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A Subspace-based Coil Combination Method for Phased-array Magnetic Resonance Imaging

Derya Gol Gungor¹ and Lee C. Potter¹

¹Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH, 43210, USA

Abstract

Purpose—Coil-by-coil reconstruction methods are followed by coil combination to obtain a single image representing a spin density map. Typical coil combination methods, such as square-root sum-of-squares and adaptive coil combining, yield images that exhibit spatially-varying modulation of image intensity. Existing practice is to first combine coils according to a signal-to-noise criterion, then post-process to correct intensity inhomogeneity. If inhomogeneity is severe, however, intensity correction methods can yield poor results. The purpose of this paper is to present an alternative optimality criterion for coil combination; the resulting procedure yields reduced intensity inhomogeneity while preserving contrast.

Theory and Methods—A minimum mean squared error criterion is adopted for combining coils via a subspace decomposition. Techniques are compared using both simulated and in vivo data.

Results—Experimental results for simulated and in vivo data demonstrate lower bias, higher signal-to-noise ratio (about 7×) and contrast-to-noise ratio (about 2×), compared to existing coil combination techniques.

Conclusion—The proposed coil combination method is non-iterative and does not require estimation of coil sensitivity maps or image mask; the method is particularly suited to cases where intensity inhomogeneity is too severe for existing approaches.

Keywords

coil combination; parallel MRI; phased-array coils; intensity inhomogeneity correction; bias reduction; contrast enhancement

Introduction

Magnetic resonance imaging (MRI) is a powerful non-invasive technique that is widely used for visualization of soft tissues. In parallel MRI an array of surface coils, rather than a single body coil, is used to accelerate data acquisition, and the data from the several coils must be combined. If coil sensitivities introduced by the surface coils are known, then signal-to-noise ratio (SNR) optimal combination of full field of view (FOV) coil images may be

obtained via pixel-by-pixel sum of coil images, each weighted by the complex conjugate of its corresponding sensitivity map (1). However, in parallel MRI the data are sub-sampled, and the sensitivity map for each surface coil is unknown. One approach is to first estimate the coil sensitivities from acquired data and then use the SENSE method (2) for linear least-squares image reconstruction. Other approaches, such as GRAPPA (3) and SPIRiT (4), bypass the coil sensitivity estimation step, interpolate the sub-sampled k -space data, and then use a coil combining method, such as square-root sum-of-squares (SoS) or adaptive coil combining (5).

SoS is widely adopted in clinical devices, but produces low-contrast images due to unbalanced noise characteristics between regions of low and high SNR in the reconstructed image (5, 6, 7). In (8), the SoS combining is generalized to the p^{th} root of the sum of p^{th} powers in an attempt to yield more uniform recovery of the image intensity; however, the resulting reconstruction tends to have lower SNR than the SoS image. The low contrast has been mitigated by the adaptive coil combining method (5) via an adaptive matched filter (9); the procedure maximizes image SNR at each pixel, under mild assumptions. Although the resulting image possesses improved SNR compared to the SoS coil combination, the bias due to an inhomogeneous intensity modulation remains in both the adaptive matched filter and SoS approaches. A coil combined image may be post-processed to partially correct for intensity inhomogeneity (10); however, the post-processing may result in reduced SNR and image contrast and may fail to correct severe intensity inhomogeneity. Motivated by the reconstruction bias due to intensity modulation, we propose an alternative optimality criterion to the adaptive matched filter. We adopt a subspace-based coil combination formulation and a minimum mean square error (MMSE) criterion. The MMSE criterion yields a non-iterative procedure that does not require estimation of coil sensitivity maps. The proposed procedure yields images with significantly reduced bias while maintaining the noise characteristics of the adaptive matched filter.

Apart from SoS and adaptive matched filter coil combining, other techniques in the literature are generally based on the estimation of coil sensitivities for use in coil combination. To estimate the coil sensitivities, an approximation of the correct image can be used; this information can be obtained from a body coil image (11, 12, 13, 14, 15), which is typically more uniform in intensity than surface coil images due to the larger size of the coil. However, this approach requires that an additional scan be performed and can be sensitive to motion and B0 field inhomogeneities. As a common alternative, an auto-calibration signal (ACS) is collected by densely sampling the low-frequency portion of k -space; an SoS combination of low-resolution images obtained from the ACS data is then used to estimate low-resolution coil sensitivity maps (16, 17) via pixel-wise division of the individual coil images by the SoS image. More computationally intensive methods for recovery of coil sensitivity maps have been proposed, as well (18, 19, 20, 21, 22, 23). Transform domain techniques obtain smooth coil sensitivities via polynomial or Gaussian fitting applied to either Radon transforms (18) or Wavelet transforms (19) of coil images. NLINV (20) and JSENSE (21) use non-convex optimization to iteratively and jointly estimate both the coil sensitivities and the image. Recently proposed subspace methods, such as ESPIRiT (22) and LOTUS (23), use eigen-decomposition of the ACS data to obtain an estimate of the coil

sensitivities. However, all these coil sensitivity estimation techniques either necessitate additional scans, or give only biased coil sensitivities with non-uniform intensity modulation, or are computationally intensive.

There exists an extensive literature on post-processing algorithms to correct intensity inhomogeneity; reviews of post-processing methods are found in (10, 24, 25). Here, we give a brief and necessarily incomplete overview of these methods, which are based on filtering, segmentation or histogram sharpening (10). Filtering methods treat inhomogeneities as low-frequency artifacts (26, 27, 28). Two most commonly used filtering methods are homomorphic filtering (27) and homomorphic unsharp masking (HUM) methods (28). The inhomogeneity map is calculated from low-pass filtered versions of the distorted image, and then divided from the input image. Although filtering methods are fast and effective, they may also reduce the contrast of the images with high dynamic range. The performance of image segmentation methods is highly affected by the intensity inhomogeneities (10). Therefore, intensity correction is an indispensable piece of segmentation algorithms. These methods assume a probabilistic model on the signal intensities and use Maximum Likelihood (ML) or maximum-a-posteriori (MAP) methods (29, 30). The model parameters are estimated using the Expectation-Maximization (EM) algorithm. It is reported in (10) that the EM based approaches are dependent on the initialization and are computationally expensive. Fuzzy-C-means (FCM) clustering is commonly used in image segmentation and inhomogeneity correction (31, 32). This method is based on partitioning the pixel values into fuzzy (soft) clusters. The standard FCM method is sensitive to inhomogeneities, and so, modified objective functions are used to handle intensity inhomogeneities (33).

The inhomogeneity correction techniques involve histogram-based methods, which directly use the histogram or functions of the histogram of pixel intensity. They typically do not require initialization or prior knowledge and so can be fully automatic. The N3 (34, 35) histogram method, for example, is a nonparametric iterative algorithm and is based on estimating the distribution of the true intensities by deconvolution of Gaussian distributions applied to the distribution of the distorted image. In (36), an improved version has been proposed. Other histogram-based approaches, such as (37, 38), minimize an entropy function to compensate intensity inhomogeneity. Most of the methods described above require an accurate knowledge of a mask that specifies the non-zero signal locations of the image. Automatic threshold extraction methods such as (39) may fail when there is severe inhomogeneity; thus, inaccurate detection of the mask adversely affects the performance of many inhomogeneity correction methods.

