Spherical grating based x-ray Talbot interferometry

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Purpose: Grating interferometry is a state-of-the-art x-ray imaging approach, which can acquire information on x-ray attenuation, phase shift, and small-angle scattering simultaneously. Phase-contrast imaging and dark-field imaging are very sensitive to microstructural variation and offers superior contrast resolution for biological soft tissues. However, a common x-ray tube is a point-like source. As a result, the popular planar grating imaging configuration seriously restricts the flux of photons and decreases the visibility of signals, yielding a limited field of view. The purpose of this study is to extend the planar x-ray grating imaging theory and methods to a spherical grating scheme for a wider range of preclinical and clinical applications.

Methods: A spherical grating matches the wave front of a point x-ray source very well, allowing the perpendicular incidence of x-rays on the grating to achieve a higher visibility over a larger field of view than the planar grating counterpart. A theoretical analysis of the Talbot effect for spherical grating imaging is proposed to establish a basic foundation for x-ray spherical gratings interferometry. An efficient method of spherical grating imaging is also presented to extract attenuation, differential phase, and dark-field images in the x-ray spherical grating interferometer.

Results: Talbot self-imaging with spherical gratings is analyzed based on the Rayleigh–Sommerfeld diffraction formula, featuring a periodic angular distribution in a polar coordinate system. The Talbot distance is derived to reveal the Talbot self-imaging pattern. Numerical simulation results show the self-imaging phenomenon of a spherical grating interferometer, which is in agreement with the theoretical prediction.

Conclusions: X-ray Talbot interferometry with spherical gratings has a significant practical promise. Relative to planar grating imaging, spherical grating based x-ray Talbot interferometry has a larger field of view and improves both signal visibility and dose utilization for pre-clinical and clinical applications. © 2015 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4933195>]

Key words: x-ray diffraction, Rayleigh–Sommerfeld diffraction, Talbot effect, spherical grating

1. INTRODUCTION

X-ray phase shift cross sections of low-Z elements are about a thousand times greater than the attenuation counterparts over the diagnostic energy range, suggesting a higher signal-to-noise ratio and a higher contrast resolution for biological tissue imaging.¹ X-ray phase-contrast imaging is invaluable to observe subtle structural features of biological tissues, such as vessels, lesions, microcalcification, and fibrous tissues.²

X-ray phase imaging methodology includes analyzer-based phase-contrast imaging, free-space propagation, and grating interferometric methods. Analyzer-based methods require high-precision crystals and a highly parallel and monochromatic x-ray beam.³ Propagation-based methods demand high spatial coherence of x-rays, only available with synchrotron facilities at a high cost or microfocus x-ray sources with a low flux.⁴ In contrast, grating-based phase imaging is the most promising with much less constraint. Coherent wavelets through a phase grating produce a Talbot self-image of the grating in the near field and can be utilized for reconstruction of an x-ray phase-contrast image. In 2003, a grating interferometric imaging system with a synchrotron x-ray source

was set up in the hard x-ray region using a phase grating and an absorption grating to demonstrate the feasibility of x-ray phase-contrast imaging.⁵ In 2006, based on the x-ray interferometric imaging principle, the synchrotron x-ray source was replaced with a low-brilliance x-ray tube coupled with a source grating to create an array of line sources, which emit x-ray beamlets each of which is spatially coherent but incoherent with others.⁶ The tomographic reconstruction was performed using a filtered backprojection algorithm to yield quantitative images of real and imaginary parts of the refractive index.^{6,7} This novel system architecture enables x-ray phase-contrast imaging with a common x-ray source and has a great potential for biomedicine, industrial nondestructive testing, and security applications.

However, the field of view (FOV) of a typical x-ray grating interferometer is on the order of a few centimeters, which can only cover small objects or small regions of interest. A major reason for a limited field of view is the use of flat gratings, which demands parallel-beam imaging geometry. Flat-grating-based imaging suffers from a limited flux of photons due to oblique incidence of x-rays, since an x-ray tube is actually treated as a point source, which emits a divergent beam of x-rays known as a fan or cone beam. The introduction

of curved gratings allows nearly perpendicular incidence of x-rays on the gratings and generates a higher visibility over a larger field of view than a conventional interferometer with flat gratings.^{8–10}

X-ray grating interferometric imaging is based on the Talbot effect, which is a self-imaging phenomenon under spatially coherent illumination. The Talbot effect with planar periodic objects was studied assuming the Fresnel diffraction. Motivated by the absence of a theoretical investigation on spherical periodic objects, in this paper, we formulate the self-imaging phenomenon with spherical transmission gratings based on the Rayleigh–Sommerfeld diffraction and perform a numerical simulation study to demonstrate the feasibility of spherical grating x-ray interferometry.

