Published in final edited form as:

IEEE Trans Robot. 2015 April ; 31(2): 246–258. doi:10.1109/TRO.2015.2394331.

Concentric Tube Robots as Steerable Needles: Achieving Follow-the-Leader Deployment

Hunter B. Gilbert [Student Member, IEEE],

Department of Mechanical Engineering, Vanderbilt University, Nashville, TN 37235 USA

Joseph Neimat, and

Department of Neurological Surgery, Vanderbilt University Medical Center, Nashville, TN 37232 USA

Robert J. Webster III [Senior Member, IEEE]

Department of Mechanical Engineering, Vanderbilt University, Nashville, TN 37235 USA

Hunter B. Gilbert: hunter.b.gilbert@vanderbilt.edu; Joseph Neimat: joseph.neimat@vanderbilt.edu; Robert J. Webster: robert.websterg@vanderbilt.edu

Abstract

Concentric tube robots can enable new clinical interventions if they are able to pass through soft tissue, deploy along desired paths through open cavities, or travel along winding lumens. These behaviors require the robot to deploy in such a way that the curved shape of its shaft remains unchanged as the tip progresses forward (i.e., “follow-the-leader” deployment). Follow-the-leader deployment is challenging for concentric tube robots due to elastic (and particularly torsional) coupling between the tubes that form the robot. However, as we show in this paper, follow-the-leader deployment is possible, provided that tube precurvatures and deployment sequences are appropriately selected. We begin by defining follow-the-leader deployment and providing conditions that must be satisfied for a concentric tube robot to achieve it. We then examine several useful special cases of follow-the-leader deployment, showing that both circular and helical precurvatures can be employed, and provide an experimental illustration of the helical case. We also explore approximate follow-the-leader behavior and provide a metric for the similarity of a general deployment to a follow-the-leader deployment. Finally, we consider access to the hippocampus in the brain to treat epilepsy, as a motivating clinical example for follow-the-leader deployment.

Index Terms

Concentric tube robot; continuum robot; medical robotics; needle steering; surgical robotics

This paper was recommended for publication by Associate Editor R. S. Dahiya and Editor B. J. Nelson upon evaluation of the reviewers' comments.

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

I. Introduction

The desire to avoid critical structures and reach previously unreachable targets during needle biopsy and needle-delivered therapy has spurred the development of many kinds of steerable needles. Designs include bevel-tipped needles [1], needles with a pre-bent tip [2], needles that extend a curved stylet [3], and actively controlled needles [4], among others (for reviews, see [2] and [5]). Early motivations for developing concentric tube continuum robots (also called “active cannulas” due to their usefulness in medicine; see Fig. 1) were 1) to generalize the steerability of steerable needles beyond soft tissues, to air- or liquid-filled cavities; and 2) to use this dexterity to create needle-diameter tentacle-like robot manipulators [6], [7]. A review of concentric tube robot history and applications can be found in [8]. While a great deal of the recent research in concentric tube robots has focused on the latter of these two motivations (see, e.g., [9], [10]), efforts have also been made to address the former through motion planning (choosing actuator sequences to keep the shaft of the robot within anatomical bounds during deployment [11]) and in using a special case with one curved tube and two straight tubes to hit targets in soft tissues (see, e.g., [12] and many references in [8] including commercial products dating back to the 1980s).

This special case is useful because it can be deployed in a perfect follow-the-leader manner, wherein the shaft of the device exactly follows the path through space traced out by the tip at all times during insertion. The term “follow the leader” for deployment of a highly articulated robot was coined by Choset and Henning in 1999 [13], and Ikuta *et al.* proposed it for endoscope deployment in 1988 under the name “shift control” [14]. It has also been used advantageously in other highly articulated robots (see, e.g., [15]). The term “follow the leader” was perhaps first applied to concentric tube robots in 2006 by Sears and Dupont [16], who provided design heuristics that enable approximate follow-the-leader behavior and showed that under the assumption of infinite torsional rigidity, general collections of tubes are able to deploy in a follow-the-leader manner.

It was later observed that torsion is typically significant in these robots in practice and that torsional deformation precludes follow-the-leader deployment, even for constant precurvature tubes. Models were subsequently developed that include the effects of torsion [6], [7], [17], [18]. These models were applied to the use of concentric tube robots as manipulators in many contexts. They have also been used to produce design heuristics [7] and motion planners [11] for approximate follow-the-leader deployment. However, the analysis of follow-the-leader behavior in concentric tube robots has not been revisited in light of them, which is the purpose of this paper.

In soft tissues, a benefit of using concentric tube robots in comparison with other steerable needle technologies is that concentric tube robots rely on internal forces rather than tip-tissue forces to bend. This makes them able to steer through open or liquid-filled cavities and through soft tissue with minimal deflection of the needle based on needle-tissue interaction forces. In contrast, the properties of bevel-steered needles (shaft stiffness, tip design, etc.) must be matched exactly to tissue properties to achieve appreciable curvature, and coping with the sensitivity of the needle’s behavior to small changes in tissue properties is one of the major current challenges in needle steering research (see [2] and references

therein). Thus, both deployment through open cavities and reducing sensitivity to tissue properties during deployment through soft tissues motivate the question we seek to answer in this paper: Can concentric tube robots deploy in a follow-the-leader manner?

In this paper, we describe exact solutions to the follow-the-leader deployment problem and examine the model-predicted deviation from follow-the-leader behavior in approximate cases for concentric tube robots. Our primary contributions are the development of necessary and sufficient conditions for follow-the-leader behavior, the special case precurvatures we describe (including helical shapes not previously considered), a metric for measuring the similarity of a general deployment to a follow-the-leader deployment, and our neurosurgical illustration. A preliminary version of some portions of this study appeared in conference form in [19]. Extensions in this archival paper beyond the initial conference version include the nondimensional analysis in Section VI-A, the discussion of approximate follow-the-leader deployment for helically precurved tubes in Section VI-F, the physical experiment in Section VII, and the neurosurgical example in Section VIII.

II. Follow-The-Leader Insertion

Before exploring special cases and approximations of follow-the-leader behavior, it is useful to have a mathematical description for follow-the-leader deployment of the robot. We proceed in this section by describing the follow-the-leader constraint on the space curve, which describes the robot shape. The following sections will then connect this constraint to the mechanics model, which will reveal the resulting restrictions on both robot design and actuation sequences. We describe the shape of the robot using a time-varying arc-length parameterized transformation $\mathbf{g}(s, t) \in SE(3)$, which assigns a position $\mathbf{p}(s, t) \in \mathbb{R}^3$ and orientation $\mathbf{R}(s, t) \in SO(3)$ to each arc length $s \in [0, L(t)]$ along the centerline of the tubes. The domain of s depends on t , but the parameters s and t are independent of one another, and wherever mixed partial derivatives with respect to s and t occur, we assume their symmetry. We assign our frames such that the columns of the matrix $\mathbf{R}$ can be considered to be

$\begin{bmatrix} \mathbf{x}(s, t) & \mathbf{y}(s, t) & \mathbf{z}(s, t) \end{bmatrix}$. The function $L(t)$ represents the exposed length of the robot, which increases during a deployment, and hence is a function of time. The differential kinematic equations describing the evolution of the transformation are given as $\mathbf{g}'(s, t) = \mathbf{g}(s, t)\hat{\xi}(s, t)$, where $\xi \in \mathbb{R}^6$ contains the body frame twist coordinates $\xi(s, t) = \begin{bmatrix} \mathbf{v}^T & \mathbf{u}(s, t)^T \end{bmatrix}^T$ and the prime denotes the partial derivative $\partial / \partial s$. The $\hat{\cdot}$ operator converts a vector in $\mathbb{R}^6$ to an element of the Lie algebra $\mathfrak{se}(3)$. The vector $\mathbf{u}$ may be thought of as the curvature or “angular velocity” (with respect to arc length) of the frame, and $\mathbf{v}$ as the “linear velocity” (with respect to arc length) of the frame $\mathbf{g}$. We assume that the $\mathbf{z}$ axis of $\mathbf{R}$ is tangent to the curve, and the transformation propagates with unit velocity along the $\mathbf{z}$ -axis, meaning that $\mathbf{v} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Equivalently, the components of the transformation $\mathbf{g}$ can be expressed as

$$\mathbf{R}'(s, t) = \mathbf{R}(s, t)[\mathbf{u}(s, t)]_{\times} \quad (1)$$

$$\mathbf{p}'(s, t) = \mathbf{z}(s, t) \quad (2)$$

where we also assume initial conditions $\mathbf{R}(0, t) = \mathbf{R}_z(\psi_0(t))$ (a rotation about the $\mathbf{z}$ -axis by angle ψ_0), and $\mathbf{p}(0, t) = \mathbf{0}$. These initial conditions model the physical robot, where a position and tangent vector are prescribed at one point of the robot centerline. The operation $[\cdot]_\times$ is the isomorphism between a vector in $\mathbb{R}^3$ and its skew-symmetric cross product matrix.