In this paper, we turn to a subspace formulation of multi-channel imaging with unknown coil sensitivities; we propose that coil combining can be achieved by a sum of eigenimages computed from coil data using a k -space construction. The proposed coil combination is shown to optimize a minimum mean square error criterion. Performance is evaluated quantitatively via simulation and qualitatively using in vivo data from cardiac and spine imaging, where inhomogeneity can be severe. The simple non-iterative coil combining approach is observed to reduce the bias from inhomogeneous intensity modulation while preserving or improving the SNR characteristics of existing approaches; the new approach does not require knowledge of an image mask.

Theory

In parallel imaging, the fully-sampled k -space data for channel i are given by

$$y_i = \mathcal{F}^*(\rho \cdot c_i) + e_i, \quad i=1, \dots, K, \quad [1]$$

for K coils where ρ represents spin density of the imaged object, c_i is the coil sensitivity for coil i , and $\mathcal{F}^*$ is the inverse of the unitary discrete Fourier transform (DFT) matrix. Here, “ $\cdot$ ” denotes point-by-point multiplication, “ * ” denotes conjugate transpose, and e_i is complex-valued additive noise. Because multiplication in the image domain corresponds to convolution in k -space, [1] can also be written as

$$y_i = x \otimes h_i + e_i, \quad i=1, \dots, K, \quad [2]$$

where $x = \mathcal{F}^* \rho$, $h_i = \mathcal{F}^* c_i$, and $\otimes$ represents the linear convolution operation. Due to the smoothness of coil sensitivities in the image domain, h_i may be approximated as a Finite Impulse Response (FIR) filter using a small support of size $M \times M$. Then, the noisy coil image for the i^{th} coil can be obtained from y_i by computing the DFT:

$$x_i = \mathcal{F} y_i = \rho \cdot c_i + F e_i. \quad [3]$$

SoS and Walsh combining methods

The square-root sum of squares method is a widely used approach for coil combining in parallel MRI. In the noiseless case, the SoS combination is directly given by

$$\sqrt{\sum_{i=1}^K |x_i|^2} = \sqrt{\sum_{i=1}^K |\rho \cdot c_i|^2} = |\rho| \cdot \sqrt{\sum_{i=1}^K |c_i|^2}. \quad (4)$$

Thus, if the coil sensitivities give a nearly uniform coverage, i.e., $\xi = \sqrt{\sum_i |c_i|^2}$ is nearly spatially invariant, then SoS returns a scaled version of the desired image, in the absence of noise. However, for many coil geometries the scaling ξ is spatially variant; further, the SoS image can exacerbate noise. As reported in (5, 6, 7), the SoS image has good SNR properties in high-signal regions, but suffers poor SNR characteristics in the low-signal regions, thus reducing image contrast.

To address this loss of contrast, the Walsh adaptive matched filter method (5, 9) selects complex-valued weights for linear combination of channels to maximize the SNR at each pixel. The governing assumptions are two approximations: additive white noise on image pixels, and coil sensitivities that are constant over small neighborhoods. Across all K channels, a neighborhood of the r^{th} pixel is selected and written as a matrix, X_r , of size $N_w^2 \times K$, where N_w^2 is the number of pixels in the neighborhood of size $NW \times NW$. A set of combining coefficients, $w_r \in \mathbb{C}^K$, is sought to maximize $w_r^* X_r^* X_r w_r$ subject to $\|w_r\|^2 = 1$.

Consequently, w_r is the eigenvector of $X_r^* X_r$ corresponding to the largest eigenvalue. A phase correction is applied, pixel-by-pixel, to force the first coefficient in w_r to have zero phase.

The Walsh combining may instructively be viewed in terms of the singular value decomposition (SVD) (40), $X_r = U_r S_r V_r^*$. In the noise-free case, under the assumption that coil sensitivities are constant across the small neighborhood, X_r is a rank-1 matrix. Let $S_r = \text{diag}\{s_{r,1}, \dots, s_{r,K}\}$ be the diagonal matrix of nonnegative singular values in descending order, $V_{r,1}$ be the first column of V_r , and $U_{r,1}$ be the first column of U_r . Because $s_{r,2} = \dots = s_{r,K} = 0$ in the noise-free case, $X_r = s_{r,1} U_{r,1} V_{r,1}^*$, and the entries of the right singular vector, $V_{r,1}(i)$, give the conjugates of the relative coil sensitivities at pixel r , i.e.

$c_i^*(r) / \sqrt{\sum_{i=1}^K |c_i(r)|^2}$. And, the magnitude of Walsh coil combining may be written as

$$W_r = \left| \sum_{i=1}^K y_i(r) V_{r,1}(i) \right| = |\rho_r| \left| \sum_{i=1}^K c_i(r) V_{r,1}(i) \right| = |\rho_r| \sqrt{\sum_{i=1}^K |c_i(r)|^2}, \quad [5]$$

revealing that an intensity modulation remains if $\sqrt{\sum_{i=1}^K |c_i(r)|^2}$ is spatially varying, just as with the SoS combining. By the orthonormality of the columns of V_r , it is easy to show that the coil combination is directly provided by the principal left singular vector,

$$W_r = |s_{r,1} U_{r,1}(1)|, \quad [6]$$

where $U_{r,1}(1)$ is the element of the principal left singular vector corresponding to the center pixel in the neighborhood. We note that the Walsh method provides the maximum likelihood estimator (MLE) of the image at pixel r for additive white Gaussian noise (41). In summary, the Walsh method processes a neighborhood of pixels to retain only one singular value. Thus, compared to the SoS method, the Walsh approach suppresses noise by discarding the noise present in $s_{r,2}, \dots, s_{r,K}$; however, the bias due to a spatially varying SoS modulation is still observed in the combined image.

A subspace approach

For coil combining, we turn to a subspace view of the blind multichannel deconvolution task developed in the signal processing literature (42, 43, 44) and applied in MRI for the estimation of coil sensitivities (22, 23, 45, 46, 47). In this section, we provide a formal basis for the proposed coil combining technique; a more intuitive justification is presented in Appendix A. We use calligraphic font to represent convolution matrices. For a k -space representation y_i of size $N \times N$, $\mathcal{C}_A\{y_i\}$ denotes a matrix that performs circular convolution of y_i with a k -space image of size $A \times A$. Thus, $\mathcal{C}_A\{y_i\}$ is a circular convolution matrix of size $N^2 \times A^2$ and exhibits a Toeplitz-block Toeplitz structure. For linear convolution matrices, we use the notation $\mathcal{C}_A\{y_i\}$, which has size $(N + A - 1)^2 \times A^2$.

