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Mathematical Models of College Myopia

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Abstract

Experimental design phase of a pilot study at Annapolis is described, using reading glasses, +1.5 D. to +3.0 D. to alleviate college myopia. College students often become 1.0 to 2.0 diopters more myopic, so reading glasses were explored to partially cancel the effects of the study environment. N = 25 different sets of (+)Add lenses are evaluated, for required adjustment period and reading comfort. Three computer models are developed to predict refraction versus time. Basic control system equations predict exponential myopia shift of refractive state R(t) with time constant $t_0 = 100$ days. Linear, exponential and Gompertz computer results are compared calculating refraction R(t) during the college years, showing correlation coefficients $|r| = 0.96$ to 0.97 , accurate ± 0.31 D. over a 14 year interval. Typical college myopia rate is -0.3 to -0.4 D/yr. Reading glasses may be a simple, practical solution to stabilize college myopia.

Keywords

emmetropia; progressive myopia; feedback control theory; time constants; progressive add lenses (PALs); bifocals; reading glasses; refraction

Introduction

We helped design an experimental study at Annapolis, using reading glasses, to reduce college myopia. Navy pilots at Annapolis are required to have 20/20 vision in order to fly. Many become myopic and therefore must quit the program. Gmelin, 1976 reports that approximately 50% of Cadets start as myopic, but the fraction rises to two-thirds at

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graduation. In addition to the challenge of just getting the pilots to graduate, while still maintaining 20/20, we have received reports of a related phenomenon called “cockpit myopia”, whereby, after close work at the instrument clusters, maps, instruction manuals, etc., the pilots and co-pilots find distance objects are blurred. In terms of motivation, some of the successful graduating Ensigns from Annapolis may, if they are lucky, go on to fly the vertical take-off and landing (VTOL) AV-8B “Sea Harrier”, or the F-18 “Super-Hornet”, or the even new F-35 VTOL “Lightning II”. It is difficult enough, trying to land on a pitching carrier deck, much more so if your vision is reduced to a 20/50 transient myopia during the sortie. Nearwork induced transient myopia (N.I.T.M.) is reported by some pilots, more so the navigators, as expected. These various problems, although well defined, currently have no practical solution. Helmet display and optical design have become an integral part of aircraft design in recent years (Jenkins & Gallimore, 2008). Our approach involved (+) Add reading glasses, +1.5 to +3.0 diopters, to be used during long hours of college study to lessen focusing effort (Cheng et al., 2011). In engineering terms, (+)Add reading glasses, Fig. 1, are considered “optical-offset distance compensators”, designed to optically shift a book or computer at 13” - 20” to infinity, thereby easing the focusing work-load on the eye. Reading glasses may be a simple, practical, solution to stabilize college myopia and pilot myopia.

(+)Add studies require large numbers of students to improve significance level, discriminating the average diopter difference dR between experimental and control groups. Although $dR = 2.00$ D is possible, this difference is not always as pronounced as we might hope (Cheng et al., 2014). Fulk et al., 2000 report $N = 42$, $dR = 0.25$ D., $p = 0.046$, using a +1.50 D. add. Gwiazda et al., 2003 report $N > 450$, $dR = 0.20$ D, $p < 0.004$, using a +2.00 D. add. Yang et al., 2009, using a +1.50 D. add, report $N = 149$ subjects, $dR = 0.25$ D, $p = 0.01$. Leung & Brown, 1999 report $dR = 0.7$ D., $p < 0.0001$, using +1.50 D and +2.00 D, $N = 36$. Cheng et al., 2014 report $dR = 1.05$ D., $N = 135$, using +1.5 D Add and +1.5D with prism, $p < 0.001$. Oakley & Young, 1975 report $dR = 1.0$ D, $N = 216$, Fig. 2, using +1.5 D and +2.0 D Add, $p < 0.001$ for each of 10 different age brackets. Cheng, Woo & Schmid, 2011 and Goss, 1994 review the literature on (+)Add studies, including bifocal and multi-focal lenses.

