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Evaluation of the transverse oscillation method using the Cramer-Rao Lower Bound

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Abstract

The transverse oscillation method enables lateral displacement tracking by generating an oscillation orthogonal to the conventional RF signal. The widely varying methods used in the field to create such oscillations and perform displacement estimation make it difficult to compare the expected performance of alternative techniques. We derive closed-form expressions for the oscillating pressure fields produced by two common apodization functions – the rectangular and bi-lobed Gaussian apodizations – after heterodyning demodulation is applied to separate the orthogonally-oscillating signals. With these fields and spectra we present a form of the Cramer-Rao Lower Bound for ultrasonic signals that contains a spectrum shape term, allowing theoretical prediction of relative performance across different techniques and parameter choices. Field II simulations show good agreement with the trends predicted by the theoretical results for the chosen class of aperture functions. The simulations demonstrate the importance of frequency-space analysis in devising a transverse oscillation scheme and suggest that the study of other classes of aperture functions and field formation techniques should be continued in order to further improve the accuracy of lateral displacement tracking.

I. Introduction

Displacement tracking is an important tool in ultrasound imaging for use in blood flow, cardiac function, and tissue elasticity measurement. The high-frequency oscillation in the axial dimension allows for precise motion estimation in the direction of the ultrasound beam with sub-wavelength precision. Conventional techniques for estimating motion in the lateral dimension use block-matching methods to track the speckle pattern and rely on interpolation for subsample estimation. It has been demonstrated that it is possible to introduce transverse oscillations to provide a similar high-frequency sinusoid in the direction orthogonal to the beam, enabling phase-based estimation for lateral displacement tracking [1], [2].

Previous work has discussed experimental or simulation-based optimization of the synthesized transverse oscillation signals [3]–[5]. However, these works are constrained to

particular aperture shapes and implementations. We propose the use of the Cramer-Rao Lower Bound in comparing and optimizing transverse oscillation signals irrespective of the techniques used to produce them. Section II describes the diverse methods used to create a laterally-oscillating field and to track lateral displacements. Section III introduces a form of the Cramer-Rao Lower Bound for use with arbitrarily-shaped lateral spectra. Section IV provides closed-form expressions for the signals from two common transverse oscillation aperture functions – the bi-lobed Gaussian and rectangular apodizations. Section V evaluates the proposed theory in simulation, highlighting cases where the technique succeeds in predicting relative performance and where refinements need to be made.

II. Transverse Oscillation Method

A. Transverse oscillation field formation

The foundation of the transverse oscillation technique is the creation of a laterally oscillating point spread function (PSF). Using the Fraunhofer approximation, the PSF in either transmit or receive is proportional to the Fourier transform of the respective aperture function and the total PSF is equal to the product of the two [6]. It is possible to create the necessary transverse oscillation using either the one-way (receive-only) or two-way (transmit and receive) PSF [7]. While using the two-way PSF produces higher lateral spatial frequencies, the technique can be limiting because the transmit focus in a conventional image is fixed to a single point unless synthetic aperture methods are used [8]. We will limit further discussion to one-way focusing where a plane wave transmit pulse has been used to approximate an unfocused, infinite transmit aperture.

The Fourier transform of the aperture function $w(\xi)$ with respect to spatial frequency $f_x = \frac{x}{\lambda_y z}$, where λ_y is the transmitted pulse wavelength and z is the focal depth, gives the lateral part of the two-dimensional one-way PSF $h(x, y)$. The axial part is given by the transmitted pulse, in this case a Gaussian-weighted sinusoid with width σ_y and wavelength λ_y .

$$h(x, y) = \mathcal{F}\{w(\xi)\} \times e^{-\pi\left(\frac{2y}{\sigma_y}\right)^2} \cos\left(4\pi\frac{y}{\lambda_y}\right) \quad (1)$$

