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A Kindergarten Number-Sense Intervention With Contrasting Practice Conditions for Low-Achieving Children

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Abstract

The efficacy of a research-based number-sense intervention for low-achieving kindergarten children was examined. Children ($N = 126$) were randomly assigned to 1 of 3 conditions: a number-sense intervention followed by a number-fact practice session, an identical number-sense intervention followed by a number-list practice session, or a business-as-usual control group. The interventions were delivered in a small-group setting over 24 half-hour lessons. Both intervention groups performed better than controls on measures of number sense, arithmetic fluency, and general mathematics calculation achievement at immediate posttest. However, the number-fact practice condition gave children an additional advantage over the number-list practice condition on the outcomes at delayed posttest 8 weeks later. The number-fact practice condition was especially effective for producing gains in English learners.

Keywords

Arithmetic practice; English learners; Fluency; High-risk students; Intervention; Low-achieving students; Number list; Number sense

Early mathematics difficulties lead to long-term educational problems. In the absence of effective interventions, many children who enter first grade with mathematics delays stay behind throughout their school careers (Morgan, Farkas, & Wu, 2009). Even more troubling, low-income children enter kindergarten far behind their middle-income counterparts and are often poorly prepared for formal mathematics instruction (Jordan, Kaplan, Locuniak, & Ramineni, 2007; Jordan, Kaplan, Oláh, & Locuniak 2006; National Mathematics Advisory

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Panel, 2008; National Research Council, 2009; Starkey, Klein, & Wakeley, 2004). According to a recent Nation's Report Card (U.S. Department of Education, Institute of Education Sciences, National Center for Educational Statistics, 2013), low-income children comprise 76% of fourth graders who scored in the lowest 25th percentile for mathematics, an increase of 2 percent from earlier reports. Although fourth- and eighth-grade mathematics scores are rising for all children, an income gap remains. There is a similar gap in performance between English Learners (ELs) and non-ELs, regardless of income. These statistics suggest that effective intervention for mathematics difficulties is a matter of equity for children in poverty, for ELs, and especially for children who find themselves in both categories.

Early number competencies predict levels of mathematics achievement (Jordan, Kaplan, Ramineni, & Locuniak, 2009) as well as the presence of mathematics learning disabilities (Mazzocco & Thompson, 2005). Kindergarten mathematics achievement has also been shown to be the best predictor of later mathematics and reading achievement (Claessens, Duncan, & Engel, 2009; Claessens & Engel, 2013; Duncan et al., 2007; Duncan & Magnuson, 2011). Understanding of number concepts and relations helps children perform arithmetic operations and can be applied to other mathematical domains such as measurement, data analysis, and geometry (National Research Council, 2009). Children with poor number competencies overrely on rote memorization, which results in weak problem-solving skills (Robinson, Menchetti, & Torgesen, 2002).

Kindergarten is a critical instructional period with increasing achievement expectations in mathematics. The U.S. Common Core State Standards in Mathematics (CCSSM) specify that by the end of kindergarten, children should be able to count to 100 by ones and tens, write numbers from 0 to 20, understand one-to-one correspondence and cardinality, compare numbers, solve addition and subtraction problems with objects, solve word problems, and fluently add and subtract within five, among other skills (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). To promote the success of all students, evidenced-based interventions aimed at improving core number competencies must be available to teachers and others working with children (Dowker, 2005; Gersten, Jordan, & Flojo, 2005).

The purpose of the present study was two-fold. First, the study tested the effectiveness of a revised number sense intervention that targets fundamental understandings of number, number relations, and number operations for low-achieving kindergartners; second, it examined the relative effectiveness of two types of practice, one that supported fluency in number facts and one that supported facility using a number list. Both practice conditions were delivered immediately after the intervention.

Developing Number Competencies

Early Numerical Development

Clements and Sarama (2009) asserted that early numerical development depends on four interrelated foundations: (a) subitizing, (b) counting with conventional number words in a stable order, (c) enumerating collections of objects, and (d) numbering (cardinal number)

skills. Children move from acquiring this core knowledge of number, to understanding relations between numbers, to operating with numbers—with the progression repeating as counting skills increase and larger numbers are learned (Clements & Sarama, 2007; National Research Council, 2009). Young children might know that 5 is 2 after 3 but still have trouble determining what number is 2 after 7.

Subitizing refers to the ability to recognize and name the exact quantities of small collections without counting and is fundamental to the development of number knowledge (Baroody, 1987). Most kindergartners can perceptually subitize or determine exact representations (without counting) for up to three objects (National Research Council, 2009). With larger quantities, children use approximate number system representations, which become less precise as magnitudes increase (De Smedt, Noel, Gilmore, & Ansari, 2013). Many young children can indicate which of two sets of objects has more, although difficulty level varies according to the numerical distance between the two sets (i.e., differentiating between 12 and 20 objects is easier than 12 and 15). Quantities also can be conceptually subitized (Clements, 1999). In *conceptual subitizing*, children recognize smaller sets within a larger one to determine the number of objects. Conceptual subitizing leads to facility with arithmetic combinations (Fuson, 1992). For example, children can “see” on a die that five is made up of four dots and one more. Using a five-frame arrangement of dots of two different colors helps children apprehend various combinations of five (Clements, 1999; Fuson, 1992).

Counting scaffolds the child’s understanding of relationships between numbers (Baroody, 2006; Baroody & Dowker, 2003; Baroody, Eiland, & Thompson, 2009). For example, knowing the next number in a count helps children see how much “one more” is, and knowing the order of the count sequence allows them to find the greater of two numbers (Clements, 1984). Children must learn to enumerate objects to determine cardinality or how many are in the collection, which depends on knowledge of one-to-one correspondence and the stable order principle (Gelman & Gallistel, 1978). Counting also helps children expand their quantitative knowledge to larger numbers (Baroody, 2006; Baroody & Dowker, 2003; Baroody et al., 2009).

Children develop subitizing and counting abilities with smaller quantities first, sometimes independently of each other (Clements & Sarama, 2009). Although most master these skills by kindergarten, some children with less exposure to counting or with learning disabilities come to school without these foundations (Dyson, Jordan, & Glutting, 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). Although children can conceptually subitize more easily when dots are arranged in a pattern, the easiest way to present objects for young children to count is in a straight line (Clements & Sarama, 2009). This representation provides an obvious beginning and end to the count, and discourages children from counting items more than once. Linear representations also help children see the linearity of numbers; that is, each number is exactly one less than the number after it and one more than the number before it—no matter how big the number (Baroody et al., 2009).

Number Operations and Arithmetic Fluency

Early facility with number combinations suggests that children have internalized core number skills. Conceptual subitizing and counting support fluency with combinations (Clements & Sarama, 2009; Fuson, 1992). For example, if children see that 1 and 2 are interchangeable “partners” for 3, they more easily learn the combinations $2 + 1$, $1 + 2$, $3 - 1$, and $3 - 2$. Kindergartners who count on their fingers to solve addition and subtraction problems achieve fluency faster than those who do not use their fingers (Jordan, Kaplan, Ramineni, & Locuniak, 2008). Although this assertion may seem counterintuitive, early finger use scaffolds adaptive strategies (Fuson & Kwon, 1992), such as counting on from an addend in addition, and leads to meaningful associations with correct answers (Jordan, Huttenlocher, & Levine, 1992; Jordan, Levine & Huttenlocher, 1994; Siegler & Shrager, 1984). Operational fluency lessens the load of other cognitive processes (e.g., working memory) and allows children to focus their attention on the problems being solved (Geary, 1994). In particular, fluency deficits characterize children with mathematics difficulties (e.g., Jordan, Hanich, & Kaplan, 2003; Jordan & Montani, 1997).

Although the importance of instruction in arithmetic operations is obvious, the value of fluency training is subject to debate. However, extended practice rooted in understanding seems to help children form long-term representations of combinations that will be available for future use (Baroody, 1985; Fuchs et al., 2013; Fuson & Kwon, 1992; Gersten & Chard, 1999; Jordan, 2007). Fuchs et al. (2013) contrasted the efficacy of two different practice conditions during the last 5 minutes of an established mathematics intervention for first graders with serious mathematical difficulties (MD). The practice conditions included a speeded fact retrieval activity and a nonspeeded number board game. The nonspeeded condition encouraged multiple strategies and thinking relevant to number combinations; the speeded condition supported fast fact retrieval as well as efficient counting strategies, thus creating many instances of correct answers to reinforce memory. Children with MD who received the speeded condition performed closer to normally achieving peers at the end of the study than MD children in the nonspeeded condition, suggesting that speeded practice with combinations may help reduce the achievement gap in this population.