Consider a matrix $\mathcal{Y}$ that represents a sum of coil-by-coil convolutions with the K data channels, $\{y_i\}_{i=1}^K$ for a kernel size, $E \times E$. Referring to [2], we define

$$\begin{aligned}\mathcal{H} &= [\overline{\mathcal{C}}_E\{h_1\} \overline{\mathcal{C}}_E\{h_2\} \cdots \overline{\mathcal{C}}_E\{h_K\}], \\ \mathcal{X} &= \mathcal{C}_{M+E-1}\{\bar{x}\}, \\ \mathcal{Y} &= [\mathcal{C}_E\{y_1\} \mathcal{C}_E\{y_2\} \cdots \mathcal{C}_E\{y_K\}],\end{aligned}\quad [7]$$

where $\bar{x}$ is a zero-padded version of x having the same size as y . Each coil yields a data channel, and h_i is alternatively referred to as a *channel filter* for the i^{th} channel. Thus, in the noiseless case [2] may be rewritten

$$\mathcal{Y} = \mathcal{X} \mathcal{H}, \quad [8]$$

where $\mathcal{Y}$ has size $N^2 \times KE^2$, $\mathcal{X}$ has size $N^2 \times (M + E - 1)^2$, and $\mathcal{H}$ has size $(M + E - 1)^2 \times KE^2$. We assume that $\mathcal{X}$ has full column rank. A small example showing the construction of $\mathcal{Y}$, $\mathcal{H}$ and $\mathcal{X}$ is given in Appendix B. The matrix $\mathcal{Y}$ admits singular value decomposition

$$\mathcal{Y} = USV^*. \quad [9]$$

Here U is a unitary matrix of size $N^2 \times N^2$, V is a unitary matrix of size $KE^2 \times KE^2$, and S is the diagonal matrix of non-negative singular values sorted in descending order (40). The matrix $\mathcal{Y}$ has low-rank, and the rank is $R = (M + E - 1)^2$ in the ideal noiseless case (22, 23, 48). Defining, U_1 to be the first R columns of U , then U_1 is a $N^2 \times R$ matrix that spans the column space of $\mathcal{Y}$, which is typically referred to as the *signal subspace*. Equations [8] and [9] imply that the image, ρ , is in the signal subspace; i.e., $\rho = \mathcal{F}U_1a$ for some $R \times 1$ list of coefficients, a . Yet, the vector a is unknown and made uncertain by both the noise in the k -space measurements used to construct $\mathcal{Y}$ and by the unknown channel filter orders. If we model a as a stochastic unit length complex-valued vector uniformly distributed on the unit sphere in $\mathbb{C}^R$, then the *minimum mean squared error* (MMSE) estimator of a non-negative image is

$$\hat{\rho}_{\text{BCC}} = \alpha \sqrt{\sum_{i=1}^R |\mathcal{F}u_i|^2}, \quad [10]$$

where u_i is the i^{th} column of U and α is a constant depending only on R . Following (49), direct computation yields that $\alpha = \Gamma(3/2)R^{-1/2}$ asymptotically as R grows large. Proof of [10] is given in Appendix C. The MMSE coil-combined image, $\hat{\rho}_{\text{BCC}}$, admits a familiar interpretation: it is the square-root sum of squares of the signal space left singular vectors transformed to the image domain.

A non-stochastic formulation likewise leads to the same estimator. Define the set T of all possible nonnegative images constructed from the signal subspace as $|\mathcal{F}U_1a|$ for $\|a\|^2 = 1$. Then, $\hat{\rho}_{\text{BCC}}$ is the *barycenter* (or centroid) of the set T . Consequently, we refer to the MMSE estimate $\hat{\rho}_{\text{BCC}}$ as the Barycentric Coil Combination (BCC).

By virtue of the singular value truncation in [10], the BCC suppresses noise much like the Walsh method, improving contrast-to-noise ratio vis-a-vis SoS combining. Additionally, by virtue of the MMSE property, the BCC mitigates the spatial inhomogeneity encountered in both the Walsh and SoS methods.

A Generalization

A generalization of the BCC estimator is to use a *weighted* sum of squares of the principal left singular vectors

$$\sqrt{\sum_{i=1}^R |w_i \mathcal{F} u_i|^2}. \quad [11]$$

By analogy to [6], a heuristic, yet intuitive, choice is to use the singular values, $s_1, \dots, s_R$ as the weights (see brief development in Appendix A). This results in the Singular-value Coil Combination (SCC) and can be written

$$\hat{\rho}_{\text{SCC}} = \sqrt{\sum_{i=1}^R |s_i \mathcal{F} u_i|^2}, \quad [12]$$

The resulting image, $\hat{\rho}_{\text{SCC}}$, exhibits the intensity inhomogeneity similar to Walsh and SoS images. By a straightforward computation beginning with [7] and [12], the SoS coil combination can be viewed as a special case of $\hat{\rho}_{\text{SCC}}$ with $E = 1$ and $R = K$.

Computationally efficient surrogate

Computational complexity of the proposed combining method is dominated by the singular value decomposition of the matrix $\mathcal{V}$. Because we work with column space vectors, only R left singular vectors need to be calculated. If QR decomposition is used in the SVD calculation (44, 50), then the computational complexity is $\mathcal{O}(N^2 R^2)$ for the proposed method, $\mathcal{O}(N^2 N_w^2)$ for the Walsh method, and $\mathcal{O}(N^2)$ for the SoS method. For reduced computational complexity, BCC or weighted subspace combination may be computed on a low-frequency (autocalibration sample, or ACS) region of k -space to generate an intensity compensation map. Specifically, if $\langle I \rangle$ represents a low-resolution version of the image I , then the computationally efficient surrogate is

$$I_{\text{BCC}}^C = \frac{\langle I_{\text{BCC}} \rangle}{\langle I \rangle} |I|. \quad [13]$$

Here, $\langle I_{\text{BCC}} \rangle$ is a low-resolution image computed as in [10] (or [11]) but using only the ACS region of k -space. Calculating $\langle I_{\text{BCC}} \rangle$ requires only $\mathcal{O}(N_{\text{acs}}^2 R^2)$ computations, where N_{acs}^2 represents the number of samples in the ACS region. The speed-up is $\mathcal{O}(N^2 / N_{\text{acs}}^2)$. However, the noise characteristics of I_{BCC}^C are inherited from I ; thus, for example, in low-

SNR applications an L1-regularized reconstruction may be used for I in Eq. [13], rather than SoS, in conjunction with the I_{BCC}^C fast intensity compensation.

Methods

We implemented the proposed BCC method of [10] in MATLAB (Mathworks, Natick, MA). Implementation includes the following five steps: 1) Construct $\mathcal{Y}$ as in [7] for a specified kernel size, $E \times E$; 2) compute the SVD of $\mathcal{Y}$ to obtain left singular vectors, $\mathcal{U}$, and singular values, $\mathcal{S}$; 3) select the first R columns of $\mathcal{U}$ to form $\mathcal{U}_1$; 5) compute the DFT and calculate pixel-wise square-root sum-of-squares to obtain the combined image.

