Function projective synchronization of memristor-based Cohen–Grossberg neural networks with time-varying delays

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Abstract This paper deals with the problem of function projective synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying delays. Based on the theory of differential equations with discontinuous right-hand side, some novel criteria are obtained to realize the function projective synchronization of addressed networks by combining open loop control and linear feedback control. As some special cases, several control strategies are given to ensure the realization of complete synchronization, anti-synchronization and the stabilization of the considered memristor-based Cohen–Grossberg neural network. Finally, a numerical example and its simulations are provided to demonstrate the effectiveness of the obtained results.

Keywords Memristor · Cohen–Grossberg neural network · Function projective synchronization · Time-varying delay

Introduction

Since the successful fabrication of physical memristive device by the scientists at Hewlett-Packard Labs in 2008 (Strukov et al. 2008), with its existence was firstly predicted by Leon Chua in 1971 (Chua 1971), various types of

models of networks based on memristor have been designed and analyzed (Itoh and Chua 2010; Oskoe and Sahimi 2011; Corinto et al. 2011; Buscarino et al. 2012; Pershin and Ventra 2012; Pershin et al. 2013; Yang et al. 2014; Qi et al. 2014). Especially, the memristor-based neural networks has been one of the most active research areas and has attracted the attention of many researchers (Itoh and Chua 2010; Pershin and Ventra 2012; Yang et al. 2014; Qi et al. 2014; Chandrasekar et al. 2014; Wan and Cao 2015). Memristor-based neural network can remember its past dynamical history, store a continuous set of states, and be “plastic” according to the pre-synaptic and post-synaptic neuronal activity (Strukov et al. 2008; Qi et al. 2014), an ideal tool to mimic the functionalities of the human brain.

As we all know, synchronization is a wide variety of phenomena in nature and plays a very important role in many different contexts, ranging from biological systems that include animal gates, descriptions of the heart, and fireflies in the forest to chemistry, nonlinear optics, and meteorology (Stilwell et al. 2006). Since Pecora and Carroll (1990) proposed a method to synchronize two identical systems with different initial values, the problem of synchronization in chaotic systems has been extensively investigated over the past few decades owing to their potential applications in many engineering areas, ranging from secure communications to modeling brain activity, even to optimization of nonlinear system performance (Ojalvo and Roy 2001). In application, there are many types of concepts of synchronization, for example, complete synchronization (Pecora and Carroll 1990), phase synchronization (Rosenblum et al. 1996), anti-synchronization (Kim et al. 2003), generalized synchronization (Rulkov et al. 1995), function projective synchronization (Runzi 2008; Abdurahman et al. 2014), etc.

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As an important neural network model, the Cohen–Grossberg neural network is very general and includes several well-known neural networks, such as Hopfield neural networks, cellular neural networks, recurrent neural networks and bi-directional associative memory networks with or without delays. As a result, Cohen–Grossberg neural networks have attracted considerable research attention and many good results related have been reported on the boundedness, stability, convergence, synchronization and other oscillatory properties of Cohen–Grossberg neural networks (Cao and Liang 2004; Zhang et al. 2014; Lisena 2011; Zhu and Cao 2010; Abdurahman and Jiang 2015). In addition, it has been reported that the memristor-based neural networks can exhibit some complicated dynamics and even chaotic behavior, and synchronization of chaotic neural networks based on memristor has also become an important area of study (Yang et al. 2014; Zhang et al. 2013; Wu et al. 2012; Wen et al. 2013; Li and Cao 2015; Zhang and Shen 2013, 2014; Abdurahman et al. 2015; Bao and Cao 2015).

In Yang et al. (2014), by nonlinear transformation, the authors studied the exponential synchronization of memristor-based Cohen–Grossberg neural network by using novel discontinuous feedback controller. In Qi et al. (2014), by employing the differential inclusion theory and the Lyapunov method, the authors investigated the stability problem for a class of general memristor-based neural networks with time-varying delay and time-varying impulses. In Rakkiyappan et al. (2015), by using Banach contraction principle, differential inclusion and framework of Filippov solution, the authors concerned the stability problem of a class of memristor-based fractional-order neural networks with time delay and different types of memductance functions. In Zhang et al. (2013), the anti-synchronization of memristive recurrent neural networks with time-varying delays was studied by using differential inclusions theory and Lyapunov functional method. In Wu et al. (2012), Wen et al. (2013), Li and Cao (2015), Zhang and Shen (2013), Zhang and Shen (2014), based on the theory of differential equations with discontinuous right-hand side as introduced by Filippov, the authors investigated the complete synchronization of various types of memristor-based neural networks with or without delays. In Abdurahman et al. (2015), the finite-time synchronization of memristor-based neural networks was studied by using finite time stability theory and differential inclusions theory. Very recently, by combining a fractional-order differential inequality, the projective synchronization of fractional-order memristor-based neural networks was considered in Bao and Cao (2015). However, to the best of our knowledge, there are very few or even no results on the function projective synchronization for memristor-based Cohen–Grossberg neural networks, while the Cohen–

Grossberg neural network plays a key role in many fields such as image processing, associative memories, classification of patterns, quadratic optimization and so on.

The type of synchronization concerned in this paper is function projective synchronization. Function projective synchronization, which bridges a gap from chaos control to chaos synchronization, is characterized that the drive and response systems could be synchronized up to a scaling function $\alpha(t)$, i.e., $y(t) \rightarrow \alpha(t)x(t)$, $t \rightarrow \infty$. Clearly, complete synchronization, anti-synchronization and projective synchronization are special cases of function projective synchronization. As compared with completed synchronization, function projective synchronization has many advantageous due to the unpredictability of the scaling function can additionally enhance the security of communication (Runzi 2008; Abdurahman et al. 2014). In addition, the proportional feature can be used to extend binary digital to variety M-nary digital communications for achieving fast communication (Bai et al. 2012).