Addressing the Needs of English Learners

ELs, or children who come from homes where English is not the primary language, are a growing population in the United States. Schools in low-income communities often contain a high percentage of English learners (Dyson et al., 2013), which makes attention to this group’s needs important to consider when developing interventions for children at risk. Although it has been argued that language is relatively less important for mathematics, young children actually learn most of their early number concepts through oral language rather than through numerical symbols (Clements, Baroody, & Sarama, 2013). Cummins (2001) stressed the importance of building and activating background knowledge when working with ELs. Children may not be able to recognize the connection between what they already know in their first language and what they are being taught in a second language. Moschkovich (2012) pointed out, however, that components of effective math instruction for children in general should also be applied to ELs (e.g., explicit instruction in mathematical concepts and the connections among those concepts; Hiebert & Grouws, 2007).

An Evidence-Based Number-Sense Intervention

Number competencies can be taught effectively to most young children (Frye et al., 2013; National Research Council, 2009). In two randomized controlled studies (Dyson et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012), our research team showed that a number-sense intervention improved key number competencies as well as general mathematics skills in randomly selected kindergartners from low-income communities. Following the National Research Council (2009), we define *number sense* as basic skills in number (counting, number recognition—both quantity and numeral), number relations (bigger and smaller, before and after), and number operations (numbers are sets that can be broken into smaller sets and combined to make bigger sets). The small-group intervention targets these number-sense skills and uses consistent linear representations of number and carefully introduced mathematics vocabulary. Although the intervention succeeded overall, children who started the study with the lowest number competencies made small gains. Analysis of individual differences revealed that these children did not know some of the most basic number concepts at the start of the intervention, making it difficult for them to keep pace with the lessons even in a small group. For example, they could not immediately recognize two objects as “two” or three object as “three,” count to 10, or name single-digit numerals. We saw the need for an intervention for children (regardless of income level) who initially scored below their peers (below the mean) on a number-sense screener that successfully predicts future mathematics achievement (Jordan & Glutting, 2012). Although we preserved many of the instructional techniques of our previous trials (Dyson et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012), we recalibrated the intervention to meet the needs of some of the highest risk learners. In short, we intensified our focus on understanding of number, relations, and operation with numerosities of 5 or less and added emphasis on fluency (being able to quickly find the sum or difference within 5) as recommended by the CCSSM.

Overview of the Present Study

The present study built on earlier findings in two ways. First, we addressed whether a revised 24-lesson kindergarten number-sense intervention, one that targets fundamental understandings of number, number relations, and number operations, would be effective for low-achieving learners. Second, we addressed the relative effectiveness of two types of practice: one that emphasized fluency in number facts versus one that emphasized proficiency using a number list.

Children, all low achievers, received the same number-sense intervention for 25 minutes but were randomly assigned to one of two practice conditions, which occurred during the final portion of the session. Both practice conditions were provided in game formats and supported by the intervention instruction. In addition to a business-as-usual, low-achieving control group, a business-as-usual control group of higher achieving children was included to determine whether the intervention and practice narrowed the kindergarten achievement gap. Pretest and posttest measures included a number-sense screener that is highly sensitive to change in kindergartners, an arithmetic fluency assessment, and a standardized test of mathematics calculation achievement. In addition to testing children immediately after the

intervention, a delayed posttest (about 8 weeks later) allowed us to examine the durability of any observed gains.

In addition, we examined the unique contribution of cognitive processes, attention, and reading to children's performance. This enabled us to identify broader predictors of mathematics learning as well as to capture a more in-depth view of children with mathematics learning difficulties. Young children's ability to reason with nonverbal representations (Jordan et al., 2003) and to transform objects spatially is associated with their mathematics success (Gunderson, Ramirez, Beilock, & Levine, 2012; Mazzocco & Myers, 2003). Attention has also been found to be a strong predictor of children's ability to solve calculation problems in first grade (Hassinger-Das, Jordan, Glutting, Irwin, & Dyson, 2014). Because reading is often linked to school success more generally and is related to verbal ability, a measure of letter and word recognition was also included in our assessments.

We predicted that both intervention groups would outperform the control group on all three outcomes. In light of Fuchs et al. (2013) findings, we also expected that the children in the number-fact practice condition would outperform children in both the control and the number-list condition in calculation fluency and mathematics calculation achievement and that they would perform equally as well as children in the number-list condition in number competencies.

Method

Participants

Participants were recruited from all kindergarten classes in four schools within two school districts in a medium-size metropolitan area. Schools were chosen because of their demographics—each had a large population of low-income children. The percentage of children enrolled in the free or reduced lunch program ranged from 48% to 93% in each school with a mean of 70% across schools. Informed consent letters were sent home with every kindergartner in each school as follows: School 1—69 kindergartners; School 2—131 kindergartners; School 3—75 kindergartners; School 4—177 kindergartners. All consenting children ($N = 276$) were screened on a version of the number-sense screener (Jordan, Glutting, & Dyson, 2012; Jordan, Glutting, Ramineni, & Watkins, 2010), which is described in the Measures section. The mean raw score for the entire primarily low-income sample was 21.1 ($SD = 8.7$) out of a possible 44. This score was used to divide children into two groups: those who scored below the mean raw score (< 21 , $n = 148$) and those who scored at or above this average (≥ 21 , $n = 128$). Children in our low-scoring group achieved below the 25th percentile, based on common items from an earlier version of the number-sense screener that has kindergarten norms from a range of social classes (Jordan, Glutting, & Dyson, 2012).

Of the children who were identified as higher achieving, half ($n = 64$) were selected at random as a contrast group for an analysis of whether the intervention may have reduced the achievement gap. (These children were not part of the intervention experiment but completed all testing.) The mean for the number-sense screener for this subset of higher

achievers remained the same as the original group (mean of original 128 higher achieving students = 28.84 [$SD = 5.46$]; mean of random selection = 28.92 [$SD = 5.60$]). This uneven representation did not present a problem because it is desirable to compare with children in higher performing schools when looking at the possibility of closing an achievement gap.

A sample size of 126 was necessary for the planned analyses. We determined this by setting $\alpha = .05$ for nine predictors (age, gender, status as an English learner, nonverbal reasoning, spatial ability, attentive behavior, reading achievement, pretest score, intervention condition), with power equal to .85 and given a medium effect size of $f^2 = .15$ (according to Cohen's [1988] criteria of $f^2 = .02$ as small, $f^2 = .15$ as medium, and $f^2 = .35$ as large). Because children were assigned to groups of four for the intervention instruction, 11 groups were needed for each of our three conditions ($n = 44$ per condition).

Randomization was at the student level. So that each of the three conditions (number-fact practice, number-list practice, and business as usual) would be equally represented in each classroom in each school, we randomly assigned low-achieving children in each classroom to one of the three conditions. For example, if classroom X had seven low-achieving children, two were randomly assigned to the number-fact condition, two to the number-list condition, and two to the business-as-usual control. The seventh child was not included in the experiment so that each classroom had the same number of children in each condition. Parents were informed prior to consent that not all children would be selected for participation and that some would be assigned to the business-as-usual control group.

After children were assigned to a condition, children in each of the intervention conditions were randomly assigned to groups of four by school, independent of their classroom. Thus, children from the same class typically were not in the same small group. This avoided classroom effects that may have occurred if children from the same classroom composed a small group. Only children who completed all three rounds of testing (pretest, posttest, and delayed posttest) were included in the final experimental sample ($N = 126$ out of 132). Six children did not complete testing; one child in the number-list practice condition refused to participate after 2 weeks, another in the number-list practice condition withdrew because of an instructional conflict, and two children each in the number-list practice and control conditions moved out of the area. The six children were distributed across the four schools: one from School 1, two from School 2, two from School 3, and one from School 4. Their mean number-sense pretest score was 14.67, consistent with the final low-achieving sample. Additionally, two children in the higher achieving comparison group moved before all tests were completed, leaving 62 out of 64 children.

The four schools were represented in the final sample proportionally to their response to participate as follows (low achieving/higher achieving): School 1 (23/4), School 2 (46/21), School 3 (22/13), School 4 (35/24). School 1 had 2 groups of each of the three conditions, School 2 had 4 groups of each condition, School 3 had 2 groups of each condition, and School 4 had 3 groups of each condition, making a total of 11 groups of each condition.