The number of coils, K , and the FIR filter size of h_i , $M \times M$, should be considered in selection of the kernel size, $E \times E$. We can derive a lower bound from the condition $KE^2 > (M + E - 1)^2$ for a given M and K (42, 51); so, E may decrease with increasing K . Thus, when there exists a large number of coils, we can use smaller kernel sizes and still obtain good reconstructions. Therefore, we selected $E = 2$ for the datasets with 32 coils to reduce the number of computations, and we chose $E = 5$ in the remaining datasets. To select the number, R , of singular values to retain, we adopt the threshold $0.05(s_{\max} - s_{\min}) + s_{\min}$, where $s_{\max}$ and $s_{\min}$ are the largest and smallest singular values of $\mathcal{Y}$, respectively. For the Walsh method, we selected the neighborhood size as [9, 9] in all experiments.

In order to test the proposed method, experiments were performed using both simulated and in vivo data.

Simulations

In the simulated experiments, we used Biot-Savart law coil sensitivities¹ for $K = 12$ coils placed symmetrically in a circle. Coil parameters were selected as FOV = 40 cm, 20 cm distance of a coil to the image center, and 3 cm coil radius. For the test image, we used the Shepp-Logan phantom. After point-wise multiplying a test image by the coil sensitivities, we computed the DFT to obtain simulated, fully sampled k -space data; circularly symmetric complex Gaussian noise was added to the k -space samples.

For the Shepp-Logan test image, simulated data were calculated for a matrix size $256 \times 256 \times 12$ to represent frequency encoding, phase encoding and coil directions, respectively. We computed the proposed BCC method in [10] for $\alpha = 1$. We then measured both the intensity modulation and SNR maps of the resulting images. For the intensity modulation map, we divided, pixel-wise, the resulting image by the true test image, then subtracted 1, and took the absolute value of the result. This non-negative measure provides an easy visualization of the absolute modulation, and mathematically is given as

$$M(r) = 100 \left| \frac{I_{\text{est}}(r)}{I_{\text{true}}(r)} - 1 \right|. \quad [14]$$

¹The code for Biot-Savart was obtained from <http://bigwww.epfl.ch/algorithms/mriphantom/>

A value of $M(r) = 0$ corresponds to no bias in the intensity of the coil-combined image at pixel r . For the pixel-by-pixel maps of SNR, in each case we computed root mean squared error (RMSE) between the noisy and noise-free reconstructions for 300 independent noise realizations. For the numerator in the SNR measure, we applied each coil combination method to noise-free data. For quantitative analysis, we measured peak-signal-to-noise ratio (PSNR), contrast-to-noise ratio (CNR), mean modulation (MOD) and foreground contrast (FCN) values. For CNR, we used the definition $CNR = |\mu_{sig} - \mu_{bg}|/\sigma_{bg}$, where μ_{sig} is the mean value of a region where the signal exists, μ_{bg} is the mean value of the background region, and σ_{bg} is the standard deviation of the background region. PSNR is defined as

$$PSNR = \frac{\max(I_{true})^2}{\frac{1}{N^2} \sum_{r=1}^{N^2} (I_{est}(r) - I_{true}(r))^2} \quad (15)$$

For a quantitative value of intensity inhomogeneity or modulation artifact (MOD), we calculated the L2-norm of the modulation map defined in Eq. [14]. Smaller values in MOD refer to less inhomogeneity artifact. FCN is a metric to quantitatively measure the foreground contrast calculated from the ratio of mean values of two neighboring regions from the inside of the phantom; for the metric we subtract 1 from the ratio, take the absolute value, and multiply by 100. The original phantom has FCN = 50. To measure the background noise reduction, we selected the background region via thresholding and then calculated the average sum of squared values. The images are scaled to have their L2-norm equal to 1 before the measurements except for SNR maps.

In vivo experiments

For qualitative assessment using in vivo data, we tested the proposed coil combining method using two cardiac datasets and one spine dataset. Firstly, we tested the proposed BCC coil combining method using a spine dataset that is available in the PULSAR-Toolbox² (52). This dataset was acquired on a 3T whole-body GE scanner (GE Healthcare, Waukesha, WI) from a healthy male volunteer using a four-channel CTL spine array with fast spoiled gradient-echo sequence. Parameters were: TE = 12 ms; TR = 300 ms; receiver bandwidth 62.5 kHz; matrix size 256×256 ; flip angle 15 degrees; and, FOV 320×320 mm². Fully sampled data were retro-respectively subsampled by a factor of 2, and 40 central lines were used for auto-calibration. We used GRAPPA with a kernel size [5, 4]. For coil combination, we used Walsh, BCC, and efficient BCC methods applied to the GRAPPA interpolated k -space data.

The first cardiac dataset was breath-hold segmented SSFP cine data taken from a healthy volunteer. Data were collected on a 1.5T Siemens scanner (Siemens Healthcare, Erlangen, Germany) equipped with 32 coils. Acquisition parameters were: FOV 360×293 mm²; matrix size 384×156 ; temporal resolution 31 ms; 15 frames; slice thickness 8 mm; TE = 1 ms; TR = 2.6 ms; receiver bandwidth per pixel 1371 Hz/pixel; 12 lines per segment; and,

²Download for PULSAR is available at <http://www.ece.tamu.edu/~jimji/software/pulsarweb/index.htm>.

flip angle 75 degrees. We employed fully sampled k -space samples, and for coil combination, we used Walsh and proposed BCC methods. We also applied NP, HUM, LSS and FCM postprocessing methods for inhomogeneity correction to Walsh images (details for these methods are provided in the following section).

The second cardiac dataset was breath-hold segmented SSFP cine data acquired on a 3T Siemens Tim Trio scanner (Siemens Healthcare, Erlangen, Germany) using a 32-channel coil array. The acquisition parameters were: TE = 1.45 ms; TR = 1.87 ms; FOV 256×256 mm²; slice thickness 6 mm; spatial resolution 1×1 mm²; temporal resolution 11.2 ms; flip angle 57 degrees; and, receiver bandwidth per pixel 977 Hz/pixel. This dataset was acquired in 38 cardiac phases with 42 segments per phase and 6 lines per segment. It was retrospectively sub-sampled in a uniform interleaved fashion by a factor of $R = 2$. L1-regularized SENSE method using FISTA (53) and 3D wavelets was applied in the reconstruction. Coil sensitivities were estimated from the average of all frames using ESPIRiT (22). Equation [13] was applied to the reconstructed image using low-resolution images obtained from the 40×40 central region of the time-averaged data. We also applied NP, HUM, LSS and FCM post-processing methods for inhomogeneity correction to the L1-SENSE reconstructed image.

Additionally, the proposed BCC coil combining was applied to the ten dynamic cardiac imaging datasets that comprised the 2013 ISMRM Challenge³; software and .gif files are available online as supplemental material at https://github.com/deryagol/pMRI_coil_combination.

Inhomogeneity Correction Algorithms

Many commercial scanners use body-coils for inhomogeneity correction at the expense of an additional scan. Consistent with the acceleration sought in parallel imaging, here we compare the proposed method to retro-respective methods that do not require body coil scans. Specifically, we compared the results of the proposed coil combining approach to four different intensity inhomogeneity correction algorithms applied to the Walsh combined and L1-SENSE reconstructed images.

