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Myocardial Perfusion: Characteristics of Distal Intramyocardial Arteriolar Trees

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Abstract

A combination of experimental, theoretical, and imaging methodologies is used to examine the hierarchical structure and function of intramyocardial arteriolar trees in porcine hearts to provide a window onto a region of myocardial microvasculature which has been difficult to fully explore so far. A total of 66 microvascular trees from 6 isolated myocardial specimens were analyzed, with a cumulative number of 2,438 arteriolar branches greater than or equal to 40 μ m lumen diameter. The distribution of flow rates within each tree was derived from an assumed power law relationship for that tree between the diameter of vessel segments and flow rates that are consistent with that power law and subject to conservation of mass along hierarchical structure of the tree. The results indicate that the power law index increases at levels of arteriolar vasculature closer to the capillary level, consistent with a concomitant decrease in shear stress acting on endothelial tissue. These results resolve a long standing predicament which could not be resolved previously because of lack of data about the 3D, interconnected, arterioles. In the context of myocardial perfusion, the results indicate that the coefficient of variation of flow rate in pre-capillary distal arterioles is high, suggesting that heterogeneity of flow rate in these arterioles is not entirely random but may be due at least in part to active control.

Keywords

Shear stress; Murray's law; Cube law; Microvasculature; Pre-capillary arterioles

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INTRODUCTION

Microvascular trees at the pre-capillary levels of vasculature constitute the final hierarchical branching stages of the arterial tree where the general branching structure is still in place before the very different branching structure of the capillary bed is reached. The geometrical characteristics of these microvasculature trees, particularly within the ventricular walls, provide an important window onto the mechanics of myocardial perfusion. To date, only a hypothetical determination of these characteristics has been possible, based on extrapolation from central levels of the arterial tree and microscopic visualization of individual arterioles. The aim of this study was to explore this vasculature deeper than has been possible in the past by using a combination of imaging and theoretical constructs. While we have succeeded in reaching vessel diameters below 10 μm lumen diameter, the results we report are based on only vessels that are 40 μm lumen diameter or larger to conform with the resolution limitations of imaging process. Thus, this is the sense in which the term “pre-capillary” is used here and in what follows.

The most important unknown at the pre-capillary levels of vasculature is the shear stress acting on endothelial tissue because it is generally believed to be the main mechanism by which the endothelial cells sense and respond to changes in flow rate⁶. In the intramyocardial coronary circulation in particular, the shear stress acting on the endothelium in pre-capillary arterioles is likely to play a key role in the mechanisms and control of myocardial perfusion^{2, 12, 15}. To date there has been no definitive measurement or theory to indicate what the level or distribution of shear stress at this level of the coronary vasculature might be, and how these relate to the implementation and control of myocardial perfusion.

Direct measurement of shear stress is fraught with obvious experimental difficulties, and in the microvasculature these difficulties are severely compounded because of the micro scale of and therefore the lack of direct access to these vessels within the myocardium. Vascular imaging methods with spatial resolutions equivalent to micro-CT, such as optical coherence tomography and intravascular ultrasound, provide high resolution detail of the local arterial wall but involve introducing an intravascular catheter into the artery of interest (which in our case is less than 1 mm in lumen diameter) and other methods such as transcutaneous ultrasound and clinical Computed Tomography and Magnetic Resonance Imaging have insufficient resolution to allow measurement of intramyocardial arterioles. Hence, micro-CT has been the most promising alternative avenue for providing access to the arteriolar vasculature within intact myocardium because it images volumes of the order of cm^3 with resolution down to the micrometer level. In our particular application we used 20 μm voxel resolution. The main challenge here has been to improve the fidelity of vascular imaging to the point where it can provide accurate quantitative description of the geometrical parameters of vascular branching at the pre-capillary scale.⁹

In this study we explore this avenue and present geometrical data relating to trees within the left ventricular wall of porcine hearts and propose a theoretical basis of using these data to obtain information about the level and distribution of shear stress within these vessels.

Theoretical considerations of the distribution of shear stress at different levels of the arterial tree are faced with a predicament which as yet has not been resolved. Based on the classical assumptions of Poiseuille flow in a tube, the shear stress τ is given by²⁴

$$\tau = \frac{32\mu}{\pi} \frac{q}{d^3} \quad (1)$$

where μ is the viscosity of the fluid, d is the tube diameter and q is the flow rate. The distribution of shear stress at different levels of the arterial tree based on the assumption of Poiseuille flow, therefore, depends on the ratio of flow rate to the cube of tube radius.