Demographics for the final sample by group are summarized in Table 1. Of the total sample of 188 children (including the higher achieving comparison group), 93 were girls (49.5%)

and 95 were boys (50.5%). Forty-five of the students were identified as African American (23.9%), 80 as Hispanic Caucasian (42.6%), 50 as non-Hispanic Caucasian (26.6%), and 13 as other ethnicities (6.9%), all by teacher report. Sixty-six of the students (35.1%) were identified as ELs. ELs were 100% Hispanic Caucasian, 100% eligible for free or reduced lunches, 54.5% male, 45.5% female, and did not differ from the general population in average kindergarten start age. The average kindergarten start age overall was 66 months and children ranged in age from 57 months to 72 months.

Dependent Variables

Number competencies—Basic number competencies were measured using a version of the Number Sense Screener first developed by Jordan, Glutting, Ramineni, and Watkins (2010). It is composed of 44 questions that assess children's number knowledge in six areas: counting, number recognition, number comparisons, nonverbal calculation, story problems, and number combinations. Every item is presented orally to the child, and the child gives an oral response. The measure is untimed and takes approximately 20 minutes to complete. The screener is internally consistent with a coefficient alpha of at least .90 in kindergarten. Furthermore, the number-sense assessment predicts mathematics growth and achievement in first through third grades (Jordan, Glutting, & Dyson, 2012; Jordan, Glutting, & Ramineni, 2010; Jordan, Glutting, Ramineni, & Watkins, 2010; Jordan, Kaplan, Ramineni, & Locuniak, 2009).

Arithmetic fluency—Arithmetic fluency was measured using a 16-item timed task containing eight addition items ($3 + 1$, $2 + 1$, $2 + 2$, $1 + 2$, $2 + 3$, $1 + 4$, $6 + 1$, $5 + 3$) and eight subtraction items ($3 - 2$, $4 - 1$, $5 - 2$, $4 - 3$, $5 - 1$, $5 - 3$, $6 - 1$, $7 - 3$). In an activity adapted from Jordan and Montani (1997), children were presented with a visually displayed combination (i.e., $2 + 1$, $3 - 2$) and asked, "How much is two plus one (or three take away two)?" Children were told to answer as quickly as possible. One nonscored addition ($1 + 1$) and one subtraction item ($3 - 1$) were given for practice. Children received credit for items answered correctly within three seconds with a total possible score of 16. Examiners were trained to use a simple "one one thousand, two one thousand, three one thousand" count to track response time cutoff. Average coefficient alpha in kindergarten is .85.

Mathematics calculation achievement—General mathematics achievement was assessed with the Woodcock–Johnson III Tests of Achievement (WJ-III) Standard Test Book Form A: Calculation subtest (Woodcock, McGrew, & Mather, 2007). The WJ-III is normed from preschool through adult and uses stop points based on students' errors. Unlike the Number Sense Screener, the test is primarily a paper-and-pencil measure involving written calculations. Internal reliability for children ages 4–7 is .87 (McGrew, Schrank, & Woodcock, 2007).

Background Variables

Nonverbal reasoning—Nonverbal reasoning was assessed using the Matrix Reasoning Subtest from the Wechsler Preschool and Primary Scale of Intelligence (WPPSI; Wechsler, 2002). This subtest is designed for use with children from 4 to 7 years of age. Children were presented a 2×2 matrix with pictures in three of the four cells. They then were asked to

complete the matrix with the related picture by selecting from a choice array of four or five picture options. Internal reliability for kindergarten is .90.

Spatial ability—The Children’s Mental Transformation Task (CMTT) Form A was used to assess children’s spatial ability. The CMTT is a 32-item multiple-choice test developed by Levine, Huttenlocher, Taylor, and Langrock (1999) to assess the two-dimensional mental transformation skills of children ages 4–7 years old (internal reliability, $\alpha = .52$).

Inattentive behavior—Kindergarten teachers completed the SWAN Rating Scale (Swanson et al., 2006) for each participant in an experimental condition group. Teachers were presented with nine statements, such as “Listens when spoken to directly,” and asked to rate the child’s ability on a 7-point Likert scale ranging from -3 to $+3$. Scores reflect the level of inattention. Therefore, a positive score indicates an above average level of inattention, and a negative score indicates a below average level of inattention. Internal consistency is high ($\alpha = .92$) according to Lakes, Swanson, and Riggs (2012).

Reading achievement—Children’s reading skills were assessed with the WJ-III Standard Test Book Form A: Letter-Word Identification subtest (Woodcock et al., 2007). Children are asked to identify letters and read words. Internal reliability for children ages 4–7 is .92.

Testing Procedures

All measures were administered individually. A pretest (November), immediate posttest (March), and 8-week delayed posttest (May) design was used. Measures of general cognitive abilities were administered only at one time point because they assess relatively fixed abilities. We also were mindful of keeping the testing to a minimum. As such, the measure of nonverbal reasoning was given at pretest, and the measure of spatial abilities was given at delayed posttest. Teachers filled out the SWAN inventory at the beginning of the intervention. Reading was administered at pretest as a measure of general school ability. All other measures of the outcome variables were administered at all three time points. Children were assessed in approximately a 1-week window for each assessment period. Testers were generally blind to children’s group status. Although group assignment was not given to the testers, a few children might have inadvertently revealed their group status at posttest by talking about their “math group.” However, the objective nature of the measures reduced the possibility of bias on the part of the examiners.

Number-Sense Intervention

Children’s regular mathematics classroom instruction was not interrupted by the study. The curriculum used in School 4 was *Investigations in Number, Data, and Space* (Russell et al., 2012), and the other schools used the *Math Connects* (Altieri et al., 2009) curriculum. With the exception of School 1, children in the control group received the school’s own form of 30-minute mathematics intervention time in addition to their regular mathematics instruction period. Our intervention groups took place during this same intervention time. School 1 had no specific time set aside for mathematics intervention, so children were taken during a nonacademic instructional period.

The experimental interventions occurred over 8 weeks between January and March. Twenty-four 30-minute lessons were carried out in small groups of four in a quiet area close to children's classrooms. Trained undergraduate or graduate students taught the scripted lessons. In order to minimize instructor effect, instructors taught one group of each condition. Because the largest school required two extra instructors, it was necessary to have two instructors teach only one condition each.

Based on previous iterations of our number-sense interventions (Dyson et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012), the set of intervention lessons was specifically geared to reach lower performing children while still engaging those who might be higher performing. The lessons were designed to improve number-sense skill through targeted instruction in number, number relations, and number operations. Representations of number were consistently linear, and vocabulary was carefully chosen, introduced, and practiced. The scope and sequence of the intervention and practice are provided in Table 2.

Children were encouraged to subitize perceptually and conceptually in three contexts: (a) dots in various configurations presented on a card, (b) fingers held up, and (c) dots presented on five and subsequently ten frames. For example, for 3, children were briefly shown (for approximately one second) a card with one, two, or three dots in various configurations and asked "How many dots?"; made and recognized 3 on their fingers without counting; and recognized 3 when presented on a five frame (Figure 1). On numbers 6 through 10, children used only fingers and ten frames; numbers were presented as 5 and 1, 2, 3, 4, or 5 more. For example, for 7, children practiced making (and recognizing) 7 on their fingers without counting as a set of five (one hand) and two (on the other hand) and practiced recognizing 7 on a ten frame as a full row of 5 and 2 more (Figure 2).

In previous interventions, lessons began with the numbers one through 10, and simpler concepts, such as that each counting number is exactly one more than the previous number, were introduced first. Then, in subsequent lessons, we used pairs of numbers from 1 to 10 to ask, "Which number is bigger/smaller?" Finally, we looked at partners for numbers (decomposing numbers into two parts), and children solved story problems and number sentences. In the present intervention, we addressed all these ideas about number in the first lesson, but with only the numbers zero, one, and two. Although zero is not a counting number, it was included to provide a solution for a number sentence such as $2 - 2 = 0$. Some of the concepts included in Lesson 1 were the following:

- Practice subitizing the quantities zero, one, and two.
- Build "two" with interlocking cubes.
- Two is "one more" than one.
- Counting to two on fingers.
- Making one and two on fingers without counting fingers (automaticity).
- One and one are partners for two.
- Number sentences and corresponding story problems modeled with objects ($1 + 1 = 2$, $2 - 1 = 1$, $2 - 2 = 0$).

- Solving number sentences and corresponding story problems on fingers.

