)} \dot{u}_1(\theta) d\theta]
\end{aligned} \tag{24}$$

Now, it is easy to get the following inequalities by using Lemma 2.3,

$$\begin{aligned}
& -2\tau_s \xi^T(t) N \int_{t-\tau(t)}^{t-\tau_m} \dot{u}_1(\theta) d\theta \\
& \leq (\tau(t) - \tau_m) \tau_s \xi^T(t) N R_2^{-1} N^T \xi(t) \\
& + \tau_s \int_{t-\tau(t)}^{t-\tau_m} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta
\end{aligned} \quad (25)$$

$$\begin{aligned}
& -2\tau_s \xi^T(t) M \int_{t-\tau_0}^{t-\tau(t)} \dot{u}_1(\theta) d\theta \\
& \leq (\tau_0 - \tau(t)) \tau_s \xi^T(t) M R_2^{-1} M^T \xi(t) \\
& + \tau_s \int_{t-\tau_0}^{t-\tau(t)} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta.
\end{aligned} \quad (26)$$

Combining (24) and (26), we get

$$\begin{aligned}
\dot{V}(t, u(t)) \leq & \xi^T(t) \left[\Pi + \Xi_1 + (\tau(t) - \tau_m) \tau_s N R_2^{-1} N^T \right. \\
& \left. + (\tau_0 - \tau(t)) \tau_s M R_2^{-1} M^T \right] \xi(t).
\end{aligned} \quad (27)$$

When $i = j = 1$, it can be deduced from Schur complement that (12) is equivalent to

$$\Pi + \Xi_1 + \tau_s^2 N R_2^{-1} N^T < 0. \quad (28)$$

Similarly, when $i = 1$ and $j = 2$, (12) is equivalent to

$$\Pi + \Xi_1 + \tau_s^2 M R_2^{-1} M^T < 0. \quad (29)$$

It can be seen from (28) and (29) that there exists a positive scalar $c_1 > 0$ such that

$$\begin{aligned}
\Pi + \Xi_1 + \tau_s^2 N R_2^{-1} N^T & < -c_1 I, \\
\Pi + \Xi_1 + \tau_s^2 M R_2^{-1} M^T & < -c_1 I.
\end{aligned} \quad (30)$$

By using Lemma 2.5, (30) is equivalent to

$$\begin{aligned}
\Pi + \Xi_1 + (\tau(t) - \tau_m) \tau_s N R_2^{-1} N^T & + (\tau_0 - \tau(t)) \\
\times \tau_s M R_2^{-1} M^T & < -c_1 I, \quad (\tau_m \leq \tau(t) \leq \tau_0).
\end{aligned} \quad (31)$$

Combining (27) and (31), we can conclude that

$$\dot{V}(t, u(t)) \leq -c_1 \|u(t)\|^2. \quad (32)$$

Case II. For $t \in \eta_2$, i.e., $\tau(t) \in (\tau_0, \tau_M]$. It can be seen from Lemma 2.4 that

$$\begin{aligned}
-\tau_s \int_{t-\tau_0}^{t-\tau_m} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta \leq & \begin{bmatrix} u(t - \tau(t)) \\ u(t - \tau_0) \end{bmatrix} \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \\
& \times \begin{bmatrix} u(t - \tau(t)) \\ u(t - \tau_0) \end{bmatrix}.
\end{aligned} \quad (33)$$

Combining (33) and (22) and using Leibniz formula, we get

$$\begin{aligned}
\dot{V}(t, u(t)) \leq & \xi^T(t) \Pi \xi(t) - \tau_s \int_{t-\tau_M}^{t-\tau_0} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta \\
& + \begin{bmatrix} u(t - \tau(t)) \\ u(t - \tau_0) \end{bmatrix} \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} u(t - \tau(t)) \\ u(t - \tau_0) \end{bmatrix} \\
& + 2\tau_s \xi^T(t) U \left[u(t - \tau_0) - u(t - \tau(t)) \right. \\
& \left. - \int_{t-\tau(t)}^{t-\tau_0} \dot{u}_1(\theta) d\theta \right] + 2\tau_s \xi^T(t) V \left[u(t - \tau(t)) \right. \\
& \left. - u(t - \tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{u}_1(\theta) d\theta \right].
\end{aligned} \quad (34)$$

