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Stable Same-Sex Friendships with Higher Achieving Partners Promote Mathematical Reasoning in Lower Achieving Primary School Children

Dawn DeLay¹, Brett Laursen², Noona Kiuru³, Anna-Maija Poikkeus³, Kaisa Aunola³, and Jari-Erik Nurmi³

¹Arizona State University, PO Box 873701, Tempe, Arizona, USA, 85287

²Florida Atlantic University, 777 Glades Road, Boca Raton, Florida, USA, 33431

³University of Jyväskylä, PO Box 35, Jyväskylä, Finland, 40014

Abstract

This study is designed to investigate friend influence over mathematical reasoning in a sample of 374 children in 187 same-sex friend dyads (184 girls in 92 friendships; 190 boys in 95 friendships). Participants completed surveys that measured mathematical reasoning in the 3rd grade (approximately 9 years old) and one year later in the 4th grade (approximately 10 years old). Analyses designed for dyadic data (i.e., longitudinal Actor-Partner Interdependence Models) indicated that higher achieving friends influenced the mathematical reasoning of lower achieving friends, but not the reverse. Specifically, greater initial levels of mathematical reasoning among higher achieving partners in the 3rd grade predicted greater increases in mathematical reasoning from 3rd grade to 4th grade among lower achieving partners. These effects held after controlling for peer acceptance and rejection, task avoidance, interest in mathematics, maternal support for homework, parental education, length of the friendship, and friendship group norms on mathematical reasoning.

Keywords

mathematics; mathematical reasoning; primary school children; friend influence

Friends have considerable influence over one another. Much attention is given to the negative consequences of peer pressure. The benefits of peer influence are often overlooked, particularly in the academic arena. Early theorists suggested that the simple act of working with a peer should promote learning (Piaget, 1932/1965) and more recent views emphasize the cognitive gains that accrue within the context of friendship (Hartup, 1996). However, it is not evident that all students profit equally from peer influence on academic achievement. Lower achieving students benefit from affiliating with higher achieving peers, but it is not clear whether higher achieving students gain or lose in this situation (Wentzel, 2009). The present study is designed to examine friend influence over mathematical reasoning during

primary school. We focus on mathematical reasoning (measured in terms of number series induction), because its instruction requires sustained cooperative exchanges that are more likely to occur among friends than among nonfriends. Friendships that were stable from one academic year to the next were identified from a large, community-based sample of Finnish students to determine the degree to which relatively higher and relatively lower achieving partners influence one another's mathematical reasoning from the 3rd to the 4th grade.

Piaget (1932/1965) initially advanced the argument that the simple act of working with a peer could promote cognitive development, Vygotsky (1978) elaborated on this theme, proposing that interactions with more advanced peers promote the cognitive development of less advanced peers by working within the latter's zone of proximal development. In this view, influence flows only in the direction of improved performance: New skills arise when more advanced children work with their less advanced counterparts. This contrasts with Piaget's position that collaboration can be beneficial or detrimental, depending on the ability level of the peer interaction partner.

Empirical studies on collaborative learning environments have tested these competing perspectives. Consistent with Vygotsky's position, there is evidence that higher achieving students promote cognitive development in lower achieving partners, but lower achieving students do not have an adverse influence on higher achieving partners (see Topping, 2005). However, other research indicates that peer collaborations can actually diminish cognitive gains in problem solving skills (Tudge, 1992; Tudge & Winteroff, 1993). The contradiction in these findings may depend on the social context of the collaborative interaction, such that high quality interactions improve performance and low quality interactions may worsen performance (Cheng, Lam, & Chan, 2008). Taken together, the empirical evidence is mixed. Most studies seem to suggest that, if peer influence occurs, it will be toward cognitive improvement. Nevertheless, there are also findings to suggest that, under certain circumstances, cognitive decline may occur.