Linear representations of quantities (dots in a line) were used for every activity (with the exception of subitizing cards) in our approach. For each lesson, the next counting number was added, and similar ideas were investigated. For the first nine lessons, children were presented with a cardinality chart (Figure 3), which allowed them to see the quantity accumulating as they counted up the number list. Each day the children “built” the numbers from one to the new number using interlocking cubes and placed them on the cardinality chart—seeing that adding one more block made the next number. Children would recite, “One and one more is two, two and one more is three, three and one more is four, etc.,” as the instructor moved his or her finger up the “steps” created by the blocks. After children reached the number 10, a number list with the numbers one through 10 (Figure 4) replaced the cardinality chart. This representation helped children see the quantities nested within other quantities. As children counted using the number list, they could see the quantities accumulating. Children also investigated numbers bigger and smaller, before and after, and the $n + 1/n - 1$ rule (each number is one more than the number before it and one less than the number after it using the number list). The $n + 1/n - 1$ rule is foundational for solving addition and subtraction problems through counting. Counting objects with a number list present also helped children connect quantity to numeral. For example, the instructor would place one token on the number list, and say, “How many dots?” When the children said “1,” the instructor would point to the numeral 1 right above the token and say, “That’s right, one dot. ... What number is this?” and the children would say, “1.” The instructor would add another dot and say, “1 and 1 more is ...” and wait for the response “2.” This would continue until the every dot on the number list contained a token.

Although children used a variety of physical models and drawings to solve number combinations and story problems, the focus of strategy instruction was adaptive use of fingers. For sums up to five, children first produced each addend on separate hands. Children were encouraged to subitize (perceptually or conceptually) the answer. For sums from 6 to 10, children counted-on to the larger addend using the “in hand” method (Fuchs et al., 2013), that is, finger counting by holding the larger addend “in hand” and then “counting on” the smaller. This activity was introduced by having children hold actual tokens representing the larger addend in the one hand and representing the smaller addend with fingers on the other hand. Children said the number “in hand” and then counted up the fingers on the other hand. Eventually they were instructed to “pretend” the tokens were in their hand and count in the same manner.

In order to meet the needs of the many ELs, we assumed minimal conventional knowledge of English number words. We believed the new format of the intervention would help ELs in two ways. First, their nonverbal number sense could be activated in a familiar context by beginning with small numbers they may have already mastered in their first language. Second, the relatively slow pace of the lessons and the “math talk” with consistent physical models would allow ELs access to both the numerical content and the vocabulary of mathematics simultaneously.

Practice conditions—Children in both conditions received the same 25-minute number-sense intervention. Lessons differed for the two intervention groups only in the last 5 minutes of the 30-minute lesson as follows.

Number-list practice: In this condition, each child played a version of the Great Race Game (Ramani & Siegler, 2008), which was also used in our previous interventions (e.g., Dyson et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). The game built directly on the number-list instruction in the lessons. In our version of this game, the child spins a spinner and moves $+1/-1$ or $+2/-2$ spaces on a colorful 1–10 number list while saying a number sentence. For example, if a child was on the number 3 and spun $+2$, the child would say, “Three and 2 more is 4, 5 ... is 5!” while moving the game piece on the board. The instructor helped model the movement and prompted children who struggled. Because of the nature of the game, one number of the combination was a number 1–9, and the second number of the combination was 1 or 2 (the numbers on the spinner).

For the first five lessons, the instructor demonstrated on a large game board and the children individually duplicated the move on their own small game boards. This allowed the instructor an opportunity to hear each child repeat the move. For the next five lessons, the teacher no longer demonstrated, but the children took turns spinning and being the first to move (i.e. they took turns demonstrating the move on the board) while the other children duplicated the move on their own game boards. In subsequent lessons, instructors worked with each child one-on-one for approximately 1 minute while the other children completed and colored a number activity paper. The number activity paper consisted of tracing numerals, writing numerals to match quantities, and writing the solution to combinations with small numbers that reflected the lesson material. By the end of the intervention, during their 1-minute session with the instructor, children were completing between four and eight moves per game, depending on their skill level.

Number-fact practice: In this condition, children participated in an activity that encouraged quick answers to addition and subtraction combinations that had been taught in the lesson that day or in previous lessons. Because the CCSSM recommends that kindergartners be able to fluently add and subtract within 5, the combinations in the practice were limited to these number facts as well as $n + 0$, $n + 1$, and $n - 1$ in which n was 1–10. Children were shown a combination printed horizontally on a card (from a random pile of 20 cards) while the instructor said, for example, “How much is two plus one?” They were encouraged to answer each combination quickly, but if they could not, or answered incorrectly, they were asked to use their fingers to work out the answer using finger strategies taught in the lesson. The children were also encouraged to mentally move up and down the number list to solve $+1$ and -1 problems.

As with the number-list game, children worked together for the first 10 lessons, going around the group, each taking a turn saying a number fact. Then, as in the number-list condition, instructors worked with each child individually for 1 minute while the other children completed and colored the same number activity paper as the children in the number-list condition. By the end of the study, children ranged from 5 to 10 facts per minute, depending on their skill level.

The one-on-one time with the instructor for each condition was to maximize attention and to ensure that children were given individual feedback on their responses.

Fidelity of implementation—Lessons were scripted to ensure fidelity of implementation. To be sure instructors followed the scripts, all lessons were audio recorded. After the 24-week intervention was concluded, researchers listened to eight randomly selected lessons of each instructor (i.e., 1/3 of the lessons). Activities completed were recorded on a chart of activities required by the lesson scripts. Out of 900 possible activities recorded over the eight lessons, instructors completed an average of 97% of the activities.

Data Analysis

Preliminary analysis: A set of ANOVAs was run to test the difference between the three experimental groups with respect to demographics. There was no significant difference between these groups with respect to gender, ethnicity, language, or income status. However, there was a significant group effect with respect to age, $F(2, 123) = 4.23, p = .017$. Post hoc tests using the Bonferroni correction to make pairwise comparisons revealed a significant difference only between the control group and the number-fact practice group, $p = .013$. Therefore, age was added as a covariate in the ANCOVAs, which were conducted to evaluate the effect of the intervention.

We also checked to see if there was a significant difference in the groups with respect to background variables (inattentive behavior, nonverbal reasoning, spatial ability, and reading achievement). There was no significant difference between the groups on any background variable. Finally, we checked to see if the groups varied significantly with respect to the dependent variables (number sense, arithmetic fluency, and mathematics calculation achievement) at pretest. Again, there was no significant difference between the groups with respect to any dependent variable.

Children in the study were nested within intervention groups. However, for all but two groups, the instructors taught one of each intervention condition, counterbalancing the experiment. As an extra precaution, the intra-class correlation (ICC), or the proportion of group-level variance versus the total variance (Raudenbush & Bryk, 2002), was calculated for each dependent variable at post-test and delayed posttest. All correlations were less than .10, indicating that there was no significant variance at the intervention group level.

Effect of the intervention—An experimental pretest, immediate posttest, and 8-week delayed-posttest design was used. Dependent variables were (a) number sense, (b) arithmetic fluency, and (c) general mathematics calculation achievement. To test the overall effectiveness of the intervention, a series of one-way analyses of covariance (ANCOVA) were run using raw scores with the related pretest and age (because there was a significant difference between the number-fact practice and control group) serving as the covariates. Covariates minimize the confounding factor of children's previous knowledge and reduce unexplained variance; as such, they increase the ability of the analyses to detect effects of the intervention (Field, 2009; Maxwell & Delaney, 2004). Post hoc tests were conducted using the Bonferroni correction to make pairwise comparisons.

The ANCOVA also produces an estimated marginal (adjusted) mean. Using the covariates, the estimated marginal mean is an estimate of what the mean would have been if all the children in the three experimental conditions had begun with the same mean pretest score. Although the ANCOVA also produces p values, p values can be unrelated “in any way to the magnitude or importance of the effect” (Lipsey et al., 2012, p. 3). What is more informative is to compare the effect size of the intervention to the effect size of other educational interventions. *What Works Clearinghouse: Procedures and Standards Handbook* (Institute of Education Sciences [IES], 2014) recommends using Hedge’s g as a measure of effect size. Therefore, Hedge’s g was used for each pairwise comparison of groups and is labeled ES for Effect Size. Hedge’s g is calculated by taking the difference of the adjusted means and dividing them by the pooled standard deviation of the sample. The handbook defines Hedge’s g of 0.25 or more as indicative of effective educational practice.

For a more in-depth analysis, the data were analyzed with direct entry multiple regression analyses (MRAs) for each dependent measure (number sense, arithmetic fluency, and calculation). Multiple regression analyses allowed us to examine the unique contributions of background variables, such as status as an English learner (ELs vs. non-ELs), nonverbal reasoning, spatial ability, attention, and reading achievement, in our models. For each MRA, we checked the model for assumptions of normality, linearity, and multicollinearity with the current sample, and no assumptions were violated.

We followed up the MRAs with a set of ANOVAs to compare the progress of ELs to non-ELs within each group. Finally, we looked at gain scores to determine how effective the interventions versus normal instruction were in closing the achievement gap between lower and higher performing kindergartners.