Inspired by the above discussions, in this paper, we are concerned with the function projective synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying delays. The main contribution of this paper lies in the following aspects. First, the definition of function projective synchronization for chaotic memristor-based Cohen–Grossberg neural networks is introduced. Then, based on the switching open-loop control and linear feedback control, some novel and useful conditions which ensure the exponential function projective synchronization of addressed network is provided. Especially, when the projective function $\alpha(t)$ is appropriately chosen, the obtained results can ensure the realization of exponentially anti-synchronization, exponentially complete synchronization, and exponentially stabilization of considered memristor-based Cohen–Grossberg neural networks. Finally, an example with its numerical simulations is given to demonstrate the validity of the obtained results.

The rest of the paper is organized as follows. In “Preliminaries” section, the drive-response memristor-based Cohen–Grossberg neural networks systems are introduced. In addition, some useful definitions and lemmas needed in this paper are given. Next section is devoted to investigate the function projective synchronization of the considered networks. In “Numerical simulations” section, an examples and its numerical simulations are provided to illustrate the feasibility of the obtained results.

Preliminaries

In this paper, we consider a class of memristor-based Cohen–Grossberg neural networks with time-varying delays described by following equation

$$\begin{aligned} \dot{x}_i(t) = & a_i(x_i(t)) \left[-b_i(x_i(t))x_i(t) + \sum_{j=1}^n c_{ij}(x_i(t))f_j(x_j(t)) \right. \\ & \left. + \sum_{j=1}^n d_{ij}(x_i(t))g_j(x_j(t - \tau_j(t))) + I_i \right], \\ & i \in \mathcal{J} \triangleq \{1, 2, \dots, n\}, \end{aligned} \quad (1)$$

where $n \geq 2$ denotes the number of neurons in the neural network; x_i corresponds to the voltage of the capacitor $\mathbf{C}_i$; $a_i(\cdot)$ represents an amplification function; $f_j(\cdot)$ and $g_j(\cdot)$ are the feedback functions; $0 \leq \tau_j(\cdot) \leq \tau_j$ denotes the transmission time-varying delay; I_i is the external bias on the i th unit; $b_i(\cdot)$, $c_{ij}(\cdot)$ and $d_{ij}(\cdot)$ are the memristor-based weights given by

$$\begin{aligned} b_i(x_i(t)) &= \frac{1}{\mathbf{C}_i} \left[\sum_{j=1}^n (\mathbf{M}_{ij}^f + \mathbf{M}_{ij}^g) \times \text{sgn}_{ij} + \frac{1}{\mathbf{R}_i} \right], \\ c_{ij}(x_i(t)) &= \frac{\mathbf{M}_{ij}^f}{\mathbf{C}_i} \times \text{sgn}_{ij}, \\ d_{ij}(x_i(t)) &= \frac{\mathbf{M}_{ij}^g}{\mathbf{C}_i} \times \text{sgn}_{ij}, \quad \text{sgn}_{ij} = \begin{cases} 1, & i = j, \\ -1, & i \neq j, \end{cases} \end{aligned}$$

where $\mathbf{M}_{ij}^f$ and $\mathbf{M}_{ij}^g$ denote the memductances of memristors $\mathbf{W}_{ij}^f$ and $\mathbf{W}_{ij}^g$, respectively. $\mathbf{W}_{ij}^f$ represents the memristor between the feedback function $f_j(x_j(t))$ and $x_i(t)$, $\mathbf{W}_{ij}^g$ represents the memristor between the feedback function $g_j(x_j(t - \tau_j(t)))$ and $x_i(t)$, and $\mathbf{R}_i$ represents the parallel-resistor corresponding to the capacitor $\mathbf{C}_i$.

According to the feature of memristor and the current characteristics (Strukov et al. 2008; Yang et al. 2014), we apply following mathematical model of memristance

$$\begin{aligned} b_i(x_i(t)) &= \begin{cases} \hat{b}_i, & |x_i(t)| \leq T_i, \\ \check{b}_i, & |x_i(t)| > T_i, \end{cases} \\ c_{ij}(x_i(t)) &= \begin{cases} \hat{c}_{ij}, & |x_i(t)| \leq T_i, \\ \check{c}_{ij}, & |x_i(t)| > T_i, \end{cases} \\ d_{ij}(x_i(t)) &= \begin{cases} \hat{d}_{ij}, & |x_i(t)| \leq T_i, \\ \check{d}_{ij}, & |x_i(t)| > T_i, \end{cases} \end{aligned}$$

where $T_i \geq 0$ are switching jumps, and $\hat{b}_i, \check{b}_i, \hat{c}_{ij}, \check{c}_{ij}, \hat{d}_{ij}$ and $\check{d}_{ij}$ are constant numbers.

Assume that $\mathbb{R}^n$ be the space of n -dimensional real column vectors. For any $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$, $\|x\|_\infty$ denotes a vector norm defined by $\|x\|_\infty = \max_{i \in \mathcal{J}} |x_i|$.

The initial conditions associated with system (1) are given by

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \quad (2)$$

where $\tau = \max_{j \in \mathcal{J}} \{\tau_j\}$, $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T \in C([-\tau, 0], \mathbb{R}^n)$, which denotes the Banach space of all

continuous functions mapping $[-\tau, 0]$ into $\mathbb{R}^n$ with a norm defined by $\|\varphi\|_\infty = \sup_{s \in [-\tau, 0]} \{\max_{i \in \mathcal{J}} |\varphi_i(s)|\}$.

In this paper, solutions of all systems considered in the following are intended in Filippov's sense. Given set $E \in \mathbb{R}^n$, by $\bar{E}$ we mean the closure of E and by $\mu(E)$ we mean the Lebesgue measure of E in $\mathbb{R}^n$. Moreover, $K[E]$ denotes the closure of the convex hull of set E . $co\{a, b\}$ denotes the closure of the convex hull generated by real numbers a and b . If $x_0 \in \mathbb{R}^n$ and $r > 0$, $\mathcal{B}(x_0, r) = \{x \in \mathbb{R}^n : \|x - x_0\|_\infty < r\}$ denotes the ball of radius r about x_0 . For $i, j \in \mathcal{J}$, we let $\underline{b}_i = \min\{\hat{b}_i, \check{b}_i\}$, $\tilde{b}_i = \max\{\hat{b}_i, \check{b}_i\}$, $\tilde{c}_{ij} = \max\{|\hat{c}_{ij}|, |\check{c}_{ij}|\}$ and $\tilde{d}_{ij} = \max\{|\hat{d}_{ij}|, |\check{d}_{ij}|\}$. Then it is not difficult to obtain that

