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Image Reconstruction in K-Space from MR Data Encoded with Ambiguous Gradient Fields

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Abstract

Purpose—In this work, the limits of image reconstruction in k-space are explored when non-bijective gradient fields are used for spatial encoding.

Theory—The image space analogy between parallel imaging and imaging with non-bijective encoding fields is partially broken in k-space. As a consequence, it is hypothesized and proven that ambiguities can only be resolved partially in k-space, and not completely as is the case in image space.

Methods—Image-space and k-space based reconstruction algorithms for multiple RF-channel data acquisitions are programmed and tested using numerical simulations as well as in-vivo measurement data.

Results—The hypothesis is verified based on an analysis of reconstructed images. It is found that non-bijective gradient fields have the effect that densely sampled auto-calibration data, used for k-space reconstruction, provide less information than a separate scan of the receiver coil sensitivity maps, used for image space reconstruction. Consequently, in k-space only the undersampling artifact can be unfolded, whereas in image space, it is also possible to resolve aliasing that is caused by the nonbijectivity of the gradient fields.

Conclusion—For standard imaging, reconstruction in image space and in k-space is nearly equivalent, whereas there is a fundamental difference with practical consequences for the selection of image reconstruction algorithms when non-bijective encoding fields are involved.

Keywords

Magnetic resonance imaging; k-space; nonlinear; gradient; reconstruction; PatLoc

Introduction

In the past few years, nonlinear gradient fields and their capability to perform spatial encoding have regained a focal research interest, see e.g. [1 - 4]. Such nonlinear spatial encoding magnetic fields (SEMs) offer new encoding options, especially for localized imaging, which make this imaging technology potentially interesting for organ-specific applications, for example in neurology or oncology.

With nonlinear gradient fields, the need to develop new methods for reliable reconstruction of anatomical images has been a topic of intensive research initiatives, e.g. [5 - 8]. Conventional gradient fields have imperfect linearity, but they are bijective, which means spins at each unique spatial location see a unique magnetic field and can be spatially resolved. For non-bijective gradient fields, spins at two different spatial locations may see exactly the same magnetic field and have the same Larmor frequency, and thus will be aliased and can only be resolved when additional information from RF-sensitivity encoding is available. It was shown in [9] that the aliasing resulting from non-bijective encoding (demonstrated with two orthogonal quadrupolar gradient fields) can be resolved using a modified version of the image-space based SENSE [10] algorithm.

While previous work has usually taken an image-space perspective, this paper addresses whether it is also possible to apply k-space based image reconstruction methods to non-bijective encoding schemes. K-space based methods have proven to be very robust and these methods avoid undesirable sharp SNR boundaries in the reconstructed images [11]. It is therefore useful to investigate if these benefits persist when non-bijective SEMs are used. A representative k-space based image reconstruction method, GRAPPA (= GeneRalized Autocalibrating Partially Parallel Acquisitions [12]), is used in this work.

It seems natural to assume that GRAPPA would result in a feasible reconstruction because of the close relationship between GRAPPA and SENSE [5, 13-16]. However, it is

demonstrated here that with non-bijective encoding, it is not valid to draw this conclusion. In PatLoc (= Parallel Imaging Technique Using Localized Gradients [1]), such non-bijective SEMs are used in combination with multiple receiver coils. With this imaging modality, partial unfolding is possible with GRAPPA, but it is problematic to acquire sufficient auto-calibration signal (ACS) data needed for complete GRAPPA image reconstruction. It turns out that while in image space the two aliasing effects – resulting from the non-bijectiveness of the SEMs and undersampling – can be treated (and generally have to be treated) simultaneously, in k-space the two effects are separated and only aliasing that results from undersampling can be resolved.

Part of this work has already been published in abstract form [17] and as part of a doctoral dissertation [5]. Based on an analogy between parallel imaging (PI) and non-bijective encoding, it is shown that GRAPPA can be applied without any modification to undersampled PatLoc imaging data, thereby partially resolving aliased image voxels. The remaining aliasing can then be resolved using previously described image reconstruction methods. A practical algorithm is tested with numerical simulations and measurement data; finally, it is discussed under what circumstances image reconstruction in k-space might not only be performed to partially resolve ambiguities, but to fully reconstruct an unaliased image.