2. SELF-IMAGING WITH SPHERICAL GRATINGS

A spherical phase grating consists of a large number uniformly spaced slits, which is periodic in term of the polar angle θ_g on the spherical surface with a radius of r_g . An x-ray tube is located at the center of the spherical grating and assumed as a point source to emit a spherical wave, which can be described by¹¹

$$S(\mathbf{r}) = \frac{A}{\|\mathbf{r} - \mathbf{r}_s\|} \exp(ik\|\mathbf{r} - \mathbf{r}_s\|), \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number for an x-ray wavelength λ , A is the amplitude, $\mathbf{r}$ is the observation position, and $\mathbf{r}_s = (x_s, y_s, 0)$ is the source position. For planar grating interferometric imaging, there would be heavy shades in the moiré pattern on the detector plane due to the divergence of the x-ray beams and a high aspect ratio of the absorption grating, significantly reducing the field of view. With a spherical grating imaging system as shown in Fig. 1, the incoming wave field makes the normal incidence on the spherical grating, effectively minimizing the x-ray shadow of the grating and maximizing the field of view in cone-beam geometry.

When a spherical wave propagates through a spherical phase grating of a period p_θ , the transmission function of the

FIG. 1. Spherical grating imaging geometry.

wave on the spherical surface of the grating can be expressed as

$$T(\mathbf{r}_g) = S(\mathbf{r}_g)G(\mathbf{r}_g), \quad (2)$$

where $G(\mathbf{r}_g)$ is the transmission function of the spherical phase grating. According to the Rayleigh–Sommerfeld diffraction integral formula, which accurately describes the wave propagation in free space without the paraxial approximation limitation, the observation wave field at a position $\mathbf{r}_o$ is formulated by¹²

$$Q(\mathbf{r}_o) = \frac{r_o - r_g}{i\lambda} \iint_{\Sigma} T(\mathbf{r}_g) \frac{\exp(ik\|\mathbf{r}_o - \mathbf{r}_g\|)}{\|\mathbf{r}_o - \mathbf{r}_g\|^2} d\mathbf{r}_g, \quad (3)$$

where Σ is the surface of the spherical phase grating of a radius r_g , a point $\mathbf{r}_g = (x_g, y_g, z_g)$ is on Σ , and $d = \|\mathbf{r}_o - \mathbf{r}_g\|$ is the distance between the point $\mathbf{r}_g$ on the spherical phase grating and a location $\mathbf{r}_o = (x_o, y_o, z_o)$ at the observation spherical surface of a radius r_o centered at the source position. Making a variable change from Cartesian to spherical coordinates $(x_g, y_g, z_g) \rightarrow (\theta_g, \phi_g, r_g)$ and $(x_o, y_o, z_o) \rightarrow (\theta_o, \phi_o, r_o)$: $x_g - x_s = r_g \cos(\theta_g) \sin(\phi_g)$, $z_g = r_g \cos(\theta_g) \cos(\phi_g)$, $y_g - y_s = r_g \sin(\theta_g)$, $-\pi/2 \leq \theta_g \leq \pi/2$, and $-\pi/2 \leq \phi_g \leq \pi/2$. $x_o - x_s = r_o \cos(\theta_o) \sin(\phi_o)$, $z_o = r_o \cos(\theta_o) \cos(\phi_o)$, $y_o - y_s = r_o \sin(\theta_o)$, $-\pi/2 \leq \theta_o \leq \pi/2$, and $-\pi/2 \leq \phi_o \leq \pi/2$. The distance d can be reduced to

$$d = \sqrt{r_o^2 + r_g^2 - 2r_g r_o (\cos(\theta_o) \cos(\theta_g) [\cos(\phi_g - \phi_o) - 1] + \cos(\theta_g - \theta_o))},$$

and Eq. (3) can be expressed in spherical coordinates,

$$Q(\theta_o, \phi_o, r_o) = \frac{Ar_g(r_o - r_g)}{i\lambda} \exp(ikr_g) \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} G(\theta_g, \phi_g, r_g) \frac{\exp(ikd)}{d^2} \cos(\theta_g) d\theta_g d\phi_g. \quad (4)$$

