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Beamforming-Enhanced Inverse Scattering for Microwave Breast Imaging

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Abstract

We present a focal-beamforming-enhanced formulation of the distorted Born iterative method (DBIM) for microwave breast imaging. Incorporating beamforming into the imaging algorithm has the potential to mitigate the effect of noise on the image reconstruction. We apply the focal-beamforming-enhanced DBIM algorithm to simulated array measurements from two MRI-derived, anatomically realistic numerical breast phantoms and compare its performance to that of the DBIM formulated with two non-focal schemes. The first scheme simply averages scattered field data from reciprocal antenna pairs while the second scheme discards reciprocal pairs. Images of the dielectric properties are reconstructed for signal-to-noise ratios (SNR) ranging from 35 dB down to 0 dB. We show that, for low SNR, the focal beamforming algorithm creates reconstructions that are of higher fidelity with respect to the exact dielectric profiles of the phantoms as compared to reconstructions created using the non-focal schemes. At high SNR, the focal and non-focal reconstructions are of comparable quality.

Index Terms

Beamforming; biomedical imaging; breast imaging; inverse problems; inverse scattering; mammography; microwave imaging; phantoms

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I. Introduction

Microwave inverse scattering involves reconstructing the unknown dielectric profile of a target region from measured scattered field data. Numerous recent studies have investigated the application of inverse scattering techniques to breast imaging. The significant dielectric contrast between the different types of tissues constituting the breast [1], [2] allows for discrimination between tissue structures in a microwave image and thus serves as the physical basis for a microwave based approach. Microwave breast imaging shows promise for breast density evaluation (e.g., [3]), cancer screening (e.g., [4]–[6]), and treatment monitoring (e.g., [7]).

Microwave breast imaging via inverse scattering typically requires solving an ill-posed, underdetermined system of linear equations in order to form an estimate for the unknown dielectric properties throughout the breast volume. The solution to the system is generally non-unique. Corruption of the measured scattered fields by noise, interference from sources or clutter outside the imaging region, and mismatch between the actual and assumed properties of the propagation environment outside the target region may cause the imaging algorithm to converge to a poor solution.

The linear system to be solved in the distorted Born iterative method (DBIM) [8] can be written in the form $\mathbf{b} = \mathbf{A}\mathbf{x}$. The vector $\mathbf{x}$ contains the unknown contrast between the true dielectric profile of the imaging volume and an assumed background profile. The matrix $\mathbf{A}$ is formed by discretizing the electric field integral equation. The vector $\mathbf{b}$ contains the scattered electric field data recorded by an antenna array. Typically, each element of $\mathbf{b}$ is the scattered field at one receiving antenna due to excitation of one transmitting antenna, and the full vector $\mathbf{b}$ is formed from multiple transmit-receive antenna pairs. In previous work, [3], we have included all bistatic channels in $\mathbf{b}$, excluding reciprocal antenna pairs.

An alternative scenario incorporates transmit-receive beamforming into the imaging algorithm. Transmit beamforming is performed by modifying the magnitude and phase of the signal fed to each antenna and exciting each antenna simultaneously, whereas receive beamforming is accomplished by modifying the signal received by each antenna and then summing the modified signals. By appropriately choosing the magnitude and phase modifications for each transmit and receive channel, that is, by designing the transmit and receive beamforming weight vectors, desired improvements to the performance of the imaging algorithm may be achieved. The transmit-receive beamforming enhancement alters not only the formation of $\mathbf{b}$, but also the formation of $\mathbf{A}$, due to changes in the electric field integral equation.

A beamforming scheme applied to the Gauss-Newton method, which is equivalent to the DBIM [9], for the purpose of decreasing the computational cost of the imaging algorithm has been previously reported [10]. This scheme used the singular vectors of the matrix of scattered fields as beamforming weight vectors. The size of the linear system to be solved was decreased by discarding beamformed data obtained from using weight vectors corresponding to the smallest singular values of the scattered field matrix. Discarding

enough data also led to an improvement in the performance of the algorithm when applied to noisy simulated data.