While it seems intuitive, as well as evident from direct observations, that in the hierarchical structure of the arterial tree the caliber of an artery is related to the flow rate which the artery carries, and more specifically that smaller flow rates would be carried by vessels with smaller caliber, the exact relationship between flow rate and caliber at different hierarchical levels (i.e., generations) of the arterial tree is not known. Classical studies by Murray¹⁴ posed this question in the context of an arterial bifurcation and assumed a power law relation of the form

$$q \propto d^x \quad (2)$$

where x is an unknown index, and then found that for optimal efficiency of flow through the junction, $x = 3$. This was based on minimizing the energy required to drive the flow through a vascular junction at which typically a parent vessel divides into two smaller branches. Considering the vast number of vascular junctions within the cardiovascular system, this so called “optimality principle” was well justified and subsequently well supported by a considerable body of physiological data.^{5, 13, 17, 19, 22} This value of the power law index ($x = 3$) has also become known as the “cube law”. The validity of the cube law at all the hierarchical levels of the arterial tree soon came into question, however, and other values of x based on different optimality principles were considered by other authors.^{18, 19, 21} The underlying predicament was that if $q \propto d^3$, the shear stress as determined by Eq. (1) would become independent of the vessel radius and thus be the same at all levels of the arteriolar tree, including the pre-capillary levels. The notion that the shear stress acting on endothelial tissue in large vessels such as the aorta is the same as that acting at a pre-capillary arteriole has been questioned on physiological grounds. It is generally believed that the shear stress must be lower at peripheral levels of the vascular tree, which implies (Eq. 1) that the value of the power law index x must be higher at peripheral levels of the arterial tree. While this seems reasonable on physiological grounds, however, to date it has not been tested because of lack of data within the intact myocardium. The main aim of our study was to obtain values of x , for the first time we believe, based on microvascular imaging of distal arteriolar trees in porcine hearts.

MATERIALS AND METHODS

Experimental Methods

Animal experimental protocol—This protocol was approved by the Institutional Animal Care and Use Committee of the Mayo Clinic. It involved 30 kg pigs which were

sedated with Telazol (4.4 mg/kg) / Ketamine (2.2 mg/kg) / Xylazine (2.2 mg/kg) (TKX) IM and maintained with isoflurane at 1–3%.

Prior to catheterization, 10,000 units of heparin was injected IV then a catheter was advanced percutaneously via the carotid artery into the left anterior descending (LAD) coronary artery and another via the jugular vein into the superior vena cava (SVC). The animal then underwent a whole body CT scan [Siemens “flash” dual source] during injection of a bolus of non-ionic contrast agent (Omnipaque™ 350, 350 mg/iodine/ml) into the SVC so that the myocardial perfusion phase could be recorded.

After completing the CT scans the jugular vein was injected with an additional 10,000 units of heparin following which the animal was euthanized. The heart was removed and the left main coronary artery cannulated so that the blood could be flushed out with heparinized saline and then injected with Microfil® (a silicon-based polymer doped with lead; Flow Tech. Inc., Carver Massachusetts) at 100 mm Hg infusion pressure. Once the Microfil® emerged from the Coronary sinus the cannula was clamped and the heart left over night in a refrigerator and then placed in 10% formalin the next day. Two transmural “biopsies” (each 2 cm long and 1 cm × 1 cm in cross section) were cut out, one from the circumflex perfusion territory and one from the LAD perfusion territory. These specimens were then patted dry and placed in a tightly fitting thin-wall plastic cylinder and immersed in low melting point paraffin and were then scanned in our micro-CT scanner. The isolated heart was then scanned in the whole-body CT scanner so that the anatomic location of the myocardial “biopsies” could be identified relative to the epicardial coronary vessels. Each of the six animals studied supplied two myocardial “biopsies”. The number of trees within each of the two specimens for each animal (except one in which one “biopsy” was inadvertently destroyed) were (6,5), (3,9), (5,7), (6,6), (8,3), (8). As the number of trees per “biopsy” and per animal are quite comparable we assume that there is no skewing of the data towards one animal or “biopsy”.

Micro-CT imaging—Microvascular imaging requires much higher fidelity than other applications for several reasons. First, the image resolution must be consistent with the micro scale of the vessels involved. Second, the image quality must be sufficiently high to preserve the 3D hierarchical integrity of the vascular tree being imaged. Third, the contrast levels must be sufficiently high to provide accurate measures of the diameters of the individual microvessel segments comprising the tree.