A natural choice for the receive apodization is the convolution of an arbitrary function with two delta functions such that the Fourier transform produces a windowed sinusoid [2]. Although many windowing functions have been proposed [2], [3], [9], the most ubiquitous is the Gaussian function. The resulting function is denoted the “bi-lobed Gaussian” apodization. However, it is also possible to use a conventional aperture, using a constant apodization, to produce a transverse oscillation [1], [10]. These two choices of apodization are studied in detail below. While beyond the scope of this paper, previous work has shown that, especially at shallow imaging depths, techniques such as back-propagation [11], pulsed plane wave decomposition [12] or least-squares optimization [13] should be used to select an apodization for a desired pressure field rather than using the Fraunhofer approximation. In either case, it is possible to determine aperture functions that create pairs of “spatial

quadrature” signals that are 90 degrees out of phase in the lateral dimension for use in phase estimation [1].

It is also possible to create the desired oscillations in post-processing without access to the individual receive channel data. Filtering can be performed in either the time or frequency domain to directly alter the frequency content present in the signals, creating oscillations similar to those produced using apodization [14]. This technique can be applied to either RF or envelope-detected data.

B. Displacement estimation

Given two or more sequential images, there are numerous ways to estimate the motion between the frames. We limit the discussion here to contrasting two common categories of techniques used in ultrasound – correlation-based and phase-based methods.

Correlation-based techniques are common to other imaging fields and rely only on the similarity of the images without regard to their content. The speckle texture present in ultrasound images gives a unique multi-dimensional feature to match in otherwise uniform tissue or blood, making the technique broadly applicable [15], [16]. Using a reference kernel and a search region based on the expected motion, the images are searched using cross-correlation to find an optimal shift of the reference kernel to maximize the similarity. The process can be performed on either radio frequency (RF) or envelope detected data depending on whether improved performance or a lower sampling rate is desired. The technique is flexible and can be used to track motion in any number of dimensions, achieving precision in each based on the PSF [17]. Some parameter optimization is required to choose the kernel and search region sizes for the desired resolution and maximum velocity. Additionally, interpolation is required to compute sub-sample displacement. Although the technique can be computationally expensive, simplifications to the cross-correlation search have been proposed that reduce the number of computations required without a significant loss in performance [18]. It is also possible to use only simple 1-D correlations to perform multi-dimensional displacement estimates by beamforming lines in the local motion direction throughout the image [19].

Phase-based techniques instead analyze changes in spectral content with displacement and are frequently used in axial motion tracking due to the presence of a sinusoid in the axial RF signal. The displacement of the sinusoidal signal, if treated as monochromatic, can be expressed as a phase shift. The complex cross-correlation of kernels from the two signals provides an estimate of the phase difference, which is expected to be a linear function of displacement with slope determined by the oscillation frequency [20]. Due to local differences in effective pulse center frequency and center frequency downshift through depth, techniques that use an estimated center frequency or search for the zero-phase point iteratively provide improved estimates [21], [22]. The computational efficiency of these methods make them attractive for real-time displacement estimation, although they are subject to aliasing unlike the correlation-based methods. Multi-resolution estimation techniques can help to overcome the problem of aliasing by first tracking a course motion and refining the estimate using higher-frequency signals [23].

C. Heterodyning demodulation

Heterodyning demodulation is the process of mixing the complex representations of the signals to be tracked to produce (in two dimensions) an axially- and a laterally-oscillating signal, each with double the center frequency of the original oscillation [24]. Each can then be processed using conventional phase-based estimation techniques using the known center frequencies. In the case where the original axial RF signal is accessible, this method is only necessary to isolate the lateral component.

It is important to note that the following process is one example implementation and can be accomplished with many variations. The complex signals can be created by the Hilbert transform in either the time or frequency domain [10], [25] or steering the oscillating PSF to create a 90 degree phase shift [2]. The combination of phase terms can be performed by multiplying the complex signals [10], [24] or, more efficiently, directly combining the phases [25].