In this paper, we propose a focal beamforming enhancement to the DBIM for the purpose of decreasing the algorithm's sensitivity to noise. This strategy involves designing beamforming weights in order to emphasize scattered signal return from a series of focal locations in the imaging region. Using this approach, the noise in each receive channel is summed non-coherently, while the scattered signals are summed coherently. We compare the noise performance of this algorithm to that of two different non-focal DBIM schemes. The first comparison scheme is equivalent to the DBIM as formulated in [3], with the exception that scattered field data from reciprocal transmit-receive antenna pairs are averaged. This scheme performs similarly to the case when all transmit-receive combinations of antennas are included in $\mathbf{b}$, but it halves the size of the linear system that must be solved. It also represents an intermediate step towards the focal beamforming scheme, as scattered field data is averaged across only two transmit-receive channels as opposed to across the entire array. In the second comparison scheme, reciprocal channels are instead discarded, so that no data averaging occurs.

We apply all three versions of the DBIM to electric field data from simulated array measurements of two anatomically realistic MRI-derived numerical breast phantoms. These phantoms allow us to evaluate and compare the performance of the algorithms in a testbed environment in which the reconstructed profiles may be directly and unambiguously compared to the known dielectric profiles of the phantoms. We evaluate the algorithms for a wide range of signal-to-noise ratios (SNR) and show that, for low SNR, the focal beamforming scheme (FBS) generates the highest-fidelity reconstructions with respect to the exact phantom profiles. The reciprocal-pair averaging scheme (RPAS) performs second best, and the reciprocal-pair discarding scheme (RPDS) performs the worst.

The rest of this paper is organized as follows. Section II describes the numerical testbeds used in this study. Section III presents our imaging technique and Section IV reports the results from the numerical study. Concluding statements are given in Section V.

Boldface Roman type is used to denote vector and matrix quantities. Superscripts T , H , and $*$ represent transpose, complex conjugate transpose, and complex conjugate, respectively. The symbol $\odot$ represents pointwise vector multiplication.

II. Data Acquisition via Simulated Array Measurements of Numerical Breast Phantoms

The numerical testbeds used for this study consist of finite-difference time-domain (FDTD) computational electromagnetics models of heterogeneous, anthropomorphic breast phantoms derived from MR scans of human subjects [11], surrounded by an array of dipole antennas. The frequency-dependent and tissue-specific dielectric properties in the breast phantoms are based on values reported by Lazebnik *et al.* [1], [2]. The frequency dependence is modeled using a single-pole Debye model with voxel-specific Debye parameters ϵ , ϵ_∞ , and σ_s , and a common (voxel-independent) relaxation constant $\tau = 15$ ps. We choose two different

breast phantoms; one phantom represents Class 2 (scattered fibroglandular) in terms of BIRADS density, and the other represents Class 3 (heterogeneously dense). These phantoms allow us to evaluate the performance of the imaging algorithms for both a relatively sparse and a relatively dense fibroglandular tissue distribution.

The antenna array, which is identical to that reported by Shea *et al.* [3] (see [3, Fig. 2]) consists of 40 dipoles of total length 14 mm. The dipoles are distributed across five elliptical rings of eight elements each. The dimensions of each ring are scaled so that there is a minimum 1-mm spacing between each element and the breast surface, leading to major and minor radii between 5 and 6 cm. Adjacent rings are rotated by 22.5° . The dipoles are oriented vertically, that is, perpendicular to the planes of the elliptical rings. The phantoms and the arrays are immersed in a lossless, non-dispersive coupling medium with dielectric constant of 2.6.

Transmitting antennas in the FDTD model were gap-fed with a current source. Time-domain electric fields at the receiving antennas were recorded and converted to phasor form via discrete Fourier transform (DFT) at 1, 1.5, 2, and 2.5 GHz. These frequencies were chosen in order to obtain the benefits of algorithm stability at lower frequencies and increased resolution at higher frequencies [12] while maintaining adequate penetration depth.