Results

The Effect of the Intervention

Table 3 presents the mean raw scores and standard deviations by group for each of the dependent variables at each time point. It also presents the estimated marginal (adjusted) means and the standard deviations for posttest and delayed posttest as derived from the ANCOVAs with pretest of the related dependent variable and age (because there was a significant difference in age) as the covariate. (The estimated marginal means are not reported for the higher achieving group, who were not a part of the ANCOVA.)

Table 4 presents p values and effect sizes for the ANCOVA analyses. The ANCOVAs examined the differences between the children in the number-fact practice, number-list practice, and business-as-usual control groups with pretest of the related dependent variable and age as covariates. There was a significant main effect for group at each time point and for each measure. As suggested by the aforementioned Institute of Education Sciences’ (2014) handbook, we used Hedge’s g to calculate ES for each pairwise comparison.

For number sense, the number-list condition outperformed the control group (although not always significantly) at both posttest and delayed posttest with effect sizes greater than .25 ($ES = .32$, $ES = .26$). The effect sizes for the number-fact practice versus control were more

than twice those of the number-list practice at both time points ($ES = .82$, $ES = .56$). The effect of the number-fact condition over the number-list condition produced an effect size even greater than the effect of the number list over the control ($ES = .42$).

For arithmetic fluency, at posttest, the effects of the two practice conditions were similar to each other but still slightly favoring number-fact practice (number list $ES = .69$, number fact $ES = .78$). The difference is seen at the delayed posttest, where the ES of the number-fact condition was again more than twice that of the number-list condition (number list $ES = .27$, number fact $ES = .58$). For mathematics computation achievement, once again, at posttest ES s were similar (number list $ES = .58$, number fact $ES = .60$), but at delayed posttest, the effect of the number-list condition had washed out whereas the ES of the number-fact condition remained well above .25 (number list $ES = .12$, number fact $ES = .49$).

Response Bias Analysis

Sometimes children respond to addition arithmetic combinations with a response bias such as always adding one to the first or second addend. Because the fluency measure has many +1 combinations, an error analysis and response bias analysis was conducted to look for possible false positives. Only one child showed a response bias of +1 and had two possible false positives at posttest on the fluency measure. However, this child was in the control group, so there was no chance of inflating the effect of the intervention. Some children showed more simplistic response biases, such as stating the first or second addend, which cannot produce a false positive.

Contributions of Background Variables

Table 5 presents the mean scores for the background measures of nonverbal reasoning, spatial ability, inattentive behavior, and reading achievement. The correlations between all measures for the experimental groups are shown in Table 6.

Tables 7 through 9 show the results of the analyses for each outcome (number sense, arithmetic fluency, and calculation achievement). The relative unique contribution of the independent variables to outcomes and effect sizes can be evaluated through the interpretation of standardized beta coefficients (Cohen, Cohen, West, & Aiken, 2003; Keith, 2006; Pedhazur, 1997). To account for differences in initial abilities, pretest scores were always included as an independent variable. Preliminary analyses revealed no significant effects or interactions involving gender, age, and cognitive abilities (nonverbal reasoning and spatial ability). Thus, these variables were excluded from the analyses. The MRAs predicting number sense (Table 6) were significant at posttest, $F(6,119) = 25.009$, $p < .001$, and at delayed posttest, $F(6,119) = 18.197$, $p < .001$. As would be expected, pretest score was an important predictor of posttest and delayed-posttest number-sense scores. With respect to background variables, inattentive behavior, reading, and status as an English learner (favoring ELs) uniquely predicted number-sense performance both at posttest and delayed posttest. Most important, when controlling for the background variables, the children in the intervention group performed better than children in the control group at both posttests, but the difference was not significant for the number-list group at delayed posttest. The intervention condition with number-fact practice predicted over twice the variance as

the number-list practice condition at posttest (.323/.152) and about 1.6 times as much at delayed posttest (.223/.136).

The MRAs predicting arithmetic fluency (Table 7) also were significant, both at posttest, $F(6,119) = 6.012, p < .001$, and delayed posttest, $F(6,119) = 6.454, p < .001$. Pretest fluency performance was a unique predictor at both posttests; status as an English learner (favoring ELs) and reading were unique predictors only at delayed posttest. Controlling for background variables, at posttest, the number-fact practice condition predicted 1.3 times the variance of the number-list practice condition (.348/.267) and, at delayed posttest, predicted over twice the variance of the number-list practice condition (.242 vs. .104, respectively).

For mathematics calculation achievement (WJ-III) (see Table 8), the MRAs were significant both at posttest, $F(6,119) = 6.853, p < .001$, and at delayed posttest, $F(6,119) = 6.284, p < .001$. Inattentive behavior uniquely predicted calculation performance at posttest; at delayed posttest, inattentive behavior, reading, and status as an English learner (favoring ELs) status were unique predictors. Number-fact and number-list intervention groups (versus control) predicted approximately equal amounts of variance at posttest (.282 vs. .268, respectively), but only the number-fact practice condition was a unique and significant predictor at delayed posttest (.190 vs. .032, respectively). Calculation pretest performance was not a significant predictor at either time point, most likely because of floor effects (where the group means raw scores were 1 point or fewer).

English Learner follow-up analyses—The advantage of ELs on the mathematics outcomes, as noted above, was of interest. In our previous version of the intervention, no language status effects were found. In an effort to pinpoint the effect for ELs in the present study, ANCOVAs were run for each group (number fact, number list, and control) and outcome (number sense, arithmetic fluency, and mathematics calculation achievement) at posttest and delayed posttest with status as an English learner (ELs vs. non-ELs) as the fixed factor and with age and pretest as the covariates. In other words, we were comparing the ELs to the non-ELs in their group for each dependent variable at each time point, controlling for age and pretest differences.

At posttest, for number sense, there was a significant difference between ELs and non-ELs for the number-fact practice group: number sense, $F(1, 40) = 5.195, p = .029$, $ES = .76$, and arithmetic fluency, $F(1, 40) = 5.145, p = .035$, $ES = .51$, and approaching significance in calculation, $F(1, 40) = 3.845, p = .057$, $ES = .61$. In other words, growth in these measures was significantly different for the ELs versus the non-ELs—in favor of the ELs—but only in the number-fact condition. There were similar significant and substantial language status effects favoring ELs in the number-fact practice condition at delayed posttest for number sense, $F(1, 40) = 10.283, p = .003$, $ES = .89$, arithmetic fluency $F(1,40) = 10.170, p = .003$, $ES = .95$, and mathematics calculation achievement, $F(1, 42) = 10.657, p = .002$, $ES = .98$. It is possible that the intervention allowed ELs to better access their classroom instruction and perform closer to their first language mathematics potential—resulting in their growth at an even faster rate, posttest to delayed posttest. However, it is unclear why the number-fact condition was more effective for the ELs.

Item analysis by kindergarten standards—Because the intervention aligned with the CCSSM for kindergarten, we looked at children’s performance on individual items on our number-sense screener with respect to these standards (arithmetic fluency and the ability to do written calculations were addressed with separate measures). Table 10 shows the CCSSM addressed by the intervention alongside the items in the number-sense screener that assess that standard. Because delayed-posttest results reflect performance close to end of kindergarten, the percentage of items correct at delayed posttest is presented and broken down by intervention and comparison groups. Listed are only those items for which the intervention groups came within 10% of the higher achieving group and the control group did not.

Most children successfully counted to 20 and 30. However, counting to higher numbers and numeral recognition beyond 10 continued to be relatively weak for all of the children in the study, compared to the higher achieving group. This is likely a function of the intervention focusing on smaller numerosities. For Story Problems and Number Combinations, with the exception of one combination ($7 - 3$), both intervention groups were more accurate than the control group. On several items, one or both of the intervention groups performed similarly to the higher achieving group, showing the intervention’s potential for narrowing the gap in these important skills.

More specifically, the CCSSM assert that children should understand that each successive number name refers to a quantity that is one larger. In our number-sense measure, the combination $2 + 1$ was presented to children in three contexts: nonverbal, verbal story problem, and verbal number combination. All groups performed at ceiling on the nonverbal item ($> 90\%$). On the $2 + 1$ story problems, the intervention groups performed similarly to the higher achieving group (81% correct; i.e., number-fact practice 80% and number-list practice 73%), but the control group had only 45% accuracy. A similar pattern was observed for the corresponding number combination (higher achieving 98%, number fact 93%, number list 80%, and control 74%).