$$\begin{aligned} K[b_i(x_i(t))] &= \begin{cases} \hat{b}_i, & |x_i(t)| < T_i, \\ co\{\hat{b}_i, \check{b}_i\}, & |x_i(t)| = T_i, \\ \check{b}_i, & |x_i(t)| > T_i, \end{cases} \\ K[c_{ij}(x_i(t))] &= \begin{cases} \hat{c}_{ij}, & |x_i(t)| < T_i, \\ co\{\hat{c}_{ij}, \check{c}_{ij}\}, & |x_i(t)| = T_i, \\ \check{c}_{ij}, & |x_i(t)| > T_i. \end{cases} \\ K[d_{ij}(x_i(t))] &= \begin{cases} \hat{d}_{ij}, & |x_i(t)| < T_i, \\ co\{\hat{d}_{ij}, \check{d}_{ij}\}, & |x_i(t)| = T_i, \\ \check{d}_{ij}, & |x_i(t)| > T_i. \end{cases} \end{aligned}$$

Now, we introduce the following definitions about set-valued map and differential inclusion (Clarke et al. 1998; Filippov 1988).

Definition 1 Let $E \subset \mathbb{R}^n$, $x \mapsto F(x)$ is called a set-valued map from $E \mapsto \mathbb{R}^n$, if for each $x \in E$, there corresponds a nonempty set $F(x) \subset \mathbb{R}^n$. A set-valued map F with nonempty values is said to be upper semi-continuous at $x_0 \in E \subset \mathbb{R}^n$ if, for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact).

Definition 2 For the system $\dot{x} = f(x)$, $x \in \mathbb{R}^n$, with discontinuous right-hand sides, a set-valued map is defines as follows:

$$\mathbb{F}[f(x)] = \bigcap_{r > 0} \bigcap_{\mu(N)=0} K[f(\mathcal{B}(x, r) \setminus N)],$$

where $K[E]$ is the closure of the convex hull of set E , $\mathcal{B}(x, r) = \{y : \|y - x\|_\infty \leq r\}$, and $\mu(N)$ is Lebesgue measure of set N .

Definition 3 Let $E_V \subset \mathbb{R}^n$ denote the set of points witch function V fails to differentiable. The generalized gradient of $\partial V : \mathbb{R}^n \mapsto \mathcal{B}(\mathbb{R}^n)$ of V is defined by $\partial V(x) = co\{\lim_{i \rightarrow \infty} \nabla V(x_i) : x_i \rightarrow x, x_i \notin S \cup E_V\}$, where

$co(\cdot)$ denotes the convex hull and $S \subset \mathbb{R}^n$ is a set of measure zero.

By applying the theories of set-valued maps and differential inclusions (Clarke et al. 1998; Filippov 1988), the memristor-based neural network (1) can be written as the following differential inclusion:

$$\dot{x}_i(t) \in a_i(x_i(t)) \left[-K[b_i(x_i(t))]x_i(t) + \sum_{j=1}^n K[c_{ij}(x_i(t))]f_j(x_j(t)) + \sum_{j=1}^n K[d_{ij}(x_i(t))]g_j(x_j(t - \tau_j(t))) + I_i \right],$$

for a.e. $t \geq 0, i \in \mathcal{I}$. (3)

From Clarke et al. (1998), Filippov (1988), we know that the differential inclusion (3) means that there exist $^*b_i \in K[b_i(x_i(t))]$, $^*c_{ij} \in K[c_{ij}(x_i(t))]$ and $^*d_{ij} \in K[d_{ij}(x_i(t))]$ such that

$$\dot{x}_i(t) = a_i(x_i(t)) \left[-^*b_i x_i(t) + \sum_{j=1}^n ^*c_{ij} f_j(x_j(t)) + \sum_{j=1}^n ^*d_{ij} g_j(x_j(t - \tau_j(t))) + I_i \right],$$

(4)

for a.e. $t \geq 0, i \in \mathcal{I}$.

To obtain our main results, throughout this paper, we also need the following hypotheses for the addressed memristor-based Cohen–Grossberg neural networks (1):

H₁: For each $i \in \mathcal{I}$, function $a_i(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^+$ is continuous and there exist positive constants $\underline{a}_i$ and $\bar{a}_i$ such that $\underline{a}_i \leq a_i(u) \leq \bar{a}_i$ for all $u \in \mathbb{R}$.

H₂: The activation functions $f_j(\cdot)$ and $g_j(\cdot)$ are general Lipschitz continuous. That is, for any $j \in \mathcal{I}$, there exist real numbers $\underline{L}_j^f, \bar{L}_j^f, \underline{L}_j^g$ and $\bar{L}_j^g$ such that

$$\underline{L}_j^f \leq \frac{f_j(u) - f_j(v)}{u - v} \leq \bar{L}_j^f, \quad \underline{L}_j^g \leq \frac{g_j(u) - g_j(v)}{u - v} \leq \bar{L}_j^g,$$

for all $u, v \in \mathbb{R}, u \neq v$.

Remark 1 It is obvious that for $i \in \mathcal{I}$, the set-valued map $x_i(t) \mapsto a_i(x_i(t))[-K[b_i(x_i(t))]x_i(t)$

$$+ \sum_{j=1}^n K[c_{ij}(x_i(t))]f_j(x_j(t)) + \sum_{j=1}^n K[d_{ij}(x_i(t))]g_j(x_j(t - \tau_j(t))) + I_i]$$

has nonempty compact convex values. Furthermore, it is upper semi-continuous (Clarke et al. 1998). Then the local existence of a solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ with initial conditions $x(t) = \varphi(t) \in C([- \tau, 0], \mathbb{R}^n)$ of (3) is

obvious (Bai et al. 2012). Moreover, under the condition **H₁** and **H₂**, this local solution $x(t)$ can be extended to the interval $[0, +\infty)$ in the sense of Filippov (1988).