It is easy to verify that

$$\begin{aligned}
-2\tau_s \xi^T(t) U \int_{t-\tau(t)}^{t-\tau_0} \dot{u}_1(\theta) d\theta & \leq (\tau(t) - \tau_0) \tau_s \xi^T(t) U R_3^{-1} U^T \xi(t) \\
& + \tau_s \int_{t-\tau(t)}^{t-\tau_0} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta,
\end{aligned} \quad (35)$$

$$\begin{aligned}
-2\tau_s \xi^T(t) V \int_{t-\tau_M}^{t-\tau(t)} \dot{u}_1(\theta) d\theta & \leq (\tau_M - \tau(t)) \tau_s \xi^T(t) V R_3^{-1} V^T \xi(t) \\
& + \tau_s \int_{t-\tau_M}^{t-\tau(t)} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta.
\end{aligned} \quad (36)$$

Combining (35) and (36), we get

$$\begin{aligned}
\dot{V}(t, u(t)) \leq & \xi^T(t) \left[\Pi + \Xi_2 + (\tau(t) - \tau_0) \tau_s U R_3^{-1} U^T \right. \\
& \left. + (\tau_M - \tau(t)) \tau_s V R_3^{-1} V^T \right] \xi(t).
\end{aligned} \quad (37)$$

When $i = 2$ and $j = 1$, by using Schur complement Eq. (12) can be simplified to

$$\Pi + \Xi_2 + \tau_s^2 U R_3^{-1} U^T < 0. \quad (38)$$

Similarly, when $i = 2$ and $j = 2$, (12) is equivalent to

$$\Pi + \Xi_2 + \tau_s^2 V R_3^{-1} V^T < 0. \quad (39)$$

Then there exists a small positive scalar $c_2 > 0$ such that

$$\begin{aligned}
\Pi + \Xi_2 + \tau_s^2 U R_3^{-1} U^T & < -c_2 I, \\
\Pi + \Xi_2 + \tau_s^2 V R_3^{-1} V^T & < -c_2 I.
\end{aligned} \quad (40)$$

Now, using Lemma 2.5, (40) is equivalent to

$$\begin{aligned}
\Pi + \Xi_2 + (\tau_0 - \tau(t)) \tau_s U R_3^{-1} U^T & + (\tau_M - \tau(t)) \\
\times \tau_s V R_3^{-1} V^T & < -c_2 I, \quad (\tau_m \leq \tau(t) \leq \tau_M).
\end{aligned} \quad (41)$$

Combining (37) and (41), we can conclude that

$$\dot{V}(t, u(t)) \leq -c_2 \|u(t)\|^2. \quad (42)$$

From Case I and Case II, it can be seen that for all $t \in \mathbb{R}^+$ with $i = 1, 2$ and $j = 1, 2$ Eq. (12) holds. Now, we need to construct the change of V at impulse times. Firstly, it follows from (13) that

$$\begin{aligned} \begin{bmatrix} P & (I - E_k)P \\ * & P \end{bmatrix} &\geq 0 \\ \iff \begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} \begin{bmatrix} P & (I - E_k)P \\ * & P \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & P^{-1} \end{bmatrix} & \quad (43) \\ \iff \begin{bmatrix} P & I - E_k \\ * & P^{-1} \end{bmatrix} &\geq 0 \\ \iff P - (I - E_k)^T P (I - E_k) &> 0 \end{aligned}$$

in which the last equivalent relation is obtained by Lemma 2.7. Secondly, from model (4), it can be obtained that

$$\begin{aligned} u(t_k) - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta & \\ = u(t_k^-) - E_k \left\{ u(t_k^-) - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta \right\} & \\ - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta & \\ = (I - E_k) \left[u(t_k^-) - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta \right]. & \quad (44) \end{aligned}$$

