Two-stage radio-frequency interferometer sensors

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We show that simple radio-frequency (RF) interferometers can have slow-wave positive group delay (PGD) or negative group delay (NGD), as well as superluminal propagation (SP) regions, due to a destructive interference process. These properties are easily tunable, which makes RF interferometers unique among systems that have NGD and SP regimes. A two-stage interferometer arrangement, which includes a first stage interferometer in the material-under-test path of a second stage, has significantly improved sensitivity in comparison with a one-stage reference interferometer. With a power divider based first stage and at its maximum NGD frequency, the frequency sensitivity improvement is as high as 7 times. With a quadrature based first stage, the sensitivity is increased by as much as 20 times. Sensitivity improvements are also observed at PGD and SP frequency regions. © 2015 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4934622>]

Radio-frequency (RF) interferometers have been demonstrated for high sensitivity detection and analysis of single cells,^{1,2} particles,³ DNAs,⁴ glucose, methanol, and 2-propanol^{5–7} in solution. Their design flexibility, tunable sensitivity, and frequency coverage are attractive properties for various sensing applications. Nevertheless, for high sensitivity measurements, the required system dynamic range is exceptionally large, e.g., ~ 120 dB for an effective quality factor of $\sim 3 \times 10^6$.⁷ As a result, such systems are difficult to use in rugged environments. To help address this problem, different filters and resonators^{8–10} are proposed and demonstrated to improve sensitivity for a given dynamic range. With engineered spectrum and dispersion relationship, the probing RF waves in these devices are slowed down with enhanced field intensities, which lead to longer and stronger interactions between RF fields and material-under-test (MUT). The group delays, which are defined as $t_g = -d\varphi/d\omega|_{\omega_0}$ with φ is the frequency dependent phase shift and ω_0 is the operating frequency, of these filters and resonators are positive. Corresponding group velocities, v_g , are lower than the speed of light, c , in the medium, i.e., $v_g < c$.

In this work, we show that an RF interferometer can be incorporated into a second one to form a two-stage interferometer to achieve up to 20-times higher frequency sensitivity, compared with one-stage interferometers. The improvements are better than that with filters and resonators.^{8–10} We also show that a single stage interferometer has negative group delays (NGDs), i.e., negative group velocities (NGVs), and superluminal propagation (SP) regimes. Such abnormal electromagnetic properties have been reported in a few other structures and circuits.^{11–14} In RF interferometers, these properties occur around the operating frequency points and are caused by destructive interference of two traveling waves at the output port. The process is similar to that at the dark port of a Mach–Zehnder interferometer.¹³ A simple model shows that loss or reflection is not necessary for the RF interferometer to exhibit those properties. At the same time, large positive group delays (PGDs) can be easily obtained. The achievable NGD, slow wave PGD, and SP levels can be easily tuned. Such tunability is not available in other passive

circuits. Furthermore, these abnormal properties can be exploited for high sensitivity sensor applications.

Fig. 1(a) shows the schematic of a two-stage interferometer, in which the first (i.e., inner) stage uses two commercial power dividers. No tuning components, such as R and Φ in the second (outer) stage, are used for the first stage to simplify the design and operation of the two-stage interferometer. Simple microstrip lines (MLs) on Rogers RO4003C substrate are built and used as high frequency electrodes for MUT detection. A second one-stage interferometer at ~ 2 GHz, Fig. 1(b), is also designed and built as the first stage in Fig. 1(a). The use of quadrature hybrids may enable some measurements not available from power divider settings.¹⁵

Figs. 2(a) and 2(b) show the measured $|S_{21}|$ and group delay, respectively, of the first stage in Fig. 1(a). The constant positive group delay, ~ 4 ns, is associated with the physical delay of the circuit. Around ~ 1.1 GHz and 3.2 GHz, which are the fundamental and 1st harmonic operating frequency, respectively, NGD, slow waves and SPs exist. For slow waves, we have $0 < |v_g| < c$, while for SP, $|v_g| > c$, where c is the speed of RF waves along the uniform microstrip line. The NGD indicates negative group velocities, $v_g < 0$.¹⁴ Simulation results with Advanced Design System (ADS) agree with the measured results in Fig. 2(b) when $|S_{21}|_{\min}$ is tuned to fit the measured $|S_{21}|_{\min}$ in Fig. 2(a). Nevertheless, simulations also show PGD operations in Fig. 2(b) at different R . A simple analysis can help understand the key factors that determine the observed main features.