All the correction algorithms tested here require knowledge of an image mask to separate background pixels from foreground pixels. In the case of strong inhomogeneity, the automated threshold selection algorithms tend to fail. In our experiments, we manually determined the threshold for best results and to avoid mask failure in competing methods.

Homomorphic Unsharp Masking (HUM)—HUM is a fast and effective filtering-based inhomogeneity correction method, where the modulation correction field is obtained by low-pass filtering the input image. We used the mean-based low-pass filtering method proposed in (38). Once a threshold is determined to separate background (noise) pixels from the histogram of the image, these pixels are filled with the mean of the non-noise pixels. The resulting image is then filtered by a Gaussian filter to obtain a modulation correction field. The input image is then corrected via pixel-wise division by the correction field.

³www.ismrm.org/challenge/node/53

Nonparametric (NP)—Entropy-based intensity correction algorithms are based on the assumption that the images distorted by intensity inhomogeneities have higher entropy than the original versions. We used Manjon et al.'s method (28), which is a non-parametric iterative algorithm that minimizes a multidimensional entropy function computed on the histogram of a gradient image. In (28), the image is first low-pass filtered to eliminate noise, and a mask is automatically obtained using Otsu's method (39). The bias field is modeled as a linear combination of low-frequency cubic B-spline basis functions. The number of basis functions used in surface fitting is adaptively adjusted. Since the Otsu's method was not effective in the datasets considered below, we manually determined the threshold.

Level Set Segmentation (LSS)—Level-set methods for image segmentation are based on modeling a contour in an image as a cross-section of a three (or more) dimensional function, called *level set function*. In (54), for example, a K-means clustering method is employed for local intensity clustering. The method requires user-defined tuning of a neighborhood size to yield best intensity correction.

Fuzzy C-Means (FCM)—We also applied the modified Fuzzy C-means segmentation method proposed in (32). The technique requires user-defined tuning of class prototypes, the vector of gray-scale means for all segment classes.

Results

Figures 1, 2 and 3 display simulation results with synthetic noise yielding 50 dB SNR. Reconstruction results and modulation are displayed in the first and second rows, respectively.

In Fig. 1, coil combination results for SoS, Walsh and proposed SCC and BCC methods are illustrated. As seen in the figure, SoS, Walsh and SCC methods exhibit similar intensity inhomogeneity, and the intensity modulation is significantly mitigated in the proposed BCC approach. In Fig. 2, we show the Walsh image and the inhomogeneity corrected Walsh images obtained after using NP, HUM, LSS and FCM methods. In Fig. 2a, the Walsh image before modulation correction is displayed. As seen in Fig. 2, NP gave the best image with inhomogeneity mitigation and less distortion in the image contrast among the tested post-processing methods. Similarly, we applied the retro-respective modulation correction methods to the BCC image shown in Fig. 1e. Resulting images are provided in Fig. 3. Again, we obtained the best results with NP method. HUM, LSS and FCM methods were also effective in the suppression of inhomogeneity but also reduced the contrast and resulted in shading artifacts around the edges of the phantom (55).

Noise characteristics for the simulated dataset are illustrated in Fig. 4 for synthetic noise yielding 20 dB SNR. As reported in the literature (5), the SoS method is seen to provide images with increased noise levels in low signal regions. This well-known effect is not prominent in Fig. 4, due to the strong inhomogeneity; however, the SNR map (bottom row) in Fig. 4b nonetheless shows the reduced SNR of SoS in the low-signal locations. Walsh and proposed SCC and BCC methods are effective in noise suppression in the dark locations as seen in the reconstructions shown in Fig. 4c–e. With the BCC approach we obtained high

contrast images with significant inhomogeneity mitigation. The SNR maps provided in the second row of Fig. 4 demonstrate that Walsh and SCC methods produce similar SNR maps at image pixels on the support of the object; and the BCC method is effective, especially in low-signal locations. We also display the Walsh image with NP post-processing and corresponding SNR map in Fig. 4f. In this example, NP provided significant inhomogeneity reduction but also reduced SNR at low-signal pixels. Note that low-signal regions near the phantom center are not eliminated in the masking process required for NP; therefore, noise enhancement is observed in these locations caused by the NP inhomogeneity correction. Horizontal and vertical line profiles of the SNR maps taken from the central points are given in Fig. 5, where SNR reduction in SoS and NP methods can be clearly observed. Line plots suggest that BCC performs the best in the low-signal locations.

Table 1 presents quantitative image quality results for the examined methods applied to the simulated data. We measured average peak-signal-to-noise ratio (PSNR), contrast-to-noise ratio (CNR), MOD as a measure of intensity modulation and foreground contrast (FCN) values for SoS, Walsh and proposed SCC and BCC methods in addition to post-processed Walsh and BCC images. Standard deviations from 300 realizations are written in parentheses. PSNR values for BCC exceed PSNR values for the existing coil combining approaches. Maximum PSNR is observed in BCC with NP post-processing. Likewise, contrast is improved, with CNR values for the proposed SCC and BCC methods exceeding the existing approaches in all cases. Mean modulation (MOD) is the L2-norm of the modulation image given in Eq. [14]. Small values indicate less inhomogeneity artifacts. The smallest values are obtained in BCC with NP post-processing and in Walsh with FCM post-processing. Measured FCN values suggest that Walsh and SCC have high foreground contrast with respect to BCC and Walsh post-processed images. Also, the results show NP post-processing well-preserved or even improves the FCN of the uncorrected image, but other post-processing methods give images with highly reduced FCN. BCC also experienced low FCN values because it tends to balance the intensity values throughout the phantom but not as aggressively as HUM, LSS and FCM.

In the experiments reported above, we used $E = 5$, and the threshold rule resulted in $R = 48$ at 20dB SNR, $R = 53$ at 30dB SNR and $R = 60$ at 50 dB SNR. In Fig. 6, we demonstrate the change of background noise power and intensity inhomogeneity in the BCC image from 12-channel simulated data at 30dB SNR, with respect to R (number of signal space vectors) and E (kernel size). Increased kernel size results in less background noise in the BCC images but also less inhomogeneity correction at small R values and higher computational complexity. Because of noise enhancement in the BCC images, the MOD parameter, a measure of intensity modulation, increases for large values of R . The dependence of R on E can be clearly observed from the minimum values of the MOD plot.

We next display results for the in vivo data sets; background regions are cropped, and only one slice is displayed for the cardiac cine images. The coil combination results for the spine data set are shown in Fig. 7 for the Walsh, BCC and efficient BCC methods. We also show the intensity correction field used to calculate the efficient BCC image from the SoS combined image. It can be clearly observed that the intensity inhomogeneity is significantly reduced in BCC images. Also, BCC and efficient BCC methods provided similar images

although the execution time for BCC was 9.6 seconds versus 1.48 seconds for the efficient method.