Therefore from Eq. (15), we have

$$\begin{aligned} V(t_k, u(t_k)) &= \left[u(t_k) - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta \right]^T P \\ &\times \left[u(t_k) - C \int_{t_k - \sigma}^{t_k} u(\theta) d\theta \right] \\ &+ 2 \sum_{i=1}^n \int_0^{u_i(t_k)} g_i(s) ds \\ &+ \int_{t_k - \tau(t_k)}^{t_k} u^T(s) Q_1 u(s) ds \\ &+ \int_{t_k - \tau(t_k)}^{t_k} g^T(u(s)) Q_1 g(u(s)) ds \\ &+ \int_{t_k - \tau_m}^{t_k} u^T(s) R_1 u(s) ds \\ &+ \int_{t_k - \tau_0}^{t_k - \tau_m} \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix}^T \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \\ &\times \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix} ds \\ &+ \tau_s \int_{t_k - \tau_0}^{t_k - \tau_m} \int_s^{t_k} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta ds \\ &+ \tau_s \int_{t_k - \tau_0}^{t_k - \tau_m} \int_s^{t_k} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta ds \\ &+ \tau_s \int_{t_k - \tau_M}^{t_k - \tau_0} \int_s^{t_k} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta ds \end{aligned}$$

$$\begin{aligned} &+ \sigma \int_{-\sigma}^0 \int_{t_k + s}^{t_k} u^T(\theta) S u(\theta) ds \\ &+ \tau_m \int_{t_k - \tau_m}^{t_k} \int_s^{t_k} \dot{u}_1^T(\theta) Q_3 \dot{u}_1(\theta) d\theta ds \\ &= \left[u(t_k^-) - C \int_{t_k^- - \sigma}^{t_k^-} u(\theta) d\theta \right]^T P \\ &\times \left[u(t_k^-) - C \int_{t_k^- - \sigma}^{t_k^-} u(\theta) d\theta \right] \\ &+ 2 \sum_{i=1}^n \int_0^{u_i(t_k^-)} g_i(s) ds \\ &+ \int_{t_k^- - \tau(t_k^-)}^{t_k^-} u^T(s) Q_1 u(s) ds \\ &+ \int_{t_k^- - \tau(t_k^-)}^{t_k^-} g^T(u(s)) Q_1 g(u(s)) ds \\ &+ \int_{t_k^- - \tau_m}^{t_k^-} u^T(s) R_1 u(s) ds \\ &+ \int_{t_k^- - \tau_0}^{t_k^- - \tau_m} \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix}^T \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \\ &\times \begin{bmatrix} u(s) \\ u(s - \tau(s)) \end{bmatrix} ds \\ &+ \tau_s \int_{t_k^- - \tau_0}^{t_k^- - \tau_m} \int_s^{t_k^-} \dot{u}_1^T(\theta) R_2 \dot{u}_1(\theta) d\theta ds \\ &+ \tau_s \int_{t_k^- - \tau_M}^{t_k^- - \tau_0} \int_s^{t_k^-} \dot{u}_1^T(\theta) R_3 \dot{u}_1(\theta) d\theta ds \\ &+ \sigma \int_{-\sigma}^0 \int_{t_k^- + s}^{t_k^-} u^T(\theta) S u(\theta) ds \\ &+ \tau_m \int_{t_k^- - \tau_m}^{t_k^-} \int_s^{t_k^-} \dot{u}_1^T(\theta) Q_3 \dot{u}_1(\theta) d\theta ds \\ &\leq \left[u(t_k^-) - C \int_{t_k^- - \sigma}^{t_k^-} u(\theta) d\theta \right]^T P \left[u(t_k^-) - C \int_{t_k^- - \sigma}^{t_k^-} u(\theta) d\theta \right] \\ &= V(t_k^-, u(t_k^-)). \end{aligned}$$

Therefore, by using Lyapunov stability theorem, the network model in (4) is globally asymptotically stable. This completes the proof of the theorem.