The transmission function of the spherical phase grating is periodic in term of the polar angle θ_g and can be expressed in a Fourier series with a period p_θ ,

$$G(\theta_g, \phi_g, r_g) = \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{i2\pi m \theta_g}{p_\theta}\right). \quad (5)$$

Substituting Eq. (5) into Eq. (4), we have

$$Q(\theta_o, \phi_o, r_o) = \frac{Ar_g(r_o - r_g)}{i\lambda} \exp(ikr_g) \sum_{m=-\infty}^{\infty} c_m \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} \exp\left(\frac{i2\pi m \theta_g}{p_\theta}\right) \frac{\exp(ikd)}{d^2} \cos(\theta_g) d\theta_g d\phi_g. \quad (6)$$

Changing the variables $\varphi = \theta_g - \theta_o$ and $\phi = \phi_g - \phi_o$ in Eq. (6), we obtain

$$\begin{cases} Q(\theta_o, \phi_o, r_o) = \frac{A r_g (r_o - r_g)}{i \lambda} \exp(i k r_g) \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{i 2 \pi m \theta_o}{p_\theta}\right) \int_{-\frac{\pi}{2}-\theta_o}^{\frac{\pi}{2}-\theta_o} \int_{-\frac{\pi}{2}-\phi_o}^{\frac{\pi}{2}-\phi_o} \exp\left(\frac{i 2 \pi m \varphi}{p_\theta}\right) \frac{\exp(i k d)}{d^2} \cos(\varphi + \theta_o) d\varphi d\phi, \\ d = \sqrt{r_o^2 + r_g^2 - 2 r_o r_g [\cos(\theta_o) \cos(\varphi + \theta_o) (\cos(\phi) - 1) + \cos(\varphi)]}. \end{cases} \quad (7)$$

From Eq. (7), the intensity of the wave field at (θ_o, ϕ_o, r_o) is the superposition of diffraction waves of different orders. In fact, major contributions are from low-order x-ray diffractions, which correspond to small diffraction angles φ and ϕ . Furthermore, the quadratic approximation to the cosine function is very accurate for a small angle. For example, with $\varphi = 30^\circ$, the relative error of the approximation is about 0.36%. The polar and azimuthal angles of $< 30^\circ$ generally meet our geometrical requirements for biomedical imaging. Hence, using the approximation $\cos(\varphi) = 1 - \varphi^2/2$, $\cos(\phi) = 1 - \phi^2/2$, $\cos(\varphi + \theta_o) \approx \cos(\theta_o)$, and changing variables, Eq. (7) can be simplified to the following form:

$$\begin{cases} Q(\theta_o, \phi_o, r_o) = \frac{A(r_o - r_g)}{i \lambda r_o} \exp(i k r_g) \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{i 2 \pi m \theta_o}{p_\theta}\right) \\ \quad \times \int_{-(\frac{\pi}{2}+\theta_o)\sqrt{r_o r_g}}^{(\frac{\pi}{2}-\theta_o)\sqrt{r_o r_g}} \int_{-(\frac{\pi}{2}+\phi_o)\sqrt{r_o r_g} \cos(\theta_o)}^{(\frac{\pi}{2}-\phi_o)\sqrt{r_o r_g} \cos(\theta_o)} \exp\left(i 2 \pi \frac{m}{p_\theta \sqrt{r_o r_g}} \varphi\right) \frac{\exp(i k d)}{d^2} d\varphi d\phi, \\ d = \sqrt{(r_o - r_g)^2 + \varphi^2 + \phi^2}. \end{cases} \quad (8)$$

Because the grating period p_θ is small relative to the ranges of azimuthal and polar angles, with a change of variables, the integration intervals in Eq. (8) would approach minus infinity to plus infinity, and Eq. (8) can be approximated by a 2D Fourier transform with respect to variables φ and ϕ . According to Wely's formula,^{11,13}

$$\begin{aligned} & -\frac{1}{2\pi} \iint \exp(i 2 \pi (\alpha \varphi + \beta \phi)) \\ & \quad \times \frac{\partial}{\partial z} \left(\frac{\exp\left(i \frac{2\pi}{\lambda} \sqrt{z^2 + \varphi^2 + \phi^2}\right)}{\sqrt{z^2 + \varphi^2 + \phi^2}} \right) d\varphi d\phi \\ & = \exp\left(i 2 \pi z \sqrt{\left(\frac{1}{\lambda}\right)^2 - \alpha^2 - \beta^2}\right), \end{aligned} \quad (9)$$