The micro-CT scanner⁷ we used consists of an x-ray source (Molybdenum anode with Zirconium filter so as to generate 18 ± 1 keV x-ray photons), a fluorescent crystal plate (CsI(Tb)) that converts the x-ray shadow image of a trans-irradiated specimen into a light image. An optical lens projects that light image (within the crystal) onto a Peltier-cooled Charge Coupled Device (CCD) imaging array. Each pixel in the CCD array generates an electrical signal proportional to the intensity of the light and thereby of the x-ray in the shadow image. The scanner x-ray and optical magnifications were adjusted so that each pixel sampled an x-ray beam through the specimen had a 20 μ m on a side square area. The specimen was trans-irradiated until a good quality shadow image was generated on the CCD, at which time the CCD pixel signals were transferred via an analog to digital

converter to a computer memory. The specimen was then rotated by one degree and the process repeated until the specimen had been rotated through 360°.

Those projection images were then subjected to a modified Feldkamp³ tomographic reconstruction process to generate a 3D image of the specimen. These three-dimensional CT images consist of a 3D array of voxels (20 µm on a side cubic). Each voxel has a gray-scale value that is proportional to the x-ray attenuation coefficient of the small piece of tissue represented by that voxel.

The average x-ray attenuation of the specimen soft tissue (mostly myocardium) was of the order of 0.5/cm whereas that of the Microfil[®] injected into the coronary circulation had a value of the order of 3/cm. This big difference in attenuation coefficients permitted discrimination of the radiopaque microvasculature from the muscle tissue it is embedded in.

Image analysis—Analysis of the reconstructed micro-CT data was carried out using a dedicated imaging software package (Analyze 11.0, Biomedical Imaging Resource, Mayo Clinic College of Medicine, Rochester, MN, USA).¹⁶ Microvascular trees were extracted from this vasculature with the criterion that only intact trees that lie entirely within the space of this specimen were selected (Fig. 1). This was done by selecting an initial vascular segment and tracing all its branches and sub-branches until all its terminal segments were reached, using a “connect” feature within the Analyze imaging software. If all the terminal segments were within the space of the specimen, then the initial vascular segment was declared as the root of a selected tree. The microvasculature within the 3D images is distinguishable from the muscle in which it is embedded by virtue of the high x-ray attenuation coefficient of the Microfil[®] contrast agent relative to the low attenuation coefficient of the muscle tissue itself. Unfortunately, to isolate the microvasculature by simple gray-scale thresholding the image at a gray-scale just greater than that of muscle will not work well for the smaller micro-vessels as they approach voxel size in diameter, as explained below. This is because the 3D tomographic image is blurred which results in reduced gray-scale and broadening of the cross-sectional diameter of the blood vessel lumen.

One reason for the blurring of the x-ray shadow image as it is projected onto the CCD imaging array, which is a function of the Modulation Transfer Function (MTF) of the imaging system. A perfect MTF means that the gray-scale is correctly conveyed even for the smallest objects within the image. In reality, and as it relates to microvascular imaging, the gray-scale progressively decreases as the size of the vessel decreases. For our micro-CT scanner the grayscale of a 20 µm object is 10% of what it should be. Thus, if the intrinsic variation (i.e., noise) between detector pixels is of the order of 1%, an object would still be clearly detectable even at 10% modulation. However, along with the reduced gray-scale of the contrast agent the small-diameter vessels the imaged diameter of the vessel is broadened so that the vessel diameter appears greater than it is in fact.

Another reason for reduced grey scale is partial volume effect due to the pixelization and voxelization of the projection and CT images respectively. If a 20 µm object happens to be completely superimposed on a 20 µm pixel (or voxel) then the gray-scale value of the pixel

(voxel) would be equal to that of the object (ignoring the MTF blurring effect). However, if the object straddles two adjacent pixels (or in 3D, four contiguous voxels) then those adjacent pixels (voxels) could have reduced gray-scale values depending on the fraction of the object within the pixel (voxel) and the apparent diameter of the vessel lumen is doubled. We largely overcome this problem by “undoing” the MTF effect as explained in more detail previously⁷ and by considering only microvessels greater than or equal to two voxels in diameter. Therefore, because the voxel size in this study was 20 μm , we have excluded microvessels with diameters less than 40 μm as explained in the next section.