Materials & Methods

The design team consisted of 5 consultants, from various universities, ages 30-60, plus 2 assistants ages 21-30 yrs. Comfort level, feasibility, and endurance factors were evaluated for the 25 different (+) Add lens combinations, Table I. In terms of mathematical theory, 3 computer models of refractive-state shift over an 11 to 14 year interval are developed, including *linear* (Goss & Jackson, 1993), *exponential* (Medina & Fariza, 1993), and *Gompertz* (Thorn, Gwiazda & Held, 2005). Age matched data sets, Fig. 2 and Fig. 5, are used to optimize model parameters. Results are calculated using Basic V. 3.2 and Excel programs. Cheng et al., 2011 and Goss, 1994 review plus lenses, bifocal, and PAL studies. Typical college myopia rate is $R' = -0.3$ to -0.4 D/yr. Our objective was to reduce the rate to 0.0 D/yr for 4 years. These myopia rates are typical (Lee et al., 2013; Sun et al., 2012; Lin et al., 1996; Yang et al., 2009) and also apply to students at the graduate level, some medical schools reporting myopia prevalence rates greater than 90% to 95% (Lin et al. 1996).

Recently, several (+) Add research studies have been published (Gwiazda et al., 2003; Leung & Brown, 1999; Fulk et al., 2000; Oakley & Young, 1975; Yang et al., 2009; the COMET Group, 2013; Cheng et al., 2011, 2014) with encouraging results, i.e. the progressive myopia rate can be attenuated by 50% or more (Holden et al., 2014) using various (+) Add technologies, i.e. bifocals and progressive addition lenses (P.A.L.'s), Fig. 2. Under proper supervision, we evaluated an assortment of (+)Add lenses, with powers ranging from +0.5 D. to +4.0 D., Table 1. Tenets of the Helsinki declaration and the internal review board were adhered to. Comfort level for these reading glasses was such that they are still in use by most members of the design team.

Results

Mathematical models of progressive myopia involve exponential and/or linear functions. The refractive state as a function of time, $R(t)$ [dioptr], of the eye responds, readjusting to the near-point optical demands of a new environment $\langle E \rangle$ [dioptr.], typically $\langle E \rangle = -1$ to -2 D. During one semester, refractive state can become negative, about -0.3 diopters, with a time constant $t_0 = 100$ days as:

$$R(t) = -1.1 \text{ D} + 0.8 \text{ D} \exp[-t/t_0] \quad \text{Eq. (1)}$$

Similar exponential functions are used by Medina & Fariza, 1993, and Greene et al., 1996

In terms of theory, we have explored 3 possibilities in detail: 1. linear regression (Goss & Jackson, 1993) ($N = 12$) 2. exponential progression ($N = 367$) (Medina & Fariza, 1993) and 3. the Gompertz function (Thorn et al., 2005), $N = 32$ iterations. The Gompertz 4-parameter double-exponent is given by :

$$R(t) = R_e + R_c \cdot (0.07295)^{a^{(t-t_0)}} \quad \text{Eq. (2)}$$

where $R(t)$ = refraction at time t . For one subject, we use parameters $R_e = -0.75$ D initial refraction, $R_c = -5.25$ D amplitude, onset age $t_0 = 12$ yrs., and $a = 0.70$ optimal shape factor, accurate within ± 0.31 diopters. The results for *linear regression* show average diopter rate $\langle R' \rangle = -0.47$ D/yr., age at stabilization 22 years, correlation coefficient $r = -0.96$. For the *exponential model* time constant is $t^* = 3.2$ to 4.4 yrs., correlation coefficient $r = 0.97$, accurate over an 11 to 14 year interval, Figs. 2, 4 and 5. Mathematically, the *regression model* is the easiest to use, the *Gompertz model* is the most difficult, but most accurate, and the *exponential model*, of intermediate difficulty, Figs. 2, 4 and 5, has the ability to predict the slower myopia drift after college, (for instance, Fledelius, 2000 reports that an additional 25% of the students become myopic during graduate school).