Another possibility is that influence varies according to how close the individual is to the comparison target (Suls, Martin, & Wheeler, 2002). Assimilative outcomes arise when an individual has an affectionate, warm relationship with the comparison target. Children usually select friends as comparison targets in school (Guay, Boivin, & Hodges, 1999), implying that friend influence over academic outcomes is typically assimilative. Empirical evidence supports claims that friends influence school grades, classroom engagement, and classroom motivation during primary school (Altermatt & Pomerantz, 2003) and middle school (Kindermann, 2007). Furthermore, both high school students (Frank, Muller, Schiller, Riegler-Crumb, & Mueller, 2008) and college students (Lomi, Snijders, Steglich, & Torlo, 2011) are susceptible to influence from friends on achievement. Lower achievers working with a higher achiever should improve the performance of the lower achieving partner because higher achieving friends, particularly those who are above average in terms of performance, model success and encourage achievement (Blanton, Buunk, Gibbons, & Kuyper, 1999) and, by the same token, higher achievers working with a lower achiever should inhibit improvement. Empirical evidence seems to suggest that academic comparisons may impact academic achievement during high school (Blanton, Buunk, Gibbons, & Kuyper, 1999; Huguet, Dumas, Monteil, & Genestoux, 2001). Unfortunately

previous research on mathematical reasoning has not distinguished higher achieving from lower achieving friends, so we do not know if simultaneous influence toward improved as well as diminished mathematics achievement are possible within friendship dyads.

The present study considers both upward and downward comparisons, distinguishing friends on the basis of their mathematical reasoning achievement. We define social comparisons broadly, as a measure of the difference in the initial achievement of two friends. It is operationalized as a characteristic of the individual within the dyad that implies (but does not specifically assess) social cognitive processes. Upward comparisons reflect the degree to which higher achieving friends influence the mathematical reasoning of lower achieving friends across the 3rd and 4th grade and downward comparisons reflect the degree to which lower achieving friends influence the mathematical reasoning of higher achieving friends across the same time period. Thus, the current study moves beyond a more traditional correlational approach by disentangling upward from downward social comparisons. A challenge to the present study, however, is the fact that data from friends are not independent (Laursen, 2005). The Actor-Partner Interdependence Model (APIM; Kenny, Kashy, & Cook, 2006) is designed to overcome these obstacles, partitioning variance across partners into variance that uniquely describes associations within and between partners. Moreover, by accounting for associations within and between partners the APIM model also allows for a less biased estimate of peer influence than is typical in correlational research because initial similarities, that may have resulted from peer selection effects (Kandel, 1978), are partitioned out of peer influence estimates. A longitudinal APIM for distinguishable dyads (DeLay, Hartl, Laursen, Werner, Campe, & Ortiz, 2014; Popp, Laursen, Kerr, Stattin, & Burk, 2008) was used to examine friend influence over mathematical reasoning.

Mathematical reasoning involves the ability to solve unfamiliar (non-routine), complex, and multistep arithmetic problems that require logical, systematic thinking (Mullis et al., 2009). Inductive reasoning, or inferring a general rule or relationship based on patterns and regularities (Liang, Jia, Taagen, Zhong, & Li, 2014), is one central aspect of mathematical reasoning and has traditionally been considered one of the most important components of intelligence (Spearman, 1923; Liang et al., 2014). Recent research suggests inductive reasoning is an important predictor of math achievement and algebraic reasoning (Lee et al., 2011). Nevertheless, while teachers may explicitly teach mathematics, they may not regularly provide specific instruction in inductive reasoning (de Koning, Hamers, Sijtsma, & Vermeer, 2002). If teachers do not as readily teach inductive reasoning in the classroom, perhaps students must learn inductive reasoning skills, in part, through peer interactions. We know, for example, that, even in the absence of direct instruction, students can work collaboratively and, when effective, collaborative learning can elaborate and enhance student learning (Biggs, 1999). With less direct instruction in inductive reasoning skills, children may be particularly susceptible to friend influence that occurs during informal collaborative learning activities with friends. Thus, susceptibility to friend influence on mathematical reasoning may result from both targeted collaboration on mathematical reasoning tasks and from many other types of peer interactions occurring in the classroom (e.g., problem solving, conflict resolution) that foster cognitive development through the development of inductive reasoning.