When there is no time delay in the leakage term in system (4), that is, $\sigma = 0$, we get the system as follows:

$$\begin{aligned} \dot{u}(t) &= -Cu(t) + Ag(u(t)) + Bg(u(t - \tau(t))) \\ &\quad + D \int_{t - \tau(t)}^t g(u(s)) ds, \quad t \neq t_k, \\ \Delta u(t_k) &= u(t_k^+) - u(t_k^-) = J_k(u(t_k^-), u_{t_k^-}) \\ &= -E_k \left\{ u(t_k^-) - C \int_{t_k}^t u(\theta) d\theta \right\}, \quad t = t_k, \end{aligned} \quad (45)$$

Then the following corollary is derived by changing $V_1(t, u(t)) = u(t)^T P u(t)$ and setting $S = 0$ in the proof of Theorem 3.1.

Corollary 2 Suppose that Assumptions 1 and 2 hold. For given scalars $0 \leq \mu < 1$, τ_m and τ_M with $0 \leq \tau_m \leq \tau_M$, the system (45) is globally asymptotically stable if there exist positive symmetric matrices P , Q_i ($i = 1, 2, 3$), R_i ($i = 1, 2, 3$), positive diagonal matrices Λ , Γ_i ($i = 1, 2$) and matrices N_l, M_l, U_l and V_l ($l = 1, 2, \dots, 9$) of appropriate dimensions such that for all $i = 1, 2$ and $j = 1, 2$ the following LMIs hold:

$$\begin{bmatrix} \hat{\Pi} + \Xi_i & \Theta_{ij} \\ * & -R_i \end{bmatrix} < 0 \quad (i, j = 1, 2), \quad (46)$$

$$\begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix} \geq 0 \quad \text{and} \quad \begin{bmatrix} P & (I - E_k)P \\ * & P \end{bmatrix} \geq 0, \quad k \in \mathbb{Z}_+$$

with

$$\hat{\Pi} = \begin{bmatrix} \hat{\Pi}_{11} & R_2 & 0 & 0 & 0 & \hat{\Pi}_{16} & \Pi_{17} & \Pi_{18} & \Pi_{19} \\ * & \Pi_{22} & 0 & X_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -X_3 & -X_2^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & \Pi_{58} & 0 \\ * & * & * & * & * & \hat{\Pi}_{66} & \hat{\Pi}_{67} & \hat{\Pi}_{68} & \hat{\Pi}_{69} \\ * & * & * & * & * & * & \Pi_{77} & \Pi_{78} & \Pi_{79} \\ * & * & * & * & * & * & * & \Pi_{88} & \Pi_{89} \\ * & * & * & * & * & * & * & * & \Pi_{99} \end{bmatrix}$$

and $\hat{\Pi}_{11} = -C^T P - PC + Q_1 + R_1 + \tau_s^2 C^T (R_2 + R_3) C + \tau_m^2 (-C^T Q_3 - Q_3 C) - 2\Gamma_1 \Sigma_1 - R_2$, $\hat{\Pi}_{16} = 0$, $\hat{\Pi}_{66} = 0$, $\hat{\Pi}_{67} = 0$, $\hat{\Pi}_{68} = 0$, $\hat{\Pi}_{69} = 0$, $\Pi_{17}, \Pi_{18}, \Pi_{19}, \Pi_{22}, \Pi_{44}, \Pi_{55}, \Pi_{58}, \Pi_{77}, \Pi_{78}, \Pi_{79}, \Pi_{88}, \Pi_{89}, \Pi_{99}, \Xi_1, \Xi_2, \Theta_{11}, \Theta_{12}, \Theta_{21}, \Theta_{22}$ are defined as the same in Theorem 3.1.

Proof The proof of this corollary is similar to that of Theorem 3.1 and so we omitted it here.