Experimental study group versus a higher achieving comparison group—We examined whether participation in the interventions reduced the achievement gap between the children who began the study with a low score on the number-sense screener and those who began with a higher score (i.e., higher achieving group). One way to examine the power of an intervention to close an achievement gap is to use the effect size of the gap in the population being studied as a benchmark (IES, 2012). Then, the effect size of the intervention can be compared to see what percentage of the gap can be closed. For the present study, we collected data for a group of children (higher achieving) who scored above the cut score for the intervention. The effect size of the gap between the higher achieving control group and the lower achieving control group reflects the gap that would exist without intervention. Delayed-posttest scores were used, as they reflect the gap at the end of kindergarten. Again, the effect size was calculated by taking the difference of the means of the low-achieving and higher achieving groups at delayed posttest and dividing by the pooled standard deviation.

For number sense, the effect size of the gap was -1.27 , meaning that, at delayed posttest, the low-achieving group was 1.27 standard deviations below the higher achieving group. The effect of the intervention for the number-list group at delayed posttest was .26 (see Table 3). This suggests that the intervention with the number-list practice has the potential of closing 20% of the gap. Likewise, the effect size for the number-fact practice group ($ES = 0.56$) has the potential of reducing 44% of the gap.

For arithmetic fluency, the effect size of the gap was -1.08 . The effect size of the number-list intervention group ($ES = 0.27$) has the potential of reducing 25% of the gap, whereas the number-fact practice group ($ES = 0.58$) has the potential of reducing 54% of the gap. For mathematics calculation achievement, the effect size of the gap was -0.82 . The effect size of the number-list intervention group ($ES = 0.12$) has the potential of reducing 15% of the gap, whereas the number-fact practice group ($ES = 0.49$) has the potential of reducing 60% of the gap.

Discussion

Our intervention, which targeted basic understandings of number, number relations, and number operations, significantly improved mathematics outcomes in children who entered kindergarten with relatively low number competencies. These findings expand previous results showing that early number skills can be supported effectively through targeted interventions (e.g., Baroody et al., 2009; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Dyson et al., 2013; Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). Designed specifically for children who come to kindergarten behind their peers, the present intervention began with the smallest counting numbers and used those numbers to reinforce verbal subitizing, counting and cardinality, magnitude comparisons, and adding and subtracting (Clements et al., 2013; Frye et al., 2013). Repeating the activities with increasingly large numbers helped children gradually build their number knowledge, especially with respect to number operations. Following the advice of the *What Works Clearinghouse: Procedures and Standards Handbook* (IES, 2014), we used effect sizes (Hedge's g) to evaluate the effect of the intervention. Except for the number-list condition with respect to mathematics computation achievement at delayed posttest, the intervention groups outperformed the control groups at both posttest and delayed posttest for all three dependent variables (number sense, arithmetic fluency, and mathematics calculation achievement) with effect sizes greater than .25 (a marker of an effective intervention).

Another goal of the study was to see if we could enhance children's competencies further through meaningful practice with number combinations. We were especially interested in improving children's performance with simple operations, as proficiency with addition and subtraction is a hallmark of primary school mathematics and predicts later achievement (Jordan et al., 2009). In the number-list condition, children solved number combinations by counting up and down on a number-list game board. The contrasting number-fact condition encouraged children to respond right away to printed number combinations within five; if they did not know an answer, they were instructed to solve the problem quickly on their fingers or use the $+/-$ strategy, methods that were presented in the intervention lessons.

Although both number-list and number-fact counting strategies were taught and practiced during the core lessons, the varied forms of short focused practice produced significant differences between the two intervention practice groups. As predicted, number-fact practice produced a stronger boost for children than number-list practice. Careful, targeted instruction combined with speeded but meaningful practice of facts seems to be optimal for kindergartners with the most serious delays. It should be noted, however, that a number list was not provided to the children on the timed arithmetic fluency and the arithmetic computation measures. Fuson and Secada (1986) and Baroody, Tiilikainen, and Tai (2006) showed that concrete counting-on (such as counting on a number list) does not easily transfer to more abstract counting-on (counting on using only a verbal count). Although we taught all intervention children how to use their fingers and they were free to do so on all of the measures, children who practiced counting on a number list might have been less able to make use of these strategies to help them calculate quickly on the fluency measure or to solve written arithmetic computations on the achievement test. Interestingly, both the number-list and the number-fact practice groups performed at similar levels on the story problems and number combinations sections of the number-sense screener. Recall that on the number-sense screener, children were given a number list as well as pencil and paper and were told they could use their fingers or any of the materials to solve the problems.

Our results build on those obtained by Fuchs et al. (2013), who found that number-fact practice is generally more effective than a game-board approach for improving first-graders' performance on calculation. We also found, as did Fuchs, that spending time on number-fact practice rather than on practice designed to strengthen number knowledge did not penalize the children in the area of number knowledge. Children in the number-fact condition outscored children in the number-list condition on the number-sense screener, which tests all aspects of number knowledge, not just computation. The CSSM analysis showed that more children in the number-fact condition than in the number-list condition were able to count to 20, and slightly more could count to 30. In the number-fact practice group, 20% more of the children could recognize the number 16, and over 17% more could recognize the number 13. This is surprising because the number-fact practice did not involve numbers over 5.

It is encouraging that the practice in the present study led to improved gains with minimal time investment (3 times per week over 8 weeks with 5 minutes of practice at the end of each session and each child receiving 1 minute of one-on-one practice) in an easy to implement small-group setting. This type of setting and time constraint is typical of a small-group intervention that would occur in schools. As we noted, most of the schools had a 30-minute time set aside for mathematics intervention. Children in the Fuchs et al. (2013) study received more intensive tutorial help than was provided here. They received one-on-one tutoring 3 times per week over 16 weeks with 5 minutes of practice at the end of each session. Many teachers would not be able to provide such intensive one-on-one training.

It was also encouraging that children's gains in the present study not only held but also continued to grow over time. Except for the number-list group for mathematics computation achievement, all effect sizes at delayed posttest remained well above .25. Fuchs and colleagues (2013) did not assess their children at a delayed posttest, yet "fade out" has been observed in other studies with disadvantaged children (Clements et al., 2013). We believe

that the intervention set a foundation for important concepts of number and mathematics vocabulary that allowed the children access to their classroom curriculum, facilitating continuous growth even after the intervention was withdrawn. Not all children will need long-term intervention; some may respond to a shorter intervention that gets them on the right track. Teachers may also employ these sorts of activities in whole-class instruction, making explicit the relationships between numbers, building fluency in the recognition of quantities, and building fluency in the computing of combinations through practice and use of strategies. Such a universal design for instruction could help catch children at risk for mathematics difficulties before they begin to struggle in mathematics.

We caution the reader against concluding that our findings show that a small amount of number-fact practice is all it takes to improve in basic number competencies. We underscore the importance of the 25 minutes of instruction in number sense to prepare the children to make the most out of the number-fact practice time (Baroody, Bajwa, & Eiland, 2009; Baroody, Eiland, & Thompson, 2009; Fuchs et al., 2013). We are also unsure what aspect of the number-fact practice contributed most to the advantage of these children in number sense, fluency, and mathematics calculation achievement. It is possible that it was the extra opportunity to practice using finger strategies that gave the advantage and not the number-fact practice per se. In the future, instructors should record what strategy, if any, each child uses to solve each fluency and calculation problem (these strategies are already recorded for number sense). It would be interesting to see how often children used their fingers to solve the problem.

Regarding the contribution of the background variables to mathematics outcomes, classroom attention made a significant and unique contribution for most outcomes, supporting findings from other studies (e.g., Fuchs et al., 2013; Hassinger-Das et al., 2014). Inattentive behavior affects young children's availability for and engagement in learning, even in small groups. The effectiveness of the focused practice, which ensures engagement, attests to this. Reading ability also was a unique predictor, suggesting that the ability to decode words and possibly phonological processing plays some role in learning mathematics (Robinson et al., 2002). Nonverbal abilities, age, and gender were not independent predictors of posttest mathematics outcomes.

A particularly interesting finding was that among the low-achieving children in our study, status as an English learner (favoring ELs) uniquely predicted mathematics outcomes. Possibly, some ELs scored low on our measures at pretest because of language differences rather than basic weaknesses in number knowledge. Because our intervention started with the most basic concepts of number, ELs learned mathematics vocabulary in a content area where they had prior understanding. Moreover, our highly visual intervention with consistent linear representations of number (cardinality chart, number list, partner cards, etc.) seemed to be valuable to ELs over the other children. ELs grew at a rate greater than their peers who are not ELs after the intervention was completed, suggesting that the intervention helped them access classroom instruction. Further analysis, however, revealed a distinct difference in growth between the ELs and non-ELs children in the number-fact practice condition. It is unclear why the number-fact practice condition was advantageous to ELs and less so to the non-ELs. Possibly, the game format of the number-list great race

game involved more verbal explanation, whereas the number-fact game that emphasized number combinations did a better job of reinforcing basic number skills.