In the paper, consider system (1) as the drive system and the corresponding response system is gives as follows:

$$\dot{y}_i(t) = a_i(y_i(t)) \left[-b_i(y_i(t))y_i(t) + \sum_{j=1}^n c_{ij}(y_i(t))f_j(y_j(t)) + \sum_{j=1}^n d_{ij}(y_i(t))g_j(y_j(t - \tau_j(t))) + I_i \right] + u_i(t), \quad t \geq 0.$$

(5)

Similarly

$$\dot{y}_i(t) \in a_i(y_i(t)) \left[-K[b_i(y_i(t))]y_i(t) + \sum_{j=1}^n K[c_{ij}(y_i(t))]f_j(y_j(t)) + \sum_{j=1}^n K[d_{ij}(y_i(t))]g_j(y_j(t - \tau_j(t))) + I_i \right] + u_i(t),$$

for a.e. $t \geq 0$. (6)

Or equivalently, there exist $\overset{\circ}{b}_i \in K[b_i(x_i(t))]$, $\overset{\circ}{c}_{ij} \in K[c_{ij}(x_i(t))]$ and $\overset{\circ}{d}_{ij} \in K[d_{ij}(x_i(t))]$ such that

$$\dot{y}_i(t) = a_i(y_i(t)) \left[-\overset{\circ}{b}_{iy_i}(t) + \sum_{j=1}^n \overset{\circ}{c}_{ij} f_j(y_j(t)) + \sum_{j=1}^n \overset{\circ}{d}_{ij} g_j(y_j(t - \tau_j(t))) + I_i \right] + u_i(t),$$

(7)

for a.e. $t \geq 0, i \in \mathcal{I}$, with initial conditions $y_i(s) = \phi_i(s)$ for all $s \in [-\tau, 0]$, where $\phi(s) = (\phi_1(s), \phi_2(s), \dots, \phi_n(s))^T \in C([- \tau, 0], \mathbb{R}^n)$, and $u_i(t)$ is the appropriate control input that will be designed.

Definition 4 The drive-response networks (1) and (5) are said to achieve exponentially function projective synchronization if there exist $M \geq 1$ and $\delta > 0$ such that

$$\|y(t) - \alpha(t)x(t)\|_\infty \leq M\|\phi - \alpha\phi\|_\infty e^{-\delta t} \quad \text{for all } t \geq 0,$$

where $x(t)$ and $y(t)$ are the solutions of drive-response systems (1) and (5) with initial conditions $\varphi(s) = (\varphi_1(s), \dots, \varphi_n(s))^T$ and $\phi(s) = (\phi_1(s), \dots, \phi_n(s))^T$, respectively.

Remark 2 If the scaling function $\alpha(t) \equiv 1$ or $\alpha(t) \equiv -1$, then the synchronization problem will be reduced to the complete synchronization or anti-synchronization. If the scaling function $\alpha(t) \equiv 0$, then the synchronization problem will be turned into a chaos control problem.

Suppose that $z(t) : [0, +\infty) \mapsto \mathbb{R}^n$ is absolutely continuous on any compact subinterval of $[0, +\infty)$. The following lemma gives a chain rule for computing the time derivative of a composed function $V(z(t)) : [0, +\infty) \mapsto \mathbb{R}$.

Lemma 1 (Chain Rule (Clarke et al. 1998)) *Suppose that $V(z) : \mathbb{R}^n \mapsto \mathbb{R}$ is C-regular, and that $z(t) : [0, +\infty) \mapsto \mathbb{R}^n$ is absolutely continuous on any compact interval of $[0, +\infty)$. Then, $V(z(t)) : [0, +\infty) \mapsto \mathbb{R}$ are differential for a.e. $t \in [0, +\infty)$, and we have*

$$\frac{dV(z(t))}{dt} = \xi(t)\dot{z}(t), \quad \forall \xi(t) \in \partial V(z(t)).$$

Main results

In this section, we will derive some criteria to guarantee the exponential function projective synchronization of systems (1) and (5). First, letting $e_i(t) = y_i(t) - \alpha(t)x_i(t)$, where $\alpha(t)$ is a continuously differentiable function, and designing the controller $u_i(t)$ in response system (5) as follows:

$$\begin{cases} u_i(t) = u_{i1}(t) + u_{i2}(t) + u_{i3}(t) + u_{i4}(t), \\ u_{i1}(t) = \sum_{j=1}^n \left[\alpha(t)c_{ij}^a(x_i(t))f_j(x_j(t)) - c_{ij}^a(y_i(t))f_j(\alpha(t)x_j(t)) \right], \\ u_{i2}(t) = \alpha(t)x_i(t)[b_i^a(y_i(t)) - b_i^a(x_i(t))] + [\alpha(t)a_i(x_i(t)) - a_i(y_i(t))]I_i + \dot{\alpha}(t)x_i(t), \\ u_{i3}(t) = \sum_{j=1}^n \left[\alpha(t)d_{ij}^a(x_i(t))f_j(x_j(t - \tau_j(t))) - d_{ij}^a(y_i(t))f_j(\alpha(t - \tau_j(t))x_j(t - \tau_j(t))) \right], \\ u_{i4}(t) = -\eta_i[y_i(t) - \alpha(t)x_i(t)]. \end{cases} \quad (8)$$

where $b_i^a(z) = b_i(z)a_i(z)$, $c_{ij}^a(z) = c_{ij}(z)a_i(z)$, $d_{ij}^a(z) = d_{ij}(z)a_i(z)$ and η_i is a positive constant determined in later.

Remark 3 In fact, the control scheme (8) is a hybrid switching control, $u_{i1}(t)$, $u_{i2}(t)$ and $u_{i3}(t)$ are switching open-loop controls and $u_{i4}(t)$ is a linear feedback control.

Letting $e_i(t) = y_i(t) - \alpha(t)x_i(t)$, where $\alpha(t)$ is a continuously differentiable function. Then, from system (1) and system (5), the error system can be described by

$$\begin{aligned} \dot{e}_i(t) = & -a_i(y_i(t))b_i(y_i(t))e_i(t) + \sum_{j=1}^n a_i(y_i(t))c_{ij}(y_i(t))\bar{f}_j(e_j(t)) \\ & + \sum_{j=1}^n a_i(y_i(t))d_{ij}(y_i(t))\bar{g}_j(e_j(t - \tau_j(t))) - \eta_i e_i(t), \end{aligned} \quad (9)$$

where $\bar{f}_j(e_j(t)) = f_j(y_j(t)) - f_j(\alpha(t)x_j(t))$ and $\bar{g}_j(e_j(t - \tau_j(t))) = f_j(y_j(t - \tau_j(t))) - f_j(\alpha(t - \tau_j(t))x_j(t - \tau_j(t)))$.