III. Beamforming-Enhanced DBIM for Image Reconstruction

We assume an array of L antennas located at $\mathbf{r}_1 \dots \mathbf{r}_L$ surrounds a region V of unknown dielectric properties. The total electric field, $\mathbf{E}^t$, represents the electric field in the presence of the unknown complex permittivity in V , denoted ϵ^* . The incident field, $\mathbf{E}^i$, represents the electric field in the presence of a known background permittivity, denoted ϵ_b^* . The scattered electric field is then $\mathbf{E}^s = \mathbf{E}^t - \mathbf{E}^i$. The scattered electric field at antenna location $\mathbf{r}_{\text{rec}}$ due to transmission from antenna location $\mathbf{r}_{\text{src}}$ can be written as

$$\mathbf{E}^s(\mathbf{r}_{\text{src}}, \mathbf{r}_{\text{rec}}) = k_0^2 \int_V \bar{\mathbf{G}}^b(\mathbf{r}', \mathbf{r}_{\text{rec}}) \mathbf{E}^t(\mathbf{r}_{\text{src}}, \mathbf{r}') o(\mathbf{r}') d^3 \mathbf{r}' \quad (1)$$

where k_0 is the free-space wavenumber, $\bar{\mathbf{G}}^b$ is the dyadic Green's function of the background profile, and $o(\mathbf{r}') = (\epsilon^*(\mathbf{r}') - \epsilon_b^*(\mathbf{r}')) \epsilon_0^{-1}$ is the dielectric contrast in V .

We linearize (1) by replacing the total field in the integrand with the incident field. This is known as the Born approximation and is valid for small contrasts. Assuming co-polarized antennas, the scattered field reduces to a scalar. We also reduce the dyad and vector field quantities in the integrand to scalars, an approximation that assumes that the co-polarized field component dominates in V . Equation (1) is then written

$$E^s(\mathbf{r}_{\text{src}}, \mathbf{r}_{\text{rec}}) = k_0^2 \int_V G^b(\mathbf{r}', \mathbf{r}_{\text{rec}}) E^i(\mathbf{r}_{\text{src}}, \mathbf{r}') o(\mathbf{r}') d^3 \mathbf{r}'. \quad (2)$$

A. Transmit-Receive Beamforming

Each antenna has associated with it a complex channel transmit weight that modifies the phase and amplitude of input current $i(\omega)$ before transmission. The channel transmit weights are represented in vector form as $\mathbf{w}_t$. The m th component of $\mathbf{w}_t$ is a complex scalar representing the channel weight for the m th antenna. Let the vector of incident fields in V from each antenna be

$$\mathbf{e}_i(\mathbf{r}') = [E^i(\mathbf{r}_1, \mathbf{r}') E^i(\mathbf{r}_2, \mathbf{r}') \dots E^i(\mathbf{r}_L, \mathbf{r}')]^T. \quad (3)$$

Let $\mathbf{g}_b(\mathbf{r}')$ be the analogous vector of Green's functions.

The incident field due to transmit beamforming with weight vector $\mathbf{w}_t$, denoted $\tilde{E}^i(\mathbf{r}')$, is given by

$$\tilde{E}^i(\mathbf{r}') = \mathbf{w}_t^H \mathbf{e}_i(\mathbf{r}'). \quad (4)$$

Using (2) and (4), the scattered field at $\mathbf{r}_{\text{rec}}$ due to transmit beamforming with $\mathbf{w}_t$ is then

$$\tilde{E}^s(\mathbf{r}_{\text{rec}}) = k_0^2 \int_V G^b(\mathbf{r}', \mathbf{r}_{\text{rec}}) \mathbf{w}_t^H \mathbf{e}_i(\mathbf{r}') o(\mathbf{r}') d^3\mathbf{r}'. \quad (5)$$