Design team data

From the $N = 7$ consultants on the design team, 5 are myopes with nominal spherical equivalent refraction (SER) from -5 to -8 D., two are emmetropes. 4 of the myopes try and like reading glasses, using various types of (+) Add lenses for an extended period of time. Myopia can continue to progress beyond the college years to age 40-45, rarely mentioned in reports (Bullimore et al., 2002; COMET Group, 2013). One myope progresses from -6 to

–8 diopt. after college, one myope progresses from –7 to –8 diopt. after college, 3 myopes remain stable. Bullimore, et al., 2002 confirm that 36% of adults continue to progress at a rate of –0.75 diopt. per 5-year interval, (ages > 28 yrs., N = 197). Note that the *exponential model*, Fig. 4, predicts a slow continuing myopia drift after the college years, approximately –0.50 D. over a 5 year interval, consistent with the results of Bullimore, et al., 2002 and Fledelius, 2000.

One myope uses progressive (+) Add lenses for 10 yrs., 1 uses bifocals for 20 years, 2 use (+)Add reading glasses for more than 30 years, 1 of the 2 emmetropes becomes presbyopic, 1 myope has surgery for cataracts, and 1 myope develops a macular problem in one eye. After starting to use the (+) Add lenses, +1.5 to +2.5 D., all 4 myopes remain stable in terms of refraction, all are older than age 25. Subject confidentiality is maintained by deleting subject I.D. from the reported records. In summary, 2 of 3 of our high myopes ($|SER| > 6$ D.) develop serious vision problems, as is often reported (Holden et al., 2014; Wong et al., 2014; Goldschmidt, 2003), 4 of 5 employ (+) Add lenses for 10 to 30 years, with refraction remaining stable. This post-graduate age bracket corresponds to that of pilots, so ordinary reading glasses may be a simple, practical solution to stabilize pilot myopia.

Discussion

There is a natural tendency of the eye to become myopic with long hours focusing at a near-point environment, confirmed by laboratory experiments ($dR = -1.7$ D, $t = 1$ yr., $N = 7$, Young, 1963). In terms of the (+) Add lenses, to determine appropriate power level, 25 sets were tried by the design team, +0.50 D to +4.00 D., Table 1. After a variable adjustment period, most find +2.0 D is comfortable and practical, although the basic equations do suggest stronger values. Typically, bifocals and PALs are used with progressing myopes with refraction $R = -1.0$ D to -6.0 D, diopter rates $R' = -0.2$ to -0.8 D/yr thereby slowing these myopia rates by 50 % or more (Holden et al., 2014). PAL and similar studies use multifocal (+) Add lenses, Fig. 1, for progressing myopes, of strength +1.00 D. to +2.00 D.

Even the best 35-mm camera, although having excellent range of focus, would have trouble with the focusing demands of a typical engineering student. In order to photograph a textbook, a computer screen, an oscilloscope display, or a blueprint on the drawing board, special close-up so-called “portrait” lenses are required, typically available in powers of +1.0 D, +2.0 D, and +3.0 D (see Fig. 3) exactly the same (+) Add values discussed herein. For distances the order of 1.0 to 1.5 feet, the portrait lenses can be added in series, in clip-on mode, to achieve +4.0 D, +5.0 D, or +6.0 D. Usually, there are not enough threads on the lens barrel, to focus up close, so (+) Add lenses are necessary equipment, even for the best of cameras.

Ultimately, the goal would be to develop and provide optical equipment to attenuate the progressive myopia problem. This R & D effort is quite complicated, far from proven. To date, the only solution has been to exclude myopes entry into the Academy, beyond a certain level. Herein we report only the practical hardware details found by the design team evaluating $N = 25$ (+) Adds, Table 1. From this list of 25 possibilities, the most promising

are (+) Adds in the range +2.0 to +3.0 diopters, including single vision, bifocal, and progressive addition PALs.

Conclusions

The purpose of this report is to relate our experiences to clinicians, investigators, professors, and students, of the various parameters and factors that must be taken into account when designing a study of this type, using (+) Add technology for the control of college myopia. Across the board, our most difficult problem has been teaching and explaining the rationale and strategy of this new optical technique. Unless you have tried (+) Add reading glasses personally, it remains an abstract concept. Basically, this is a matter of prescribing reading glasses, normally used by those of age 40+, to students age 20. Eight successful (+) Add studies reported here (Gwiazda et al. 2003; Leung & Brown, 1999; Fulk et al., 2000; Oakley & Young, 1975; Yang et al., 2009; COMET Group, 2013 ; Cheng et al., 2011, 2014) provide a considerable data base for ages 6 – 18. Experiments and theory help predict *what* we may expect to happen, and *when*, during the college years. Often overlooked, as an important design parameter, is the level of the transition line between the distance and near correction in bifocals and PALs, usually set at 50% of frame height, but to guarantee using the (+) Add segment at near, it is suggested at the 60 to 70% level, (Prof. Young, personal communication).