Given a two-dimensional PSF $h(x, y)$ that oscillates in both the axial and lateral dimensions, the partial Hilbert transform in the y dimension is added as an imaginary component to the real PSF to create a function, $r_{even}(x, y)$, analytic in axial frequency. A second signal $r_{odd}(x, y)$ is formed by taking the partial Hilbert transform in the x dimension:

$$r_{even}(x, y) = h(x, y) + i\mathcal{H}_y(h(x, y)), \quad (2)$$

$$r_{odd}(x, y) = \mathcal{H}_x(r_{even}(x, y)). \quad (3)$$

The spatial indices (x, y) are omitted in subsequent discussion for readability. Two single-quadrant functions r_1 and r_2 are formed by combining r_{even} and $\pm r_{odd}$ as the real and imaginary parts respectively of a complex signal:

$$r_1(x, y) = r_{even}(x, y) + ir_{odd}(x, y), \quad (4)$$

$$r_2(x, y) = r_{even}(x, y) - ir_{odd}(x, y). \quad (5)$$

The axial heterodyned signal h_{ax} is the product of r_1 and r_2 . The lateral signal h_{lat} is the product of r_1 and r_2^* , where $*$ is the complex conjugate operator. Their k-space representations or spatial frequency spectra [26], H_{ax} and H_{lat} , are found using the two-dimensional Fourier transform:

$$h_{ax}(x, y) = r_1(x, y) \times r_2(x, y), \quad (6)$$

$$H_{ax}(u, v) = \mathcal{F}\{h_{ax}(x, y)\}, \quad (7)$$

$$h_{lat}(x, y) = r_1(x, y) \times r_2^*(x, y), \quad (8)$$

$$H_{lat}(u, v) = \mathcal{F}\{h_{lat}(x, y)\}. \quad (9)$$

A lateral shift x produces a change in phase angle that can be measured using phase-based displacement techniques:

$$\theta(\Delta x) = \arg(h_{lat}(x + \Delta x, y)) - \arg(h_{lat}(x, y)). \quad (10)$$

III. Cramer-Rao Lower Bound

The Cramer-Rao Lower Bound (CRLB) predicts the minimum attainable standard deviation of the jitter of the displacement estimates made by an unbiased estimator. This jitter error represents small displacements of the cross-correlation peak that are not caused by a systematic bias but inherent uncertainties in tracking. Walker and Trahey derived an expression for the CRLB of time delay estimation in ultrasound based on tracking echoes with kernel length (T), normalized signal correlation (ρ), signal-to-noise ratio (SNR), center frequency (f_0) and bandwidth (B) [27]. In order to provide a closed-form expression for the jitter as a function of these parameters, the authors assumed flat, bandlimited spectra for both the signal and noise terms. While this simplification is useful for guiding parameter selection in axial delay estimation, the effects of using different transverse oscillation formation methods can only be appropriately summarized by their individual spectra.

We propose using the Cramer-Rao Lower Bound to compare and optimize transverse oscillation techniques with an expression that retains the frequency spectrum term. The signal model of [27] is used here except that the signal power spectrum shape G_{ss} is not assumed to be constant. As in that work, the signal power spectrum shape G_{ss} and noise power spectrum shape G_{nn} are scaled so their integrals are equal to one and their amplitudes are captured by the mean squared signal amplitudes σ_s^2 and σ_n^2 , where the amplitudes for the pre- and post-displacement signals are the same. The noise spectrum is assumed to have the same extent as the signal spectrum and constant amplitude. The additive noise term contains both the uncorrelated noise and noise-signal cross terms produced by heterodyning, leading to cross-correlation spectra that are non-zero but negligible compared to the noise and signal power spectra.