Theory

Analogy between SENSE and Cartesian PatLoc reconstruction

Consider Fig. 1. The top row shows the situation for standard PI, the bottom row the situation for PatLoc imaging, assuming orthogonal quadrupolar encoding fields. An object (a) weighted with the receiver coil sensitivity (b) appears aliased if accelerated by factor 2 (c) or factor 4 (d). SENSE is then used to resolve the ambiguities (e). An equivalent situation occurs in PatLoc imaging. This equivalence becomes visually obvious if the object is represented in a different space representation, the so-called PatLoc-encoding space. Note that the PatLoc encoding space is completely equivalent to the natural image space representation of the object; there is a 1-to-1 relationship, the object simply appears distorted due to the spatially varying voxel size and orientation [9]. Fig. 1b and Fig. 1g (apart from an intensity variation) or also Fig. 1e and Fig. 1j show the same object. The nonbijectiveness of the SEMs leads to two-fold aliasing (h) and two-fold undersampling leads to in total an aliasing factor of $2 \cdot 2 = 4$ (i). SENSE (plus correction for an intensity variation) is then used to unfold the image (j). In PatLoc encoding space, the aliased locations lie equidistantly on a line, which is also the case in standard PI.

Fig. 1 illustrates that there is indeed an analogy between SENSE and Cartesian PatLoc imaging, a result that has been found previously based on mathematical arguments [9]. However, there is one feature that needs to be considered: In SENSE, aliasing is exclusively caused by undersampling; in PatLoc imaging, however, aliasing is also caused by ambiguities of the encoding fields. Therefore, a SENSE situation with an acceleration factor of R is fully equivalent to a PatLoc situation with an acceleration factor of R/L , when L denotes the degree of ambiguity that is generated by the non-bijective SEMs, i.e. the maximum number of spatial locations that experience exactly the same local magnetic field

gradients during the process of spatial encoding. The situation shown in Fig. 1g has no direct k-space counterpart in PatLoc imaging. A fully acquired k-space results in the aliased image shown in Fig. 1h, the figure in Fig. 1g might be said to belong to a “virtual” k-space. This difference between PI and PatLoc imaging is important and causes problems for image reconstruction in k-space as will be discussed later.

Partially resolving ambiguities with GRAPPA in PatLoc imaging

It is shown here that the GRAPPA algorithm can be applied to undersampled PatLoc imaging data without any modification to resolve the ambiguities that result from undersampling. This property is not obvious because one might object:

As a result of the SEM-ambiguities, each signal consists in fact of a superposition of two signals (for $L=2$). Why should the GRAPPA weights be the same for *both* signals?

It is shown here that this argument is unfounded. In the previous paragraph, it was shown that an R -times subsampled PI dataset is fully equivalent to an R/L -fold subsampled PatLoc dataset. Recall that for GRAPPA, a densely sampled k-space center is used to find the GRAPPA weights necessary to reconstruct missing k-space lines of an R -times subsampled dataset. It is to be proven that this fact is also valid in PatLoc imaging. Using the above established analogy, it is therefore sufficient to prove that it is possible to use an L -times subsampled ACS-dataset in standard PI to reconstruct an L -times subsampled image from an $(R \cdot L)$ -times subsampled dataset. This rather unusual situation of using subsampled ACS-data for image reconstruction is depicted in Fig. 2, and it is compared to the equivalent situation in PatLoc. In [18] it has been proven, that subsampled ACS-data can indeed be used to reconstruct a subsampled dataset in standard PI. This shows that, as long as a sufficient amount of densely sampled calibration data are acquired, the standard GRAPPA algorithm can be applied to undersampled PatLoc imaging data to resolve the ambiguities that result from undersampling. The PatLoc image, reconstructed with GRAPPA, will still exhibit aliasing that is caused by the non-bijection of the SEMs. It can therefore be concluded that with GRAPPA, aliasing can indeed be partially unfolded, but not completely. However, the Cartesian PatLoc reconstruction method presented above can then be used to complete the image reconstruction.