Each antenna also has associated with it a complex channel receive weight that modifies the phase and amplitude of the received signal resulting from the scattered field in (5). Let the receive channel weight vector be denoted $\mathbf{w}_r$. The modified received signals are summed to form the beamformer output signal s , given by

$$s = k_0^2 \int_V \mathbf{w}_r^H \mathbf{g}_b(\mathbf{r}') \mathbf{w}_t^H \mathbf{e}_i(\mathbf{r}') o(\mathbf{r}') d^3\mathbf{r}' = \mathbf{w}_t^H \mathbf{D} \mathbf{w}_r^*, \quad (6)$$

where $\mathbf{D}$ is the $L \times L$ bistatic scattering matrix given by $D_{mn} = E_s(\mathbf{r}_m, \mathbf{r}_n)$. Equation (6) shows that transmit-receive beamforming can be accomplished by acquiring $\mathbf{D}$ on a channel-by-channel basis, designing the weights $\mathbf{w}_t$ and $\mathbf{w}_r$, and multiplying. Thus, the beamformer output signal for a given choice of beamformer weights may be obtained entirely in post-processing, without need for additional hardware. If desired (e.g., [3]), monostatic data may be removed from (6) by replacing the diagonal components of $\mathbf{D}$ with zeros and subtracting the term

$$\zeta(\mathbf{w}_t, \mathbf{w}_r, \mathbf{r}') = [\mathbf{w}_t \odot \mathbf{w}_r]^H [\mathbf{g}_b(\mathbf{r}') \odot \mathbf{e}_i(\mathbf{r}')] \quad (7)$$

from the integral kernel.

B. Imaging Algorithm

In this study, the imaging region, V , comprises all points interior to the skin of the breast. The skin layer is assumed to be known and thus part of the background profile. This is not an unreasonable assumption, since techniques have been developed for estimating the

location of the breast surface [13]–[16] and the dielectric properties of skin are well-characterized [17].

Following data acquisition, the incident field and Green's function quantities are obtained by simulating the background profile using FDTD. This step in the DBIM is known as the forward solution. The computed field quantities are used to form the matrix system $\mathbf{b} = \mathbf{A}\mathbf{x}$ upon discretization of the electric field integral equation. The system is then solved for an estimate of the unknown contrast at each voxel in V . The system is typically undetermined and ill-posed, necessitating a regularized optimization technique, in this case, conjugate gradient least-squares (CGLS), in order to find a solution. This step in the DBIM is known as the inverse solution. The background profile is then updated by adding the contrast estimate. The process is repeated using the new background profile. The DBIM iterates between forward and inverse solutions until convergence in the simulated and measured scattered field is achieved.

Details of the implementation of the standard DBIM without the focal beamforming enhancement, including the initial guess, regularization technique, and solution constraints, are given in [3]. The difference between the standard formulation and the beamforming-enhanced formulation is in the creation of the matrix system $\mathbf{b} = \mathbf{A}\mathbf{x}$. The formation of the linear system for the beamforming-enhanced case is performed as follows.

After each forward solution, $\mathbf{D}$ is formed for each frequency of interest by subtracting the simulated incident field at the antenna terminals from the measured total field (where 'measured' in this case refers to the simulated data acquisition from the exact profile) for all transmit-receive antenna combinations. We also form $\mathbf{e}_i(\mathbf{r}')$ from the forward-solution-computed incident fields at each voxel in V , and $\mathbf{g}_b(\mathbf{r}')$, which is computed from the incident fields as

$$\mathbf{g}_b(\mathbf{r}') = \frac{j\mathbf{e}_i(\mathbf{r}')}{\omega\mu_0\epsilon_0\ell_s i(\omega)} \quad (8)$$

where ℓ_s is the length of the current source of the transmit antenna.

The beamformer output signal s is computed for each of a total of M pairs of transmit and receive weight vectors. Let the M transmit and receive weight vectors be denoted $\mathbf{w}_{t1} \dots \mathbf{w}_{tM}$ and $\mathbf{w}_{r1} \dots \mathbf{w}_{rM}$, respectively. For the FBS, M foci are placed in V with a uniform focal spacing of ℓ_f in all three Cartesian coordinates. An example distribution of foci in one cross-section of the Class 2 phantom is shown in Fig. 1. Let the locations of the foci in V be given by $\mathbf{r}'_{f1} \dots \mathbf{r}'_{fM}$. The transmit and receive weight vectors for the m th focus are chosen to be identical and to satisfy the maximum SNR criterion for white noise [18], given by

$$\mathbf{w}_{tm} = \mathbf{w}_{rm} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{g}_b(\mathbf{r}'_{fm}) \mathbf{g}_b^H(\mathbf{r}'_{fm}) \mathbf{w}}{\mathbf{w}^H \mathbf{w}}. \quad (9)$$