with $\alpha = m/(p_\theta \sqrt{r_o r_g})$, $\beta = 0$, and $z = r_o - r_g$, Eq. (8) has a closed-form expression,

$$\begin{aligned} Q(\theta_o, \phi_o, r_o) &= \frac{A}{r_o} \exp(i k r_g) \sum_{m=-\infty}^{\infty} c_m \exp\left(\frac{i 2 \pi m \theta_o}{p_\theta}\right) \\ & \quad \times \exp\left(i 2 \pi (r_o - r_g) \sqrt{\left(\frac{1}{\lambda}\right)^2 - \frac{m^2}{p_\theta^2 r_o r_g}}\right). \end{aligned} \quad (10)$$

Applying a binomial expansion to the square root in Eq. (10), $\sqrt{1 - (\lambda m)^2/(p_\theta^2 r_o r_g)} \approx 1 - (\lambda m)^2/(2 p_\theta^2 r_o r_g)$, we obtain

$$\begin{aligned} & \exp\left(i 2 \pi (r_o - r_g) \sqrt{\left(\frac{1}{\lambda}\right)^2 - \frac{m^2}{p_\theta^2 r_o r_g}}\right) \\ & = \exp\left(i 2 \pi \frac{r_o - r_g}{\lambda}\right) \exp\left(i \pi m^2 \left(\frac{1}{r_g} - \frac{1}{r_o}\right) \frac{\lambda}{p_\theta^2}\right). \end{aligned} \quad (11)$$

From Eq. (11), a characteristic distance can be defined to satisfy the following equation:

$$\left(\frac{1}{r_g} - \frac{1}{r_o}\right) = \frac{n p_\theta^2}{\lambda}, \quad (12)$$

where n is referred to as the fractional Talbot order in this case. Therefore, the wave field on the observation spherical surface of a radius r_o does show a Talbot self-imaging phenomenon at distances defined by Eq. (12).

3. EXTRACTION OF CHARACTERISTICS INFORMATION

When an object is put in front of a spherical phase grating, the wave field on a spherical observation surface will be modulated by $\exp(-(\mu(\mathbf{r})/2) + i\Phi(\mathbf{r}))$, reflecting the variation in attenuation $\mu(\mathbf{r})$ and phase $\Phi(\mathbf{r})$. Because the m -th diffraction beam will be deflected by an angle $m\lambda/(r_g p_\theta)$ from the primary beam direction, the wave function at the observation location a Talbot distance away is computed as^{5,14}

$$\begin{aligned} Q(\theta_o, \phi_o, r_o) &= \frac{A}{r_o} \exp(i k r_g) \exp\left(-\frac{\mu(\theta_o, \phi_o, r_o)}{2}\right) \sum_{m=-\infty}^{\infty} c_m \\ & \quad \times \exp\left(\frac{i 2 \pi m \theta_o}{p_\theta} + i \Phi\left(\theta_o - \frac{m \lambda}{r_g p_\theta}, \phi_o, r_o\right)\right). \end{aligned} \quad (13)$$

Assuming a smooth spatial variation in $\mu(\mathbf{r})$ and $\Phi(\mathbf{r})$, Eq. (13) can be reduced to

$$\begin{aligned} & Q(\theta_o, \phi_o, r_o) \\ & = \frac{A}{r_o} \exp(i k r_g) \exp\left(-\frac{\mu(\theta_o, \phi_o, r_o)}{2} + i \Phi(\theta_o, \phi_o, r_o)\right) \\ & \quad \times \sum_{m=-\infty}^{\infty} c_m \exp\left[\frac{i 2 \pi m}{p_\theta} (\theta_o - \varphi_\theta(\theta_o, \phi_o, r_o))\right], \end{aligned} \quad (14)$$

where $\varphi_\theta(\theta_o, \phi_o, r_o) = (\lambda/2\pi r_g)(\partial\Phi/\partial\theta_o)(\theta_o, \phi_o, r_o)$. From Eq. (14), the intensity of the wave field is expressed as

$$I(\theta_o, \phi_o, r_o) = \left(\frac{A}{r_o}\right)^2 \exp(-\mu(\theta_o, \phi_o, r_o)) \times \sum_{l=-\infty}^{\infty} b_l \exp\left[\frac{i2\pi l}{p_\theta}(\theta_o - \varphi_\theta(\theta_o, \phi_o, r_o))\right],$$