For convenience, we denote

$$\xi_i = \underline{a}_i \underline{b}_i - \omega_i - \sum_{j=1, j \neq i}^n \bar{a}_i \bar{c}_{ij} L_j^f, \quad \gamma_i = \sum_{j=1}^n \bar{a}_i \bar{d}_{ij} L_j^g.$$

where $L_j^f = \max\{|\underline{L}_j^f|, |\bar{L}_j^f|\}$, $L_j^g = \max\{|\underline{L}_j^g|, |\bar{L}_j^g|\}$, $\omega_i = \max\{\hat{\omega}_i, \check{\omega}_i\}$, here $\hat{\omega}_i$ and $\check{\omega}_i$ are defined as

$$\hat{\omega}_i = \begin{cases} \bar{a}_i \hat{c}_{ii} \bar{L}_i^f, & \text{if } \hat{c}_{ii} \geq 0, \\ \underline{a}_i \hat{c}_{ii} \underline{L}_i^f, & \text{if } \hat{c}_{ii} < 0. \end{cases} \quad \check{\omega}_i = \begin{cases} \bar{a}_i \check{c}_{ii} \bar{L}_i^f, & \text{if } \check{c}_{ii} \geq 0, \\ \underline{a}_i \check{c}_{ii} \underline{L}_i^f, & \text{if } \check{c}_{ii} < 0. \end{cases}$$

Based on the above notations, we give following assumption for system parameters and control strengths.

H₃: $\xi_i + \eta_i - \gamma_i > 0$ for any $i \in \mathcal{I}$.

For each $i \in \mathcal{I}$, consider the following function

$$G_i(\varepsilon_i) = \xi_i + \eta_i - \lambda_i - \gamma_i e^{\lambda_i \tau}, \quad \lambda_i \geq 0.$$

It is easy see that $\dot{G}_i(\lambda_i) < 0$, $G_i(0) > 0$. Since $G_i(\varepsilon_i)$ is continuous and $G_i(\lambda_i) \rightarrow -\infty$ as $\lambda_i \rightarrow +\infty$. Thus, there exists a positive number λ_i^* such that $G_i(\lambda_i^*) > 0$. Let $\lambda^* = \min_{i \in \mathcal{I}} \{\lambda_i^*\}$, then for any $\lambda \in (0, \lambda^*)$, we have

$$\xi_i + \eta_i - \lambda - \gamma_i e^{\lambda \tau} > 0, \quad \text{for any } i \in \mathcal{I}.$$

Based on the hybrid switching controller (8), the following result can be derived.

Theorem 1 *Suppose that assumptions **H₁**, **H₂** and **H₃** hold, then the memristor-based drive-response systems (1) and (5) are exponentially function projective synchronized under hybrid switching controller (8).*

Proof According to the switching feature of memristor, the following two cases can be happen for error system (9).

Case 1 For $|y_i(t)| \leq T_i$, the error system can be reduced to the following form

$$\begin{aligned} \dot{e}_i(t) = & -\hat{b}_i a_i(y_i(t)) e_i(t) + \sum_{j=1}^n \hat{c}_{ij} a_i(y_i(t)) \bar{f}_j(e_j(t)) \\ & + \sum_{j=1}^n \hat{d}_{ij} a_i(y_i(t)) \bar{g}_j(e_j(t - \tau_j(t))) - \eta_i e_i(t). \end{aligned}$$

Case 2. For $|y_i(t)| > T_i$, the error system can be written as

$$\begin{aligned} \dot{e}_i(t) = & -\check{b}_i a_i(y_i(t)) e_i(t) + \sum_{j=1}^n \check{c}_{ij} a_i(y_i(t)) \bar{f}_j(e_j(t)) \\ & + \sum_{j=1}^n \check{d}_{ij} a_i(y_i(t)) \bar{g}_j(e_j(t - \tau_j(t))) - \eta_i e_i(t). \end{aligned}$$

Calculating the the Clarke's generalized gradient of absolute value function $|e_i(t)|$ by Definition 3, we get

$$\frac{d}{dt}|e_i(t)| = \partial|e_i(t)|\dot{e}_i(t) = \vartheta_i(t)\dot{e}_i(t), \quad \text{for a.e. } t \geq 0. \quad (10)$$

where

$$\vartheta_i(t) = \mathbb{F}[\text{sign}(e_i(t))] = \begin{cases} -1 & e_i(t) < 0, \\ [-1, 1] & e_i(t) = 0, \\ 1 & e_i(t) > 0. \end{cases}$$

It is easy to see that $\vartheta_i(t)e_i(t) = |e_i(t)|$ for $i \in \mathcal{I}$.

For Cases 1 and 2, from the hypotheses $\mathbf{H}_1$ and $\mathbf{H}_2$ we have

$$\begin{aligned} \frac{d}{dt}|e_i(t)| \leq & (\omega_i - \underline{a}_i \underline{b}_i - \eta_i)|e_i(t)| + \sum_{j=1, j \neq i}^n \bar{a}_i \tilde{c}_{ij} L_j^f |e_j(t)| \\ & + \sum_{j=1}^n \bar{a}_i \tilde{d}_{ij} L_j^g |e_j(t - \tau_j(t))|. \end{aligned} \quad (11)$$

Denote

$$\rho = \max_{i \in \mathcal{I}} \left\{ \sup_{-\tau \leq s \leq 0} |\phi_i(s) - \alpha(s)\varphi_i(s)| \right\}, \quad V_i(t) = e^{\lambda t} |e_i(t)|,$$

where $t \geq -\tau$ and $i \in \mathcal{I}$. Let $Q_i(t) = V_i(t) - h\rho$, where $h > 1$ is a constant. From the definitions of $V_i(t)$ and ρ , it is easy to check that

$$Q_i(t) < 0, \quad \text{for all } t \in [-\tau, 0] \quad \text{and } i \in \mathcal{I}. \quad (12)$$