This result implicitly shows that the objection that was presented above is simply based on a wrong – image space oriented – intuition about how GRAPPA works. In fact, GRAPPA is a k-space technique, where it is a regular feature that the signal consists of superimposed components. An MR scanner cannot immediately detect the spatial distribution of the spin density; the scanner is only sensitive to frequencies. When non-bijective SEMs are used, a single superimposed signal is detected. As usual, the receiver coil sensitivities induce small k-space shifts that cause correlations in k-space. GRAPPA makes use of these correlations – in this regard, the principles of standard parallel imaging and PatLoc imaging do not differ from each other.

Mathematical Treatment

The above proof was based on purely logical arguments. The result can also be underpinned with mathematical formulas as shown here. For reasons of clarity, the treatment is in 1D only; extension to higher dimensions is straightforward. It is considered that non-bijective SEMs have in fact two properties: they are (i) nonlinear and (ii) non-bijective. Both effects can be treated separately, thereby clarifying the difference of PatLoc imaging and conventional imaging with gradient fields of imperfect linearity.

To (i)—Consider a nonlinear bijective SEM that is generated by a constant current. Then, the spins resonate constant in time at the Larmor frequency $\omega(x) = \gamma B(x)$, where γ is the gryromagnetic ratio and $B(x)$ is the magnetic field strength of the SEM. Define the k-space variable $k(t) := t$. Then, it is straightforward to show that the signal $s_\alpha(t)$ of RF receive channel $\alpha \in [1, N]$ with sensitivity $c_\alpha(x)$ is given by: $s_\alpha(k) = \int c_\alpha(x) \rho(x) e^{-ik\omega(x)} dx$. Transforming the spatial variable x to the space of spatial frequencies $x \rightarrow \omega$ results in $s_\alpha(k) = \int \tilde{c}_\alpha(\omega) \tilde{\rho}(\omega) e^{-ik\omega} d\omega$. Apart from the substitution of x by ω , the equation is formally equivalent to the standard imaging equation known from parallel imaging. Nonlinearity therefore has the effect that application of GRAPPA or SENSE results in a distorted and intensity-modulated image $\tilde{\rho}(\omega)$. After correcting distortion and intensity variation, the correct spin density is recovered, $\tilde{\rho}(\omega) \rightarrow \rho(x)$. It can be concluded that feature (i), i.e., the nonlinearity of SEMs, does not destroy the uniformity of GRAPPA and SENSE. This is different for feature (ii), i.e., the non-bijective nature of SEMs.

To (ii)—Consider a non-bijective SEM. In this case there are L locations which have the same Larmor frequency. Without loss of generality, $L = 2$ is assumed, so one has $\omega(x_1) = \omega(x_2)$ for two different locations with spin densities $\tilde{\rho}^1, \tilde{\rho}^2$ and coil sensitivities $\tilde{c}_\alpha^1, \tilde{c}_\alpha^2$. This ambiguity and undersampling cause a combined aliasing artifact (see Fig. 1i). Below, it is recalled that SENSE can be used to completely unfold aliasing; then, it is shown that GRAPPA can resolve the undersampling artifact, but not the ambiguities of the SEM.

Unfolding with SENSE—As a result of non-bijectivity, the signal consists of a superposition of two signals: $s_\alpha = s_\alpha^1 + s_\alpha^2 = \int (\tilde{c}_\alpha^1(\omega) \tilde{\rho}^1(\omega) + \tilde{c}_\alpha^2(\omega) \tilde{\rho}^2(\omega)) e^{-ik\omega} d\omega$. By constructing a sensitivity matrix $\mathbf{C} = \begin{pmatrix} \tilde{c}_1^1 \cdots \tilde{c}_N^1; \tilde{c}_1^2 \cdots \tilde{c}_N^2 \end{pmatrix}$ and inverting this matrix, one can resolve the spin density $\tilde{\rho}(\omega)$ using standard SENSE. In case k-space is undersampled, simply more pixels are aliased. A sensitivity matrix can still be constructed and inverted using SENSE [9].