Potential confounds will also be considered. We know that educational achievement varies as a function of parent education and maternal support for schoolwork (Sameroff, Bartko, Baldwin, Baldwin & Seifer, 1998). We know that higher status peers tend to be more influential than lower status peers (Laursen, Hafen, Kerr, & Stattin, 2011). We also know that the effects of social comparison vary as a function of motives and interests (Wentzel, 2009). Presumably more change will take place in newer, as compared to more established, relationships (Tambs & Moum, 1992), because there is a ceiling as to how similar relationship partners can become. We also know that the norms represented in a group of peers may affect the nature and strength of friend influence (Berkowitz, 2003). Supplemental analyses will be conducted that control for the contribution of each of these potential confounds. In sum, the analyses of the current study will address the hypothesis that relatively higher and relatively lower achieving partners influence one another's mathematical reasoning from the 3rd to the 4th grade, independent of the contribution of peer acceptance and rejection, task avoidance, interest in mathematics, maternal support for homework, parental education, length of the friendship, and friendship group norms on mathematical reasoning. The detection of friend influence over mathematical reasoning would support a Piagetian perspective that cognitive change occurs in the context of collaboration with peers. However, we take the current investigation one step further by considering if influence is more likely to flow from higher achievers toward lower achievers or from lower achievers toward higher achievers. A Vygostkian perspective would hold that influence flows only in the direction of improved performance, so that higher achieving friends influence the mathematical reasoning of their lower achieving counterparts, but not necessarily the reverse.

Method

Participants

The current study focuses on a sample of 374 children in stable, reciprocated, same-sex friendship dyads (190 boys and 184 girls) who differed from one another on mathematical reasoning ($n=187$ friendship dyads). Children attended public schools in two municipalities in Central Finland, one in Western Finland, and one in Eastern Finland. Most participating students came from families with two biological parents (83%). About 50% of parents had a high school or vocational school degree and the rest attended some form of college.

Procedure

Participants were drawn from the ongoing First Steps Study. A total of 2,299 students were invited to participate. Parent consent and child assent were required for participation. Of this total, 86.8% ($n=1,995$) completed questionnaires in Grade 3 (2009) and (84.2%) 1,936 completed questionnaires in Grade 4 (2010). Of the 1,936 students who completed questionnaires in both Grade 3 and Grade 4, there were 981 children involved in stable reciprocated friendships across Grade 3 and Grade 4. Of this total, 18.7% ($n=183$ participants) were eliminated from analyses because their friends were members of other reciprocated friendship dyads. Two other-sex friendships were also eliminated ($n=4$ participants). This left a sample of 794 children (410 girls and 384 boys) in 397 stable, unique, reciprocated, same-sex friendship dyads. However, to heighten the distinctions

between friends, we created a subsample 187 friend dyads in which partners differed on mathematical reasoning (i.e., friends differed from one another by at least 1 standard deviation in mathematical reasoning).

Trained researchers administered tests to students during regular school hours. Mothers completed questionnaires at home and returned them by mail. All participants had complete data on mathematical reasoning. There were no greater than chance differences on demographic or control variables between those who did and those who did not participate in both waves of data collection, between those with and those without stable mutual friends, and between those who did and those who did not differ from their friends by at least 1 standard deviation on mathematical reasoning.

Measures

Stable Friendships—In the spring of 3rd and 4th grade participants were given a roster of all students in the classroom and asked to circle the names of up to three classmates whom they “most liked to spend time with.” The current sample represents 107 classrooms with 10 to 32 students per classroom in Grade 3 and 107 classrooms with 7 to 32 students per classroom in Grade 4. In almost all cases, children retained the same classmates from one year to the next.

Friends were defined as children who reciprocated most-liked nominations. *Stable friends* were dyads in which the same children reciprocated most-liked nominations in both the 3rd and the 4th grade. Stable friends were overwhelmingly same-sex (99%), so we restricted our study to these relationships. Some participants had more than one stable same-sex friendship. In the event of a tie, preference was given to friendships that maximized the sample size.

On the basis of a within dyad difference of at least 1 standard deviation in 3rd grade mathematical reasoning scores, each partner in each stable friend dyad was classified as either the *relatively higher achieving* or the *relatively lower achieving* member of the friendship dyad. These classifications did not change from the 3rd to the 4th grade (i.e., the same friend was classified as the higher achieving partner at both time points).

Mathematical Reasoning—In the spring of the 3rd and 4th grades, children completed the 30-item Arithmetic Reasoning Test (Räsänen, 2000). *Mathematical reasoning* was measured by number series induction tasks in which children were asked to continue a series of three numbers (e.g., 3, 5, 7) by adding a fourth number that best fit the series (e.g., 9). For each task, children were provided four response alternatives. One point was given for each correct answer. Internal reliability was acceptable ($\alpha = .78$ at Grade 3; $.79$ at Grade 4). The mathematical reasoning variable was significantly correlated with other variables that assessed performance in arithmetic word problems ($r = .59, p < .05$) and written computation ($r = .54, p < .05$).