where

$$b_l = \sum_{n=-\infty}^{\infty} c_{n+l} c_n. \quad (15)$$

A Talbot interferometer needs an analyzer transmission grating to extract x-ray characteristics information. A spherical analyzer grating has a transmission function,

$$G2(\theta_o, \phi_o, r_o) = \sum_{m=-\infty}^{\infty} s_m \exp\left(\frac{i2\pi m \theta_o}{p_\theta}\right), \quad (16)$$

and is placed at a Talbot distance. Due to the modulation of the analyzer grating, the intensity image observed immediately downstream of the analyzer grating is formulated as

$$\begin{aligned} & I(\theta_o, \phi_o, r_o) \times G2(\theta_o, \phi_o, r_o) \\ &= \left(\frac{A}{r_o}\right)^2 \exp(-\mu(\theta_o, \phi_o, r_o)) \\ & \times \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} b_m s_n \exp\left[\frac{i2\pi}{p_\theta}((m+n)\theta_o - m\varphi_\theta(\theta_o, \phi_o, r_o))\right]. \end{aligned} \quad (17)$$

Generally, a pixel of the detector is substantially larger than the period of the moiré pattern, and the grating structure will be averaged out over the moiré pattern due to the integration,

$$\begin{aligned} J(\theta_o, \phi_o, r_o) &= \int_{\theta_o - \frac{p_\theta}{2}}^{\theta_o + \frac{p_\theta}{2}} I(\theta, \phi_o, r_o) \times G2(\theta, \phi_o, r_o) d\theta \\ &= \left(\frac{A}{r_o}\right)^2 \exp(-\mu(\theta_o, \phi_o, r_o)) \sum_{m=-\infty}^{\infty} b_{-m} s_m \\ & \times \exp\left[\frac{i2\pi m}{p_\theta} \varphi_\theta(\theta_o, \phi_o, r_o)\right]. \end{aligned} \quad (18)$$

In Talbot interferometric imaging, the fringe scanning method is commonly used to extract x-ray characteristics information. Specifically, the analyzer grating (or the phase grating) is shifted by a step size $\Delta\theta$ along the direction of the polar angle multiple times to acquire modulated intensity images. Then, the intensity signal in each detector pixel would oscillate as a function of the total shift $l\Delta\theta$,

$$\begin{aligned} J_l(\theta_o, \phi_o, r_o) &= \left(\frac{A}{r_o}\right)^2 \exp(-\mu(\theta_o, \phi_o, r_o)) \\ & \times \sum_{m=-\infty}^{\infty} b_{-m} s_m \exp\left[\frac{i2\pi m}{p_\theta}(\varphi_\theta(\theta_o, \phi_o, r_o) + l\Delta\theta)\right]. \end{aligned} \quad (19)$$

Since the major contribution to the intensity image in Eq. (19) comes from low order x-ray diffractions, Eq. (19) can be simplified to

$$\begin{aligned} J_l(\theta_o, \phi_o, r_o) &\approx a_0(\theta_o, \phi_o, r_o) + a_1(\theta_o, \phi_o, r_o) \\ & \times \cos\left(\frac{2\pi}{p_\theta}(\varphi_\theta(\theta_o, \phi_o, r_o) + l\Delta\theta)\right), \end{aligned} \quad (20)$$

where $a_i(\theta_o, \phi_o, r_o)$ ($i = 1, 2$) are amplitudes and $\varphi_\theta(\theta_o, \phi_o, r_o)$ is the corresponding phase shift. From Eq. (20), the individual intensity image $J_l(\theta_o, \phi_o, r_o)$ can be used to extract x-ray attenuation, differential phase shift, and dark-field signals, respectively. Hence, the key measurement formulas are as follows:

$$a_0(\theta_o, \phi_o, r_o) = \frac{1}{M} \sum_{l=0}^{M-1} J_l(\theta_o, \phi_o, r_o), \quad (21)$$

$$\varphi_\theta(\theta_o, \phi_o, r_o) = \frac{p_\theta}{2\pi} \tan^{-1} \left(\frac{\sum_{l=0}^{M-1} J_l(\theta_o, \phi_o, r_o) \sin\left(\frac{2\pi l \cdot \Delta\theta}{p_\theta}\right)}{\sum_{l=0}^{M-1} J_l(\theta_o, \phi_o, r_o) \cos\left(\frac{2\pi l \cdot \Delta\theta}{p_\theta}\right)} \right), \quad (22)$$