The resulting expression for the CRLB as a function of lateral displacement error, simplified from (17) in [27], is:

$$\sigma(\Delta x - \Delta \hat{x}) = \left(\int_{-\infty}^{\infty} \frac{4\pi^2 T f^2}{\frac{1}{\rho^2} \left(1 + \text{SNR}^{-2} \frac{G_{nn}(f)}{G_{ss}(f)} \right)^2 - 1} df \right)^{-\frac{1}{2}}. \quad (11)$$

For the purpose of this equation, kernel length and displacement are expressed in units of distance and frequency is expressed in units of inverse distance. SNR is expressed as a fraction, $\frac{\sigma_s}{\sigma_n}$, not in decibels. The signal power spectrum used in this calculation can be

obtained using a number of methods including, but not limited to, analytical expressions, numerical simulation or computation, and experimental analysis.

IV. Theoretical Spectra

In order to evaluate the use of this equation in predicting the relative performance of lateral tracking methods, we examine two aperture functions that are commonly used to create transverse oscillation signals – the bi-lobed Gaussian and the rectangular apodization. For each aperture function, we present closed-form expressions for each step of the transverse oscillation process outlined in Section II using narrowband signals. The resulting expressions provide the change in phase angle produced by lateral displacement, relevant for phase-based estimation methods, and the two-dimensional frequency spectrum of the heterodyned signals.

A. Bi-lobed Gaussian apodization

The parameters of the aperture are defined in terms of the resulting PSF, specifying the lateral wavelength λ_x and envelope width σ_x . The lateral frequency should not depend on focal depth z , so aperture growth is required in the form of shifting and scaling the Gaussian functions based on focal depth. The bi-lobed Gaussian apodization is composed of two Gaussian functions displaced outward from the center of the array:

$$w(\xi) = \frac{\sigma_x}{2} \left[e^{-\pi\sigma_x^2 \left(\frac{\xi}{\lambda_y z} - \frac{1}{\lambda_x} \right)^2} + e^{-\pi\sigma_x^2 \left(\frac{\xi}{\lambda_y z} + \frac{1}{\lambda_x} \right)^2} \right]. \quad (12)$$

The infinite nature of the Gaussian function means that the aperture function $w(\xi)$ should strictly be windowed by a rectangular function. We omit this term to simplify the necessary signal manipulations and instead choose parameters such that at least 99% of the aperture function lies within the physical aperture, ensuring that the Fourier transform of the aperture function closely approximates the true pressure field produced. The PSF is given by:

$$h(x, y) = e^{-\pi \left(\frac{x}{\sigma_x} \right)^2} e^{-\pi \left(\frac{y}{\sigma_y} \right)^2} \cos \left(2\pi \frac{x}{\lambda_x} \right) \cos \left(4\pi \frac{y}{\lambda_y} \right). \quad (13)$$

Note that the axial variable y is given in terms of round-trip distance, requiring scaling by a factor of two to match the dimensions of λ_y and σ_y . The real parts of the heterodyned signals h_{ax} and h_{lat} oscillate as a cosine function in the axial and lateral dimensions respectively:

$$h_{ax}(x, y) = e^{-2\pi \left(\frac{x}{\sigma_x} \right)^2} e^{-8\pi \left(\frac{y}{\sigma_y} \right)^2} e^{8\pi i \frac{y}{\lambda_y}}, \quad (14)$$

$$h_{lat}(x, y) = e^{-2\pi \left(\frac{x}{\sigma_x} \right)^2} e^{-8\pi \left(\frac{y}{\sigma_y} \right)^2} e^{4\pi i \frac{x}{\lambda_x}}. \quad (15)$$