Supporting Figure S1 illustrates the potential failure of the automated mask determination used in post-processing for inhomogeneity correction. We applied Otsu's method (39), which is an iterative histogram-based threshold selection algorithm and is implemented in MATLAB's built-in function `graythresh` (Mathworks, Natick, MA). The resulting masks are shown for the first (left) and second (right) cardiac datasets in Sup. Fig. S1. As seen, only the bright locations are assigned as foreground regions (marked black). Once these masks are employed in the post-processing methods, we only obtain inhomogeneity correction in these partially selected regions.

The comparison of the proposed coil combined images with the post-processed images applied to Walsh method are provided in Fig. 8 for the first cardiac dataset and in Fig. 9 for the second cardiac dataset. The retro-respective intensity correction methods were applied to the Walsh image (shown in Fig. 8a) and L1-SENSE reconstructed image (shown in Fig. 9a) using manually selected image mask. We did not observe significant correction in using the NP post-processing in the first cardiac dataset. HUM and LSS methods mostly reduced the inhomogeneity but also reduced the image contrast and the intensity differences between different types of tissues (such as fat and water). FCM causes shading artifacts as shown in Fig. 8e and Fig. 9e. The proposed coil combining approach enhanced the dark locations and suppressed the bright locations, resulting in an image that appears to have a more uniform intensity weighting than the SoS and Walsh methods as shown in Fig. 8f and Fig. 9f. Additionally, the BCC method did not reduce the contrast between different types of tissues as much as HUM.

Finally, we report the computational costs for the coil combining techniques implemented in MATLAB on an Intel Core i7-2600 CPU 3.40 GHz processor. For the first cardiac dataset, for which there were $K = 32$ coils, the run times of the SoS, Walsh and proposed methods were 0.1, 18.35 and 11.33 seconds, respectively. Both SoS and I_{BCC}^C methods each required approximately 0.2 seconds for coil combining of the second cardiac dataset. For the spine data set, the run times of the SoS, Walsh, proposed BCC and efficient I_{BCC}^C methods were 0.024, 1.8, 9.6 and 1.46 seconds, respectively.

Discussion

Subspace approaches

Subspace techniques have been widely used in the signal processing literature for blind multi-channel image deconvolution. Indirect methods estimate the unknown blurs (43), whereas direct methods reconstruct the input image without explicit recovery of the blurs (44) via the relationship $U_1 U_1^* \mathcal{X} = \mathcal{X}$. Indirect methods use the right singular vectors spanning either the row-space or null-space of $\mathcal{V}$, in contrast to the direct methods, which use the left singular vectors. The indirect methods have been adopted for parallel MRI. For example, ESPIRiT (22) and LOTUS (23) use the row-space to recover the coil sensitivities, while PRUNO (56) uses the null-space vectors to interpolate missing k -space samples. In

contrast, the direct methods for blind multi-channel image deconvolution have not been employed in parallel MRI, due to a high sensitivity to noise and modeling error. However, as demonstrated here, the direct methods can be leveraged after interpolating unacquired k -space points to obtain a sum-of-squares image with low bias and high SNR.

Parameter selection

BCC—The proposed method has two parameters that must be defined by the user: kernel size, E , and number of principal singular values, R (and hence rank of the modified calibration matrix). Kernel size for the calibration matrix $\mathcal{V}$ is analogous to the SPIRiT kernel size. We observe that $E = 5$ gives good results for all kind of datasets. As we demonstrated in Fig. 6, a larger value for E provides better noise suppression in the background and dark regions of the image, but computational load increases with increasing kernel size. E can be selected smaller than 5, when the number of coils is large. For example, in datasets with 32 coils, we selected $E = 2$ and still obtained good combining results. Fig. 6 suggests that slightly better inhomogeneity correction can be achieved with small kernel sizes. To select the number of singular values to retain, we adopt the threshold $0.05(s_{max} - s_{min}) + s_{min}$, where s_{max} and s_{min} are the largest and smallest singular values of $\mathcal{V}$, respectively. An alternative thresholding rule is the L-curve method known as adaptive pruning algorithm (57).

Post-processing algorithms—The inhomogeneity correction methods applied to a single reconstructed image can effectively reduce the inhomogeneities, but performance depends significantly on selection of algorithm parameters. The first parameter is a mask; all inhomogeneity correction methods tested here require knowledge of a mask used to distinguish background pixels from signal pixels. The automatic methods fail when there exists high intensity inhomogeneity. For example, NP uses Otsu's method (39) for mask selection but for the cardiac dataset 1 shown in Fig. 8, the Otsu's method only selected the outer wall of the chest and failed to select any location inside the chest. Otsu's method also failed in simulations. Therefore, in our experiments, we manually selected the mask and used it for all the methods tested. In addition to requiring a mask, FCM requires tuning the mean values for the fuzzy clusters must be selected. HUM is sensitive to the selection of the parameters of the low-pass filter (for example, the variance of the Gaussian filter), and LSS is sensitive to the scale parameter that specifies the size of the neighborhood for local clustering. We manually optimized these parameters for each dataset in our applications.

Among the post-processing algorithms applied to the simulated data, both HUM and LSS exhibited edge artifacts. In HUM, the bias is calculated from the low-pass filtered version of the inhomogeneity distorted image, and so it includes not only the bias but also the low-pass features of the image. Therefore, the corrected image experiences edge effects (such as shading and blurring-like artifacts), as also reported in (55). The simulated phantom has discontinuities of large amplitude, compared to in vivo MRI images; therefore, we observe the edge effect artifacts to be more aggressively present in the simulated images than in the in vivo images. In addition, the post-processing to correct inhomogeneity tends to reduce contrast.

Stability

Given noisy coil images and the associated convolution matrix, $\tilde{\mathcal{Y}} = \mathcal{Y} + \mathcal{E}$, the subspace decomposition yields a perturbed estimate, $\tilde{U}_1$, of the singular vectors spanning the column space of $\mathcal{Y}$. The noise matrix, $\mathcal{E}$, has a Toeplitz-block-Toeplitz structure. By construction, the barycenter coil combination depends on the subspace spanned by $\tilde{U}_1$, rather than on the individual singular vectors comprising the columns $\tilde{U}_1$. The stability of the barycenter estimate in noise follows from Wedin's theorem (58). Bounds on the error in the subspace $\tilde{U}_1$ are given by Nekrutkin (59) and Vu (60).

From [10], for perturbations in U_1 modeled as Gaussian, the BCC estimate has a non-central chi distribution. However, the perturbation $\mathcal{E}$ is itself a structured matrix with entries arising from both measurement error on acquired k -space points and interpolation error at non-acquired samples. For Gaussian measurement errors at acquired k -space locations, the marginal and joint distributions of $\mathcal{E}$ are open problems and depend on the k -space reconstruction algorithm employed.