Further, when there are no impulsive disruptions in system (4), then it can be rewritten in the following form:

$$\begin{aligned} \dot{u}(t) = & -Cu(t - \sigma) + Ag(u(t)) + Bg(u(t - \tau(t))) \\ & + D \int_{t-\tau(t)}^t g(u(s)) ds, \end{aligned} \quad (47)$$

Corollary 3 Suppose that Assumptions 1 and 2 hold. For given scalars $0 \leq \mu < 1$, τ_m and τ_M with $0 \leq \tau_m \leq \tau_M$, the system (47) is globally asymptotically stable if there exist positive symmetric matrices P , Q_i ($i = 1, 2, 3$), R_i ($i = 1, 2, 3$), S , positive diagonal matrices Λ , Γ_i ($i = 1, 2$) and matrices N_l, M_l, U_l and V_l ($l = 1, 2, \dots, 9$) of appropriate dimensions such that for all $i = 1, 2$ and $j = 1, 2$ the LMI (12) in Theorem 3.1 hold.

Proof The proof of this corollary is similar to that of Theorem 3.1 and so we omitted it here.

Remark 3 In He et al. (2007), developed the stability problem for neural networks with time-varying interval delay. Further, in Qiu et al. (2009), investigated the new robust stability criterion for uncertain neural networks with interval time-varying delays. The authors Kwon et al. in Kwon et al. (2008) established the robust stability for uncertain neural networks with interval time-varying delays. Recently, Zhang et al. (2009) proposed a new delay dependent stability criterion of neural networks with interval time-varying delay by using piecewise delay method. However, in this paper, we provide a new set of delay-dependent stability conditions to ensure the global asymptotic stability of the considered neural network (4) with time delay in the leakage term, interval time-varying delays and impulsive perturbations. The stability criterion is derived by using the appropriate model transformation that shifts the equilibrium point to the origin by translation, suitable Lyapunov–Krasovskii functional and some inequality techniques. In contrast to the above mentioned literature, the derived stability criteria are dependent on both the upper bound of the leakage delays and the interval time-varying delays.

Remark 4 Our proposed main results deal with the asymptotic stability problem for a class of NNs with interval time-varying delay. To obtain the stability criteria first we have to construct a Lyapunov function $V(t, u(t))$ as shown in (15). Then by checking the variation derivatives of $V(t, u(t))$ for the considered cases $\tau(t) \in [\tau_m, \tau_0]$ or $\tau(t) \in (\tau_0, \tau_M]$, respectively, some new set of delay-dependent stability criteria are derived which can guarantee $V(t, u(t)) < 0$. The obtained stability criterion can be readily checked by resorting the set of LMIs to the Matlab LMI Control toolbox.

Numerical examples

In this section, we have given a numerical example and their simulations to demonstrate the effectiveness and applicability of our developed method.

Example 1 Consider a second-order delayed neural network (4) with the following parameters:

$$C = \begin{bmatrix} 7 & 0 \\ 0 & 6 \end{bmatrix}, A = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.6 & -0.1 \\ -1.2 & -0.8 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.4 & -0.3 \\ 0.8 & 0.2 \end{bmatrix},$$

Here the time-varying delay and the activation functions are taken to be

$$\tau(t) = 0.5, \quad g_1(u) = \tanh(0.7x) - 0.1 \sin x,$$

$$g_2(u) = \tanh(0.4x) + 0.2 \cos x.$$

Further, it satisfies Assumption 1 with $\sigma_1^- = -0.1$, $\sigma_1^+ = 0.8$, $\sigma_2^- = -0.2$, $\sigma_2^+ = 0.6$, and hence

$$\Gamma_1 = \begin{bmatrix} -0.08 & 0 \\ 0 & -0.12 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0.35 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

By solving the LMIs in Theorem 3.1 via MATLAB LMI Control toolbox, we can obtain a set of feasible solution, but due to the limited length of this paper, we do not give such solutions here. The above result shows that all the conditions stated in Theorem 3.1 have been satisfied and

hence system (4) with the above given parameters is globally asymptotically stable in the mean square.

In addition, we have calculated the upper bounds of interval time-varying delays as shown in Table 1, which describes the allowable upper bounds for different values of σ , μ and τ_m . From this table, it is evidently proved that the delay-dependent stability criterion obtained in our paper is finer and less conservative than some existing results in the sense of upper bound technique.