In the following, we will prove that

$$Q_i(t) < 0 \quad \text{for all } t \in [0, +\infty) \quad \text{and } i \in \mathcal{I}. \quad (13)$$

Otherwise, there exist a $\ell \in \mathcal{I}$ and $t^* \in [0, +\infty)$ such that

$$Q_\ell(t^*) = 0, \quad Q_j(t^*) \leq 0, \quad j \in \mathcal{I} \setminus \{\ell\}, \quad (14)$$

$$\frac{dQ_\ell(t^*)}{dt} \geq 0 \quad (15)$$

and for any $i \in \mathcal{I}$,

$$Q_i(t) < 0 \quad \text{for } t \in [0, t^*). \quad (16)$$

From (12) and (16), for any $i \in \mathcal{I}$,

$$Q_i(t) < 0 \quad \text{for all } t \in [-\tau, t^*). \quad (17)$$

Calculating the time derivative of $Q_\ell(t)$ at t^* along the solution trajectories of the system (11), by the chain rule in Lemma 1, and using (14), (15) and (17), leads to

$$\begin{aligned} \frac{dQ_\ell(t^*)}{dt} = & \lambda e^{\lambda t^*} |e_\ell(t^*)| + e^{\lambda t^*} \frac{d}{dt} |e_\ell(t^*)| \\ \leq & \lambda e^{\lambda t^*} |e_\ell(t^*)| + e^{\lambda t^*} (\omega_\ell - \underline{a}_\ell \underline{b}_\ell - \eta_\ell) |e_\ell(t^*)| \\ & + \sum_{j=1, j \neq i}^n \bar{a}_\ell \tilde{c}_{\ell j} L_j^f e^{\lambda t^*} |e_j(t^*)| \\ & + \sum_{j=1}^n \bar{a}_\ell \tilde{d}_{\ell j} L_j^g e^{\lambda t^*} |e_j(t^* - \tau_j(t^*))| \\ \leq & (\lambda + \omega_\ell - \underline{a}_\ell \underline{b}_\ell - \eta_\ell) V_\ell(t^*) + \sum_{j=1}^n \bar{a}_\ell \tilde{c}_{\ell j} L_j^f V_j(t^*) \\ & + \sum_{j=1}^n \bar{a}_\ell \tilde{b}_{\ell j} L_j^g V_j(t^* - \tau_j(t^*)) e^{\lambda \tau_j} \\ \leq & (\lambda + \omega_\ell - \underline{a}_\ell \underline{b}_\ell - \eta_\ell) h\rho + \sum_{j=1}^n \bar{a}_\ell \tilde{c}_{\ell j} L_j^f h\rho \\ & + \sum_{j=1}^n \bar{a}_\ell \tilde{d}_{\ell j} L_j^g h\rho e^{\lambda \tau_j} \\ \leq & (\lambda - \eta_\ell - \xi_\ell + \gamma_\ell e^{\lambda \tau}) h\rho \\ < & 0, \end{aligned}$$

which leads to a contradiction with (15). (Hence the inequality (13) holds).

Let $h \rightarrow 1$ in $Q_i(t)$, then from (13) and the definition of $V_i(t)$, we obtain

$$|e_i(t)| \leq \rho e^{-\lambda t},$$

for any $t \geq 0$ and $i \in \mathcal{I}$, which shows that

$$\|y(t) - \alpha(t)x(t)\|_\infty \leq \rho e^{-\lambda t} = \|\phi - \alpha\varphi\|_\infty e^{-\lambda t},$$

for any $t \geq 0$. Hence, the drive-response Cohen–Grossberg neural networks (1) and (5) are exponentially function projective synchronized under the hybrid switching controller (8). The proof of Theorem 1 is completed. $\square$

In system (1), if the amplification function $a_i(u) \equiv 1$ for $u \in \mathbb{R}$ and $i \in \mathcal{I}$, then system (1) becomes

$$\begin{aligned} \dot{x}_i(t) = & -b_i(x_i(t))x_i(t) + \sum_{j=1}^n c_{ij}(x_i(t))f_j(x_j(t)) \\ & + \sum_{j=1}^n d_{ij}(x_i(t))g_j(x_j(t - \tau_j(t))) + I_i, \end{aligned} \quad (18)$$

Definitely, the hypothesis $\mathbf{H}_1$ is satisfied in this case. Accordingly, the response system (5) is reduced to following form

$$\begin{aligned} \dot{y}_i(t) = & -b_i(y_i(t))y_i(t) + \sum_{j=1}^n c_{ij}(y_i(t))f_j(y_j(t)) \\ & + \sum_{j=1}^n d_{ij}(y_i(t))g_j(y_j(t - \tau_j(t))) + I_i + u_i(t), \end{aligned} \quad (19)$$

where $t \geq 0$ and $i \in \mathcal{J}$, $u_i(t)$ is given by

$$\begin{cases} u_i(t) = \bar{u}_{i1}(t) + \bar{u}_{i2}(t) + \bar{u}_{i3}(t) + \bar{u}_{i4}(t), \\ \bar{u}_{i1}(t) = \sum_{j=1}^n [\alpha(t)c_{ij}(x_i(t))f_j(x_j(t)) - c_{ij}(y_i(t))f_j(\alpha(t)x_j(t))], \\ \bar{u}_{i2}(t) = \alpha(t)x_i(t)[b_i(y_i(t)) - b_i(x_i(t))] + [\alpha(t) - 1]I_i + \dot{\alpha}(t)x_i(t), \\ \bar{u}_{i3}(t) = \sum_{j=1}^n [\alpha(t)d_{ij}(x_i(t))f_j(x_j(t - \tau_j(t))) - d_{ij}(y_i(t))f_j(\alpha(t - \tau_j(t))x_j(t - \tau_j(t)))], \\ \bar{u}_{i4}(t) = -\eta_i[y_i(t) - \alpha(t)x_i(t)]. \end{cases} \quad (20)$$