For follow-the-leader deployment, we require that the position at any arc length does not change in time, or mathematically that

$$\dot{\mathbf{p}}(s, t) = \frac{\partial \mathbf{p}(s, t)}{\partial t} = \mathbf{0}. \quad (3)$$

The overdot will continue to denote the partial derivative with respect to time. Intuitively, the above assumptions result in a constraint on the curvature $\mathbf{u}(s, t)$. Since the space curve cannot change except to telescopically extend, the curvature function $\mathbf{u}(s, t)$ must not physically change except to allow growth in the domain of s . More precisely, we will prove that the follow-the-leader criterion of (3) is equivalent to the following: for the angular displacement function $\psi(s, t)$ which satisfies $\psi' = u_z$ and $\psi(0, t) = \psi_0(t)$, the projections of the curvature vector $u_x(s, t) = \mathbf{e}_1 \cdot \mathbf{u}(s, t)$ and $u_y(s, t) = \mathbf{e}_2 \cdot \mathbf{u}(s, t)$, with $\mathbf{e}_i$ the i th standard basis vector, satisfy

$$\frac{\partial}{\partial t} \begin{bmatrix} u_x(s, t) \\ u_y(s, t) \end{bmatrix} = \begin{bmatrix} 0 & \dot{\psi}(s, t) \\ -\dot{\psi}(s, t) & 0 \end{bmatrix} \begin{bmatrix} u_x(s, t) \\ u_y(s, t) \end{bmatrix}. \quad (4)$$

First, we show that (3) $\Rightarrow$ (4). From (2) and (3), we can use the fundamental theorem of calculus and the commutativity of the partial derivatives to yield

$$\frac{\partial \mathbf{p}}{\partial t}(s, t) = \int_0^s \frac{\partial}{\partial s} \frac{\partial \mathbf{p}}{\partial t}(\sigma, t) d\sigma = \int_0^s \frac{\partial \mathbf{z}}{\partial t}(\sigma, t) d\sigma = \mathbf{0} \quad (5)$$

which holds for all values of s if and only if

$$\frac{\partial \mathbf{z}}{\partial t}(s, t) = \mathbf{0}. \quad (6)$$

This equation holds if and only if the time variation of $\mathbf{R}$ has the form

$$\frac{\partial \mathbf{R}}{\partial t}(s, t) = \mathbf{R}[\omega(s, t)\mathbf{e}_3]_\times \quad (7)$$

where $\omega(s, t)$ is the scalar angular velocity function, which is unknown except at $s = 0$, where $\omega(0, t) = \dot{\psi}(0, t)$.

Equating the mixed partial derivatives of $\mathbf{R}$ by taking the partial derivative of (7) with respect to s and the partial derivative of (1) with respect to t yields

$$\frac{\partial \mathbf{R}}{\partial t} [\mathbf{u}]_{\times} + \mathbf{R} \left[\frac{\partial \mathbf{u}}{\partial t} \right]_{\times} = \frac{\partial \mathbf{R}}{\partial s} [\omega \mathbf{e}_3]_{\times} + \mathbf{R} \left[\frac{\partial \omega}{\partial s} \mathbf{e}_3 \right]_{\times}. \quad (8)$$

Substituting again from (1) and (7), premultiplying by $\mathbf{R}^T$, applying the identity $[\mathbf{a} \times \mathbf{b}]_{\times} = [\mathbf{a}]_{\times} [\mathbf{b}]_{\times} - [\mathbf{b}]_{\times} [\mathbf{a}]_{\times}$, and taking the inverse of the $[\cdot]_{\times}$ operator on the entire equation yield that (8) is equivalent to the condition

$$\frac{\partial \mathbf{u}}{\partial t} = -\omega \mathbf{e}_3 \times \mathbf{u} + \frac{\partial \omega}{\partial s} \mathbf{e}_3. \quad (9)$$

The third vector component of this equation is $\omega' = u_z'$, and from the definition of $\psi(s, t)$, we thus have $\omega' = \psi'(s, t)$. The initial condition for $\mathbf{R}$ implies that $\omega(0, t) = \psi(0, t)$ so that we can identify $\omega(s, t) = \psi(s, t)$. Then, the first two vector components of (9) are given by (4), and the first part of the proof is complete.

We now show that (4) $\Rightarrow$ (3). We start with only (1), (2), (4), and the existence of the function $\psi' = u_z'$. We now seek the time variation of $\mathbf{R}$, which is given by

$$\frac{\partial \mathbf{R}}{\partial t}(s, t) = \mathbf{R}(s, t) [\boldsymbol{\omega}(s, t)]_{\times} \quad (10)$$

for an unknown vector function $\boldsymbol{\omega}(s, t)$. By again equating the mixed partial derivatives and making the necessary substitutions, we find a differential equation

$$\frac{\partial \boldsymbol{\omega}}{\partial s} = \boldsymbol{\omega} \times \mathbf{u} + \begin{bmatrix} \dot{\psi} u_y & -\dot{\psi} u_x & \frac{\partial^2 \psi}{\partial t \partial s} \end{bmatrix}^T. \quad (11)$$

By the initial condition $\mathbf{R}(0, t) = \mathbf{R}_z(\psi_0(t))$, we have the initial condition $\boldsymbol{\omega}(0, t) = \psi(0, t) \mathbf{e}_3$. Therefore

$$\boldsymbol{\omega}(s, t) = \frac{\partial \psi}{\partial t}(s, t) \mathbf{e}_3 \quad (12)$$

is the unique solution to this differential equation, which in turn implies that $\psi(s, t) = 0$, which is equivalent to the follow-the-leader condition of (3). Thus, (4) is a necessary and sufficient condition for follow-the-leader behavior. In the next section, this equation is connected to a mechanics-based model for concentric tube robots.

III. Mechanics-Based Model Review

The curvature vector $\mathbf{u}$ from the previous section is defined by a mechanics-based model that results in a set of differential equations, which define a boundary value problem [6], [7], [20], which is reviewed in this section. Each component tube of the robot is grasped at its

proximal end where translation and rotation are applied. We denote with $\beta_i(t)$ the positive length between the point where $s = 0$ and the point where $\sigma_i = 0$ for the i th tube, which is where an actuator will grasp the tube, and with $\alpha_i(t)$ the rotational actuation (see Fig. 2). Let the variables $\psi_i(s, t)$ be the angular rotations of each tube, which may be thought of as the angular displacement about the local z-axis between the material frame and the zero-torsion

or rotation-minimizing Bishop frame. The curvature $\mathbf{u}_i \begin{bmatrix} u_{ix} & u_{iy} & u_{iz} \end{bmatrix}^T$ of each tube is written in the material frame of the respective tube. Here, $i \in 1, \dots, N$ denotes the tube number with tube 1 being the tube with the smallest diameter. The precurved shape of each tube is expressed in the Frenet–Serret convention as $\mathbf{u}_i^*(\sigma_i) = \begin{bmatrix} u_{ix}^*(\sigma_i) & 0 & u_{iz}^*(\sigma_i) \end{bmatrix}^T$, where we will refer to u_{ix}^* as the curvature and u_{iz}^* as the torsion of the frame. We use σ_i for arc length here to clarify that this length is measured with respect to a fixed point on the tube, not with respect to the fixed point in space where $s = 0$.

A diagram of the variables of interest is shown in Fig. 3. From this point forward, as in the previous section, for notational brevity, the explicit dependence on both s and t will be suppressed except where not clear from context.

The multipoint boundary value problem for a concentric tube robot with no applied external loads is given in [20] as

$$\begin{aligned} \psi_i' &= u_{iz} \\ u_{iz}' &= u_{iz}^*(s + \beta_i) + \frac{1}{EI} \frac{E_i I_i}{G_i J_i} \sum_{k=1}^N E_k I_k u_{ix}^*(s + \beta_i) u_{kx}^*(s + \beta_k) \sin(\psi_i - \psi_k) \end{aligned} \quad (13)$$

for $i = 1, \dots, N$, with boundary conditions

$$\begin{aligned} \psi_i(0, t) &= \psi_{i0}(t), & i=1, \dots, N \\ u_{iz}(\ell_i, t) &= u_{iz}^*(\ell_i + \beta_i), & i=1, \dots, N. \end{aligned} \quad (14)$$

Note that the precurvature functions u_{ix}^* and u_{iz}^* are evaluated at an arc length offset by the time-varying values $\beta_i(t)$, which accounts for possible translations of the tubes. Here, $\psi_{i0}(t)$ specifies the initial z-axis rotation of tube i , which is produced by controlling an actuator α_i at some proximal location β_i , and the latter boundary conditions on u_{iz} result from the fact that each tube must have zero torsional internal moment at its endpoint $s = \ell_i$ (i.e., the sum $\ell_i + \beta_i$ is constant). The values of $\psi_{i0}(t)$ are related to the rotational actuation variables $\alpha_i(t)$ through $\psi_{i0}(t) = \alpha_i(t) - \beta_i(t) u_{iz}(0, t)$. We use a linear constitutive law for the internal moment, $\mathbf{m}_i = \mathbf{K}_i(\mathbf{u}_i - \mathbf{u}_i^*)$, where $\mathbf{K}_i$ is a stiffness matrix. Here, $\mathbf{K}_i = \text{diag}(E_i I_i, E_i I_i, G_i J_i)$, E_i is Young's modulus, and I_i is the area moment of inertia of the tube cross section, which is assumed to be annular, about the x - or y -axes. The term EI is the sum over all the stiffnesses,

$EI = \sum_{k=1}^N E_k I_k$. Similarly, G_i is the shear modulus, and J_i is the area moment of inertia about the tangent axis. In some cases, it may be useful to specify Poisson's ratio, which relates E and G by the relation $G = E/(2(1 + \nu))$. It is important to realize that the sum is

performed only over those tubes which exist at the arc length being evaluated, and that the equations are only valid for tube i over the length that it exists.