The solution to (9) is $\mathbf{w}_{tm} = \mathbf{w}_{rm} = \alpha \mathbf{g}_b(\mathbf{r}'_{fm})$, where α is an arbitrary constant. We choose α in order to obtain unit norm weights, leading to $\mathbf{w}_{tm} = \mathbf{w}_{rm} = \mathbf{g}_b(\mathbf{r}'_{fm}) / \|\mathbf{g}_b(\mathbf{r}'_{fm})\|_2$.

The RPDS is equivalent to choosing the standard real basis vectors as beamforming weights. Let the h th standard basis vector be denoted $\mathbf{v}^h = [v_1 \ v_2 \ \dots \ v_L]^T$, where $v_{i=h} = 1$, $v_{i \neq h} = 0$. For a channel formed by transmitting at the h th source and receiving at the i th source, $\mathbf{w}_t = \mathbf{v}^h$, while $\mathbf{w}_r = \mathbf{v}^i$. For the RPAS, the corresponding weights are $\mathbf{w}_t = \mathbf{w}_r = (\mathbf{v}^h + \mathbf{v}^i) / \sqrt{2}$. Both cases yield $M = L(L - 1)/2$.

Let $\mathbf{r}'_1 \dots \mathbf{r}'_K$ be the locations of all voxels in V . Then, the $3K \times 1$ contrast vector $\mathbf{x}$ formulated for a single-pole Debye parameter reconstruction is given by

$$\mathbf{x} = [\mathbf{o}_{\varepsilon\infty}^T \ \mathbf{o}_{\varepsilon}^T \ \mathbf{o}_{\sigma}^T]^T \quad (10)$$

where $\mathbf{o}_{\varepsilon\infty}$, $\mathbf{o}_{\varepsilon}$, and $\mathbf{o}_{\sigma}$ are each of size $K \times 1$ and contain the unknown contrast in Debye parameters ε_{∞} , ε , and σ_s at the K voxels in V .

For a single-frequency reconstruction, the vector $\mathbf{b}$ is given by

$$\mathbf{b} = \begin{bmatrix} \Re\{\mathbf{w}_{t1}^H \mathbf{D} \mathbf{w}_{r1}^*\} \\ \vdots \\ \Re\{\mathbf{w}_{tM}^H \mathbf{D} \mathbf{w}_{rM}^*\} \\ \Im\{\mathbf{w}_{t1}^H \mathbf{D} \mathbf{w}_{r1}^*\} \\ \vdots \\ \Im\{\mathbf{w}_{tM}^H \mathbf{D} \mathbf{w}_{rM}^*\} \end{bmatrix} \quad (11)$$

where $\Re$ and $\Im$ refer to the real and imaginary parts, respectively, and the monostatic data has been removed from $\mathbf{D}$ by replacing its diagonal components with zeros. Let the submatrix $\mathbf{A}^0$ be of size $M \times K$, where the m th row of $\mathbf{A}^0$ is formed by discretizing the kernel of the integral (6) for the m th transmit-receive beamforming weight combination. The component in the m th row and k th column is given by

$$\frac{A_{mk}^0}{\Delta^3 k_0^2} = \mathbf{w}_{tm}^H \mathbf{g}_b(\mathbf{r}'_k) \mathbf{e}_i^T(\mathbf{r}'_k) \mathbf{w}_{rm}^* - \zeta(\mathbf{w}_{tm}, \mathbf{w}_{rm}, \mathbf{r}'_k) \quad (12)$$

where Δ^3 is the volume of one voxel in the imaging region and ζ is as defined in (7). The $2M \times 3K$ matrix $\mathbf{A}$ is then given by