Theoretical Considerations

We use j, k notation²³ to identify vessel segments within a given tree, such that $j = 0, 1, 2, 3 \dots$ refer to the root (generally at the epicardial surface or within the myocardium) and subsequent levels (or generation numbers) of the tree, and $k = 1, 2, 3 \dots$ refer to the sequence of vessel segments at that level as shown in Fig. 2. For illustrative purposes the Table shows the branch segment diameters and lengths for a typical microvascular tree. The progression from one level of the tree to the next consists of a sequence of bifurcations at each of which a parent segment (j, k) splits into two daughter branches $(j + 1, 2k - 1)$ and $(j + 1, 2k)$. By convention, the first of these (with the odd value of k) is taken as the daughter branch with the larger diameter. Thus, the root segment of the tree will be designated as $(0, 1)$ and its two branch segments as $(1, 1)$ and $(1, 2)$, and the two branch segments of $(5, 7)$, for example, are designated as $(6, 13)$ and $(6, 14)$. While the imaged trees are actually sub-trees of the entire coronary arterial tree as a whole, which itself is a sub-tree of the systemic arterial tree, the j, k notation shall be used autonomously for each sub-tree, independently of the larger tree of which it is a part. The term “bifurcation” is generally used in a wide sense to encompass the full range of dichotomous junctions, from a symmetrical one in which two branches have equal diameters to a highly asymmetrical one in which one branch is much smaller than the other as is the case of a “side branch”. While in this case the parent trunk appears to continue unchanged past the site of the side branch, it has actually changed because it now carries only part of the flow. Mathematically this is generally presented in terms of a bifurcation index α , defined as the ratio of the two branch diameters, the smaller of the two divided by the other. The full range of bifurcations is covered by values of α between 1.0 in the case of symmetrical bifurcation and near zero in the case of a side branch.

A tree analysis program developed in-house was used to track and label the entire segments of each captured tree using a Fast Marching Method for labelling each voxel within the tree with its distance from the object surface, thus providing the position coordinates (j, k) and diameter $d_{j,k}$ of each segment within that tree. The integrity of each tree was tested to ensure that there are no “orphan” segments within the tree, specifically that each terminal segment could be traced back continuously to the root segment of the tree, a terminal segment being defined as one that did not give rise to any daughter branches. It is important to note that this definition is to be construed not in the context of anatomy but in the context of imaging resolution. Thus, a non-terminal vessel segment is deemed to be terminal if it gives rise to branch segments that are below the threshold of image resolution and are therefore excluded. It is important to note further that the exclusion of these branches segments does not upset the flow rate balance within the tree because the flow rate that would have been

carried by the excluded branch segments is accounted for by the flow rate carried by their parent segment.

In general, terminal segments may occur at different hierarchical levels of the tree, which means that tree structures are in general not uniform, that is not all vessel segments lead to the same number of descendants. Additional notation is therefore required to identify terminal segments which we selected to be greater than 40 μm in diameter. We shall use a single subscript t to identify terminal segments, with a superscript i as a running index, $i = 1, 2, \dots, N$ where N is the number of terminal branches in a given tree. Thus, the diameters of the terminal branches in a given tree shall be denoted by d_t^i and the corresponding flow rates by q_t^i .

The first step in our theoretical approach was to invoke conservation of flow rate in a given tree, namely

$$q_{0,1} = \sum_{i=1}^N q_t^i \quad (3)$$

where $q_{0,1}$ is the flow rate in the root segment of that tree. This equality was based on the assumption that any “missing” branches from that tree, whether caused by incomplete filling or by imaging threshold issues, would be represented by their surviving parents, thus flow rate is conserved despite any such missing branches.

As a corollary of the above, and to comply with the resolution limit of the micro-CT scans as discussed in the previous section, any terminal branches with diameters that are less than 40 μm were intentionally removed from the tree and its “place” taken by either a sibling or parent with diameter greater or equal to 40 μm , thereby, again, conservation of flow rate is satisfied.

The next step in this process was to invoke a power law relation between diameter and flow rate as in Eq. (2) and applying it to the hierarchy of each tree. Since in a given tree the flow rate in the root segment must equal the flow rate in all the terminal branches of that tree, the application of a power law relation in Eq. (2) gives

$$(d_{0,1})^x = \sum_{i=1}^N (d_t^i)^x \quad (4)$$

where $d_{0,1}$ is the diameter of the root segment of the tree.

Measured diameters from micro-CT images were used to solve for the value of x in Eq. (4) for each tree. This was based on the assumption that variation in the value of x over the relatively small number of generations in each of these microvascular trees was small, and that the calculated value provided a good approximation of the average.

RESULTS

Figure 3 shows an example of the segmented microvasculature within one myocardial specimen that was generated as described in *Experimental Methods* and segmented using methods described in Ref. 10.