Only partial unfolding with GRAPPA—Recall that in standard GRAPPA, the unfolded coil images $\tilde{\sigma}_\alpha(\omega) = \tilde{c}_\alpha(\omega) \tilde{\rho}(\omega)$ are reconstructed in first step, and $\tilde{\rho}(\omega)$ is only found in a second step, e.g. using a root-sum-of-squares. Application of GRAPPA resolves undersampling and one gets $\tilde{\sigma}_\alpha(\omega)$ from the undersampled signal $s_\alpha(k) = \int \tilde{\sigma}_\alpha(\omega) e^{-ik\omega}$. Consider now non-bijective SEMs. Define $\tilde{f}_\alpha(\omega) = \tilde{c}_\alpha^1(\omega) \tilde{\rho}^1(\omega) + \tilde{c}_\alpha^2(\omega) \tilde{\rho}^2(\omega)$. From the previous paragraph, it follows that $s_\alpha = \int \tilde{f}_\alpha(\omega) e^{-ik\omega} d\omega$. This equation is formally equivalent to the standard GRAPPA equation with $\tilde{\sigma}_\alpha$ replaced by $\tilde{f}_\alpha$. As a consequence,

GRAPPA cannot resolve the individual components $\tilde{c}_\alpha^1(\omega) \tilde{\rho}^1(\omega)$ and $\tilde{c}_\alpha^2(\omega) \tilde{\rho}^2(\omega)$, but only the superimposed spin density $\tilde{f}_\alpha(\omega) = \tilde{c}_\alpha^1(\omega) \tilde{\rho}^1(\omega) + \tilde{c}_\alpha^2(\omega) \tilde{\rho}^2(\omega)$. This superimposed spin density can be reconstructed with GRAPPA because, similar to standard imaging, the different coil sensitivities result in correlations of signal data from different coils at adjacent k-space locations. It can be concluded that application of GRAPPA resolves aliasing that is caused by undersampling. Aliasing that is caused by the non-bijectivity of the SEMs remains. This aliasing can be resolved in a second step using SENSE (see previous paragraph).

It is interesting to note that with SENSE an intermediate, partial unfolding is in general not possible because the sensitivity matrix that needs to be inverted cannot be separated into two smaller matrices that are inverted consecutively. This difference between GRAPPA and basic SENSE is not only valid for PatLoc imaging, but also for standard parallel imaging [18,19].

Methods

Several algorithms for the reconstruction of subsampled imaging data were tested with numerical simulations and experiments. Standard SENSE and GRAPPA algorithms were implemented and the PatLoc-SENSE algorithm was programmed as described in [9]. Different combinations of these algorithms were used in this work; these combinations are presented in Fig. 3; the algorithms depicted in Fig. 3a-d are applicable to data encoded with linear SEMs, and Fig. 3e,f illustrates reconstruction from PatLoc data encoded with quadrupolar SEMs. A detailed description of the methods is found in the figure caption.

Matlab (The Mathworks Inc., Natick, MA, USA) was used to program the algorithms and to perform simulations. Two densely sampled 256×256 datasets, one for linear SEM encoding, the other for quadrupolar SEM encoding, were generated using a Shepp-Logan head phantom and RF-coil sensitivities approximating an eight-channel coil array. White noise was added to the signal data. Subsampled data were simulated by extracting every fourth k-space line of the linear data and every second line of the PatLoc data. For the linear dataset, 32 densely sampled, and 16 subsampled ACS-lines were taken for 2-times and 4-times GRAPPA reconstruction. For the PatLoc dataset, 16 calibration lines were used. GRAPPA was based on a 7×2 -kernel to reduce subsampling by a factor of 2 as well as 4. The data were reconstructed with the algorithms illustrated in Fig. 3.