Furthermore, in He et al. (2007), Zhang et al. (2009) it can be seen that the system is stable if the difference between τ_M and τ_m is < 1.3606 and 1.7532 (i.e. $\tau_M - \tau_m \leq 1.3606$, $\tau_M - \tau_m \leq 1.7532$), respectively. Whereas, by Theorem 3.1 in this paper, we can verify that the allowable value of $\tau_M - \tau_m$ is improved to be 3.0035 .

For $\sigma = 0.2$, $\tau_m = 1$ and $\mu = 0.95$, the upper bound of time delay in He et al. (2007) which ensures and verifies that the system is globally asymptotically stable is 6.5227 . It can also be seen that in Zhang et al. (2009) the upper bound is improved to be 8.4119 . By using Theorem 3.1 in

Table 2 Upper bounds of $\tau_M - \tau_m$ for various μ

	μ		
	0.8	0.9	Unknown μ
He et al. (2007)	2.2552	1.4769	1.3606
Zhang et al. (2009)	2.8335	1.9234	1.7532
This paper	4.8735	3.1845	3.0035

Table 1 Maximum upper bounds of τ_M for various τ_m and μ

τ_m	Methods	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
$\tau_m = 0$	Liu and Chen (2007), Hua et al. (2006), He et al. (2005), He et al. (2006)	1.2281	0.8636	0.8298
	Cho and Park (2007)	1.2459	0.8827	0.8259
	Kwon et al. (2008), He et al. (2007)	1.6831	1.1493	1.0880
	He et al. (2007)	2.3534	1.6050	1.5103
	Zhang et al. (2009)	2.8654	1.9508	1.7809
	This paper	6.0124	5.2873	5.0010
$\tau_m = 1$	Kwon et al. (2008)	2.5967	2.0443	1.9621
	He et al. (2007)	3.2575	2.4769	2.3606
	Zhang et al. (2009)	3.8359	2.9234	2.7532
	This paper	7.3243	6.2903	6.0025
$\tau_m = 100$	Kwon et al. (2008)	101.5946	101.0443	100.9621
	He et al. (2007)	102.2552	101.4769	101.3606
	Zhang et al. (2009)	102.8335	101.9234	101.7532
	This paper	106.0002	105.7932	105.2252

Table 3 Maximum upper bounds of τ_M for various τ_m and μ

τ_m	Methods	$\mu = 0.95$	$\mu = 0.99$	Unknown μ
$\tau_m = 1$	Qiu et al. (2009)	–	–	3.0465
	He et al. (2007)	6.5227	4.3522	3.9112
	Zhang et al. (2009)	8.4119	5.4834	4.9471
	This paper	12.0025	9.6557	9.0757
$\tau_m = 2$	Qiu et al. (2009)	–	–	4.0324
	He et al. (2007)	7.5227	5.3135	4.8847
	Zhang et al. (2009)	9.4119	6.4377	5.9198
	This paper	13.6442	10.3366	7.7683

Fig. 1 State variable of $x_1(t)$ of the network (4) with non-impulsive and impulsive effects **Fig. 2** State variable of $x_2(t)$ of the network (4) with non-impulsive and impulsive effects

this paper, we obtain the maximum allowable upper bound is 12.0025. Furthermore, the comparisons of upper bound between the criterion in the paper and those in He et al. (2007), Qiu et al. (2009), Zhang et al. (2009) are listed in Table 2.

From Table 1, 2 and 3, it is clear that the proposed stability criteria in this paper seems to be less conservative than the existing ones in the literature.

The simulation result reveals that by taking the initial condition $[\phi_1(s), \phi_2(s)] = [3, -4]$, $s \in [-0.2, 0]$, and then Figs. 1, 2 and 3 show that the considered network (4) with and without impulsive effect leads to a stable position. However, if we take the leakage delay with $\sigma > 0.2$ for the

Fig. 3 State variable of $x_1(t)$, $x_2(t)$ of the network (4) with non-impulsive and impulsive effects

network (4), one may deduce that the conditions (LMIs) in Theorem 3.1 have not been satisfied and do not have a feasible solution. It has shown in Figs. 4, 5 the unstable and in Fig. 6 the chaotic behavior of the neural network (4) have been shown. Therefore, our proposed method cannot guarantee the stability of network (4) with $\sigma > 0.2$.