Consider an input RF signal (with unit amplitude) that is split by the first broadband power divider into two. At the inputs of the second power divider, we have

$$S_{MUT} = A_1 e^{-j\beta_1 l_1} \quad S_{REF} = A_2 e^{-j\beta_2 l_2}, \quad (1)$$

where $A_{1,2}$ are the magnitudes of the transmission coefficients, $\beta_{1,2}$ are the propagation constants, and $l_{1,2}$ are the electrical lengths of the transmission lines. The output S_{21} of the second power divider will be

$$S_{21} = A_{21} e^{-j\Phi_{21}} = \frac{A_1}{\sqrt{2}} e^{-j(\beta_1 l_1 + \pi/2)} + \frac{A_2}{\sqrt{2}} e^{-j(\beta_2 l_2 + \pi/2)}. \quad (2)$$

FIG. 1. (a) The schematic of a two-stage RF interferometer. Attenuator R and phase shifter Φ are used for sensitivity and operating frequency tuning. Two commercial power dividers are used for the first stage, and two quadrature hybrids are used for the second stage. A polydimethylsiloxane (PDMS) slab is used as MUT and is placed on one path of the first stage. The paths are formed with simple and uniform microstrip lines. (b) A first stage built with quadrature hybrids.

Assume A_1 and A_2 are frequency independent around the operating frequency, taking the negative frequency derivative of (2), $-d/d\omega$, gives

$$jA_{21}e^{-j\Phi_{21}}\tau_g - \frac{dA_3}{d\omega}e^{-j\Phi_{21}} = \frac{1}{\sqrt{2}} \left[jA_1e^{-j\Phi_1}\tau_1 - \frac{dA_1}{d\omega}e^{-j\Phi_1} + jA_2e^{-j\Phi_2}\tau_2 - \frac{dA_2}{d\omega}e^{-j\Phi_2} \right], \quad (3)$$

where $\Phi_{1,2} = \beta_{1,2}l_{1,2} - \pi/2$, τ_g is the total group delay, and $\tau_{1,2}$ are the group delays of the individual transmission lines. Equating the imaginary parts of (3) gives the total group delay as

$$\tau_g = \frac{A_1e^{-j\Phi_1}\tau_1 + A_2e^{-j\Phi_2}\tau_2}{A_1e^{-j\Phi_1} + A_2e^{-j\Phi_2}}, \quad (4)$$

while $|S_{21}|_{\min}$ occurs at frequencies where the two branches have a phase difference of

$$\beta_1l_1 - \beta_2l_2 = (2n - 1)\pi, \quad (5)$$

where $n=1$ is the fundamental operating frequency of the interferometer. Under the condition of (5) the total group delay from (4) becomes

$$\tau_g = \frac{A_1\tau_1 - A_2\tau_2}{A_1 - A_2}. \quad (6)$$

The above equation implies that if A_1 and A_2 are the same, the group delay goes to infinity. However, if they differ, the group delay can be either positive or negative, depending on which signal is larger, as shown in Fig. 2(b). When $A_1\tau_1 = A_2\tau_2$, the group delay is 0, i.e., superluminal propagation. Thus, through the summation of two traveling waves, i.e., an interference process, an RF interferometer exhibits unique dispersion characteristics with minimal reflections occurring only in the summation device. Other NGD electrical structures and circuits, such as the electromagnetic structures of a backward wave oscillator (BWO),¹⁶ the distributed transmission line filter,¹⁷ left-handed media,^{12,18} and the active circuits,^{13,19} involve complicated wave reflection and propagation processes.

FIG. 2. Experimental results of (a) $|S_{21}|$ and (b) group delay of the first stage in Fig. 1(a). Simulation results agree with measured ones while simulation also indicates that PSD regimes can exist. Furthermore, the group delay values mainly depend on the balance of the two signal paths, i.e., $|S_{21}|_{\min}$. Measured (c) $|S_{11}|$, $|S_{21}|$, and (d) group delay of the first stage in Fig. 1(b).

FIG. 3. (a) Typical measurement results of the two-stage interferometer in Fig. 1(a). (b) Typical measurement results of the reference interferometer for Fig. 1(a). (c) Measured sensitivity improvement factor vs. frequency.

Fig. 2(c) shows that at the design frequency, ~ 2 GHz, both $|S_{11}|$ and $|S_{21}|$ are small, as expected for the structure in Fig. 1(b). Fig. 2(d) shows the measured group delay, where the ~ 1 ns constant group delay is shorter than that in Fig. 2(b) due to shorter physical structure length. The -15 ns group delay indicates slower waves than those in Fig. 2(b). The slower waves are associated with lower $|S_{21}|$ values in Fig. 2(c), as indicated in Eq. (6). Both $|S_{11}|$ and $|S_{21}|$ in Fig. 2(c) indicate strong reflections within the structure due to discontinuities. Such reflections may be an issue for many signal processing applications. However, for the investigation in this work, the reflections induce group delay patterns in Fig. 2(d) and allow convenient study of group delay effects on interferometer sensitivities.