The boundary value problem determines the tube torsion, but the local $\mathbf{xy}$ -curvature is found in closed form as a weighted sum of the precurvatures rotated into the frame of tube 1

$$\mathbf{u}_{xy}(s, t) = \frac{1}{EI} \sum_{j=1}^N E_j I_j \begin{bmatrix} \cos(\psi_j - \psi_1) \\ \sin(\psi_j - \psi_1) \end{bmatrix} u_{jx}^*(s + \beta_j) \quad (15)$$

where again the arguments to the functions $\psi_i(s, t)$ have been omitted for space. It is this frame which we define to be the central axis frame (i.e., the “backbone frame”) of the robot. Thus, the final robot curve is found by integrating (1) and (2) for the variables of tube 1.

The model equations also predict phenomena of elastic instability, which are seen in concentric tube robots [7], [18], [21]. Note that the results in this paper only apply to sable configurations. This means that, for example, two circularly precurved tubes that have a bifurcation cannot be rotated so that their curvatures exactly oppose one another. For two tubes of circular precurvature, this behavior can be predicted analytically, and it is known that the model solution is unique whenever the nondimensional inequality

$$\lambda = L^2 u_{1x}^* u_{2x}^* \frac{E_1 I_1 E_2 I_2 (G_1 J_1 + G_2 J_2)}{G_1 J_1 G_2 J_2 (E_1 I_1 + E_2 I_2)} < \frac{\pi^2}{4} \quad (16)$$

is satisfied [7], [18]. L represents the arc-length of the section of the robot in which both tubes are precurved and $s > 0$. A trivial extension of the proof of (16) shows that this equation also holds for two helical tubes as long as the precurved torsion u_z^* is the same for both tubes. This result is obtained by noting that the boundary conditions on the reduced set of equations presented in [18] remain the same under those conditions. Note that (16) does not account for transmission lengths, but recent results address transmission both transmission lengths and larger numbers of tubes, for particular design cases [21]. It should be emphasized that the presence of multiple solutions does not imply elastic instability, and thus, designs that have a bifurcation may be utilized in some approximate follow-the-leader cases if the user ensures that the actuation does not give rise to elastic instability.

IV. Special Cases of Follow-The-Leader Deployment

To determine conditions for concentric tube follow-the-leader deployment, first note that the definition of $\psi_1'(s, t)$ in (13) allows $\mathbf{u}_{xy}$ and $\dot{\psi}_1$ to be substituted into the follow-the-leader condition of (4). Performing this substitution and making simplifications, two constraint equations emerge:

$$\begin{aligned} \sum_{j=1}^N E_j I_j \left[\cos(\psi_j - \psi_1) u_{jx}^* \dot{\psi}_j + \sin(\psi_j - \psi_1) u_{jx}^* \dot{\beta}_j \right] &= 0 \\ \sum_{j=1}^N E_j I_j \left[\sin(\psi_j - \psi_1) u_{jx}^* \dot{\psi}_j - \cos(\psi_j - \psi_1) u_{jx}^* \dot{\beta}_j \right] &= 0. \end{aligned} \quad (17)$$

These two differential equations, if satisfied, guarantee that an N -tube robot follows the leader. Note that all of the rotational configuration functions ψ_i appear in these conditions, as well as their time derivatives. The precurvature functions also appear, and due to actuation, their derivatives with respect to s also appear. Note that they are evaluated at $s + \beta_j$. For a follow-the-leader insertion, the differential equations (17) must be consistent with the mechanics of the robot (13).

A. Two-Tube Case with Planar Constant Precurvatures

One prototype design which is appealingly simple is that of two tubes, which have precurvatures that are constant in arc length and planar (i.e., precurvatures that are circular arcs). Since the tubes have constant precurvature, $u'_{jx} = 0$. Constraints (17) simplify to

$$\begin{aligned} E_1 I_1 u_{1x}^* \dot{\psi}_1 + E_2 I_2 \cos(\psi_2 - \psi_1) u_{2x}^* \dot{\psi}_2 &= 0 \\ E_2 I_2 \sin(\psi_2 - \psi_1) u_{2x}^* \dot{\psi}_2 &= 0. \end{aligned} \quad (18)$$

All of the variables that appear in these equations are determined by actuation and the mechanics; therefore, we seek the ways in which these equations can be satisfied through choice of precurvature and/or actuation. We first show that if both precurvatures are nonzero, then (18) implies that $\psi_2 - \psi_1 = n\pi$. To see this fact, first assume that $\psi_2 - \psi_1 \neq n\pi$. Then, the two equations of (18) imply that follow-the-leader deployment requires $\dot{\psi}_1 = \dot{\psi}_2 = 0$. However, we will show that this cannot happen due to the torsional mechanics. The robot is deploying and increasing in total arc length; therefore, the arc length ℓ_i at which the distal boundary condition in (14) is evaluated is an increasing function of time $\ell_i(t)$. The total derivative of the distal boundary condition for each tube in (14) must be equal to zero by definition, and this is expressed mathematically by

$$\frac{\partial u_{iz}}{\partial s}(\ell_i(t), t) \frac{d\ell_i}{dt}(t) + \frac{\partial u_{iz}}{\partial t}(\ell_i(t), t) = 0. \quad (19)$$

During a deployment, $d\ell_i/dt$ is positive, and from (13), we have that u_{iz}/s is nonzero as long as both precurvatures are nonzero and $\psi_2 - \psi_1 \neq n\pi$. Then, the second term in (19) must also be nonzero. This implies by (13) that $\partial u_{iz}/\partial t = \dot{\psi}_i \neq 0$, and therefore, $\dot{\psi}_i$ cannot be zero everywhere. Thus, (18) is satisfied only if $\psi_2 - \psi_1 = n\pi$. In this case, (13) indicates that the tubes experience no torsional twisting, and thus, with no applied rotational actuation, we have $\dot{\psi}_1(s, t) = \dot{\psi}_2(s, t) = 0$, which trivially satisfies (18) and ensures follow-the-leader deployment.

There are also two special cases in which (18) can hold. If one of the tubes has zero precurvature, then follow-the-leader behavior is guaranteed by simply ensuring no rotational actuation of the other precurved tube. Additionally, if $E_1 I_1 u_{1x}^* = E_2 I_2 u_{2x}^*$ and the tube rotations are chosen so that $\psi_2 - \psi_1 = n\pi$ with n odd, then the two fully overlapped tubes can be rotated with equal angular velocity $\dot{\psi}_1 = \dot{\psi}_2$.

B. Helical Precurvatures

If either of the precurved shapes is a helix, then by the arguments in the previous section, $\psi_2 - \psi_1$ will not equal $n\pi$ at all arc lengths unless additionally both tubes have the same helical torsion u_z^* . If they did not, $\psi_1'(L) = u_{1z}^*(L) \neq u_{2z}^*(L) = \psi_2'(L)$, which implies that $\psi_2(s, t) - \psi_1(s, t)$ is not constant with respect to arc length. Equal torsion in the Frenet–Serret sense is, therefore, a necessary condition for follow-the-leader deployment of two helical tubes.

C. Stability of Solutions

The odd and even solutions for n in the previous two sections represent different solutions. The odd solutions are only stable in a robot that does not have bifurcations (see [7], [17], and [18] for further information on bifurcations) in the boundary value problem of (13). As λ approaches the critical value, the odd solutions become progressively “less stable,” and therefore, any designer wanting to leverage the odd solutions in a device designed to follow the leader would likely want to maintain some margin of safety below this critical value, and the inclusion of straight transmission segments in the tubes will require an even more conservative design than predicted by (16).

D. Required Deployment Sequences

In cases of follow-the-leader insertion, the deployment procedure for two tubes of constant curvature proceeds in two stages. The first stage is the insertion of both tubes together (fully overlapping). The second, optional, stage is the continued insertion of one tube only while holding the other fixed. These tubes will be called the “moving” and “fixed” tubes, respectively, for this stage of insertion. Since the moving tube has a constant curvature design, the interaction of the two tubes will remain unchanged as long as the actuation satisfies the requirement that $\psi_i(0, t)$ is constant in time. If the moving tube has any precurved torsion, an actuator will have to rotate the tube base, which is at negative arc length, to maintain a constant angle at $s = 0$. If the fixed tube is moved at all after the moving tube has been extended, then the condition $u_{ix}^* = 0$ has been violated at the discontinuity where the fixed tube ends, and the simplifications from (17) to (18) are no longer valid. Thus, the follow-the-leader behavior will cease in this case, and the shaft of the robot will deviate from the path traced previously by its tip. For the same reason, a step change in the curvature of a tube (e.g., a tube with an initial straight transmission followed by a circularly precurved tip section) will not be able to deploy in a follow-the-leader manner if the change in curvature occurs in the length from $s = 0$ to $s = L$.