The phase angle produced by a lateral shift is a linear function with slope determined by the lateral wavelength:

$$\theta(\Delta x) = \frac{4\pi}{\lambda_x} \Delta x. \quad (16)$$

The regions of support for these functions have the same extent, but are shifted axially and laterally. The frequency space covered is a Gaussian shape dependent on the original Gaussian widths and shifted axially and laterally based on the axial and lateral wavelengths respectively:

$$H_{ax}(u, v) = \frac{\sigma_x \sigma_y}{4} e^{-\frac{\pi}{2}(u\sigma_x)^2} e^{-\frac{\pi}{8}\left(\left(v - \frac{4}{\lambda_y}\right)\sigma_y\right)^2}, \quad (17)$$

$$H_{lat}(u, v) = \frac{\sigma_x \sigma_y}{4} e^{-\frac{\pi}{2}\left(\left(u - \frac{2}{\lambda_x}\right)\sigma_x\right)^2} e^{-\frac{\pi}{8}(v\sigma_y)^2}. \quad (18)$$

B. Rectangular apodization

A conventional rectangular apodization is evaluated for an aperture of width $D\lambda_y z$, where aperture growth is implemented to keep a constant lateral PSF through depth:

$$w(\xi) = \text{rect}\left(\frac{\xi}{D\lambda_y z}\right). \quad (19)$$

Although not a pure sinusoid, the PSF produced by the rectangular apodization does contain a lateral oscillation in the form of the *sinc* function:

$$h(x, y) = \lambda_y z \frac{\sin(\pi Dx)}{\pi x} e^{-\pi\left(\frac{2y}{\sigma_y}\right)^2} \cos\left(4\pi\frac{y}{\lambda_y}\right). \quad (20)$$

The real parts of the heterodyned signals h_{ax} and h_{lat} oscillate in the axial and lateral dimension respectively. Both signals have finite limits as x approaches zero:

$$h_{ax}(x, y) = (\lambda_y z)^2 \frac{[2 - 2\cos(\pi Dx)]}{(\pi x)^2} e^{-8\pi\left(\frac{y}{\sigma_y}\right)^2} e^{8\pi i \frac{y}{\lambda_y}}, \quad (21)$$

$$h_{lat}(x, y) = -(\lambda_y z)^2 \frac{[e^{i\pi Dx} - 1]^2}{(\pi x)^2} e^{-8\pi\left(\frac{y}{\sigma_y}\right)^2}. \quad (22)$$

The phase angle produced by a lateral shift is a linear function with slope determined by the aperture width:

$$\theta(\Delta x) = \pi D \Delta x. \quad (23)$$

The regions of support for these functions have the same extent, but are shifted axially and laterally. The frequency space is axially a Gaussian function and laterally a triangle function, denoted by $A(u/U)$ for a total width U :

$$H_{ax}(u, v) = \frac{(\lambda_y z)^2 \sigma_y}{\sqrt{2\pi^2}} \wedge \left(\frac{2}{D} u \right) e^{-\frac{\pi}{8} \left(\left(v - \frac{4}{\lambda_y} \right) \sigma_y \right)^2}, \quad (24)$$

$$H_{lat}(u, v) = \frac{(\lambda_y z)^2 \sigma_y}{2\sqrt{2\pi^2}} \wedge \left(\frac{2}{D} u - 1 \right) e^{-\frac{\pi}{8} (v \sigma_y)^2}. \quad (25)$$

C. Sample aperture functions

The aperture-, space- and frequency-domain results for each aperture function are shown in Fig. 1 using the parameters given in Table I, which have been specifically chosen to demonstrate the case of matched lateral center frequencies between the two resulting signals such that $D/2 = 2/\lambda_x$. The expected phase angle as a function of lateral displacement and the lateral frequency spectrum $H_{lat}(u, 0)$ are compared for the two aperture functions in Fig. 2. Note that the wavelengths given in Table I are the parameters required for (13) and (20), not the resulting wavelengths of the heterodyned signals.

When the center frequency is matched, the phase angle produced by a lateral displacement for the two apertures is identical. However, the lateral frequency content present in the two signals differs and (11) implies that performance between the two will also differ.