We also examined the extent to which our intervention narrowed the achievement gap between children who showed low versus higher achievement at the beginning of the study. Interestingly, all of our lower achieving study groups (both experimental and control) made more gains, on average, than the higher achieving comparison group on our measure of number sense. Although this may reflect some natural regression to the mean (see Jordan, Kaplan, & Hanich, 2002), the finding also suggests that the nonstudy classroom interventions provided to control children were also effective. However, there was still a substantial difference at the end of kindergarten between the higher achieving comparison group and the low-achieving control group. The number-fact practice intervention group showed the greatest promise for reducing the achievement gap (number sense—44%, arithmetic fluency—54%, and mathematics calculation achievement—60%). The CCSSM analysis shows in more detail where the gap was reduced with respect to the number-sense screener. Most compelling is the improvement for both intervention conditions with respect to story problems and number combinations. For combinations within five, which was the focus of the intervention, the gap was nearly closed.

A potential limitation of the study was the length of time (i.e., 8 weeks) available for the intervention. However, two school-related factors constrained the intervention span in the present study. First, because children who initially show low performance at the beginning of kindergarten may catch up to their peers after a few months of instruction (Jordan et al., 2006), we waited to start our program until midyear. This enabled us to identify children who were not responding to typical instruction. Second, we needed to complete the intervention by mid-March to allow for an 8-week period for delayed posttest in kindergarten. We suspect that the intervention would have been even more effective in producing gains for the children most in need had it been continued beyond the 8-week period. For example, our item analysis suggests that intervention children would benefit from additional instruction to develop counting skills and operational knowledge with larger numbers, as stipulated by the kindergarten CCSSM.

Our study also did not address exactly what features of the 25 minutes of number-sense instruction was most effective in helping children develop number competencies. Although, when compared to previous iterations of the number-sense intervention, our results showed that the new version of the intervention was more powerful as a whole, we are not certain which activities are necessary or most effective and for what type of learner. For example, each intervention lesson contains a series of activities designed to build skill in quantity and number recognition (building the number, sequencing numeral cards, number recognition, counting on fingers, automaticity on fingers, and subitizing related quantities on fingers, dot cards, and ten frames). It is not known if a subset of these activities would be sufficient, thus shortening the length of the intervention. It is also not known if certain of these activities are better suited to some students than to others. Future research could use a similar research design to test individual activities or groups of activities for effectiveness so that interventions would (a) include only the most effective activities or (b) be tailored to individual differences.

In conclusion, our intervention approach holds promise for helping high-risk, low-achieving kindergartners keep pace with the challenges of early mathematics. The conceptually driven instruction combined with a small amount of number-fact practice approach seems to be most beneficial, especially for children who are English learners. Future studies should continue to examine combinations of instruction and practice to determine the most effective ones for different populations of children within the practical constraints of classrooms and schools.

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Figure 1.
Five frame showing the quantity 3.

Figure 2.
Ten frame showing the quantity 7.

Figure 3.

A cardinality chart shows quantities growing by one for each consecutive number in the list.

Figure 4.
The number list shows quantities accumulating linearly.

Table 1

Demographics by Group for Total Sample

	Gender		Ethnicity			Language		Income status ^d		Age in months as of Sept. 1		
	N	Male	Female	AA	H	C	Other	EL ^b	%	Eligible	%	Mean (SD)
Experimental conditions												
Control	42	22	20	10	22	8	2	18	42.9	36	85.7	64 (3.6)
Number-list practice	40	20	20	9	19	10	2	17	42.5	31	77.5	65 (3.6)
Number-fact practice	44	20	24	12	23	9	0	17	38.6	36	81.8	67 (3.0)
Total experimental	126	62	64	31	64	27	4	52	41.3	103	81.7	66 (3.5)
Higher achieving (for comparison only)	62	33	29	14	16	23	9	14	22.6	62	100	66 (3.4)
Total	188	95	93	45	80	50	13	66	35.1	165	87.8	66 (3.5)

Note. N = Number of participants, AA = African American, H = Hispanic, C = Caucasian, Other = Bi-racial or Asian, EL= English Learners, SD = Standard Deviation.

^a Income Status determined by free or reduced lunch eligibility.

^b EL students were identified by enrollment in bilingual classrooms.

Table 2

Intervention Scope and Sequence

		Lessons 1–5	Lessons 6–10	Lessons 11–15	Lessons 16–20	Lessons 21–24
Number	Oral counting	From 1, by 1s to 10/20	From 1, by 1s to 30/40	From other numbers, by 1s, to 90	From other numbers to 100, by 1s, by 10s	From other numbers to 100, by 1s, by 10s
	Finger and block counting	To 5	To 10	To 15	To 20	To 30
	Number writing	To 5	To 10	Write Number Sentences	2 Digit Numbers	2 Digit Numbers
	Recognizing quantities	0–5 Using fingers and dot cards	0–10 Using fingers and ten frames	0–10 Using fingers and ten frames	0–10 Using fingers and ten frames	0–10 Using fingers and ten frames
	Number recognition	0–5	6–10	11–15	16–20	21–100
Number Relations	Base ten understandings		Lesson 10 Make a “stick” of 10	Using interlocking blocks Build 11–15 as $10 + n$	Using interlocking blocks Build 16–19 as $10 + n$ 20 as 2 10s	Using interlocking blocks Build numbers 30–100 as sets of tens and a number of ones
	+1 principle	Using cardinality chart to build numbers 1–5	Using cardinality chart to build numbers 6–10	Sequence numbers 1–15	Sequence numbers 1–20	Sequence numbers 1–30
	Bigger/Smaller	Comparing numbers between 0 and 5	Comparing numbers between 0 and 10	Comparing numbers between 0 and 10	Comparing numbers between 0 and 10	Comparing numbers between 0 and 10
	Before/After	Numbers 0–5	Numbers 0–10	Numbers 0–10	Numbers 0–10	Numbers 0–10
	Part/Part/Whole	Find partners for numbers 2–5	Find partners for numbers 6–10			
Number-Fact Practice	Story problems	Use multiple strategies to solve story problems for facts within 5	Use multiple strategies to solve story problems for facts within 10	Use multiple strategies to solve story problems for facts within 10	Use multiple strategies to solve story problems for facts within 10	Use multiple strategies to solve story problems for facts within 10
	Combinations	Solve $+/ -$ combinations within 5	Solve $+/ -$ combinations within 10	Solve $+/ -$ combinations within 10	Solve $+/ -$ combinations within 10	Solve $+/ -$ combinations within 10
	Fluency	$+/ -$ facts within 5	$+/ -$ facts within 5	$+/ -$ facts within 5	$+/ -$ facts within $5n + 1/n - 1$, $n + 0/n - 0$ with $n = 6$ through 9/10	$+/ -$ Facts within $5n + 1/n - 1$, $n + 0/n - 0$ with $n = 6$ through 9/10
Number-List Practice	Supports understanding in number, number relations, and number operations	Practice $n + 1$ and $n + 2$ by counting up the number list	Practice $n + 1/n - 1$ and $n + 2/n - 2$ by counting up and down the number list	Practice $n + 1/n - 1$ and $n + 2/n - 2$ by counting up and down the number list	Practice $n + 1/n - 1$ and $n + 2/n - 2$ by counting up and down the number list	Practice $n + 1/n - 1$ and $n + 2/n - 2$ by counting up and down the number list

Table 3

Means and Standard Deviations of Dependent Variables by Group and Time

	Pretest			Posttest			Delayed posttest		
	<i>M</i>	<i>(SD)</i>		<i>M</i>	<i>(SD)</i>	EMM	<i>M</i>	<i>(SD)</i>	EMM <i>(SE)</i>
Number sense									
Control	14.67	(3.57)		25.74	(6.37)	25.66	29.69	(7.73)	29.52 (1.08)
Number-list practice	15.03	(4.58)		28.70	(8.69)	28.08	32.40	(9.03)	31.77 (1.09)
Number-fact practice	13.57	(4.05)		31.00	(8.00)	31.64	33.05	(7.30)	33.78 (1.04)
Higher achieving	28.92	(5.60)		36.65	(4.45)	NA	38.84	(3.52)	NA NA
Arithmetic fluency									
Control	1.95	(1.81)		4.55	(3.06)	4.50	6.29	(3.50)	6.20 (0.62)
Number-list practice	1.88	(1.87)		6.85	(3.64)	6.83	7.33	(4.43)	7.30 (0.64)
Number-fact practice	1.68	(1.65)		7.45	(4.44)	7.52	8.48	(4.51)	8.58 (0.61)
Higher achieving	6.16	(3.96)		9.65	(4.06)	NA	10.77	(3.43)	NA NA
Mathematics calculation achievement									
Control	0.98	(1.39)		3.21	(2.53)	3.31	5.17	(2.32)	5.10 (0.43)
Number-list practice	0.85	(1.33)		4.88	(3.12)	4.92	5.38	(2.90)	5.41 (0.44)
Number-fact practice	1.00	(1.49)		5.09	(3.09)	4.96	6.41	(3.11)	6.45 (0.42)
Higher achieving	3.50	(2.56)		5.89	(2.07)	NA	7.24	(2.19)	NA NA

Note. NA = Not Applicable. All scores are raw scores. EMM refers to the estimated marginal means with pretest score of the dependent variable and age as covariate. *SD* refers to standard deviation and *SE* to standard error.