E. Summary of Follow-The-Leader Cases

Based on the discussions in preceding sections, we can now describe some potentially useful cases where follow-the-leader behavior is possible for two tubes:

1. $u_{2x}^* = 0$ or $u_{1x}^* = 0$. In this case, one but not both of the tubes has zero precurvature, meaning that the final shape consists of two tangent circular or helical arcs with different curvatures. This case was identified and elaborated upon previously for circular tubes in [22].

2. $\mathbf{u}_{1z}^* = \mathbf{u}_{2z}^* = 0$. Both tubes are circular in precurvature, and we ensure that $\psi_2 - \psi_1 = n\pi$. The final shape consists of two circular arcs (which may have different radii based on tube stiffnesses and precurvatures) that lie in the same plane and are tangent to one another. For even n , these two arcs will curve in the same direction, while for odd n , they may curve in opposite directions in the plane, depending on the choice of tube stiffnesses and precurvatures.
3. $\mathbf{u}_{1z}^* = \mathbf{u}_{2z}^* \neq 0$. Both tubes are helical in shape with the same helical torsion, but not necessarily the same curvature. The interaction results in a piecewise helical final shape as long as $\psi_2 - \psi_1 = n\pi$, where again the even and odd solutions are different. The even solutions will always contain two helical segments which curve in the same direction and have equal torsion. The segments of the odd solutions may curve in the same or opposite directions.

In agreement with previous results mentioned in Section I, the follow-the-leader solutions are the ones which have no internal material torsion. This fact is evident from (13) since $\sin(\psi_i - \psi_k) = 0$ and, therefore, $u_{iz} = u_{kz}^*$ along the whole length of each tube. Note that the precurvature must be constant for all $s > 0$, which means that before insertion, this precurved segment of the tube must lie at an arc length less than zero. This requirement imposes some restrictions on the actuation mechanism, since it must either be able to accommodate the load of straightening the tube in between $s = 0$ and $s = -\beta_i$, or there must be some straight length of tube at the proximal end so that the precurved portion can be retracted into a rigid sleeve.

F. Space of Curves Enabled by Helical Precurvatures

The case of two helically precurved tubes provides a large family of overall device curves to the robot designer, even though there are only two tubes. For a visual aid, two final shapes for a single tube set are shown in Fig. 4(b). Parameters that can be selected include the handedness of the helices (whether u_z^* is positive or negative), the curvature of each tube, the overlapped insertion arc length, the nonoverlapped insertion length of one tube extending beyond the overlapped section, and a rigid rotation of the entire final shape. In a practical application involving insertion through a winding lumen or through soft tissue (as discussed in Section I), one could envision having many helical tubes premade, and allowing a planning algorithm to select the best pair of tubes, based on the target location and any relevant obstacles and tissue boundaries. A useful fact is that the solutions are easy to compute because the torsional mechanics do not need to be solved and the kinematics can be solved in closed form [17].

V. Follow the Leader With General Tube Sets

In the general case where we cannot simplify the constraints further than (17), it is less clear for which special cases of precurvature follow-the-leader deployment will be possible. Certainly, any set of circularly precurved tubes could follow the leader if the angle between every combination of two tubes is $n\pi$ for any integer n . This assumes that the insertion length is sufficiently short and the tubes have low enough curvatures that the boundary value problem associated with (13) and (14) has not undergone a bifurcation, since this would

cause the solutions with odd n to be unstable and not achievable in the physical system. The solution would be planar and could have changes in the direction of the curvature. By a similar argument, three helices with equal torsion could also follow the leader.

It is possible that other special case functions exist for which the constraints in (17) can be consistent with the torsional mechanics in (13) for more general solutions of ψ and more general nonconstant choices of precurvature. Identifying them (if they exist) and determining whether they are potentially useful in various applications remain open research questions. As with the special case solutions we have described in this paper, any new solutions will involve both specific precurvatures and specific deployment sequences, because the torsional mechanics are completely defined (i.e., the time derivatives of all ψ_i are determined completely by the mechanics model of the robot). Thus, the two additional constraints from Section IV are unlikely to be satisfied except in very special cases. Maintaining the constraints, which are infinite-dimensional, with only a finite number of actuator inputs is a challenging endeavor that will require assistance from special properties of the precurvature functions selected.

One potential way to solve this problem through design is to “key” the tubes together, which constrains their rotational motion with respect to one another. Effectively, this causes $\dot{\psi}_i = 0$ for constant-precurvature tube shapes at any initial angular displacement. One way that has been suggested to achieve this [23] (although not physically prototyped to the best of the authors’ knowledge) is to extrude tubes with a polygonal rather than annular cross section. In this case, any number of constant-precurvature tubes can be made to follow the leader by using an insertion sequence similar to that described earlier for two tubes, namely one in which all tubes advance together initially, with one or more stopping sequentially at specified arc length points and then remaining stationary after stopping.

VI. Approximate Follow-The-Leader Deployment

In some cases, approximate follow-the-leader deployment may be sufficient to accomplish a given task. In order to measure closeness to exact follow-the-leader deployment, we define an error metric that quantifies the maximum displacement of any point along the backbone:

$$E = \max_{t_1} \max_{t_2} \max_{\sigma} \|\mathbf{p}(\sigma, t_1) - \mathbf{p}(\sigma, t_2)\| \quad (20)$$

where σ denotes arc lengths along the robot which exist for both times t_1 and t_2 . That is, $\sigma \in [0, L_e]$ with $L_e = \min(L(t_1), L(t_2))$. This error metric captures the largest movement of the backbone at any arc length point during the entire deployment period and has units which are the same as $\mathbf{p}$. This error metric does not perfectly capture the amount of “stretching” that would occur in a tissue surrounding the needle path, because some component of the difference $\mathbf{p}(\sigma, t_1) - \mathbf{p}(\sigma, t_2)$ may be tangential to the path. The tangential component will often be small, however, due to the comparison occurring at equal arc lengths. Note that the error is nondecreasing with time, which for an insertion corresponds also to nondecreasing error with insertion length. Minimizing this error over the set of possible actuator trajectories would give a best-case follow-the-leader behavior for a given robot design. This error metric could also provide a bound for planning and design algorithms, which seek to

design both the properties of the tube set and the actuation sequence to be used for a specific task.

We now apply this error metric to explore two cases. The first is that of two circularly precurved tubes in the case that $\psi_2 - \psi_1 \neq n\pi$, meaning that the curvatures lie out of plane and the torsional mechanics become nontrivial. The second case is of two helical tubes with opposite handedness. We show that under certain conditions, these tubes approximately follow the leader. For all simulations, the deployment proceeds with both tubes fully overlapping and extending in length.

A. Dimensionless Model for Two Constant-Precurvature Tubes

The equations for two circularly precurved tubes may be conveniently nondimensionalized, which reduces the number of design parameters to a minimal set and allows conclusions to be drawn without regard to the overall size of a particular design. We first replace $\psi_1(s, t)$ and $\psi_2(s, t)$ with $\theta(s, t) = \psi_2(s, t) - \psi_1(s, t)$. The arc length is then transformed into normalized arc length $\tilde{s} \doteq s/L \in [0, 1]$. Let the symbol “ $\sim$ ” over the top of a variable represent the quantity as nondimensionalized and as a function of the normalized arc length. Then, the precurvatures of the tubes may be nondimensionalized as $\tilde{u}_i^* = Lu_i^*$. The torsional boundary value problem for two tubes (see [18]) is then given in nondimensional form by

$$\tilde{\theta}'' - \lambda \sin(\tilde{\theta}) = 0 \quad (21)$$

subject to the boundary conditions

$$\begin{aligned} \tilde{\theta}(0, t) &= \tilde{\theta}_0(t) \\ \tilde{\theta}'(1, t) &= \tilde{u}_{2z}^* - \tilde{u}_{1z}^*. \end{aligned} \quad (22)$$

Note that under the assumption of circularly precurved tubes and that Poisson's ratio ν is the same for both tubes, the bifurcation parameter $\lambda = \tilde{u}_1^* \tilde{u}_2^* (1 + \nu)$. The dependence on squared length is not omitted in this expression, rather it is present through the definition of $\tilde{u}_1$ and $\tilde{u}_2$. We assume for the remainder of this section that $\nu = 0.33$, the value quoted for Nitinol by the manufacturer, for both tubes. The differential kinematic equations may also be nondimensionalized as

$$\tilde{\mathbf{p}}' = \tilde{\mathbf{z}} \quad (23)$$

$$\tilde{\mathbf{R}}' = \tilde{\mathbf{R}}[\tilde{\mathbf{u}}]_{\times} \quad (24)$$

where the relations hold that $L\mathbf{p}(\tilde{s}) \doteq \mathbf{p}(L\tilde{s})$ and $\mathbf{R}(\tilde{s}) \doteq \mathbf{R}(L\tilde{s})$. The normalized curvature $\tilde{\mathbf{u}}$ is computed as $\tilde{\mathbf{u}} = L\mathbf{u}$, which is more explicitly found as

$$\begin{aligned} \tilde{u}_{xy} &= \frac{1}{1+\gamma_b} \tilde{u}_{1x}^* \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{\gamma_b}{1+\gamma_b} \tilde{u}_{2x}^* \begin{bmatrix} \cos \tilde{\theta} \\ \sin \tilde{\theta} \end{bmatrix} \\ \tilde{u}_z &= -\frac{\gamma_t}{1+\gamma_t} \tilde{\theta}' \end{aligned} \quad (25)$$

where γ_b is the dimensionless bending stiffness ratio $E_2 I_2 / E_1 I_1$, and γ_t is the dimensionless torsional stiffness ratio $G_2 J_2 / G_1 J_1$. For tubes which have the same value of Poisson's ratio, $\gamma = \gamma_t = \gamma_b$, which will be assumed with some loss of generality for the remainder of this section. Additionally, let $\eta = \tilde{u}_2^* / \tilde{u}_1^*$ represent the precurvature ratio. We then choose the following set of parameters that uniquely define the entire solution space up to a rigid transformation of the backbone: γ , η , ν , λ , and $\tilde{\theta}_0$. Fixing ν leaves only four parameters determining the forward kinematic problem.