V. Field II Simulation

A. Methods

To validate the expressions proposed in the previous section and evaluate the use of the Cramer-Rao Lower Bound in predicting relative lateral displacement tracking performance for an arbitrary aperture, Field II simulations [28], [29] were performed over a space of possible bi-lobed Gaussian apodizations and for the rectangular apodization. An aperture with 128 elements with 154 μm pitch was simulated using a transmit pulse of 5 MHz and 80% bandwidth. Uniform speckle-generating media with extent 20 mm laterally, 2.5 mm in elevation, and 10 mm axially was simulated about a depth of 40 mm using 25 scatterers per resolution volume. The simulation was repeated using 20 unique speckle realizations.

For each simulation, a single flat plane wave was used in transmit and individual receive channel data were recorded. In the first scenario presented, the medium was subjected to a rigid lateral displacement of 100 μm and imaged again, producing a second data set. Gaussian white noise with magnitude -12 dB with respect to the recorded data was added to each channel before focusing. In the second scenario presented, the medium was subjected to a rigid lateral displacement of 200 μm and noise with magnitude -20 dB relative to the

channel data was added. In each case, focusing was performed to create 1000×1000 pixel images representing a span of 16 mm laterally and 6 mm axially.

Bi-lobed Gaussian apertures were formed by weighting the individual receive channel data using (12) calculated for a single depth (40 mm) and summing the focused data across the aperture. Lateral heterodyned images were created using the procedure outlined in Section II. An iterative phase root displacement estimation method [22] was run for three iterations with a lateral kernel length of 1 mm to estimate the lateral displacement at each point in the beamformed image. The iterative estimator was used to compensate for changes in the lateral wavelength through depth caused by the fixed apodization. Estimates from the central 10 mm of the field of view, where the kernel length and finite transmit aperture extent are not expected to affect the results, were compared to the true displacement function and the root-mean-square error (RMSE) was recorded.

The weighting and tracking processes were repeated for bi-lobed Gaussian functions with parameters $1.0 \leq \lambda_x \leq 4.0$ mm and $0.1 \leq \sigma_x \leq 20.0$ mm and for the full rectangular aperture with uniform weighting. Fig. 3 presents sample apodization functions over the sampled space, with those that lie at least 99% within the physical aperture shown in white and the rest, which the closed-form expressions presented do not hold for, shown in gray. For each of the two imaging scenarios, ρ was estimated using normalized cross-correlation between sample simulated signals without additive noise, giving values of 0.994 and 0.974. SNR for each case was estimated by comparing sample simulated signals with and without additive noise, giving values of 17.4 dB and 25.4 dB. This simulation-based approach ensures maximum agreement with the simulated tracking results in order to evaluate the role of spectrum shape, although in practice values from theory or estimates must be used.

For each case, the relative performance prediction made by the CRLB and results of the Field II simulation are compared using two sets of plots. The performance over the full range of allowed parameters is shown using a 2-D plot, with the optimum performance indicated by vertical and horizontal black lines. A second plot provides a projection of the first, showing the best performance at each λ_x and comparing to the performance of the rectangular aperture. For the CRLB prediction, the aperture that would be selected to optimize performance is indicated by a circular marker. For the Field II simulation, the performance of that selected aperture is indicated by a circular marker and compared to the true optimum, indicated by an asterisk marker.

B. Results and Discussion

The results for the case of $100 \mu\text{m}$ displacement with -12 dB additive noise are shown in Fig. 4. Qualitatively, the trends shown in (a) and (c) are very similar, demonstrating improved performance as λ_x and σ_x decrease. The optimum value occurs in the “elbow” of the graph, with a balance between the separation and width of the Gaussian functions. This is a similar optimum point to the one indicated by Liebgott et al. in Fig. 2(b) of [4]. The minimum jitter values for each value of λ_x , both predicted and simulated, occur along the masked edge of the graph. This is intuitive because that line represents maximum utilization of the available aperture extent. Elsewhere, the parameters could be changed to further increase the center frequency or increase the extent of the lateral frequency spectrum

without trade-off, increasing performance. The comparison of the best aperture at each λ_x is therefore equivalently the smallest value of σ_x allowed at each λ_x .