$$\mathbf{A} = \begin{bmatrix} \Re \mathbf{A}^0 & \Re c_{\Delta} \Re \mathbf{A}^0 - \Im c_{\Delta} \Im \mathbf{A}^0 & -\Im c_{\sigma} \Im \mathbf{A}^0 \\ \Im \mathbf{A}^0 & \Im c_{\Delta} \Re \mathbf{A}^0 + \Re c_{\Delta} \Im \mathbf{A}^0 & +\Im c_{\sigma} \Re \mathbf{A}^0 \end{bmatrix} \quad (13)$$

where $c_{\Delta} = (1 + j\omega\tau)^{-1}$ and $c_{\sigma} = (j\omega\varepsilon_0)^{-1}$. For reconstructions using more than one frequency, the matrix system may be created by forming $\mathbf{b}$ and $\mathbf{A}$ for each frequency and then vertically concatenating the respective rows.

The imaging algorithm is considered to have converged in this study based on the convergence of the norm of the residual vector, $\mathbf{b}$. Let $\mathbf{b}^{(i)}$ be defined as the residual vector $\mathbf{b}$ at the i th DBIM iteration. Termination of the DBIM algorithm occurs when one of three conditions has been reached: 1) When 10 DBIM iterations are performed; 2) when $\|\mathbf{b}^{(i)}\|_2 -$

$\|\mathbf{b}^{(i-1)}\|_2 < 0.01\|\mathbf{b}^{(1)}\|_2$; or 3) when $\|\mathbf{b}^{(i)}\|_2 > \|\mathbf{b}^{(i-1)}\|_2 > \|\mathbf{b}^{(i-2)}\|_2$. In the latter case, performing further iterations is unlikely to improve the quality of the reconstruction. Termination tends to occur after around five iterations.

IV. Results AND Discussion

In this section we report the results from applying all three DBIM schemes to the synthetic data acquired from the numerical testbeds as described in Section II. We report the results by displaying quantitative images of the Debye parameter reconstructions and by using the $\cos(\phi)$ quality metric defined in [3]. Let $\mathbf{y}$ be the vector of the exact debye parameters in V , following the same convention as $\mathbf{x}$ from (10). Let the vector $\hat{\mathbf{y}}$ be the corresponding vector of estimated properties. Then,

$$\cos(\phi) = \frac{\mathbf{y}^T \hat{\mathbf{y}}}{\|\mathbf{y}\|_2 \|\hat{\mathbf{y}}\|_2}. \quad (14)$$

This quality metric is invariant to global scaling and gives a quantitative measure of the similarity between the true and estimated distributions of dielectric tissues in V . For a perfect reconstruction, $\cos(\phi) = 1$.

For each phantom, reconstructions were created after Gaussian white noise was added to the simulated array measurements. The SNR was varied from 35 dB to 0 dB in decrements of 5 dB. We define SNR here as the ratio between the total received signal power across all channels to the total noise power across all channels. Five independent noise instances were generated for both phantom classes. At each designated SNR level, each noise instance was scaled appropriately and DBIM reconstructions were performed. The focal-beamforming-enhanced reconstructions were performed for $r_f = 1, 2$, and 3 cm.

Fig. 2 shows the $\cos(\phi)$ as a function of SNR, averaged across the five noise instances, for both the Class 2 and Class 3 breast phantoms. At SNR levels above 20 dB, there is essentially no difference in performance between RPAS and RPDS, and the performance of the FBS with $r_f = 1$ cm is comparable. As the SNR decreases, the quality of the RPDS reconstruction degrades most dramatically, followed by the reconstructions from the RPAS and the FBS. The FBS with $r_f = 1$ cm significantly outperforms both of the non-focal schemes for SNR below 10 dB. When r_f is increased from 1 to 3 cm, the FBS performs progressively worse for all SNR.