The objective of the metric is to find the maximum movement of the backbone at any arclength point. Since the domain of s is $[0, 1]$ for any solution $\tilde{\mathbf{p}}$, we must take care to compare points that correspond to the same physical arc length. Note that during any particular deployment, the only term not constant in λ is L , meaning that we may find that

for arbitrary times t_1 and t_2 at which the tubes have nonzero length, $\sqrt{\lambda_2 / \lambda_1} = L_2 / L_1$. Furthermore, we may parameterize the deployment by λ rather than by time. Considering these facts, the error metric may be nondimensionalized as

$$\tilde{E}(\lambda) = \max_{\substack{\lambda_1 \\ \lambda_1 < \lambda}} \max_{\substack{\lambda_2 \\ \lambda_2 < \lambda}} \max_{\tilde{\sigma} \in [0,1]} \left\| \sqrt{\frac{\lambda_1}{\lambda}} \tilde{\mathbf{p}}(\tilde{\sigma} \sqrt{\lambda_2 / \lambda_1}, \lambda_1) - \sqrt{\frac{\lambda_2}{\lambda}} \tilde{\mathbf{p}}(\tilde{\sigma}, \lambda_2) \right\| \quad (26)$$

where we assume, without loss of generality, that $\lambda_1 > \lambda_2$. Note that to compute $\tilde{E}(\lambda)$ for an insertion sequence, we must additionally know the actuator history $\tilde{\theta}_0(\lambda)$, stiffness ratios, Poisson's ratio, and the curvature ratio, and these additional parameters have been suppressed from the notation. Once equipped with this nondimensionalized error, the error may be computed by $E(L, \lambda) = L \tilde{E}(\lambda)$ for a deployment which has final overlapped tube length L . This nondimensionalized error metric, unlike the original metric, is *not* nondecreasing. This fact may be seen from the division by $\sqrt{\lambda}$ on the right-hand side of (26) and is consistent with the fact that longer insertions may incur less error on a percentage basis of the final length.

In the sections that follow, we investigate the behavior of this error metric for varying choices of tube designs and actuation. In all cases, the model equations are solved via a nonlinear Galerkin method, using MATLAB's `fsolve` to perform the minimization of the weighted residual equations. Once this solution is obtained, the kinematic equations are integrated via `ode45` and interpolated to 2000 evenly spaced points.

B. Effect of Initial Angular Difference

In order to investigate the follow-the-leader error for varying angular differences between the tubes, we choose tubes that have circular precurvature and vary the nondimensional parameters of the model. The actuator history is given by $\tilde{\theta}_0(0, t) = \text{constant}$.

The dimensionless error is shown in Fig. 5 for the case of equal stiffnesses and equal curvatures for various initial angular differences at $s = 0$. It is seen that a maximum in the error occurs near an initial angle of 160° .

C. Effect of Tube Stiffness Ratio

Fig. 6 displays the dimensionless error for varying stiffness ratios, using the worst-case angular difference from the previous subsection of 160° and tubes of equal curvature. The maximum amount of error occurs when the stiffnesses are equal and is reduced as one tube becomes stiffer relative to the other.

D. Effect of Tube Curvature Ratio

Fig. 7 shows the dimensionless error for varying curvature ratios with equal stiffness tubes at a 160° relative angle. It is seen that the minimum occurs at unity. However, the effect of varying the curvature ratio is clearly less significant than the effect of varying stiffness ratio. Unlike varying stiffness, the error is slightly increased as the tubes deviate from unity curvature ratio.

E. Effect of Actuation Sequence

It is important to note that the proximal boundary conditions $\tilde{\theta}(0, t) = \text{constant}$ and $\mathbf{R}(0, t) = \text{constant}$ which were chosen for the previous sections imply a particular actuation sequence during the deployment, because the tubes must be grasped at a location $s < 0$ in order for the insertion to extend from $s = 0$ forward. Torsional windup in the section $s < 0$ must be compensated so that the two boundary conditions $\tilde{\theta}(0, t)$ and $\mathbf{R}(0, t)$ are held fixed. Note further that there is no guarantee that this choice of actuation provides the minimum follow-the-leader error. Rather, this choice makes the equations amenable to the preceding analysis. One subtle benefit of this choice is that the graphs in Figs. 5–7 may be labeled simply “ λ ” along the abscissa because it does not matter whether the value is taken at the end of an insertion or during the middle of an insertion.

In order to give one comparison with a different choice of actuation, however, we examine the case where one applies no rotational actuation during the insertion. In all physical prototypes built to date of which we are aware, there is a section of tube which is not precurved present before the curved section, termed a transmission length. To account for this length, if it is present, we introduce the parameter $\lambda_s = L_s^2 u_{1x}^* u_{2x}^* (1 + \nu)$, where L_s is the straight length of tube located at $s \lesssim 0$. We term this actuation with no rotational motion “uncompensated,” and $\tilde{\theta}(-d(t), t) = \alpha$, where d is the nondimensional length (nondimensionalized by the currently overlapped length $L(t)$ of the tubes) of the portion of the tubes physically located at $s \lesssim 0$ and α is the rotational actuator relative angle. The value d may then be computed as

$$d = \sqrt{\frac{\lambda_s}{\lambda(t)}} + \sqrt{\frac{\lambda_f}{\lambda(t)}} - 1 \quad (27)$$

with λ_f being the value of λ at the end of the insertion sequence, and where $\lambda(t) > 0$. Then, the proximal boundary condition changes to $\tilde{\theta}(0, t) = \alpha + d\tilde{\theta}(0, t)$, and we must also account for the change in the initial axial rotation of tube 1 at $s \approx 0$. This rotation is $\mathbf{R}(0, t) = \mathbf{R}_z(-d(t)\tilde{\theta}(0, t)\gamma/(1 + \gamma))$.

Fig. 8 shows the error metric as a function of λ_f for various choices of the ratio λ_s/λ_f , with $\gamma = 2$, $\eta = 1$, and $\tilde{\theta}(0, t) = 160^\circ$. Note that with this choice of actuation, λ is no longer the only changing parameter in the torsional boundary value problem and forward kinematic problem, because λ , λ_f , and d must all be known in order to find the correct solution to the kinematic equations part-way through an insertion. It is for this reason that Fig. 8 shows λ_f on the abscissa. For high values of λ_f , the follow-the-leader error is lower in magnitude than those for the previous compensated actuation method. However, a direct comparison is not valid since the relative angle $\tilde{\theta}(s, t)$ is significantly lower for most of the uncompensated solutions when compared with the 160° compensated solution. As the straight length increases, the torsional windup in the region $s < 0$ increases, lowering the effective relative angle for the region $s > 0$ and thus decreasing the follow-the-leader error. However, for lower values of λ_f , where most prototypes have been built to date, the error increases substantially with increasing transmission length. Thus, where it is feasible, the compensated actuation method should be preferred to this uncompensated one when follow-the-leader deployment is desired. We note that in some cases, excessive transmission length will prevent all values of θ_0 from being achievable due to bifurcation.

F. Helically Precurved Tubes

Another case that provides interesting study is that of two opposite-handed helices. For this simulation, we will choose a set of tubes and an insertion length that results in a bifurcation parameter $\lambda > \pi^2/4$. It is harder to span many values of curvature in this case because there is a two-parameter family of both torsion and curvature for each tube. For illustrative purposes, we will choose the parameters $EI_1 = EI_2 = 1$, and $u_{1z}^* = -25 \text{ m}^{-1}$ and $u_{2z}^* = +25 \text{ m}^{-1}$. When $\psi_2(0, t) - \psi_1(0, t) = 0$ is maintained and the curvature is high enough, the tubes will conform to a planar shape and “unwind” so that $\psi_2(s, t) - \psi_1(s, t)$ stays less than π over the entire length of the insertion. Fig. 9 depicts how the two helical shapes conform to a common centerline. Since no error will occur during the second stage of insertion when the inner tube continues to deploy beyond the outer one, an overall robot curve featuring a first almost-circular segment followed by a second helical segment can be achieved in a quasi-follow-the-leader manner. For three choices of curvature, which are sufficiently high, the follow-the-leader error versus insertion length is shown for the first stage of fully overlapped insertion in Fig. 10. For this simulation, the density of the discretization is again 2000 points along the length of each solution, with 1000 discrete insertion steps in order to obtain a smooth result.