Conclusion

A subspace-based approach has been presented for coil combination in parallel imaging, motivated by applications in which intensity homogeneity is severe and SNR-optimal coil combination with an intensity correction post-processing step yields poor results. Starting with a subspace construction using interpolated k -space samples, a minimum mean squared error criterion was adopted. The MMSE coil combination was found to be simply the square-root sum-of-squares of eigen-images from the signal subspace. Example image reconstructions using both simulated and in vivo data demonstrated reduced inhomogeneity in image intensity and improved contrast, compared to existing coil combination and post-processing techniques. The proposed technique does not require knowledge of image mask and is suggested for use in coil-by-coil parallel imaging when an asymmetric coil geometry results in significant intensity inhomogeneity.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

Acknowledgments

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Appendix A

The proposed estimator is presented in [10] using an estimation theoretic derivation. Here, we provide an intuition relating the BCC coil combination to the well-known square-root sum of squares (SoS) combination. To begin, we express the SoS image using an alternative, but equivalent, formulation. Let $\mathbf{x}_i$ be a column vector formed by listing the pixels of the i^{th} coil image. Then,

$$\hat{\rho}_{\text{SoS}} = \sqrt{\text{diag}(\mathbf{xx}^*)}, \quad [16]$$

where boldface $\mathbf{x}$ is the matrix $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_K \end{bmatrix}$ of size $N^2 \times K$, and “ $\text{diag}(A)$ ” selects the main diagonal elements of the matrix A . Similarly, let $\mathbf{y}$ be the matrix whose columns stack the k -space samples from each coil and observe the Fourier relationship, $\mathbf{x} = \mathcal{F}\mathbf{y}$. Thus, we can rewrite the SoS combination given in Eq.[16] in terms of the k -space samples,

$$\hat{\rho}_{\text{SoS}} = \sqrt{\text{diag}(\mathbf{xx}^*)} = \sqrt{\text{diag}(\mathcal{F}\mathbf{yy}^*\mathcal{F}^*)}. \quad [17]$$

Next generalize this formulation by replacing the data matrix $\mathbf{y}$ with the structured matrix, $\mathcal{V}$, given in Eq. [7] formed from the k -space samples. $\mathcal{V}$ is the concatenation of the circular

convolution matrices of fully sampled k-space samples for a kernel size $E \times E$. (detail is given in Appendix B). The resulting generalized version of the SoS method is defined as

$$\sqrt{\text{diag}(\mathcal{F}\mathcal{Y}\mathcal{Y}^*\mathcal{F}^*)}. \quad [18]$$

Observe that [18] is equivalent to [16] when the convolution kernel size is 1×1 .

Direct substitution of the singular value decomposition (SVD) $\mathcal{Y} = USV^*$ into Eq. [18] yields

$$\sqrt{\text{diag}(\mathcal{F}USS^*U^*\mathcal{F}^*)}. \quad [19]$$

The matrix $\mathcal{Y}$ is known to have low rank (22, 23, 48). Thus, to suppress noise in the coil-combined image, we truncate the singular values to retain only the first R . Partition U as $U = [U_1 \ U_2]$, such that U_1 includes the only first R columns of U . Thus we obtain

$$\hat{\rho}_{\text{SCC}} = \sqrt{\text{diag}(\mathcal{F}U_1S_1S_1^*U_1^*\mathcal{F}^*)} = \sqrt{\sum_{i=1}^R s_i^2 |\mathcal{F}u_i|^2}, \quad [20]$$

where s_i is the i^{th} singular value of the calibration matrix $\mathcal{Y}$ and u_i is the i^{th} left singular vector. $\hat{\rho}_{\text{SCC}}$ is denoted as Singular-value Coil Combining (SCC). The expression in [20] says to transform to the image domain the R leading left singular vectors of the calibration matrix and combine their squared magnitude via weighted sum. The weights are the squares of the singular values of the calibration matrix. An illustration of the singular images $|\mathcal{F}u_i|$ of the SoS method (i.e. $E = 1$) is given in Supporting Figure S2 for the leading twelve singular values from the first cardiac dataset. SoS calculates the weighted sum of squares of all singular-images, where the weights are the square of the corresponding singular values as in SCC. Note that the singular images corresponding to small singular values represent the noise subspace vectors and are noisier. Since these noise subspace vectors are eliminated in SCC, we obtain higher SNR images. Also, the brighter locations in the SoS image correspond to those singular-images weighted by the larger singular values. So, empirically and intuitively, the weighting by singular values leads to the spatially varying amplitude modulation. To mitigate the intensity modulation, the weights are uniformly set to α , yielding the proposed MMSE estimation (or BCC image) in [10],

$$\hat{\rho}_{\text{BCC}} = \alpha \sqrt{\sum_{i=1}^R |\mathcal{F}u_i|^2}. \quad [21]$$

Appendix B

Small Numeric Example

Here, we illustrate the construction of convolution matrices on a one-channel example with $E = 2$. One channel is shown for brevity. Let

$$x = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}, \quad h_1 = \begin{bmatrix} 17 & 19 \\ 18 & 20 \end{bmatrix}, \quad y_1 = \begin{bmatrix} 17 & 87 & 195 & 133 \\ 52 & 215 & 437 & 292 \\ 87 & 289 & 511 & 331 \\ 54 & 168 & 282 & 180 \end{bmatrix}.$$

Then, $M = 3$ and $N = 4$. Also, let $\bar{x}$ be a zero-padded version of x , having the same size as y_i .

Then, $\mathcal{C}_E\{y_1\}$, $\mathcal{C}_{E+M-1}\{\bar{x}\}$, $\bar{\mathcal{C}}_E\{h_1\}$ are given by

$$\mathcal{C}_E\{y_1\} = \begin{bmatrix} 17 & 54 & 133 & 180 \\ 52 & 17 & 292 & 133 \\ 87 & 52 & 331 & 292 \\ 54 & 87 & 180 & 331 \\ 87 & 168 & 17 & 54 \\ 215 & 87 & 52 & 17 \\ 289 & 215 & 87 & 52 \\ 168 & 289 & 54 & 87 \\ 195 & 282 & 87 & 168 \\ 437 & 195 & 215 & 87 \\ 511 & 437 & 289 & 215 \\ 282 & 511 & 168 & 289 \\ 133 & 180 & 195 & 282 \\ 292 & 133 & 437 & 195 \\ 331 & 292 & 511 & 437 \\ 180 & 331 & 282 & 511 \end{bmatrix}, \quad \mathcal{C}_{E+M-1}\{\bar{x}\} = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 & 0 & 7 & 0 & 9 \\ 2 & 1 & 0 & 0 & 0 & 0 & 8 & 7 & 0 \\ 3 & 2 & 1 & 0 & 0 & 0 & 9 & 8 & 7 \\ 0 & 3 & 2 & 0 & 0 & 0 & 0 & 9 & 8 \\ 4 & 0 & 6 & 1 & 0 & 3 & 0 & 0 & 0 \\ 5 & 4 & 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 6 & 5 & 0 & 3 & 2 & 0 & 0 & 0 \\ 7 & 0 & 9 & 4 & 0 & 6 & 1 & 0 & 3 \\ 8 & 7 & 0 & 5 & 4 & 0 & 2 & 1 & 0 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 0 & 9 & 8 & 0 & 6 & 5 & 0 & 3 & 2 \\ 0 & 0 & 0 & 7 & 0 & 9 & 4 & 0 & 6 \\ 0 & 0 & 0 & 8 & 7 & 0 & 5 & 4 & 0 \\ 0 & 0 & 0 & 9 & 8 & 7 & 6 & 5 & 4 \\ 0 & 0 & 0 & 0 & 9 & 8 & 0 & 6 & 5 \end{bmatrix}, \quad \bar{\mathcal{C}}_E\{h_1\} = \begin{bmatrix} 17 & 0 & 0 & 0 \\ 18 & 17 & 0 & 0 \\ 0 & 18 & 0 & 0 \\ 19 & 0 & 17 & 0 \\ 20 & 19 & 18 & 17 \\ 0 & 20 & 0 & 18 \\ 0 & 0 & 19 & 0 \\ 0 & 0 & 20 & 19 \\ 0 & 0 & 0 & 20 \end{bmatrix}.$$