Fig. 3 is of the same form as Fig. 2, with the exception that $\cos(\phi)$ is calculated using the corresponding 35 dB reconstruction to create the reference vector $\mathbf{y}$ for each case. As the SNR decreases, fidelity with respect to the 35 dB reconstruction degrades for all DBIM schemes. The degradation is significantly less for the FBS as compared to the RPAS or RPDS. Figs. 2 and 3 demonstrate that the focal beamforming enhancement decreases the sensitivity of the imaging algorithm to noise corruption at the cost of lower accuracy at high SNR. The degree to which accuracy degrades at high SNR is correlated with the sparsity of the focal distribution; the densest focal placement results in only a small degradation in performance, while the sparsest focal placement results in a more severe degradation.

Figs. 4 and 5 show coronal cross-sections through the 3-D exact and reconstructed dielectric profiles for the Class 2 and Class 3 phantoms, respectively, for one representative noise instance at each of the following SNR levels: 35, 25, 15, and 5 dB. The profiles are depicted in terms of Debye parameter ϵ . The three Debye parameters are highly correlated in the true profiles and are constrained to be highly correlated in the reconstructed profiles [3], so ϵ_∞ and σ_s are not shown. Low- ϵ regions in the profiles correspond to adipose tissue, while high- ϵ regions correspond to fibroglandular tissue. The left, middle, and right columns correspond to reconstructions created using the FBS with $\Delta_f = 1$ cm, the RPAS, and the RPDS, respectively.

For high SNR, all three types of reconstruction are of high fidelity with respect to the exact dielectric profiles. Reconstructed fibroglandular regions are distinguishable from adipose regions and are also accurate in regards to basic shape and location. A slightly greater degree of blurring is evident in the FBS reconstructions, which perhaps accounts for the slightly lower $\cos(\varphi)$ at 35 dB as seen in Fig. 2. As SNR decreases, the FBS reconstructions degrade less than the RPAS reconstructions, which in turn degrade less than the RPDS reconstructions. In particular, there is a significant difference in quality between the focal and non-focal reconstructions for SNR below 10 dB. For the non-focal reconstructions, the central fibroglandular region of the Class 2 phantom disappears at lower SNR, while the shape and general distribution of the reconstructed fibroglandular tissues in the Class 3 phantom change to the point where they are unrecognizable. Less change is evident in the FBS reconstructions between high and low SNR.

Figs. 2, 4 and 5 suggest that the focal beamforming enhancement makes the DBIM algorithm less susceptible to corruption of the measured signal. For a desired reconstruction quality, the required SNR for the FBS is around 5 dB lower than that for the RPAS and around 10 dB lower than that for the RPDS, assuming the densest focal placement is used.

For the Class 2 phantom, the total number of foci in V , and thus the value of M , is 500, 57, and 16 for $\Delta_f = 1, 2$, and 3 cm, respectively; for the Class 3 phantom, there are 306, 34, and 9 foci for the three focal spacings. In comparison, $M = 780$ for the non-focal schemes. The size of the linear system, and thus the computational cost of the inverse system, scales with M . Thus, each FBS reconstruction is less computationally costly than the non-focal scheme reconstructions. Fig. 6 shows cross-sections through the FBS reconstruction for the Class 2 and Class 3 phantoms at 30 dB SNR using $\Delta_f = 1, 2$, and 3 cm. As the focal placement becomes more sparse, some detail is lost in the resulting images, leading to a degradation of reconstruction accuracy. These results, along with the quantitative results from Fig. 2, demonstrate the importance of using a dense focal placement in order to create high fidelity reconstructions using the beamforming-enhanced algorithm.

While we used a lossless coupling medium in this study, the beamforming enhancement to the DBIM also applies in general to lossy coupling media. For the lossy case, the conductivity of the coupling medium affects the Green's function vector $\mathbf{g}_b(\mathbf{r}')$, and is thus reflected in the beamformer weight design.

V. Conclusion

In this study, a focal transmit-receive beamforming technique was incorporated into the DBIM algorithm for microwave breast imaging. Images of 3-D anatomically-realistic breast phantoms were reconstructed using the focal-beamforming-enhanced DBIM and compared to reconstructions created using non-focal-beamforming formulations of the DBIM algorithm. The focal-beamforming-enhanced reconstructions are less sensitive to additive Gaussian white noise. For an SNR of 15 dB or below, the focal-beamforming-enhanced reconstructions are considerably more faithful to the exact dielectric profiles. The superior performance of the focal-beamforming-enhanced algorithm in the presence of noise is significant, as the ill-posed nature of the inverse system makes it sensitive to perturbations in the measured signal.