In previous research, mathematical reasoning has been shown to predict performance in advanced mathematics, above and beyond the effect of computational skills (Lee, Ng, Bull, Pe, & Ho, 2011). Number-series or number-patterns have been widely used in assessing

mathematical or arithmetic reasoning (e.g., Mullis et al., 2009). Number series are used as indicators of inductive reasoning in many intelligence test batteries, such as the Stanford-Binet Intelligence Scales (for a review, see Liang et al., 2014).

Confounding Variables—To mitigate the contribution of potential confounds, supplemental analyses were conducted that included 3rd grade measures of peer relations, academic interest, and family characteristics. *Peer acceptance* was the sum of all within classroom liked-most peer nominations a child received, standardized by class size. *Peer rejection* was the sum of all liked-least peer nominations a child received, standardized by class size. *Task avoidance* represents the sum of affirmative responses on a 9-item task-avoidance scale (Onatsu-Arvilommi & Nurmi, 2000), describing the extent to which the child diverts attention from challenging activities (e.g., If something is difficult at school, I readily do something else). Each item was rated as true or false. Internal reliability was acceptable ($\alpha=.72$ at Grade 3; $.74$ at Grade 4). *Interest in mathematics* represents the average of 3 items (Nurmi & Aunola, 2005), describing the extent to which the child is interested in math (e.g., How much do you like mathematics?). Items were rated on a 1 (*do not like at all*) to 5 (*like very much*) scale. Internal reliability was acceptable ($\alpha=.76$ at Grade 3; $.87$ at Grade 4). *Maternal support for homework* represents the average of 7 items (Silinskas, Niemi, Lerkkanen, & Nurmi, 2012), describing the extent to which the parent assists the child with homework (e.g., Do you check your child's homework?). Items were rated on a 1 (*never*) to 5 (*always*) scale. Internal reliability was acceptable ($\alpha=.77$ at Grade 3; $.87$ at Grade 4). *Parental education* represents the highest degree completed by the child's mother or father, ranging from 1 (no study beyond basic education) to 4 (completion of a Master's or Doctoral degree). *Friendship length* represents the number of years the participants were friends prior to Grade 3 (range=0–2 years), using similar assessments of liking from grade 1 and grade 2. *Friendship group norms on mathematical reasoning* were calculated as the average mathematical reasoning score of all nominated friends excluding the score of the stable reciprocated friend.

Plan of Analysis

Longitudinal Actor Partner Interdependence Model (APIM) analyses were conducted within a path analysis framework. To determine the suitability of distinguishable dyad analyses, a χ^2 test of distinguishability (Kenny & Kashy, 2006) constrained the variances, interpersonal correlations, and intrapersonal correlations to be equal on mathematical reasoning for relatively higher achieving and relatively lower achieving friends. Significant χ^2 values revealed poor fit, indicating that friends should be distinguished on the basis of mathematical reasoning $\chi^2_{(4)}=309.60, p<.001$. Thus, we were able to reliably distinguish higher achieving friends from their lower achieving counterparts on the basis of mathematical reasoning.

Figure 1 illustrates the longitudinal APIM with two waves of data. Correlations between partners in the 3rd and 4th grade are included to model statistical nonindependence. Paths (*a1*) and (*a2*) are actor paths that represent the over-time stability for each partner on mathematical reasoning. Paths (*p1*) and (*p2*) are partner paths that represent the influence that each partner has over the other. Because the model controls for the actor's 3rd grade

level of mathematical reasoning, partner paths represent influence over change in mathematical reasoning from the 3rd to 4th grade. Thus, (*p1*) represents the influence of the higher achieving partner's 3rd grade mathematical reasoning on change from Grade 3 to Grade 4 in the lower achieving partner's mathematical reasoning; (*p2*) represents the influence of the lower achieving partner's 3rd grade mathematical reasoning on change from Grade 3 to Grade 4 in the higher achieving partner's mathematical reasoning. To compare the influence paths of the relatively higher (*p1*) and relatively lower (*p2*) achieving partners, the partner paths were constrained and a Satorra-Bentler scaled χ^2 difference test was conducted. A significant χ^2 value reveals poor fit, indicating that the paths are different. We present results for friends whose mathematical reasoning scores differed by at least 1 SD ($n=374$ individuals in 187 friendship dyads).