Lastly, there is the *adjustment period*, rarely mentioned in reports, approximately 1 – 2 hours for a +1.0 D Add, 1 – 2 days for a +2.0 D Add (20-inches), 1 – 2 weeks for a +3.0 D Add, and 1 – 2 months for a +4.0 D Add (10 inches), Table 1. As anyone who has tried contact lenses can tell you, a remarkable degree of bravery, skill, co-ordination, and persistence are required to learn this task, similar to learning to ice skate, or to ride a bicycle. (+) Add reading glasses have a similar learning-curve, in terms of required adjustment time, although considerably easier to use.

The basic strategy of the (+) Add technique is to reduce the focusing demand on the visual system during prolonged study, (n.b. - 13" reading dist. = -3.0 diopt. accommodative demand). There are several reports of myopia developing with Navy submariners, and extensive LASIK use in the Army (Hammond et al., 2005), to cure the myopia problem, more than 16,000 recruits as of 2003, a total of 26,000 recruits as of 2005. The Annapolis Navy pilots are required to be in excellent physical condition, it is a demanding job flying a Mach 2 fighter/bomber. These various (+) Add lenses effectively shift a book or computer at 13" – 20" to infinity. Various types of reading glasses, i.e. single vision, bifocals, and the new multi-focal progressive lenses (PALs), may be a practical way to stabilize college myopia and pilot myopia.

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Figure 1.

Reading glasses for a -5.00 D. college myope. (+) Add technology is used by both bifocals and progressive addition lenses, “PALs”. PALs are “no-line” bifocals.

Figure 2.

+2.0 D bifocals (N=216) can stabilize progressing myopes (N=367), as indicated by the horizontal lines. $dR = 1.0$ D., $p < 0.001$ for each of 10 age brackets. Otherwise, normal myopia refraction rates are $R'(t=8 \text{ yrs}) = -0.7$ D/yr, $R'(t=16 \text{ yrs}) = -0.4$ D/yr.

Figure 3. +3.00 D. and +1.00 D. (+) Add lenses for a 49 mm lens barrel. Photo is taken using a +2.00 D. (+) Add portrait lens, at a distance of 16-inches. These lenses can be added in sequence to the primary lens.

Figure 4.

Fig. 4 - Exponential visual system response to a -2.0 D negative step showing delayed myopia onset.

Figure 5.

Typical college student with progressive myopia, exponential time constant $t_0 = 4.4$ yrs., correlation coefficient $r = 0.97$.

Table 1

(+) Add [D]	Adapt. time	Duration	# subj.	Cost
1. +0.5	< 1 hr.	2 yrs.	1	\$200.
2. +1.0	2 hrs.	2 yrs.	1	\$200.
“	“	0.5 yr.	1	\$250.
3. +1.5 clip-on lenses	3 hrs.	n.a.	3	\$12.
+1.5	“	2 yrs.	1	\$200.
bifocal sunglasses	<1 hr.	n.a.	2	\$16.
	“	1 wk.	1	\$250.
4. +2.0	1 day	20 yrs.	2	\$200.
	“	1 wk.	1	\$250.
5. + 2.5 clip-on lenses	1 day	n.a.	3	\$12.
6. + 3.0	1 wk.	1 yr.	1	\$200.
	“	1 wk.	1	\$250.
7. +3.25	n.a.	n.a.	2	\$12.
8. +3.5	1 m.	1 yr.	1	\$200.
	“	1 m.	1	\$250.
	“	“	1	\$250.
9. +4.0	2 m.	1 yr.	1	\$200.
“flipper” lenses −2.0 to +2.0 D	0	1 hr.	1	\$25.