In this case, we denote

$$\bar{\xi}_i = \underline{b}_i - \bar{\omega}_i - \sum_{j=1, j \neq i}^n \tilde{c}_{ij}L_j^f, \quad \bar{\gamma}_i = \sum_{j=1}^n \tilde{d}_{ij}L_j^g.$$

where $\bar{\omega}_i = \max\{\hat{\omega}_i, \check{\omega}_i\}$, here $\hat{\omega}_i$ and $\check{\omega}_i$ are defined as

$$\hat{\omega}_i = \begin{cases} \hat{c}_{ii}L_i^f, & \text{if } \hat{c}_{ii} \geq 0, \\ \hat{c}_{ii}L_i^f, & \text{if } \hat{c}_{ii} < 0. \end{cases} \quad \check{\omega}_i = \begin{cases} \check{c}_{ii}L_i^f, & \text{if } \check{c}_{ii} \geq 0, \\ \check{c}_{ii}L_i^f, & \text{if } \check{c}_{ii} < 0. \end{cases}$$

Correspondingly, hypothesis $\mathbf{H}_3$ can be stated as follows:

$$\bar{\mathbf{H}}_3: \bar{\xi}_i + \eta_i - \bar{\gamma}_i > 0 \text{ for any } i \in \mathcal{I}.$$

Thus, according to Theorem 1, we can easily derive the following corollary.

Corollary 1 Suppose that $\mathbf{H}_2$ and $\bar{\mathbf{H}}_3$ hold. Then system (18) and system (19) are exponentially function projective synchronized under the hybrid switching controller (20) with the synchronization rate $\bar{\lambda}$, where $\bar{\lambda}$ satisfies $\bar{\xi}_i + \eta_i - \bar{\lambda} - \bar{\gamma}_i e^{\bar{\lambda}\tau} > 0$ for any $i \in \mathcal{J}$.

In system (1), if the memristor-based time-varying delay strength $d_{ij}(\cdot) \equiv 0$ for all $i, j \in \mathcal{J}$, then system (1) is reduced to following form

$$\dot{x}_i(t) = a_i(x_i(t)) \left[-b_i(x_i(t))x_i(t) + \sum_{j=1}^n c_{ij}(x_i(t))f_j(x_j(t)) + I_i \right], \quad (21)$$

where $t \geq 0$ and $i \in \mathcal{J}$. Accordingly, the response system (5) is degenerated to following form

$$\begin{aligned} \dot{y}_i(t) = & a_i(y_i(t))[-b_i(y_i(t))y_i(t) \\ & + \sum_{j=1}^n c_{ij}(y_i(t))f_j(y_j(t)) + I_i] + u_i(t), \end{aligned} \quad (22)$$

where $t \geq 0$ and $i \in \mathcal{J}$. In this case, $\gamma_i = 0$ and hypothesis $\mathbf{H}_3$ turns to the following condition:

$$\tilde{\mathbf{H}}_3 \quad \xi_i + \eta_i > 0 \text{ for any } i \in \mathcal{J}.$$

Then, the following results can be obtained readily from Theorem 1.

Corollary 2 Suppose that $\mathbf{H}_1, \mathbf{H}_2$ and $\tilde{\mathbf{H}}_3$ hold. Then system (21) and system (22) are exponentially function projective synchronized under the hybrid switching controller $u_i(t) = u_{i1}(t) + u_{i2}(t) + u_{i4}(t)$ with the synchronization rate $\tilde{\lambda}$, where $\tilde{\lambda}$ satisfies $\xi_i + \eta_i - \tilde{\lambda} > 0$ for any $i \in \mathcal{J}$.

In system (18), if the memristor-based time-varying delay strength $d_{ij}(\cdot) = 0$ for all $i, j \in \mathcal{J}$, then system (18) is reduced to following form

$$\dot{x}_i(t) = -b_i(x_i(t))x_i(t) + \sum_{j=1}^n c_{ij}(x_i(t))f_j(x_j(t)) + I_i, \quad i \in \mathcal{J}. \quad (23)$$

Accordingly, the corresponding response system (19) is degenerated to following form

$$\begin{aligned} \dot{y}_i(t) = & -b_i(y_i(t))y_i(t) + \sum_{j=1}^n c_{ij}(y_i(t))f_j(y_j(t)) + I_i + u_i(t), \\ & i \in \mathcal{J}. \end{aligned} \quad (24)$$

Similarly, from Corollary 1, we have a following result.

Corollary 3 Suppose that the hypotheses $\mathbf{H}_2$ and $\tilde{\mathbf{H}}_3$ hold, then the drive-response systems (23) and (24) are exponentially function projective synchronized under hybrid switching controller $u_i(t) = \bar{u}_{i1}(t) + \bar{u}_{i2}(t) + \bar{u}_{i4}(t)$ with the synchronization rate λ' , where λ' satisfies $\bar{\xi}_i + \eta_i - \lambda' > 0$ for any $i \in \mathcal{I}$.

Remark 3 In Yang et al. (2014), by nonlinear transformation, the authors studied the complete synchronization of memristor-based Cohen–Grossberg neural network by using a novel discontinuous feedback controller. In this paper, for the special case $\alpha(t) = 1$, the complete synchronization can be achieved. Also, we do not require that the activation functions are bounded, which is the main assumption of Yang et al. (2014). From these points, our results are more general.

Remark 4 In Zhang et al. (2013), the authors investigated the anti-synchronization of memristor-based recurrent neural networks. In Wu et al. (2012), Wen et al. (2013), Li and Cao (2015), Zhang and Shen (2013, 2014), based on the theory of differential inclusions, the authors studied the complete synchronization of various types of memristor-based cellular neural networks. In this paper, for the special case $\alpha(t) = -1$, the anti-synchronization can be achieved, and for the special case $\alpha(t) = 1$, the complete synchronization can be achieved. In addition, it is known that Cohen–Grossberg neural network includes some well-known neural networks such as Hopfield neural networks, cellular neural networks and recurrent neural networks as a special case. From this point, we can conclude that our results are more practical than those in Zhang et al. (2013), Wu et al. (2012), Wen et al. (2013), Li and Cao (2015), Zhang and Shen (2013), Zhang and Shen (2014).