A total of 66 microvascular trees from seven specimens were analyzed, to be henceforth referred to as “the 66 sampled trees”, with a cumulative number of 2,438 terminal branches. The range and distribution of diameters are shown in Fig. 4, and the corresponding range and distribution of flow rates are shown in Fig. 5. The flow rate in each vessel segment in a given tree was determined from the diameter of that segment, using the power law relationship in Eq. (2), with the value of x as determined for that particular tree. All flow rates in a given tree were normalized in terms of the flow rate in the root segment of that tree.

The value of the power law index x for each tree was calculated by solving Eq. (4) with the diameter data for that particular tree. As a measure of how the values of x vary along the hierarchical level of the coronary arterial tree, and in particular how they vary towards our selected most distal pre-capillary arteriolar end of that tree, the values of x are shown in Fig. 6 plotted against the root diameters of the microvascular sub-trees for which they were calculated with the view that sub-trees with smaller root diameters are closer to the pre-capillary level. Figure 7 shows the number of generations in each of the 66 arteriolar trees on which the results are based.

DISCUSSION

By a combination of experimental, theoretical, and imaging methodologies, the present study provides a window onto a region of myocardial microvasculature which has been unexplored so far. While some images of this vasculature has been available in the past, whether under the microscope or by anatomical dissection, these have not been available in the context of intact branching trees. By examining intact trees our study has extended this picture into aspects relating to function. Other studies^{4, 8} have explored these issues but the methodologies used were sufficiently different that direct comparison of the results is not possible. We believe that these studies complement each other in an areas of microvasculature to which direct access is extremely difficult and where more data are badly needed.

The *vascular* arteriolar trees which we examined range in root diameter from several hundred to less than 100 μm , and terminal microvessels down to less than 20 μm , with an average of 57 μm . While terminal microvessels with diameters less than 40 μm were excluded from the final analysis to comply with the resolution limit of the micro-CT scans on which the imaging was based, the intact microvascular trees actually did extend down to the capillary level. Again, this level of intramyocardial vasculature has not been explored before in the context of intact 3D branching hierarchy.

By considering intact vascular trees we have been able to derive information about the distribution and heterogeneity of flow rates at this level of vasculature. The results indicate a

standard deviation (SD) considerably higher than the average (M) flow rate, with a coefficient of variation (SD/M) of approximately 2.07 (0.056/0.027). The spatial heterogeneity of myocardial blood flow distribution has been examined by a number of investigators using different methodologies and spatial resolutions. Malyar et al¹¹ showed that arterioles 30 μm in lumen diameter perfused myocardial volumes of 0.027 mm^3 , 100 μm diameter 0.95 mm^3 and 200 μm 7.6 mm^3 . As shown by Bassingthwaite et al² the coefficient of variation of myocardial perfusion (measured by radiolabelled microspheres) is 0.1 for myocardial volumes of 10 cm^3 and increases by 0.1 for every decade reduction in myocardial sample volume so that for a volume of 0.01 mm^3 it is 0.5. Van Bavel et al²⁰ used a mathematical model to extend their microscope-based measurements of the coronary arterial tree to show that the coefficient of variation of computed flow heterogeneity as a function of number of 10 μm diameter arterioles was about 1.7 and for a cluster of 100 such arterioles (which we estimate emerge from an arteriole about 60 μm in diameter) the coefficient of variation is about 0.6. The study by Austin et al,¹ using radiolabelled microspheres, examined the spatial heterogeneity of myocardial perfusion and flow reserve. Their data do not explicitly provide a measure of coefficient of variation in the fully dilated coronary vasculature, but using their data from one of the dogs (for which a detailed flow-frequency histogram is provided) we get a coefficient of variation 1.08 (SD/M = 5.56/5.16) for myocardial tissue volumes of 106 mm^3 . Thus there appears to be a considerable range of estimated flow heterogeneities, which may in part be due to the different methodologies used. In the context of myocardial perfusion, this high value of the coefficient of variation suggests that heterogeneity of flow rate at the pre-capillary level is not entirely random but may be at least in part due to active control.

Perhaps the most important results of our study is that of providing estimated values of the power law exponent x and, by implication, estimated values of the shear stress at this level of myocardial vasculature. The results in Fig. 6 provide the first confirmation of a long standing hypothesis/expectation on physiological grounds that at the pre-capillary end of the arterial tree the value of x must be higher than 3.0 and, by implication, the level of shear must be lower than it is at the central (larger vessels) end of the tree.