Results

Preliminary Analyses

Table 1 describes zero-order intercorrelations and basic descriptive information for all study variables in Grade 3 and Grade 4. There were statistically significant correlations between friends' mathematical reasoning scores in Grade 3 ($r=.56$) and in Grade 4 ($r=.54$). **APIM Analyses for Friends Distinguished on the Basis of Relative Mathematical Reasoning**

Figure 1 depicts the results. The higher achieving partner's 3rd grade mathematical reasoning was positively associated with change in the lower achieving partner's mathematical reasoning from the 3rd to the 4th grade when the initial level of mathematical reasoning during Grade 3 was controlled. The estimate ($\beta=.27$), 95% CI [.16, .38] ($d=.56$) indicated that greater initial levels of mathematical reasoning in higher achieving partners predicted greater increases in mathematical reasoning of lower achieving partners. In contrast, lower achieving partners' mathematical reasoning did not influence the subsequent mathematical reasoning of higher achieving partners ($\beta=-.01$), 95% CI [-.12, .10] ($d=0.02$). The longitudinal APIM model is saturated with a model fit of CFI = 1.0 and RMSEA = 0.0. To compare the influence paths of the relatively higher (*p1*) and relatively lower (*p2*) achieving partners, the partner paths were constrained to be equal. Comparison of this model with the unconstrained model revealed a statistically significant difference between influence paths, $\chi^2_{diff}(1)=7.25, p<.05$.

Figure 2 illustrates the degree (or slope) at which the lower achieving friend's mathematical reasoning changes as a function of the higher achieving friend's level of mathematical reasoning. To illustrate this we plotted the predicted slope for the lower achieving friend at three levels of the higher achieving friend's Grade 3 mathematical reasoning: (1) above average higher achieving friend (1 SD above the peer group mean); average higher achieving friend (between 1 SD above the peer group mean and 1 SD below the peer group mean); and below average higher achieving friend (1 SD below the peer group mean). The results indicated that the relatively lower achieving partners with highest achieving friends had the sharpest rate of improvement in mathematical reasoning. Thus, relatively lower achievers with the highest achieving friends profit most from friend influence over mathematical reasoning.

Differences in the rate of change in the mathematical reasoning of higher and lower achieving friends are further illustrated in an analyses of covariance (ANCOVA) with 4th grade mathematical reasoning as the dependent variable and 3rd grade mathematical reasoning as the covariate. Sex and relative achievement were the independent variables. Both lower achievers and higher achievers improved in their levels of mathematical reasoning from 3rd to 4th grade. However, differences emerged in the rate of improvement, $F(1, 371) = 18.83, p < .001$. Lower achieving youth with a higher achieving friend increased in mathematical reasoning more (Grade 3 $M = 13.57, SD = 3.46$; Grade 4 $M = 15.49, SD = 3.30$) than higher achieving youth with a lower achieving friend (Grade 3 $M = 18.99, SD = 3.12$; Grade 4 $M = 20.35, SD = 3.38$). Both boys and girls improved in their levels of mathematical reasoning from 3rd to 4th grade and there were no differences between boys and girls in rate of improvement.

Supplemental Analyses

To demonstrate the unique effect of friend influence, each partner's score for peer acceptance and rejection, task avoidance, interest in mathematics, and maternal support for homework, parental education were separately added to the model as control variables (i.e., with correlated paths to the Grade 3 predictor variable and Grade 4 outcome variable). Furthermore, because some friendships may have existed prior to Grade 3 we also used nomination data from grades 1 and 2 to control for the length of the friendship on peer influence (i.e., with correlated paths to the Grade 3 predictor and Grade 4 outcome variables). Finally, to disentangle the influence of friends from other affiliates, we included friendship group norms on mathematical reasoning, calculated as the average score on mathematical reasoning for all other friends nominated by each member of the friendship dyad (i.e., with correlated paths to the Grade 3 predictor and Grade 4 outcome variables). The same pattern of statistically significant results emerged after the inclusion of control variables. All dyads were same-sex, so sex could not be included in the model as a control variable. Instead, multiple group APIM analyses contrasted influence paths for boys and girls. A χ^2 difference test indicated that there were no sex differences in influence paths.

Discussion

The present study is unique in that it used a naturalistic classroom setting coupled with an innovative methodological design to investigate who influences whom on mathematical reasoning within stable primary school same-sex friendship dyads. The results demonstrate that children were influenced by the mathematical reasoning of their higher achieving friends, but not by the mathematical reasoning of their lower achieving friends. Thus, there was no evidence that peer influence resulted in a lower achieving friend having a negative impact on the mathematical reasoning of their higher achieving partner. These effects held after controlling for potential confounds, including peer status, child motivation and task avoidance, maternal support, parental education, length of the friendship, and friendship group norms.