Unlike the two circularly precurved tubes, the helices conforming to a plane have better tip-following behavior when the curvature is increased. This is easily explained by the fact that the higher curvature allows $\psi_2 - \psi_1$ to stay closer to zero, which would be a perfect follow-the-leader case for circularly precurved tubes. Essentially, the preset torsion of the tubes is “removed” by the material twisting when the curvature is sufficiently high. From a design perspective, this is a tradeoff where increasing the torsion allows the second segment of the insertion to more rapidly leave the plane of the first segment, but also requires higher curvature to achieve the desired behavior.

G. How to Use Nondimensional Approximate Follow-the-Leader Results in Practice

Given a physical robot design and motion plan, Figs. 5–7 can be used to predict the follow-the-leader error. For example, consider two tubes with curvatures of $u_{1x}^* = 20 \text{ m}^{-1}$ and $u_{2x}^* = 15 \text{ m}^{-1}$, where each tube is precurved for its entire final length of insertion. Consider a motion plan in which the two tubes will be deployed together for 50 mm, and then, the inner tube deploys by itself another 50 mm. Assume that the plan uses the compensated actuation method discussed in Section VI-E and prescribes an initial angle $\theta(0, t) = 90^\circ$.

We wish to know how much follow-the-leader error can be expected from these tubes and this deployment plan. First, note that since the overlapped length $L = 50 \text{ mm}$, the dimensionless parameter $\lambda = L^2 u_{1x}^* u_{2x}^* (1 + 0.33) = 1.0$. From Fig. 5, we see that at $\lambda = 1$, equal stiffness tubes at $\theta(0, t) = 90^\circ$ would be predicted to have a follow-the-leader error less than 1 % of L . Fig. 6 tells us that the effect of the stiffness ratio is to slightly reduce the error, and Fig. 7 shows us that the curvature ratio $u_{2x}^*/u_{1x}^* = 1.33$ will slightly increase the error, roughly canceling the effect of the stiffness ratio. Therefore, we predict an error of less than 1% of $L = 50 \text{ mm}$, or 0.5 mm, during the first stage of deployment, and no error during the second stage.

VII. Experimental Helical Case Demonstration

In this section, we examine the follow-the-leader behavior of a physical prototype. The experiment shows the case of two same-handed helices inserted in free space, providing a proof-of-concept demonstration of an odd solution case. We emphasize that this section is not intended to be a conclusive statistical statement about practically achievable distributions of errors. In this section, we present the results of measurement of the follow-the-leader error of a single insertion as a proof of concept for follow-the-leader deployment of helically precurved concentric tubes, which has never before been demonstrated.

A. Experimental Protocol

An outer tube and an inner wire were first independently shape-set via heat treatment into helical shapes. To create a helix with the correct precurvature, we converted u_x^* and u_z^* into the more physically intuitive helical pitch $2\pi p$ and radius r , using the following relationships:

$$r = \frac{u_x^*}{u_x^{*2} + u_z^{*2}}, \quad p = \frac{u_z^*}{u_x^{*2} + u_z^{*2}}. \quad (28)$$

Note that here a negative value for p would indicate a left-handed helix, and a positive value a right-handed helix.

To set the helical precurved shape, the tube and wire were each wrapped in a helical profile with a pitch of $2\pi p = 160\text{mm}$ around a steel cylinder 19 mm in diameter, with the proximal part of the tube/wire left free to leave the surface of the fixture in order to remain in a straight configuration. Fig. 12 shows the process. The tube and wire were placed in an air furnace at 500°C for 30 min. This heating was followed by an immediate quench in room-temperature water, followed by a second heating period at 600°C for 5 min. A second water quench was performed immediately after the second heating period. Note that the full theory of heat treating Nitinol is a complex metallurgy problem, and the interested reader is directed to [24] and [25] for further information. Broadly speaking, the first heat treatment period sets the shape, while the second moves the transition point between Austenite and Martensite back below room temperature so that the sample is superelastic at room temperature. Accounting for the diameters of the tube and wire, by (28), they were constrained around the steel cylinder with $u_{1x}^* = 13.64$, $u_{1z}^* = -33.75$, $u_{2x}^* = 13.92$, and $u_{2z}^* = -33.48$ before heat treating began, where all values are in units of rad/m.

The robot described in [26] was used to independently control the insertion and rotation of the concentric tube and wire, which passed through a rigid sheath that served to constrain the tubes to be straight at arc lengths $s < 0$. Since both tubes are helical, the bases of the tubes must be turned during insertion in order to maintain the angles $\psi_i(0)$ at a constant value. The rate of twist must be equal to the precurved helical torsion, which can be seen from the fact that the tubes undergo no material torsion at negative arc lengths. A photograph of the experimental setup is shown in Fig. 11.

All measurements were taken with a stereo camera system of two Sony XCD-X710 firewire cameras, calibrated using a rectangular 10-mm square grid and the Camera Calibration Toolbox for MATLAB. To triangulate points, a point in one image was clicked, and then, the intersection of the epipolar line and the feature of interest was selected in the second image.

The insertions were performed according to the steps outlined in Section IV-D. Before beginning the insertion, the two helices were aligned, with their tips at $s = 0$. Next, the inner wire was inserted forward while holding $\psi_1(0) = 0$. It was then rotated to $\psi_1(0) = \pi$ and, finally, retracted while holding $\psi_1(0) = \pi$. This initial setup procedure minimizes frictional effects by reducing the arc length of overlapping tube and wire during initial rotation and also causing the tubes to slide relative to one another after rotation. Since it is known that friction between the tubes induces a hysteresis between the relative base angle $\psi_2(0) - \psi_1(0)$ and the relative tip angle $\psi_2(\ell) - \psi_1(\ell)$ (see [27]) frictional effects additionally need to be considered when designing a real device, as the base actuators must be able to maintain sufficient control over the axial rotation of the tubes. The steps of Section IV-E were

implemented 0.5 cm at a time, with an insertion velocity of approximately 0.5 cm/s. After each 0.5-cm step, insertion was paused and a pair of stereo images was captured for analysis. The backbone shape was discretized in each image pair manually using the triangulation technique discussed above with a density of approximately one point every 2 to 5 mm.

B. Experimental Results

Table I lists the tube parameters as measured after heat treatment. The radius and pitch were estimated from points collected with the stereo cameras using a nonlinear regression. The results indicate that the curvature (u_{1x}^* and u_{2x}^*) relaxed approximately 40% after removal from the steel cylinder. In contrast, the torsion (in the helical parameter sense, not a material sense) did not change appreciably. This makes sense intuitively, because there is no internal torsional strain associated with wrapping the tube around the cylinder as we did before heat treating. The tube and wire after shape setting and trimming to length are shown in Fig. 12.

Experimental results for a single free space insertion are shown in Fig. 13. The graph of error versus insertion distance reveals that for an 80-mm insertion, overall error is only 2 mm. The most likely sources of error are the unmodeled effects of diametral clearances between tubes, friction between the tubes, and manufacturing error in the shapes of the tubes. Note that this demonstration is not designed to corroborate the simulation cases presented before, since the model would predict zero error for this experiment. What the experiment does demonstrate is that the modeling errors are small enough that such an insertion could be practical.

VIII. Neurosurgical Example

Consider epilepsy treatment as a motivating clinical example for follow-the-leader deployment. This application was suggested in [28] which addressed the design of a magnetic resonance imaging (MRI) compatible concentric tube robot actuation system, but did not address tube design or deployment strategy. Epilepsy is estimated to affect 1% of the global population at some point in their lives. Antiseizure medications are the first line of defense, but about 20–30% of people do not respond to them [29]. For many of these patients, surgical removal of the hippocampus is highly effective in controlling seizures, but carries with it all the risks (bleeding, inadvertent damage to nearby critical brain structures, blood vessels, etc.) typically associated with a major surgery on the brain. As discussed in [28], it would be highly desirable to perform a less-invasive needle-based treatment in which thermal energy is deployed from the tip of a needle that is inserted along the axis of the hippocampus, while monitoring brain temperatures using MRI temperature maps. Recent clinical results of minimally invasive MRI-guided laser ablation of the hippocampus indicate that straight needles are able to surgically treat on average 60% of the desired volume [30].

To explore whether helical follow-the-leader trajectories might be useful in this application, an experienced neurosurgeon first defined the bounds of a safe insertion region, using an occipital approach, for the cannula by drawing contours on an MRI image set of a patient's head. The safe insertion zone geometry the surgeon indicated is shown in blue on Fig. 14. The surgeon also segmented the target portion of the hippocampus to be ablated (shown in

brown on the figure). Next, we considered delivery of an ablator that radiates energy radially around the needle tip, such as the acoustic ablation probe in [31]. With such a probe, it would be desirable to deliver the ablator along the central axis of the hippocampus. This central axis, known as the Euclidean skeleton, was computed for a voxelized image of the hippocampus using the method of Lee *et al.* [32]. We used the mean distance between the skeleton points and the concentric tube robot trajectory as a measure of how well an ablator could thermally treat the hippocampus by radiating energy evenly in all radial directions.