In vivo imaging data from the head of a volunteer were acquired on a MAGNETOM Trio, A Tim System 3T (Siemens Healthcare, Erlangen, Germany) with a modified gradient hardware capable of generating orthogonal quadrupolar SEMs in addition to the conventional linear SEMs [20,21] following local IRB approval. A fourfold subsampled 256×64 dataset with 32 ACS-lines was measured with the linear gradients, and a twofold undersampled 256×128 dataset with 16 ACS-lines was acquired with the PatLoc coil. The data were then reconstructed using the algorithms shown in Fig. 3.

Results

Figure 4 shows simulation results as well as experimental data; depicted are reconstructed images that were encoded with the standard linear SEMs. Two 4-times subsampled k-space datasets served as input data for the four algorithms illustrated in Fig. 3a-d. The presented images verify that it is indeed possible to use subsampled calibration data to partially unfold the image in a first step, confirming [18]. In a second step, the remaining aliasing is well resolved using SENSE, or, with additional ACS-lines, using GRAPPA a second time. Comparison with the images reconstructed using SENSE and standard GRAPPA demonstrates that the two algorithms which assume subsampled calibration data lead to similar results. Close inspection shows that the two-step approaches seem to yield images with less artifacts, especially compared to pure SENSE reconstruction (see figure caption for more details). It is interesting that, at least for the simulated data, the two-step GRAPPA reconstruction yields images with less noise than the one-step GRAPPA reconstruction; this might appear counterintuitive because with the two-step approach reconstructed k-space lines with usually increased noise level are partially used as input for the second step whereas the one-step approach only uses the originally measured signal data. This aspect should be further explored in the future.

Based on this result and the analogy to PatLoc imaging, established above, it is to be expected that GRAPPA effectively resolves the fold-over artifact also for a subsampled PatLoc dataset. This is confirmed by Fig. 5. The final reconstruction is then completed using the PatLoc image-space method. The reconstruction results are compared to pure image-space reconstruction in Fig. 5. Though less prominent, but in conformity with linear encoding (Fig. 4), image quality seems to be slightly improved with the two-step approach.

Discussion

Partial unfolding in k-space

It has been shown above that aliasing is not completely removed in PatLoc imaging by simply applying the GRAPPA algorithm. However, it has been demonstrated that it is possible to use GRAPPA for partial unfolding. Theoretically, this has been proven by referring to an analogy to conventional PI. Mathematical formulas were then used to substantiate this proof.

The theoretical analysis of k-space based PatLoc reconstruction could also be verified with numerical simulations and in vivo measurement results. The findings have shown that purely image space based and k-space/image-space hybrids reconstructions give similar results. The hybrid k-space/image-space reconstruction seems to slightly outperform pure image space reconstruction, mainly because the partial usage of GRAPPA avoids sharp artifact boundaries that are a typical feature of pure image space reconstructions like SENSE or PatLoc-SENSE [9, 10]. However, also the overall artifact power seems to be reduced in conformity with GRAPPA theory [5, 13, 22]. Overall, it can be concluded that a hybrid GRAPPA-SENSE-like reconstruction can compete with, or even outperform, pure image-space reconstruction in terms of image quality.

Such a hybrid k-space/image-space method is especially interesting for accelerated non-Cartesian acquisition trajectories, where direct image space reconstruction is often problematic (the encoding matrix is dense and artifacts may occur [23]). In k-space, a GRAPPA-variant, pseudo-Cartesian GRAPPA [24], has shown to be useful in the context of conventional PI. This method or variations thereof should also be applicable to subsampled non-Cartesian PatLoc trajectories and would allow non-iterative image reconstruction in combination with the Cartesian PatLoc image space method. The presented reconstruction methods were tested with quadrupolar SEMs; however, according to the theory section, they are also applicable to SEMs of any shape. If more than two SEMs are used for 2D-encoding, iterative reconstruction methods can be used for image reconstruction [6, 8, 25 - 27].