Fig. 4(b) shows the selection of apodization with $\lambda_x = 1.60$ mm and $\sigma_x = 5.38$ mm. In the simulation, this aperture produced a RMSE of $35.76 \mu\text{m}$, compared to the true optimum value of $35.75 \mu\text{m}$. The rectangular aperture produces a RMSE of $45.1 \mu\text{m}$, slightly worse than the optimum. The shape of the plots in (b) and (d) show close agreement as a function of λ_x , showing a minimum value that represents a trade-off between λ_x and σ_x . Although the phase angle used for displacement estimation given by (16) depends only on λ_x , this result confirms the prediction of (11) that the complete shape of the lateral frequency spectrum, influenced by σ_x , determines estimation performance. Although the CRLB is predictive of the relative performance trend and location of the optimum aperture in this case, the magnitude of jitter predicted is not achieved in simulation. This result could be due to the choice of a biased estimator, where the iterative estimate of the changing lateral center frequency through depth may not converge in just three iterations, or the limitations of the additive noise model discussed in Section III.

The results for the case of $200 \mu\text{m}$ displacement with -20 dB additive noise are shown in Fig. 5. The trends shown in (a) and (c) are very similar to those seen in Fig. 4, but the optimum bi-lobed Gaussian apodization has shifted to slightly lower σ_x and slightly higher λ_x . Both (b) and (d) show a region around the optimum bi-lobed Gaussian apodization that is less sensitive to the choice of λ_x than in the previous case, giving more leniency in choosing an apodization. In this case, the CRLB in (b) predicts that the rectangular aperture will outperform the bi-lobed Gaussian apodizations. The simulation results in (d) confirm this, showing a minimum jitter of $38.8 \mu\text{m}$. However, the magnitude of the difference in simulation between the best bi-lobed Gaussian aperture ($\lambda_x = 1.82$ mm and $\sigma_x = 3.70$ mm) and the rectangular aperture is only $0.3 \mu\text{m}$, disagreeing with the significant predicted difference in (b). Unlike in the previous case where only the overall magnitude was different, here the relative performance between the bi-lobed Gaussian apertures and the rectangular aperture is inaccurate. In addition to refining the signal model, better characterizing how ρ and SNR vary with apodization may be important in more accurately predicting estimator performance in cases such as this.

While the theoretical expressions for the bi-lobed Gaussian apodizations provided here only hold for functions contained within the physical aperture, the Field II simulation is valid for windowed aperture functions. While the phase angle from (16) is likely incorrect for the windowed aperture functions, the iterative estimation technique compensates for such errors. As an example, Fig. 6 shows the lower-left region of Fig. 5(c) with the mask replaced by a dashed white line. The region below the line is not covered by the spectra investigated in this work, but the true optimum bi-lobed Gaussian apodization for this case, marked with an asterisk, lies in this region. The RMSE improves from $38.8 \mu\text{m}$ for the rectangular aperture or $39.1 \mu\text{m}$ for the best bi-lobed Gaussian aperture outside of the masked region to $30.1 \mu\text{m}$ for the true optimum. The aperture function with $\lambda_x = 1.65$ mm and $\sigma_x = 1.91$ mm differs from the previous bi-lobed Gaussian aperture mainly in its width, truncating the tails of the Gaussian functions.

Further improvements to the presented theoretical evaluation can be made in two areas – improvements to the class of transverse oscillation fields provided in (15) and (22) and improvements to the assumed signal model of the CRLB in (11). Describing transverse oscillation fields, such as those produced by the truncated bi-lobed Gaussian apodization, requires more complicated Hilbert transform expressions. Such work may be restricted to numerical computation but would allow for the analysis of fields that simulations have shown to outperform the two conventional fields presented here. The second improvement requires modifying the uncorrelated additive noise model used to derive (11) to include higher-order partially-correlated terms produced by heterodyning. Finding an absolute rather than relative performance for a given transverse oscillation field requires a complete characterization of these signals as well as for decorrelation and SNR. Although simulations remain the gold standard for predicting tracking performance due to these limitations, the proposed approximation to the CRLB can be used in exploring large parameter spaces and more intuitively understanding the trade-offs made in producing various transverse oscillation signals.