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Biographies

Matthew J. Burfeindt (S'10–M'13) received the B.S. degree in electrical engineering from the University of Nebraska-Lincoln, Lincoln, NE, USA, in 2008, and the M.S. and Ph.D. degrees in electrical engineering from the University of Wisconsin-Madison, Madison, WI, USA, in 2011 and 2013, respectively.

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Jacob D. Shea (S'05–M'14) received dual B.S. degrees in mathematics and electrical engineering in 2000, the M.S. degree in electrical engineering in 2006, and the Ph.D. degree in electrical engineering in 2010, all from the University of Wisconsin-Madison, Madison, WI, USA.

From 2000 to 2005, he was with Agilent Technologies, first in Santa Rosa, CA, USA, as an R&D Hardware Engineer and later in Columbia, MD, USA, as an Application, Software, and System Engineer. Since 2006, he has been an Engineering Consultant in the defense industry. In 2010, he founded ELEMEN LLC, an independent consulting firm. His research interests include microwave imaging, RF geolocation, and spectral surveillance.

Dr. Shea received a University of Wisconsin-Madison Graduate School Fellowship in 2005, the Best Student Paper Competition Award at the 13th International Symposium on Antenna Technology and Applied Electromagnetics (ANTEM) in 2009, and the IEEE Antennas and Propagation Society Doctoral Research Award in 2010.

Barry D. Van Veen (S'81–M'86–SM'97–F'02) was born in Green Bay, WI, USA. He received the B.S. degree from Michigan Technological University, Houghton, MI, USA, in 1983 and the Ph.D. degree from the University of Colorado-Boulder, Boulder, CO, USA, in 1986, both in electrical engineering.

In spring 1987, he was with the Department of Electrical and Computer Engineering, University of Colorado-Boulder. Since August 1987, he has been with the Department of Electrical and Computer Engineering, Wisconsin-Madison, Madison, WI, USA, where he is currently the Lynn H. Matthias Professor. His research interests include signal processing for sensor arrays and biomedical applications of signal processing. He coauthored *Signals and Systems* (1st ed. 1999, 2nd ed., 2003 Wiley).

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Figure 1.

Foci distribution in a cross-section of the Class 2 phantom with $r_f = 1$ cm. Foci are marked with a '+' sign. The colorbar refers to Debye parameter ϵ .

Figure 2.

Quality metric $\cos(\phi)$ versus SNR for the (a) Class 2 and (b) Class 3 breast phantoms. The quality metric is referenced to the exact profile and is averaged across five noise instances.

(a)

(b)

Figure 3.

Quality metric $\cos(\phi)$ versus SNR for the (a) Class 2 and (b) Class 3 breast phantoms. The quality metric is referenced to the 35 dB SNR reconstruction and is averaged across five noise instances.

Figure 4.

Cross-sections through the (a) Class 2 breast phantom and (b) 3-D reconstructed image for SNR ranging from 35 to 5 dB. The left column shows results for the focal beamforming scheme ($r_f = 1$ cm), the middle column shows results for reciprocal-pair averaging scheme, and the right column shows results for reciprocal-pair discarding scheme. The colorbar refers to Debye parameter ϵ .

Figure 5.

Cross-sections through the (a) Class 3 breast phantom and (b) 3-D reconstructed image for SNR ranging from 35 to 5 dB. The left column shows results for focal beamforming scheme ($r_f = 1$ cm), the middle column shows results for reciprocal-pair averaging scheme, and the right column shows results for reciprocal-pair discarding scheme. The colorbar refers to Debye parameter ϵ .

Figure 6.

Cross-sections through 3-D dielectric reconstructions of Class 2 (top row) and Class 3 (bottom row) phantoms with an SNR of 30 dB. From left to right, $\Delta_f = 1, 2,$ and 3 cm. The colorbar refers to Debye parameter ϵ .