$$a_1(\theta_o, \phi_o, r_o) = 2 \sqrt{\left(\frac{1}{M} \sum_{l=0}^{M-1} J_l(\theta_o, \phi_o, r_o) \sin\left(\frac{2\pi l \cdot \Delta\theta}{p_\theta}\right) \right)^2 + \left(\frac{1}{M} \sum_{l=0}^{M-1} J_l(\theta_o, \phi_o, r_o) \cos\left(\frac{2\pi l \cdot \Delta\theta}{p_\theta}\right) \right)^2}, \quad (23)$$

where M is the number of steps. The acquired signals with a sample are normalized by the signals without any sample in the x-ray beam. Finally, we obtain a transmission projection $T(\theta_o, \phi_o, r_o) = a_0^s(\theta_o, \phi_o, r_o)/a_0^r(\theta_o, \phi_o, r_o)$, a differential phase shift projection $P(\theta_o, \phi_o, r_o) = \varphi_\theta^s(\theta_o, \phi_o, r_o) - \varphi_\theta^r(\theta_o, \phi_o, r_o)$,

and a dark-field projection $D(\theta_o, \phi_o, r_o) = a_1^s(\theta_o, \phi_o, r_o) a_0^r(\theta_o, \phi_o, r_o) / a_0^s(\theta_o, \phi_o, r_o) a_1^r(\theta_o, \phi_o, r_o)$, where the superscripts s and r denote the values of $a_0(\theta_o, \phi_o, r_o)$, $a_1(\theta_o, \phi_o, r_o)$, and $\varphi_\theta(\theta_o, \phi_o, r_o)$ computed from measured data with and without a sample, respectively.¹⁵

FIG. 2. Downstream wave fields for (a) a $\phi = \pi/2$ phase grating and (b) a $\phi = \pi$ phase grating, respectively.

4. NUMERICAL SIMULATION

In the numerical simulation, a point x-ray source operated at 60 keV was simulated to emit a spherical wave toward a spherical phase grating. Binary phase gratings are used in the interferometric imaging, whose transmission function is described by

$$T(x) = \begin{cases} 1 & \text{if } 0 \leq (x \bmod p_\theta) < D \cdot p_\theta \\ Ae^{i\phi} & \text{if } D \cdot p_\theta \leq (x \bmod p_\theta) \leq p_\theta \end{cases}, \quad (24)$$

where “mod” is the modulo operator, p_θ is the period, D is the duty cycle, and A is the amplitude. The radius of the spherical grating was set to 800 mm. The phase grating had a periodic of 4 μm and a duty cycle of 0.5 (transmission line width to period). The amplitude A was set to 1. The downstream wave fields of a $\phi = \pi/2$ phase grating and a $\phi = \pi$ phase grating were computed according to the diffraction formula (8), as shown in Figs. 2(a) and 2(b), respectively. At the Talbot distances, the intensity distribution was computed which reproduced the transmission image of the grating up to a scaling factor, which verifies the Talbot self-imaging effect with spherical gratings.

5. DISCUSSIONS AND CONCLUSION

X-ray grating interferometry can acquire information on attenuation, refraction, and small-angle scattering simultaneously, and is promising for preclinical, clinical, and other applications. Impressive results were reported on grating-based phase imaging with low-energy x-ray radiation for relatively small FOV. For further development of x-ray grating interferometry, high-energy x-ray beams and large FOVs are highly desirable but these requirements have been technically challenging. A common x-ray source emits a cone beam of x-rays. This divergent imaging mode seriously restricts the flux of photons, decreases the visibility of signals, and degrades the imaging performance in the cases of planar gratings, which is constrained to work under the paraxial approximation. With our proposed use of spherical gratings, the incoming wave

field makes the normal incidence on the interaction surfaces, effectively avoiding the problems with planar gratings. The Talbot self-imaging with spherical gratings can be extended to use two-dimensional spherical gratings.^{16,17} Theory and methods of two-dimensional spherical grating imaging can be also established with similar methodology to the proposed approach.

In conclusion, we have reported a theoretical treatment for Talbot self-imaging with spherical gratings characterized by a periodic angular distribution in a polar coordinate system. It has been numerically shown that the interference field on a spherical observation surface does have a Talbot self-imaging pattern at a Talbot distance. The proposed theoretical framework of spherical grating based imaging can also be adapted to describe a cylindrical grating based imaging process to guide the grating design and system optimization. We plan to prototype spherical gratings and experimentally evaluate their imaging performance.

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