Lower achieving friends profited from affiliation with higher achieving partners. We do not know the mechanism, but several explanations are plausible. Lower achieving children may

observe and internalize the beneficial learning strategies demonstrated by higher achieving friends (Bandura, 1986). Lower achieving students may feel pressure to conform to the standards set by their higher achieving friends, motivating them to invest in mastering the material (Brown, Clasen, Eicher, 1986). It is also possible that lower achieving friends simply find academic activities more enjoyable when working with a higher achieving friend than when working alone, enhancing their academic engagement and willingness to practice new skills. Formal and informal collaborations may also benefit lower achieving partners, because higher achieving partners work in ways designed to promote their friend's success and foster positive social interactions (Azmitia, 1988).

We found no evidence that lower achieving friends adversely influenced higher achieving friends. The findings are consistent with other naturalistic investigations indicating that lower achievers do not actively determine the direction of the learning process (King, 1993). The results are also in line with experimental findings. Azmitia (1988), for example, found that assimilative outcomes in problem solving tasks were qualified by the tendency for higher ability peers to provide guidance toward the promotion of learning for lower ability peers, but not the reverse. In another experimental study, there was evidence that working collectively promotes performance more than working independently, however, this finding was also further qualified by evidence that peer collaborations generally lead to improved performance flowing from higher ability children to lower ability children, and not the reverse (Fawcett & Garton, 2005).

Some experimental findings seem to contradict our results. Results suggesting that working with a lower achieving partner appears to be a liability for a higher achieving children (e.g., Tudge, 1992, Tudge & Winterhoff, 1993; Tudge, Winterhoff, & Hogan, 1996), are perhaps best explained in terms of the configuration of the collaboration and the demands of the task. In such studies, children are not typically paired with friends, which may reduce interest in the task (Rogoff, 1990). Nonfriends do not have a history of collaboration and may find cooperation difficult (Zajac & Hartup, 1997). Furthermore, experimental studies typically entail explicit peer instruction procedures. The benefits of goal-directed instruction may be limited because the higher achieving partner lacks incentive to alter or persist in instructional activities in the face of slow progress, or a lack of motivation, in their peer partner (Tudge, 1990). Finally, parents and teachers are available to support high achieving students in a naturalistic learning environment, something that is not available to students in a laboratory procedure. Thus, lower achievers may pose more of a risk to the educational accomplishments of higher achievers in experimental settings than in a naturalistic classroom environment.

In general, there seems to be something unique about peer influence over mathematical reasoning in the context of a naturalistic classroom friendship. To consider this possibility a bit further, we turn first to the ideas of Piaget. The Piagetian tradition holds that "cognitive conflict" will seek resolution. Schwarz and Linchevski (2007) further demonstrated that it is not simply peer interaction that promotes learning, but the resolution of divergent views. It follows that dyadic friendships may be a particularly relevant learning context because (a) given the frequency of friend interactions, conflict is likely to arise and (b) unlike nonfriends, friends are particularly motivated to resolve conflict within the relationship

(Laursen & Pursell, 2009). Still, change does not happen automatically, it requires a specific set of collaborative skills (Brown & Palincsar, 1989) and, as Vygotsky suggested, partners must be working within a zone of proximal development on learning tasks in order for influence over learning to occur. Friends are typically more similar than nonfriends on academic abilities (Ryan, 2000), which makes it reasonable to assume that friends tend to work within one another's zone of proximal development. Thus, cooperative friend interactions during mathematical reasoning tasks may foster skills that are otherwise untaught during primary school.

Caveats and Conclusions

Several caveats should be noted. First, participants were drawn from homogenous communities in Finland. Although representative of the population from which they were drawn, it will be up to future scholars to determine the degree to which these findings generalize to children living in other settings. Second, longitudinal data were collected at annual intervals, limiting precision about the timing of influence. Third, primary school children involved in long-term friendships may be unique. Children who change friends with some regularity undoubtedly differ from those who keep the same friends on characteristics associated with academic achievement. Fourth, only one aspect of mathematical reasoning was assessed, namely number series induction, which increase over time for a majority of students in this sample (67% increase). Although this measure has been widely used as an indicator of reasoning skills, there are also other aspects of mathematical reasoning, such as deductive reasoning that should be given some consideration in future research on friend influence over learning. Further, learning processes that are less susceptible to change or that do not normatively increase over time may not follow the same pattern of influence. Sixth, additional research is needed to determine if influence effects will similarly hold for (a) children who are relatively higher achievers within the dyad and absolutely low achievers within the classroom and for (b) children who are relatively low achievers within the dyad and absolutely high achievers in the classroom. Finally, although we controlled for the influence of all other nominated friends, we could not rule out the possibility that other sources of peer influence may have affected mathematical reasoning. The simultaneous consideration of multiple sources of peer influence within a classroom peer network is better addressed using a complete longitudinal social network approach.