Why is there a difference between k-space and image-space based reconstruction?

From conventional imaging one is used to the fact that SENSE and GRAPPA have their strengths and weaknesses, but that if one method is applicable, the other method can in general also be used. This work has shown that this is different when non-bijective SEMs are involved. While a variant of SENSE is capable of completely reconstructing useful images, GRAPPA can only partially unfold aliased image voxels. At the same time, SENSE is not capable of differentiating between aliasing that is caused by ambiguous encoding or by undersampling, while GRAPPA naturally can. Why is this the case?

The answer can be found based on mathematical arguments [5], but the answer can also be approached by realizing that there is a fundamental physical difference for aliasing that is caused by ambiguous SEMs or by undersampling. The ambiguity of the encoding from the SEMs depends on a geometrical property of the SEMs themselves, whereas undersampling is an alteration of the temporal characteristics of the acquisition. In image space, both causes have the same effect: aliasing. Consequently, the analogy shown in Fig. 1 is perfect in image space. In k-space, however, the effects are different. This has the consequence that the analogy shown in Fig. 1 is imperfect in k-space: in conventional imaging, the fold-over artifact does not occur if k-space is not undersampled; with non-bijective SEMs, however, voxels are aliased even for a densely sampled PatLoc k-space (cf. Fig. 1). What is the consequence for GRAPPA? Fundamentally, GRAPPA uses calibration data to fill unsampled regions of k-space. If, however, k-space is densely sampled, there are no unsampled regions in k-space to be filled. Therefore, there is a real physical limitation with the effect that GRAPPA can (generally) not be applied to resolve all ambiguities in PatLoc imaging, but only those that result from undersampling.

This is different when SENSE is used, where a separate scan is performed to explicitly measure the coil sensitivity maps (and for PatLoc imaging these maps are typically acquired with the conventional linear SEMs). Therefore, full information about the RF data required for voxel unfolding is available. On the other hand, from the ACS-data in GRAPPA, only aliased sensitivity maps can be retrieved (if encoded with non-bijective SEMs), and voxel unfolding is not completely possible with GRAPPA.

Toward pure k-space based image reconstruction

It can therefore be concluded that a pure k-space based image reconstruction is only feasible if these physical limitations are overcome. One solution would be to introduce a “virtual” k-space that corresponds to the unfolded image displayed for example in Fig. 1g. A similar idea had been used before in [28], where GRAPPA is applied to reconstruct a dataset that has been encoded with linear gradients, but with undersampling along the partition-encoding direction. In contrast to the situation in [28], with non-bijective SEMs, this space is “virtual” in that sense that it is not a space to which a physical meaning can immediately be attributed, and therefore it is not (immediately) possible to acquire adequate calibration lines. Complete unfolding would, however, require such “virtual” k-space calibration data belonging to the completely unfolded image.

If it were possible to give the virtual k-space a physical background, purely k-space based reconstruction should be feasible. One possibility is to use a separate low-resolution scan with the standard gradient system to synthesize adequate calibration data. However, such a scan would be similar to the additional coil sensitivity scan required for the hybrid method, and little would be gained. It is probably more useful to supplement non-bijective SEM encoding with RF-transmit encoding such that acquisition of calibration lines in the virtual k-space becomes possible. The required shift in the virtual k-space can be achieved by generating a linear phase ramp of 2π in the unfolded PatLoc encoding space onto the object. The usefulness of such an approach still has to be shown and it should be investigated if such a pure k-space based approach will share the same benefits that k-space reconstruction has in standard PI. The behavior seems to be different with non-bijective encoding because a virtual k-space is accompanied by the artificial introduction of sharp edges in image space, such as those two horizontal cuts which are depicted in Fig. 1g.