Remark 5 In this paper, for the first time, we study the exponential function projective synchronization of memristor-based Cohen–Grossberg neural networks with time-varying delays. Because of the unpredictability of the scaling function $\alpha(t)$ in this function projective synchronization can additionally enhance the security of communication than other complete synchronization or anti-synchronization papers (Yang et al. 2014; Zhang et al. 2013; Wu et al. 2012; Wen et al. 2013; Li and Cao 2015; Zhang and Shen 2013, 2014). From this point, our results are superior and have greater applicability.

Numerical simulations

In this section, an example is given to illustrate the effectiveness of our results obtained in this paper.

Example 1 Consider the 2-dimensional memristor-based Cohen–Grossberg neural networks with time-varying delays given by

$$\dot{x}_i(t) = a_i^k(x_i(t)) \left[-x_i(t) + \sum_{j=1}^2 c_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^2 d_{ij}(x_i(t)) g_j(x_j(t - \tau_j(t))) \right], \quad (25)$$

where $i, k \in \{1, 2\}$, $a_1^1(u) = a_2^1(u) = 1.1 + 0.72 \sin(u)$, $a_1^2(u) = 0.6 + 1.4/(1 + u^2)$, $a_2^2(u) = 0.8 + 1.6/(1 + u^2)$, $f_1(u) = g_1(u) = 1.2 \sin(u)$, $f_2(u) = g_2(u) = 1.4 \tanh(u)$, $\tau_1(t) = e(t)/(1 + e(t))$, $\tau_2(t) = 0.75 - 0.25 \cos(t)$, and

$$c_{11}(v) = \begin{cases} 0.98, & |v| \leq 1, \\ 1.01, & |v| > 1, \end{cases} \quad c_{21}(v) = \begin{cases} 2.8, & |v| \leq 1, \\ 2.5, & |v| > 1, \end{cases}$$

$$c_{21}(v) = \begin{cases} 0.8, & |v| \leq 1, \\ 1, & |v| > 1, \end{cases} \quad c_{22}(v) = \begin{cases} 1.78, & |v| \leq 1, \\ 1.82, & |v| > 1, \end{cases}$$

$$d_{11}(v) = \begin{cases} -1.5, & |v| \leq 1, \\ -1.2, & |v| > 1, \end{cases} \quad d_{12}(v) = \begin{cases} 1, & |v| \leq 1, \\ 0.8, & |v| > 1, \end{cases}$$

$$c_{11}(v) = \begin{cases} 0.8, & |v| \leq 1, \\ 1, & |v| > 1, \end{cases} \quad d_{22}(v) = \begin{cases} -1.4, & |v| \leq 1, \\ -1.6, & |v| > 1, \end{cases}$$

The numerical simulation of system (25) with initial conditions $x_1(s) = 1.45$ and $x_2(s) = 0.6$ for $s \in [-1, 0]$ with respect to two forms of amplification functions are given in Figs. 1, 2. We can see that, for different amplification functions, the system (25) has different dynamical behaviors such as stable periodic solution (Fig. 1) and chaotic attractor (Fig. 2). This shows that the amplification

Fig. 1 The stable periodic solution of system (25) with amplification function $a^1(u)$

Fig. 2 The chaotic attractor of system (25) with amplification function $a^2(u)$

function has a great impact on the dynamic behavior of the system.

Consider the exponential function projective synchronization of driving system (25) with response system as follows

$$\dot{y}_i(t) = a_i^2(y_i(t)) \left[-y_i(t) + \sum_{j=1}^2 c_{ij}(y_i(t)) f_j(y_j(t)) + \sum_{j=1}^2 d_{ij}(y_i(t)) g_j(y_j(t - \tau_j(t))) \right] + u_i(t), \quad (26)$$

where the parameters $a_i^2(v)$, $c_{ij}(v)$, $d_{ij}(v)$, $f_j(v)$, $g_j(v)$ and $\tau_j(t)$ are the same as defined in system (25).

It is not difficult to check that $\tau = 1$, $\underline{a}_1^2 = 0.6$, $\bar{a}_1^2 = 2$, $\underline{a}_2^2 = 0.8$, $\bar{a}_2^2 = 2.4$, $\underline{L}_1^f = \underline{L}_1^g = -1.2$, $\bar{L}_1^f = \bar{L}_1^g = 1.2$,

Fig. 3 The evaluation of synchronization errors and curves for $\alpha(t) = -1$

Fig. 4 The evaluation of synchronization errors and curves for $\alpha(t) = 0$

Fig. 5 The evaluation of synchronization errors and curves for $\alpha(t) = 1$

$\underline{L}_2^f = \underline{L}_2^g = 0$, $\bar{L}_2^f = \bar{L}_2^g = 1.4$, $\xi_1 = -9.6640$, $\xi_2 = -8.1952$, $\gamma_1 = 6.4000$ and $\gamma_2 = 8.2560$. Choosing $\eta_1 = 16.1$ and $\eta_2 = 16.5$ and letting $\zeta_i = \xi_i + \eta_i - \gamma_i$ for $i \in \{1, 2\}$, then by simple computation we get $\zeta_1 = 0.0360 > 0$, $\zeta_2 = 0.0488 > 0$. So hypotheses $\mathbf{H}_1 - \mathbf{H}_3$ are all satisfied. Thus, according to Theorem 1, the systems (25) and (26) are exponentially function projective synchronized. In Fig. 3, the synchronization errors (left) and curves (right) between drive-response systems (25) and (26) with scaling function $\alpha(t) = -1$ are shown. Similarly, the function projective synchronization between system (25) and system (26) with scaling functions $\alpha(t) = 0$ and $\alpha(t) = 1$ are simulated in Figs. 4, 5, respectively.

Conclusion

In this paper, under the framework of Filippovs solution for differential equations with discontinuous right-hand side as introduced by Filippov, we investigated the exponential function projective synchronization for a class of memristor-based Cohen–Grossberg neural networks with time-varying delays by combining switching open-loop control and linear feedback control. It is worthwhile to note that the anti-synchronization, complete synchronization and the stabilization of the considered system can be achieved as some special cases of our main results. It is believed that our results may provide some practical guidelines for secure communication and other engineering applications.

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