VI. Conclusions

We have studied two classes of transverse oscillation apodization functions to better characterize their relative performances using the Cramer-Rao Lower Bound and demonstrated good agreement with Field II simulations. These results highlight the importance of considering the filtering effects of signal processing methods on the underlying ultrasound signal content. While a universally optimum aperture function may not exist, the CRLB provides a way to study the response of the tracking and imaging system to changes in correlation, signal-to-noise ratio, tracking kernel and post-processed signal spectrum shape without extensive experimentation or simulation work.

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Fig. 1.

(top row) Bi-lobed Gaussian aperture with parameters from Table I. (bottom row) Rectangular aperture with parameters from Table I. The same plots are shown for each aperture. Axes are not proportional. (a) Apodization profile. (b) Receive PSF. (c) Axial heterodyned PSF. (d) Lateral heterodyned PSF. (e) Frequency spectrum for axial heterodyned PSF. (f) Frequency spectrum for lateral heterodyned PSF.

Fig. 2.

(a) Lateral frequency spectrum at an axial frequency of zero. This is equivalent to a single horizontal slice of plot (f) in Fig. 1. (b) Phase angle of the lateral heterodyned signal produced by a lateral displacement. The two curves are identical for the selected parameters.

Fig. 3.

Sample aperture functions over a range of wavelength λ_x and width σ_x parameters. Note that x- and y-axis scales are non-linear. Apodization functions are windowed by the physical aperture extent. Functions that lie less than 99% within the aperture are shown in gray and cannot be properly evaluated by the provided closed-form spectra equations.

Fig. 4.

Tracking results for the simulated $100\ \mu\text{m}$ lateral displacement with $-12\ \text{dB}$ additive channel noise. (a) CRLB-predicted performance over evaluated bi-lobed Gaussian parameters, best performance denoted by black crosshairs. (b) Projection through σ_x of (a), showing best performance at each λ_x and comparing to the rectangular apodization. The aperture selected is denoted by a circular marker. (c) Field II simulation results, presented as in (a). (d) Projection through σ_x of (c). The true optimum, denoted by an asterisk, is compared to the selected aperture.

Fig. 5.

Tracking results for the simulated 200 μm lateral displacement with -20 dB additive channel noise. Figures are presented as in Fig. 4. The selected aperture in (b) is the rectangular aperture. The optimum aperture in (d) is also the rectangular aperture, although performance is very similar to the optimum bi-lobed Gaussian aperture.

Fig. 6.

(a) Simulation results for the 200 μm case without masking, zoomed in to the region of interest. The white dashed line indicates 99% within the aperture. The region below the line was previously masked. The asterisk indicates the true optimum aperture for this simulation and the circle represents the best modeled bi-lobed Gaussian aperture. The rectangular aperture, the selected aperture for this case, is not pictured. (b) Comparison of RMSE for the rectangular aperture, the best bi-lobed Gaussian aperture that lies 99% within the aperture, and the true optimum bi-lobed Gaussian aperture. (c) Aperture functions corresponding to the three apertures of (b).

TABLE I

Parameters for sample aperture functions

Property	Bi-lobed Gauss.	Rectangular
Axial wavelength (λ_y)	0.31 mm	0.31 mm
Axial Gaussian width (σ_y)	0.18 mm	0.18 mm
Focal depth (z)	40 mm	40 mm
Lateral wavelength (λ_x)	2.46 mm	–
Lateral Gaussian width (σ_x)	3 mm	–
Aperture width ($\lambda_y z D$)	–	19.7 mm