Conclusions

It was shown in this publication that a k-space oriented reconstruction of imaging data encoded with non-bijective encoding fields forms a valuable alternative to reconstruction in image space. In image space, a modification to the SENSE algorithm is required, whereas GRAPPA can immediately be applied to undersampled PatLoc imaging data. Well-known benefits of GRAPPA such as smooth SNR boundaries are also observed with PatLoc imaging; however, reliance on calibration data imposes a physical limitation to image reconstruction that does not exist when a separate scan of the RF data is performed. As a consequence, a SENSE-variant is capable of completely reconstructing anatomical images while GRAPPA can only partially unfold aliasing and still requires a final SENSE-like reconstruction step to resolve the geometrically-imposed encoding ambiguities. Future research should investigate whether these limitations may be overcome with methods that could provide “virtual” calibration data to perform the reconstruction entirely in k-space.

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Figure 1.

Analog of image space Cartesian PatLoc reconstruction and SENSE and limited analogy for pure k -space based approaches. Depicted are examples encoded with linear SEMs (top) and quadrupolar SEMs (bottom). Top row: Increasing acceleration leads to increased aliasing. The image can be unfolded using SENSE. Bottom row: The situation is similar in PatLoc. If the image is represented in PatLoc encoding space (see subfigures (g - j)); the two parts of the object left and right of the black line in (f) appear in (g) separated and one above the other), the analogy becomes perfect: In this case, aliased image locations are also equidistantly distributed as in SENSE. This analogy is not perfect in k -space: Whereas an R -fold aliased image corresponds to an R -fold subsampled k -space in conventional imaging, PatLoc k -space is only $R/2$ -fold subsampled, if R voxels are aliased. There is only a “virtual” k -space that corresponds to the unfolded PatLoc image.

Figure 2.

Equivalent GRAPPA reconstruction problems for subsampled PatLoc imaging and its linear analogue: It is valid to apply GRAPPA to 2-times subsampled PatLoc data if and only if GRAPPA can be used in conventional imaging to reconstruct 2-times aliased images from 4-times subsampled data using 2-times subsampled calibration data. It can be shown that this is indeed possible, and therefore the GRAPPA algorithm can be applied without any modification to the subsampled PatLoc dataset.

Figure 3.

Different reconstruction methods for undersampled Cartesian imaging data. (a-d) Encoded with linear SEMs. (e,f) Encoded with quadrupolar SEMs. (a) All calibration lines are used to calculate the GRAPPA weights required to reconstruct the individual missing lines. (b) In a first step, only each second calibration line is used to calculate half of the missing lines. Then, additional ACS-lines are introduced and the remaining aliasing is unfolded using GRAPPA again. (c) SENSE is used to reconstruct the image from a 4-times subsampled dataset. (d) First, GRAPPA is used as in method (b), but the remaining aliasing is resolved with SENSE and not with GRAPPA. (e) A 2-times subsampled PatLoc dataset is reconstructed using the Cartesian PatLoc image space method. (f) First, standard GRAPPA is applied to the subsampled PatLoc dataset using additional calibration lines. In a second step, the remaining aliasing is reconstructed using the Cartesian PatLoc image space method. Observe the similarity of approaches (c,e) as well as (d,f).

Figure 4.

Reconstruction results for conventional PI. One simulated, and one measured, subsampled dataset are reconstructed with the four algorithms that are illustrated in Fig. 3a-d. Some differences in image quality are visible concerning residual aliasing and SNR. The worst result concerning both artifacts is observed with the 4-times SENSE reconstruction, where the artifacts have sharp boundaries. In the SENSE simulations, RF maps were used with a circular support that exceeded the boundaries of the object. In the numerical simulations, the one-step GRAPPA reconstruction seems to be a little bit worse concerning SNR compared to the two-step reconstructions. Such a difference is not visible in the measured data. No significant difference is seen between both two-step reconstructions.

Figure 5.

Reconstruction results for PatLoc-encoded images. For a simulated and a measured dataset, pure image space reconstruction is compared to partial GRAPPA reconstruction in combination with finalizing image space reconstruction (cf. Fig. 3e,f). The two-step reconstruction seems to yields slightly better results than the one-step approach. The data were not filtered; therefore, a pronounced streaking artifact is visible (cf. [9]).