To demonstrate that our helical case can provide a more desirable result than a straight needle, we generated an optimal straight needle trajectory using principal component analysis. Using all skeleton points results in a trajectory that lies outside the insertion zone; therefore, points were gradually excluded from the tail of the hippocampus toward the posterior side of the head, until a trajectory was found that passed entirely inside the safe insertion zone marked by the surgeon. This straight needle trajectory results in a mean distance to the skeleton of 2.4 mm.

We then considered designs with two helical tubes of identical helical torsion, with the objective of staying within the safe region of the brain and minimizing the mean distance to the skeleton. A design consisting of tube curvatures and lengths was manually selected, and the resulting shape was positioned and rotated using `fminsearch` in MATLAB to minimize the mean distance to the hippocampus skeleton. This design is shown in Fig. 14. The resulting tubes have a stiffness ratio of 2.3, and curvatures of 20 and 80 m^{-1} for the inner and outer tubes, respectively. Both tubes have a helical torsion of -10 m^{-1} . The curved tubes enter through a third, straight, rigid tube, which is assumed to be stiff enough to completely straighten the curved tubes when they are retracted inside of it.

The helical tubes have a smaller mean distance to the skeleton of 1.5 mm, and they would allow ablation of tissues in the tail of the hippocampus that cannot be reached with the straight needle, which is important given that correlations exist between higher resection volume and better clinical outcomes [33]. The ability to treat the tail of the hippocampus may increase the potential for seizure freedom. Furthermore, as can be seen in Fig. 14, the helical tubes do not require a needle trajectory that is very close to the boundary of the safe zone for the entire deployment (meaning that a small registration error could potentially significantly damage the patient's brain). Using helical tubes also provides the ability to alter the insertion point without substantially reducing the covered volume by slightly altering the tube design, thus avoiding patient-specific obstacles such as sulci near the surface of the brain.

We note that in future work, automated planners could design the tubes for this application and could consider all design objectives including obstacle avoidance and desired insertion points. However, such optimization routines are an active area of research themselves (see, e.g., [34]) and are hence beyond the scope of this paper. It is also worth noting that the example described in this section shows that even a hand-selected (i.e., not optimized) helical design was capable of performing the desired surgical task better than the optimal straight needle trajectory.

IX. Conclusion

In this paper, we have explored follow-the-leader behavior as it relates to concentric tube robots, showing that it is possible, but only with appropriate precurvature selections and deployment sequences. We derived the conditions that must hold for follow-the-leader deployment in general and explored various two-tube cases in depth. Fortunately, even these “simple” circular and helical precurvature special cases provide a large design space of possible curves. Thus, we expect them to enable a number of new clinical applications where concentric tube robots act as steerable needles in both soft tissues and open or liquid-filled spaces in the human body—cases where the robot operates in a manner conceptually similar to our epilepsy treatment illustration. We anticipate that approximate follow-the-leader behavior will also be useful in applications like traversing a lumen where a tolerance exists between the robot and wall, or in tissues where some stretching is permitted. In such cases, our metric for comparing a general deployment to a follow-the-leader deployment will be useful in assessing the ability of an existing robot to achieve the surgical task at hand. The metric may also be useful in future design and motion planning studies for concentric tube robots. In summary, follow-the-leader deployment appears to be a useful attribute for concentric tube robots. It enables them to act like steerable needles that do not depend on tissue interaction to steer. This ability to travel along desired curved trajectories through both soft tissue and open space may enable a variety of future less invasive interventions to be developed.

Acknowledgments

This work was supported in part by the National Science Foundation (NSF) under CAREER Award IIS-1054331 and Graduate Research Fellowship DGE-0909667 and in part by the National Institutes of Health (NIH) under awards R01 EB017467 and R21 EB017952. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF or NIH.

References

1. Webster RJ III, Kim JS, Cowan NJ, Chirikjian GS, Okamura AM. Nonholonomic modeling of needle steering. *Int J Robot Res.* 2006; 25(5/6):509–525.
2. Reed K, Majewicz A, Kallem V, Alterovitz R, Goldberg K, Cowan N, Okamura A. Robot-assisted needle steering. *IEEE Robot Autom Mag.* Dec; 2011 18(4):35–46. [PubMed: 23028210]
3. Okazawa S, Ebrahimi R, Chuang J, Salcudean S, Rohling R. Hand-held steerable needle device. *IEEE/ASME Trans Mechatronics.* Jun; 2005 10(3):285–296.
4. Ryu SC, Quek ZF, Renaud P, Black RJ, Daniel BL, Cutkosky MR. An optical actuation system and curvature sensor for a mr-compatible active needle,” in. *Proc IEEE Int Conf Robot Autom.* 2012:1589–1594.
5. Abolhassani N, Patel R, Moallem M. Needle insertion into soft tissue: A survey. *Med Eng Phys.* 2007; 29(4):413–31. [PubMed: 16938481]
6. Rucker DC, Jones BA, Webster RJ. A geometrically exact model for externally loaded concentric-tube continuum robots. *IEEE Trans Robot.* Oct; 2010 26(5):769–780. [PubMed: 21566688]
7. Dupont PE, Lock J, Itkowitz B, Butler E. Design and control of concentric-tube robots. *IEEE Trans Robot.* Apr; 2010 26(2):209–225. [PubMed: 21258648]
8. Gilbert HB, Rucker DC, Webster RJ III. Concentric tube robots: The state of the art and future directions, submitted for publication. *Proc 16th Int Symp Robot Res.* 2013

9. Burgner J, Rucker DC, Gilbert HB, Swaney PJ, Russell PT III, Weaver KD. A telerobotic system for transnasal surgery. *IEEE/ASME Trans Mechatronics*. Jun; 2014 19(3):996–1006. [PubMed: 25089086]
10. Gosline AH, Vasilyev NV, Butler EJ, Folk C, Cohen A, Chen R, Lang N, del Nido PJ, Dupont PE. Percutaneous intracardiac beating-heart surgery using metal MEMS tissue approximation tools. *Int J Robot Res*. 2012; 31(9):1081–1093.
11. Torres LG, Alterovitz R. Motion planning for concentric tube robots using mechanics-based models. *Proc IEEE/RSJ Int Conf Intell Robots Syst*. 2011:5153–5159.
12. Burgner J, Swaney PJ, Bruns TL, Clark MS, Rucker DC, Webster RJ III. An autoclavable steerable cannula manual deployment device: Design and accuracy analysis. *ASME J Med Devices*. 2013; 6(4):041007-1–041007-7.
13. Choset H, Henning W. A follow-the-leader approach to serpentine robot motion planning. *J Aerosp Eng*. Apr; 1999 12(2):65–73.
14. Ikuta K, Tsukamoto M, Hirose S. Shape memory alloy servo actuator system with electric resistance feedback and application for active endoscope. *Proc IEEE Int Conf Robot Autom*. 1988:427–430.
15. Degani A, Choset H, Zubiate B, Ota T, Zenati M. Highly articulated robotic probe for minimally invasive surgery. *Proc Int Conf IEEE Eng Med Biol Soc*. Jan.2006 :4157–4172.
16. Sears P, Dupont P. A steerable needle technology using curved concentric tubes. *Proc IEEE/RSJ Int Conf Intell Robots Syst*. Oct.2006 :2850–2856.
17. Webster RJ III, Romano JM, Cowan NJ. Mechanics of precurved-tube continuum robots. *IEEE Trans Robot*. Feb; 2009 25(1):67–78.
18. Rucker DC, Webster RJ III, Chirikjian GS, Cowan NJ. Equilibrium conformations of concentric-tube continuum robots. *Int J Robot Res*. 2010; 29(10):1263–1280.
19. Gilbert HB, Webster RJ III. Can concentric tube robots follow the leader? *Proc IEEE Int Conf Robot Autom*. 2013:4866–4872.
20. Rucker DC, Webster RJ III. Parsimonious evaluation of concentric-tube continuum robot equilibrium conformation. *IEEE Trans Biomed Eng*. Sep; 2009 56(9):2308–2311. [PubMed: 19535313]
21. Xu R, Atashzar SF, Patel RV. Kinematic instability in concentric tube robots: Modeling and analysis. *Proc IEEE/RAS/EMBS Int Conf Biomed Robot Biomechatronics*. Aug.2014 :163–168.
22. Su H, Cardona DC, Shang W, Camilo A, Cole Ga, Rucker DC, Webster RJ III, Fischer GS. A MRI-guided concentric tube continuum robot with piezoelectric actuation: A feasibility study,” in. *Proc IEEE Int Conf Robot Autom*. 2012:1939–1945.
23. Greenblatt, EE.; Trovato, KI.; Popovic, A.; Stanton, D. Interlocking nested cannula. US Patent. 20110201887. 2011.
24. Drexel MJ, Selvaduray GS, Pelton AR. The effects of cold work and heat treatment on the properties of nitinol wire,” in. *Proc Int Conf Shape Memory Superelastic Technol*. 2006:447–454.
25. Pelton AR, Russell SM, DiCello J. The physical metallurgy of nitinol for medical applications. *J Metals*. 2003; 55(5):33–37.
26. Burgner J, Swaney PJ, Rucker DC, Gilbert HB, Nill ST, Russell PT, Weaver KD, Webster RJ III. A bimanual tele-operated system for endonasal skull base surgery,” in. *Proc IEEE/RSJ Int Conf Intell Robots Syst*. 2011:2517–2523.
27. Lock J, Dupont PE. Friction modeling in concentric tube robots. *Proc IEEE Int Conf Robot Autom*. 2011:1139–1146.
28. Comber DB, Barth EJ, Webster RJ III. Design and control of an magnetic resonance compatible precision pneumatic active cannula robot. *ASME J Med Devices*. 2013; 8(1):011003-1–011003-7.
29. Smithson, WH.; Walker, MC. *ABC of Epilepsy*. 1. Vol. 1. New York, NY, USA: Wiley; 2012. What is Epilepsy? Incidence, prevalence and aetiology; p. 1-3.
30. Willie JT, Laxpati NG, Drane DL, Gowda A, Appin C, Hao C, Brat DJ, Helmers SL, Saindane A, Nour SG, Gross RE. Real-time magnetic resonance-guided stereotactic laser amygdalohippocampotomy for mesial temporal lobe epilepsy. *Neurosurgery*. 2014; 74(6):569–585. [PubMed: 24618797]