To test two stage interferometer operations, we first tune R and Φ in Fig. 1(a) to the desired frequency, f_0 , and $|S_{21}|_{min}$ (~ -62 – -63 dB), as shown in Fig. 3(a). Then, a MUT is added to observe frequency shift, Δf , and $|S_{21}|_{min}$ change. Both parameters can be used as sensitivity indicators. A one-stage reference interferometer is also tested for sensitivity comparisons, as shown in Fig. 3(b).

The MUT used in the tests is a polydimethylsiloxane (PDMS) slab with dimensions of $4 \text{ mm} \times 2.5 \text{ mm} \times 2 \text{ mm}$. For the two-stage in Fig. 1(a), the MUT is placed on a microstrip line (MUT-ML). For one-stage reference interferometer, the same MUT-ML is used to replace the first stage of Fig. 1(a). Additional cables are added to the MUT-ML section so the electrical length is similar to that of the first stage. For the quadrature stage in Fig. 1(b), two measurements are conducted with the MUT positioned at the center of the microstrip line on path 1 or path 2.

Figs. 3(a) and 3(b) show that the two-stage interferometer has much larger Δf , i.e., frequency sensitivity. Nevertheless, the curves in Fig. 3(a) are less sharp than those in Fig. 3(b), indicating a lower effective quality factor, Q_{eff} ,

and lower frequency reading resolution. The frequency dependent signal amplitude from the first stage, i.e., dispersion in Figs. 2(a) and 2(b), is likely the main factor that affects the Q_{eff} . Nevertheless, Δf is not determined by Q_{eff} , instead, it is determined by the effective electrical length difference of the two branches as discussed in Ref. 10. Thus, the counterintuitive results are understandable.

Fig. 3(c) shows the measured sensitivity improvement factor, which is defined as $\Delta f_{two-stage} / \Delta f_{one-stage}$ at the same operating frequency ω_0 and $|S_{21}|_{min}$. Also plotted is the measured group delay. It shows that sensor sensitivity is improved when operating at or near the fundamental and 1st harmonic frequencies of the first stage interferometer, regions that also have large NGDs. Second, between the two NGD regions, $\Delta f_{two-stage} / \Delta f_{one-stage}$ is smaller than 1, which is expected. Within this region, the two-stage interferometer becomes a one-stage interferometer, but with slightly lower sensitivities since the first-stages are effectively straight transmission lines. Yet, the RF probing power for MUT detection is 3 dB lower when compared with the probing power in the reference interferometer. Although this indicates a correlation between group delay and sensor sensitivity, the mechanisms responsible for the sensitivity enhancement are not fully understood. Fig. 3(c) also indicates that the two- and one-stage interferometers are operating at the same harmonic (fundamental) frequencies, so the comparisons in Fig. 3 are fair.¹⁰

Fig. 4(a) shows typical measurement results with the quadrature hybrid stage in Fig. 1(b). MUT is placed on each of the paths. So, the curves shifted left and right. The obtained $\Delta f_{two-stage}$ is ~ 3200 kHz compared to ~ 150 kHz in the one-stage reference interferometer, which is an improvement of over 20 times. Fig. 4(b) summarizes the measured $\Delta f_{two-stage}$ from 1 GHz to 3.5 GHz. It shows that $\Delta f_{two-stage}$ follows the group delay pattern. However, the slow wave with positive group delay, Fig. 2(d), yields the highest sensitivity.

FIG. 4. (a) Typical measured results of the two-stage interferometer with the first-stage in Fig. 1(b). (b) Measured $\Delta f_{two-stage}$ vs. frequency.

In conclusion, we demonstrate that a single stage RF interferometer has NGD and SP properties. Depending on the relative power ratio of the two signal paths, frequency regions with large PGD can also exist. These abnormal frequency regimes are formed due to RF guided wave interference. Nevertheless, MUT still interacts with the traveling waves. The arrangement of two-stage interferometers enables high sensitivity operations. When the first stage is built with commercial power dividers, the measured frequency sensitivity is increased by up to 7 times, which is comparable to the improvements with filters and resonators. When using a quadrature hybrid based first stage, the frequency sensitivity improvement is up to 20 times. The improvements occur at frequencies where the signal group delay is slow, regardless if it is positive or negative. Thus, the two-stage RF interferometers, like other microwave sensors,²⁰ are attractive choices for many of biomedical, electronic, and industrial applications.

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