Fig. 4 State variable of $x_1(t)$ of the network (4) with $\sigma = 0.3$, non-impulsive and impulsive effects

Conclusions

In this paper, we have dealt with the stability criteria for a class of neural networks with interval time-varying delays in the leakage term and impulses. By using model

Fig. 5 State variable of $x_2(t)$ of the network (4) with $\sigma = 0.3$, non-impulsive and impulsive effects

transformation, constructing appropriate Lya punov–Krasovskii functional, employing piecewise delay method and some known inequality techniques, several improved delay-dependent stability criteria for the considered neural networks have been derived. The derived criterion has been obtained in LMI forms and it can be solved in MATLAB LMI Control toolbox. Finally, a numerical example have been provided to show the effectiveness and superiority of the proposed stability results.

Further, we would like to point out that, the considered model can be generalized to discrete time neural networks or more complex neural networks, such as Cohen–Grossberg NNs, BAM NNs, NNs with stochastic perturbations and Markovian jumping parameters. The corresponding results will appear in the near future.

Fig. 6 State variable of $x_1(t)$, $x_2(t)$ of the network (4) with $\sigma = 0.3$, non-impulsive and impulsive effects

Funding Quanxin Zhu’s work was jointly supported by the National Natural Science Foundation of China (61374080), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

- Arik S (2004) An analysis of exponential stability of delayed neural networks with time-varying delays. *Neural Netw* 17:1027–1031