31. Burdette EC, Rucker DC, Prakash P, Diederich CJ, Croom JM, Clarke C, Stolka P, Juang T, Boctor EM, Webster RJ III. The ACUSITT ultrasonic ablator: The first steerable needle with an integrated interventional tool. *Proc SPIE Med Imaging: Ultrasonic imaging, Tomography, and therapy*, 76290V. 2010
32. Lee TC, Kashyap RL, Chu CN. Building skeleton models via 3-D medial surface/axis thinning algorithms. *CVGIP: Graph Models Image Process*. 1994; 56(6):462–478.
33. Wyler AR, Hermann BP, Somes G. Extent of medial temporal resection on outcome from anterior temporal lobectomy: A randomized prospective study. *Neurosurgery*. 1995; 37(5):982–991. [PubMed: 8559349]
34. Burgner J, Gilbert HB, Webster RJ III. On the computational design of concentric tube robots. *Proc IEEE Int Conf Robot Autom*. 2013:1185–1190.

Biographies

Hunter B. Gilbert (S'10) received the B.S. degree in mechanical engineering from Rice University, Houston, TX, USA, in 2010. Since then, he has been working toward the Ph.D. degree in mechanical engineering with Vanderbilt University, Nashville, TN, USA, with the Medical and Electromechanical Design Laboratory, currently researching medical robotics and continuum robotics.

He received the NSF Graduate Research Fellowship in 2012.

Joseph Neimat received the A.B. degree in music and biochemistry from Dartmouth College, Hanover, NH, USA, in 1992. He completed medical training and received the Master's degree in neurobiology from Duke University, Durham, NC, USA, in 1998. He performed his internship in general surgery and residency in neurosurgery with Massachusetts General Hospital, Boston, MA, USA, and completed a fellowship in functional neurosurgery with the University of Toronto, Toronto, ON, Canada.

In 2006, he joined the Department of Neurosurgery, Vanderbilt University, Nashville, TN, USA, where he is currently an Associate Professor and the Director of Epilepsy Surgery, Psychiatric Neurosurgery and Adult Neurotrauma. His clinical interests include all aspects of neurological surgery, with emphasis on the treatment of Parkinson's disease, the surgical

treatment of epilepsy, novel surgical therapies for mental disease, and the multimodality treatment of trigeminal neuralgia. He runs a collaborative research lab directed toward the exploration of cognitive and emotional processing in cortical and subcortical structures. The lab uses the opportunity of DBS and epilepsy surgeries, where electrophysiology is already being performed to evaluate the response of neuronal ensembles to novel stimuli that task cognitive and emotional processing. Through collaborations with Vanderbilt engineering, he helps investigate the potential role of advanced imaging modalities and novel robotic approaches to impact the performance of DBS and epilepsy surgeries.

Robert J. Webster, III, (S'97–M'08–SM'14) received the B.S. degree in electrical engineering from Clemson University, Clemson, SC, USA, in 2002, and the M.S. and Ph.D. degrees in mechanical engineering from the Johns Hopkins University, Baltimore, MD, USA, in 2004 and 2007, respectively.

In 2008, he joined the Faculty of Vanderbilt University, Nashville, TN, USA, where he is currently an Associate Professor of mechanical engineering, electrical engineering, otolaryngology, neurological surgery, and urologic surgery and directs the Medical and Electromechanical Design Laboratory. His research interests include surgical robotics, image-guided surgery, and continuum robotics.

Dr. Webster received the IEEE Volz Award and the NSF CAREER Award in 2011. In 2014, he received the Vanderbilt Award for Excellence in Teaching and the IEEE Robotics and Automation Society Early Career Award.

Fig. 1.
Concentric tube robot with three tubes.

Fig. 2.

Tubes are grasped at their respective proximal ends, and the actuation variables α_i and β_i denote the proximal base rotation and translation, respectively. The length ℓ_i is the length of each tube which is extended, as measured from the point $s = 0$ to the end of the tube.

Fig. 3.

Diagram of a section of two concentric tubes showing relevant variables. The position and orientation are those of the innermost tube (tube 1), while each tube has its own angular displacement ψ . The tubes as illustrated here have a slight positive curvature about the x axis.

Fig. 4.

(a) Two circular tubes are shown. These tubes have an overall insertion length of 12 cm, with an overlapped tube length of 6 cm. The left solution has the tubes aligned, while the right solution has the inner tube rotated the opposite direction. (b) Two helical tubes with the same pre-curved torsion are shown with the same curvature as the circular tubes of (a). All of these configurations can be reached in a perfect follow-the-leader manner.

Fig. 5. Dimensionless follow-the-leader error at a stiffness ratio of $\gamma = 1$ and curvature ratio $\eta = 1$. The value $\tilde{\theta}_0$ represents the relative angle between the tubes at $s = 0$, which is held constant during the insertion.

Fig. 6. Dimensionless follow-the-leader error at an initial angle $\tilde{\theta}_0 = 160^\circ$ and curvature ratio $\eta = 1$ for varying stiffnesses.

Fig. 7. Dimensionless follow-the-leader error at an initial angle $\theta_0 = 160^\circ$ and stiffness ratio $\gamma = 1$ for varying curvature ratios.

Fig. 8.

Follow-the-leader error has a more complex pattern with respect to the straight transmission length. Here, $\tilde{\alpha}(-d)$ is set to 160° , $\gamma = 2$, and $\eta = 1$. As λ_s is increased relative to λ_f , the error at low values of λ_f increases, but the error at high values of λ_f decreases.

Fig. 9.

Two helically precurved tubes of opposite handedness and equal stiffness combine to form an almost-circular section followed by a helical section. Where the two helices interact, they twist each other into a common plane. This configuration can be reached in a quasi-follow-the-leader manner.

Fig. 10.

For the helical “unwinding” case, again error increases with insertion length, but decreases with increasing curvature.

Fig. 11.
Experimental setup: The robot grasps the tube and wire at their respective bases where rotation and translation are applied. The rigid sheath straightens the precurved portions of the tube and wire when they are retracted.

Fig. 12.

Shape setting was performed by constraining the tube and wire to the surface of a cylinder in a helix at two points, with the proximal end of the tube/wire left free. Dots indicate the locations of the constraints. The tube and wire after the shape setting process are shown after trimming to length. (a) Fixture (top view). (b) Fixture (front view). (c) Shape set tube/wire.

Fig. 13.

Experimental follow-the-leader error for two helical tubes in free space. The gray-shaded area denotes the margin of error using our estimate of 0.5-mm measurement error at any point along the insertion.

Fig. 14.

Two views (a) Sagittal plane and (b) Axial Plane showing the trajectory of a two-tube concentric tube robot with helical precurvatures. At all times, the path stays within the safe area (shown in blue) designated by an experienced neurosurgeon. The helical shape allows the path to stay nearer to the middle of the allowable area in the axial plane, while traveling approximately along the Euclidean skeleton of the hippocampus. The straight needle shown in black must pass much closer to the boundaries of the blue insertion zone in order to remain close to the skeleton. Mean distance to the skeleton is 1.5 mm for the concentric tube robot and 2.4 mm for the straight needle. The concentric tube robot enters through a straight, rigid tube.

TABLE I

Tube Parameters After the Shape-Setting Process

Parameter	Outer Tube	Inner Wire
Outer Diameter	2.18	1.58 mm
Inner Diameter	2.02	0.00 mm
Curvature (u_x^*)	8.4	9.9 m^{-1}
Torsion (u_z^*)	-31.6	-34.3 m^{-1}

The tube diameters and Poisson's ratio are taken from manufacturer's data.