Our interpretation of the results in Fig. 6 is that they point to a clear trend as shown by the dashed line in the figure, which is given by

$$x = ae^{b/(d_{1,0}-c)} \quad (5)$$

where $a = 3, b = 0.1, c = 0.085$ and $d_{1,0}$ is root diameters of the trees. Because of the scarcity of trees with root diameters less than 100 μm and terminal branches with diameters greater than 40 μm , data points in that part of the figure are correspondingly scarce. This is further compounded by the steep rise of the trend curve in that region and therefore we do not consider the trend curve as a legitimate “fit” of the data. However, for values of the power law exponent x less than 6.0 or so there are sufficient data points to actually fit with a regression line as shown in Fig. 8. The line indicates clearly that the value of x is increasing towards the pre-capillary end of the vasculature. Obtaining the required data to reach this conclusion was the main aim of our study.

The main purpose (and best hope) of our study was to determine only the *trend* of x as vessel diameters decrease, namely whether the value of x was increasing or decreasing. The only conclusion which our results support, and the only conclusion in which we have full confidence, is that x is increasing in the direction of small diameters. We do not think that we have sufficient basis for determining *how* it is increasing.

In the way of limitation, the exclusion of terminal vessel segments less than 40 μ m in diameter was necessary because of the 20 μ m voxel resolution of the micro-CT scans on which the study was based. As explained in *Theoretical Considerations*, this did not affect the ability to implement conservation of flow rate within the vascular tree because the excluded segments with diameters less than 40 μ m were replaced by their siblings or parents. Nevertheless, higher resolution CT scans would permit the inclusion of these terminal more distal segments, an issue which we hope to address in a future study.

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Figure 1.

The j, k notation used for mapping the hierarchical structure of the imaged arterial trees. The notation allows for a simple tabulation of the position of each vascular segment within the tree. (Modified with permission from Ref. 23).

Figure 2.
Example of segmented vasculature from one tissue sample, showing one tree extracted from within as described in the text.

Figure 3.
Example of vascular trees captured within one tissue sample.

Figure 4.
Diameters of terminal vascular segments within the 66 sampled trees.

Figure 5.

Flow rates in terminal vascular segments within the 66 sampled trees, based on a power law relationship between the diameter of a blood vessel and the flow rate within the vessel, coupled with conservation of flow rate within each tree.

Figure 6.

Values of the power law exponent in the 66 sampled trees, plotted against the corresponding root diameter of the trees. The rationale for the plot is that trees with smaller root diameters are hierarchically closer to the capillary bed than are trees with larger root diameters, thus the figure gives an indication of how the value of the power law index x varies towards the pre-capillary end of the vascular tree. The dashed curve is based on Eq. (5) and is intended only to indicate a “trend”. It does not represent a legitimate “fit” of the data points.

Figure 8.

Because of the scarcity of data points in the steep rising part of the curve in Fig. 6, here we consider only values of the power law exponent x less than 6.0 or so where there are sufficient data points to actually fit with a regression line. The purpose of this line is not to suggest that the relationship between the diameters and power law exponent is linear but rather to show clearly that the value of x is increasing towards the pre-capillary end of the vasculature. The line indicates clearly that the value of x is increasing towards the pre-capillary end of the vasculature.

TABLE

Table illustrates a typical data result from a “j,k” analysis of a single tree. Repeated analysis of the same tree, using the same computer program with the same analysis parameters used in analysis of all trees, would provide the same results, hence no variability of these values can be obtained

J	K	Diameter (mm)	Length (mm)	Continuing (=1)
0	1	0.094	0.606	1
1	1	0.085	0.454	1
1	2	0.050	0.776	1
2	1	0.069	1.001	1
2	2	0.052	0.632	1
2	3	0.041	0.046	1
2	4	0.026	1.061	0
3	1	0.068	0.737	1
3	2	0.022	1.474	1
3	3	0.041	0.966	1
3	4	0.020	0.173	0
3	5	0.032	0.707	0
3	6	0.032	0.055	0
4	1	0.061	0.727	1
4	2	0.052	0.572	1
4	3	0.015	0.147	0
4	4	0.011	0.010	0
4	5	0.035	0.526	1
4	6	0.030	0.480	0
5	1	0.047	0.333	1
5	2	0.024	0.309	0
5	3	0.035	0.355	0
5	4	0.026	0.674	0
5	9	0.026	0.164	0
5	10	0.025	0.155	0
6	1	0.029	0.423	0
6	2	0.019	0.134	0