- Akca H, Alassar R, Covachev V, Covacheva Z, Al-Zahrani E (2004) Continuous-time additive Hopfield-type neural networks with impulses. *J Math Anal Appl* 290:436–451
- Boyd S, Ghaoui LEI, Feron E, Balakrishnan V (1994) Linear matrix inequalities in system and control theory. SIAM, Philadelphia
- Cao J, Wang J (2003) Global asymptotic stability of a general class of recurrent neural networks with time-varying delays. *IEEE Trans Circuits Syst Part I: Fundam Theory Appl* 50:34–44
- Cao J, Li X (2005) Stability in delayed Cohen–Grossberg neural networks: LMI optimization approach. *Phys D* 212:54–65
- Cho HJ, Park JH (2007) Novel delay-dependent robust stability criterion of delayed cellular neural networks. *Chaos, Solitons Fractals* 32(3):1194–1200
- Fu X, Yan B, Liu Y (2005) Introduction of impulsive differential systems. Science Press, Beijing
- Gopalsamy K (2007) Leakage delays in BAM. *J Math Anal Appl* 325:1117–1132
- Gopalsamy K (1992) Stability and oscillations in delay differential equations of population dynamics. Kluwer Academic Publishers, Dordrecht
- Gu K, Kharitonov V, Chen J (2003) Stability of time-delay systems. Birhauser, Boston
- Han QL, Yue D (2007) Absolute stability of Lure systems with time-varying delay. *Control Theory Appl IET* 1(3):854–859
- Haykin S (1999) Neural networks. Prentice Hall, New Jersey
- He Y, Liu G, Rees D (2007) New delay-dependent stability criteria for neural networks with time-varying delay. *Neural Netw IEEE Trans* 18(1):310–314
- He Y, Liu GP, Rees D, Wu M (2007) Stability analysis for neural networks with time-varying interval delay. *Neural Netw IEEE Trans* 18(6):1850–1854
- He Y, Wang Q, Wu M (2005) LMI-based stability criteria for neural networks with multiple time-varying delays. *Phys D: Nonlinear Phenom* 212(12):126–136
- He Y, Wu M, She J (2006) Delay-dependent exponential stability of delayed neural networks with time-varying delay. *Circuits Syst II: Express Briefs IEEE Trans* 53(7):553–557
- Hua C, Long C, Guan X (2006) New results on stability analysis of neural networks with time-varying delays. *Phys Lett A* 352(45):335–340
- Ignatyev A (2008) On the stability of invariant sets of systems with impulse effect. *Nonlinear Anal* 69:53–72
- Kosko B (1992) Neural Netw Fuzzy Syst. Prentice Hall, New Delhi
- Kwon O, Park J, Lee S (2008) On robust stability for uncertain neural networks with interval time-varying delays. *Control Theory Appl IET* 2(7):625–634
- Lakshmikantham V, Bainov D, Simenov P (1989) Theory of impulsive differential equations. World Scientific, Singapore
- Li C, Huang T (2009) On the stability of nonlinear systems with leakage delay. *J. Franklin Inst.* 346:366–377
- Li X, Fu X, Balasubramaniam P, Rakkiyappan R (2010) Existence, uniqueness and stability analysis of recurrent neural networks with time delay in the leakage term under impulsive perturbations. *Nonlinear Anal Real World Appl* 11:4092–4108
- Li X, Cao J (2010) Delay-dependent stability of neural networks of neutral-type with time delay in the leakage term. *Nonlinearity* 23:1709–1726
- Liu DR (1997) Cloning template design of cellular neural networks for associative memories. *IEEE Trans Circuits Syst* 1(44):645–650
- Liu H, Chen G (2007) Delay-dependent stability for neural networks with time-varying delay. *Chaos, Solitons Fractals* 33(1):171–177
- Liu X, Teo K, Xu B (2005) Exponential stability of impulsive high-order Hopfield-type neural networks with time-varying delays. *IEEE Trans Neural Netw* 16:1329–1339
- Liu Y, Wang Z, Liu X (2006) Global exponential stability of generalized recurrent neural networks with discrete and distributed delays. *Neural Netw* 19:667–675
- Meng J, Wang XY (2007) Robust anti-synchronization of a class of delayed chaotic neural networks. *Chaos* 17(2):023113
- Mou SS, Gao HJ, Lam J, Qiang WY (2008) A new criterion of delay-dependent asymptotic stability for Hopfield neural networks with time delay. *IEEE Trans Circuits Syst* 19(3):532–535
- Peng S (2010) Global attractive periodic solutions of BAM neural networks with continuously distributed delays in the leakage terms. *Nonlinear Anal: Real World Appl* 11:2141–2151
- Qiu J, Yang H, Zhang J, Gao Z (2009) New robust stability criteria for uncertain neural networks with interval time-varying delays. *Chaos, Solitons Fractals* 39(2):579–585
- Sanchez EN, Perez JP (1999) Input-to-state stability (ISS) analysis for dynamic neural networks. *Circuits Syst I: Fundam Theory Appl: IEEE Trans* 46(11):1395–1398
- Senan S, Arik S (2007) Global robust stability of bidirectional associative memory neural networks with multiple time delays. *IEEE Trans Syst, Man Cybern, Part B* 37:1375–1381
- Song Q, Cao J (2012) Passivity of uncertain neural networks with both leakage delay and time-varying delay. *Nonlinear Dyn* 67:1695–1707
- Tan J, Li C, Huang T (2015) The stability of impulsive stochastic Cohen–Grossberg neural networks with mixed delays and reaction-diffusion terms. *Cogn Neurodyn* 9:213–220
- Wang XY, Zhao Q, Wang MJ, Yang YH (2008) Generalized synchronization of communication. *Mod Phys Lett B* 22(2):2077–2084
- Wang Z, Shu H, Liu Y, Ho DWC, Liu X (2006) Robust stability analysis of generalized neural networks with discrete and distributed time delays. *Chaos Solitons Fractals* 30:886–896
- Xu D, Yang Z (2005) Impulsive delay differential inequality and stability of neural networks. *J Math Anal Appl* 05:107–120
- Yang Z, Xu D (2005) Stability analysis of delay neural networks with impulsive effects. *IEEE Trans Circuits Syst II* 52:517–521
- Yang Z, Zhou W, Huang T (2014) Exponential input-to-state stability of recurrent neural networks with multiple time-varying delays. *Cogn Neurodyn* 8:47–54
- Yue D (2004) Robust stabilization of uncertain systems with unknown input delay. *Automatica* 41:331–336
- Zhang Y, Yue D, Tian E (2009) New stability criteria of neural networks with interval time-varying delay: a piecewise delay method. *Appl Math Comput* 208:249–259