In spite of these caveats, the current study offers some insight into how cooperative interaction between friends in the classroom may promote student learning. Much attention is given to how educators can foster an optimal learning environment. Often overlooked, however, is the role of friendship in promoting academic achievement. The present findings suggest that friends exert a positive influence over mathematical reasoning. Thus, encouraging students to work with friends can improve individual performance, which, in turn, enhances the classroom climate. Children may not always be able to work with friends, but educators can strive to structure their classrooms in ways that make use of relationship capital whenever possible.

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Figure 1. Friend Influence on Mathematical Reasoning as a Function of Relative Mathematical Achievement

Note. $n = 374$ (187 friend dyads). $* p < .01$.

Figure 2. The predicted slope of change in the lower achieving friend's mathematical reasoning as a function of the higher achieving friend's initial level of mathematical reasoning

Note. $n = 374$ (187 friend dyads). The lines represent the slope of change in the lower achieving friends mathematical reasoning from Grade 3 to Grade 4 as a function of having a relatively higher achieving friend who is initially (at Grade 3) above average, average, or below average on mathematics achievement.

Table 1
Intercorrelations Between Study Variables for Dyads Distinguished on the Basis of Mathematical Reasoning

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	M	SD
<u>Grade 3</u>																				
1. Mathematical Reasoning	—																		16.28	4.26
2. Peer Acceptance	.05	—																	2.71	1.70
3. Peer Rejection	-.06	-.23*	—																1.63	2.32
4. Task Avoidance	-.24*	-.02	-.01	—															2.75	2.13
5. Motivation for Mathematics	.18*	.06	.00	-.44*	—														3.77	1.22
6. Maternal Support	-.25*	-.03	.07	.17*	-.04	—													2.81	0.61
7. Parental Education	.23*	.09	-.06	-.07	.04	-.19*	—												4.63	1.46
8. Length of Friendship	.05	-.00	-.01	-.12*	.09*	-.13*	.02	—											0.55	0.76
9. Friendship Group Norms	.18*	.02	-.10	-.10	.20*	-.08	-.01	-.04	—										16.39	3.34
<u>Grade 4</u>																				
10. Mathematical Reasoning	.75*	.07	-.07	-.20*	.24*	-.33*	.27*	.09	.13	—									17.92	4.13
11. Peer Acceptance	.10	.43*	-.18*	-.03	.04	-.03	.08	-.06	.16*	.12*	—								2.57	1.72
12. Peer Rejection	-.19*	-.20*	.56*	-.00	-.00	.08	-.12*	-.01	-.11	-.13*	-.26*	—							1.53	2.21
13. Task Avoidance	-.31*	-.03	-.05	.51*	-.30*	.07	-.12*	-.12	-.16*	-.25*	-.03	.01	—						3.14	2.29
14. Motivation for Mathematics	.25*	.07	-.05	-.32*	.60*	.01	.08	.09	.21*	.26*	.01	-.03	-.49*	—					3.54	1.23
15. Maternal Support	-.30*	-.12*	.16*	.17*	-.02	.64*	-.11*	-.11	-.05	-.31*	-.06	.14*	.12*	.02	—				2.63	0.56
16. Parental Education	.23*	.09	-.06	-.07	.04	-.19*	1.0*	.02	-.01	.27*	.08	-.12*	.12*	.08	-.11*	—			4.63	1.46
17. Length of Friendship	.05	-.00	-.01	-.12*	.09*	-.13*	.02	1.0*	-.04	.09	-.06	-.01	.12*	.09	-.11	.02	—		0.55	0.76
18. Friendship Group Norms	.12	-.01	-.06	-.03	.09	-.09	.08	-.08	.40*	.19*	.09	-.01	-.12	.07	-.04	.08	-.08	—	17.66	3.39

Note. $n = 374$ (187 friend dyads).

* $p < .05$