Table 4

Effects of Intervention on Dependent Variables

Posttest	Number sense		Arithmetic fluency		Mathematics computation achievement	
	<i>p</i>	ES	<i>p</i>	ES	<i>p</i>	ES
Posttest						
	Main effect for group $F(2,121) = 8.396, p < .001, \eta_p^2 = .122$		Main effect for group $F(2,122) = 7.324, p = .001, \eta_p^2 = .108$		Main effect for group $F(2,122) = 4.489, p = .013, \eta_p^2 = .069$	
Pairwise comparison						
Number list vs. control	.296	.32	.016	.69	.034	.58
Number fact vs. control	< .001	.82	.001	.78	.029	.60
Number fact vs. number list	.049	.42	1.00	.17	1.00	.01
Delayed posttest						
	Main effect for group $F(2,121) = 3.788, p = .025, \eta_p^2 = .059$		Main effect for group $F(2,121) = 3.460, p = .035, \eta_p^2 = .054$		Main effect for group $F(2,121) = 2.666, p = .074, \eta_p^2 = .042$	
Pairwise comparison						
Number list vs. control	.439	.26	.680	.27	1.00	.12
Number fact vs. control	.021	.56	.029	.58	.090	.49
Number fact vs. number list	.579	.24	.463	.28	.271	.34

Table 5

Means and Standard Deviations of Background Variables

Background variable	<i>M</i>	<i>SD</i>
Nonverbal reasoning		
Control	9.62	4.16
Number-list practice	11.13	4.63
Number-fact practice	10.82	4.74
Higher Achieving	12.65	4.99
Spatial ability		
Control	14.50	3.52
Number-list practice	13.98	4.36
Number-fact practice	14.80	3.95
Higher achieving	17.27	4.42
Inattentive behavior^a		
Control	0.34	1.16
Number-list practice	0.47	1.18
Number-fact practice	0.27	0.84
Reading achievement		
Control	.98	1.388
Number-list practice	.85	1.331
Number-fact practice	1.00	1.494
Higher achieving	3.50	2.559

Note. All measures were given at pretest with the exception of spatial ability, which was given at delayed posttest.

^aTeachers did not rate children in the higher achieving control group. A negative score indicates better

Table 6

Correlations Among All Measures

Measures	1	2	3	4	5	6	7	8	9	10
1. Nonverbal reasoning	-									
2. Spatial ability	.205*									
3. Inattentive behavior	-.104	-.106								
4. Reading achievement	.127	-.023	-.477**							
5. Number sense—post	.219*	.242**	-.532**	.645**						
6. Number sense—delayed	.164	.243**	-.475**	.683**	.809**					
7. Arithmetic fluency—post	.157	.140	-.242**	.388**	.635**	.588**				
8. Arithmetic fluency—delayed	.221*	.125	-.273**	.487**	.601**	.752**	.617**			
9. Math calculation achievement—post	.209*	.172	-.368**	.479**	.700**	.687**	.695**	.603**		
10. Math calculation achievement—delayed	.164	.202*	-.351**	.542**	.621**	.777**	.573**	.711**	.647**	-

*
 $p < .05$,**
 $p < .01$,***
 $p < .001$.

Table 7
Results of Multiple Regression for Predicting Number Sense at Posttest and Delayed Posttest

Variable	Posttest			Delayed posttest		
	B	SE B	β	R ²	B	SE B
Constant	10.407	2.433***			13.203	2.681***
Background variables						
Number sense pretest	.595	.148	.305***	.060***	.724	.163
English learner status ^b	2.407	1.019	.149*	.021*	3.548	1.123
Reading achievement	.581	.161	.274***	.049***	.461	.177
Inattentive behavior	-2.511	.506	-.332***	.092***	-2.199	.557
Intervention contrast						
Number list ^c vs. control	2.591	1.211	.152*	.017*	2.357	1.334
Number fact ^d vs. control	5.378	1.202	.323***	.074***	3.779	1.324
<i>R</i>		.747				.692
<i>R</i> ²		.558				.478
<i>F</i>		25.009***				18.197***

Note. *N* = 126.

^a male = 1,

^b English learner = 1,

^c Number-list practice group = 1,

^d Number-fact practice group = 1.

* *p* < .05,

** *p* < .01,

*** *p* < .001.

Table 8
Results of Multiple Regression for Predicting Arithmetic Fluency at Posttest and Delayed Posttest

Variable	Posttest			delayed Posttest		
	B	SE B	β	B	SE B	R^2
Constant	1.544	1.285		1.833	1.367	
Background variables						
Arithmetic fluency pretest	.396	.185	.177*	.504	.197	.210* .041*
English learner status ^b	.939	.650	.118	1.729	.692	.202* .040*
Reading achievement	.183	.094	.174	.265	.100	.236** .044**
Inattentive behavior	-.576	.326	-.154	-.617	.347	-.154 .020
Intervention contrast						
Number list ^c vs. control	2.257	.790	.267**	.942	.840	.104 .008
Number fact ^d vs. control	2.868	.771	.348***	2.147	.820	.242* .043*
<i>R</i>			.482			.496
<i>R</i> ²			.233			.246
<i>F</i>			6.012***			6.454***

* $p < .05$,

** $p < .01$,

*** $p < .001$.

Table 9
Results of Multiple Regression for Predicting Math Calculation Achievement at Posttest and Delayed Posttest

Variable	Posttest			Delayed posttest		
	B	SE B	β	B	SE B	β
Constant	1.770	.942		3.010	.906**	
Background variables						
Calculation problems pretest	.192	.175	.091	.174	.168	.086
English learner status ^b	.721	.482	.120	1.281	.464	.224**
Reading achievement	.112	.070	.141	.151	.067	.201*
Inattentive behavior	-.847	.243	-.301**	-.677	.234	-.253**
Primary contrast						
Number list ^c vs. control	1.701	.586	.268**	.195	.563	.032
Number fact vs. control	1.754	.569	.282**	1.125	.547	.190*
<i>R</i>	.507			.491		
<i>R</i> ²	.257			.241		
<i>F</i>	6.853***			6.284***		

Note. *N* = 126.

^a male = 1,

^b English learner = 1,

^c Number-list practice group = 1,

^d Number-fact practice group = 1.

* *p* < .05,

** *p* < .01,

*** *p* < .001.

Table 10

Item Analysis at Delayed Posttest by Common Core State Standard Domainsf

Domain	Number-sense item Counting	Percent correct by group		
		Control	Number list	Number fact
Counting and cardinality				Higher achieving
Know number names and the count sequence	Count to 20	83.3	90	100
	Count to 30	67.7	85	88.6
Compare numbers	Number comparison			
	Smaller? 8 or 6	83.3	95.0	93.2
Number and Operations in Base 10	Number recognition			
	Recognize the number 13	69.0	67.5	84.1
Operations and algebraic thinking	Recognize the number 16	78.6	70.0	90.9
	Story problems			
Solve addition and subtraction word problems, and add and subtract within 10 (e.g., by using objects or drawings to represent the problem)	2 + 1	45.2	72.5	79.5
	3 + 2	57.1	75.0	79.5
	5 − 2	50.0	67.5	61.4
	Number combinations			
	2 + 1	73.8	80.0	93.2
Understand addition; Understand subtraction Understand that each successive number name refers to a quantity that is one larger.	4 + 3	54.8	65.0	72.7
	2 + 4	59.5	70.0	68.2
	7 − 3	54.8	60.0	50.0
	6 − 4	40.5	57.5	47.7