These matrices possess Toeplitz-block-Toeplitz (TBT) structure. Their transposed and column-flipped versions (i.e. $\text{fliplr}(A')$, in MATLAB notation, for $A = \mathcal{C}_E\{y\}$) possess Hankel-block-Hankel (HBH) structure. $\mathcal{Y}$'s transposed and column flipped version is equal to SPIRiT, ESPIRiT, PRUNO and SAKE calibration matrices applied to fully sampled k -space data with periodic boundary conditions.

Appendix C

Proof for MMSE

Let μ denote the uniform measure on the sphere in $\mathbb{C}^R$. Let e_k^* denote the k th row of $\mathcal{F}U_1$, and $\hat{e}_k$ the unit vector in the direction of e_k . The MMSE estimator is the conditional mean; therefore, we have

$$\int |e_k^* a| d\mu = \|e_k\| \int |\hat{e}_k^* a| d\mu = \|e_k\| \alpha$$

where α is a constant depending only on R (49). Thus, the MMSE image is α times the square-root of the sum of squares of the rows of the matrix $\mathcal{F}U_1$.

Figure 1.

Simulation results for 50 dB SNR. Reconstruction results are shown in the first row, and modulation maps of the corresponding reconstructions are shown in the second row for (a) original image, (b) SoS, (c) Walsh, (d) SCC, and (e) proposed BCC method. The intensity modulation for the SCC method is similar to the modulation in SoS and Walsh methods; the intensity modulation is significantly mitigated in the BCC image.

Figure 2. Inhomogeneity correction results applied to the Walsh combined image with (a) no inhomogeneity correction, (b) NP, (c) HUM, (d) LSS, and (e) FCM post-processing intensity correction methods. Reconstruction results are shown in the first row, and modulation maps are shown in the second row.

Figure 3.

Inhomogeneity correction results applied to the BCC image with (a) no inhomogeneity correction, (b) NP, (c) HUM, (d) LSS, and (e) FCM post-processing intensity correction methods. Reconstruction results are shown in the first row, and modulation maps are shown in the second row.

Figure 4.

Simulation results for 20 dB SNR. Reconstruction results and SNR maps are shown in the first and second rows respectively for (a) Reference coil image (b) SoS, (c) Walsh, (d) proposed SCC, (e) proposed BCC, and (f) NP applied to Walsh image. SNR maps are similar in SoS, Walsh and BCC methods, but lower SNR is obtained in NP post-processed Walsh images. In SNR maps, higher is better.

Figure 5.
Horizontal (top) and vertical (bottom) line profiles of SNR maps given in Fig. 4.

Figure 6.

Background noise power (left) and L2-norm of the modulation image (right) in BCC reconstruction from 12-channel simulated data with 30 dB SNR versus R (the number of signal space vectors) and E (kernel size).

Figure 7.

In vivo results from spine dataset for (a) Walsh method, (b) proposed BCC method, (c) computationally efficient BCC method, I_{BCC}^C , and (d) modulation map $\langle I_{SoS} \rangle \div \langle I_{BCC} \rangle$ obtained from SoS combined image.

Figure 8.

In vivo results from cardiac dataset 1 for (a) Walsh method, (b–e) post-processing methods applied to Walsh image and (f) proposed BCC method.

Figure 9.

In vivo results from cardiac dataset 2. (a) L1-SENSE reconstructed image, (b–e) post-processing methods applied to (a), and (f) computationally efficient BCC method, I_{BCC}^C , obtained from L1-SENSE reconstructed image shown in (a).

Image quality measures for SoS, Walsh, proposed SCC and BCC methods, and modulation corrected images (Comb. stands for coil combination and Corr. stands for inhomogeneity correction). Higher is better in all of the metrics except MOD.

Table 1

Methods		PSNR		CNR		MOD		FCN	
Comb.	Corr.	30dB	20dB	30dB	20dB	30dB	20dB	30dB	20dB
SoS	–	39.2 (0.007)	38.1 (0.026)	8.6 (0.085)	1.3 (0.138)	70.8 (0.002)	64.9 (0.077)	18.8 (0.401)	9.1 (0.804)
	–	38.8 (0.004)	38.5 (0.015)	9.6 (0.372)	1.9 (0.114)	71.7 (0.001)	71.2 (0.025)	21.2 (0.456)	19.7 (1.426)
Walsh	NP	67.5 (1.723)	68.8 (1.67)	4.8 (0.220)	0.053 (0.042)	23.3 (1.090)	44.5 (0.816)	19.6 (5.234)	22.1 (2.414)
	HUM	57.4 (0.002)	53.0 (0.078)	1.6 (0.750)	0.3 (0.053)	40.7 (0.167)	44.6 (0.090)	0.2 (0.153)	0.5 (0.351)
	LSS	40.6 (0.004)	37.4 (0.115)	2.1 (0.404)	0.1 (0.004)	56.1 (0.390)	56.6 (1.442)	2.6 (0.137)	0.4 (0.290)
	FCM	38.5 (0.008)	38.2 (0.023)	1.4 (0.016)	0.3 (0.098)	46.0 (0.038)	17.6 (0.020)	0.03 (0.021)	0.1 (0.059)
SCC	–	38.7 (0.006)	38.2 (0.016)	18.5 (0.960)	6.4 (0.781)	71.8 (0.010)	70.7 (0.028)	21.1 (0.410)	18.3 (1.132)
	–	261.7 (4.038)	123.5 (0.801)	11.9 (0.488)	2.3 (0.159)	15.6 (0.037)	21.7 (0.082)	3.54 (0.660)	4.7 (1.460)
BCC	NP	291.8 (5.755)	114.5 (2.592)	11.8 (0.483)	0.5 (0.072)	13.0 (0.057)	20.9 (0.202)	5.1 (0.722)	3.3 (1.245)
	HUM	55.0 (0.175)	52.3 (0.049)	2.9 (0.653)	0.5 (0.001)	38.8 (0.021)	41.2 (0.061)	0.2 (0.136)	0.5 (0.345)
	LSS	40.5 (0.031)	38.6 (0.630)	2.7 (0.001)	0.2 (0.002)	47.4 (0.078)	51.4 (0.157)	0.3 (0.186)	1.0 (0.618)
	FCM	66.2 (0.074)	60.9 (0.081)	4.2 (0.015)	0.1 (0.001)	34.4 (0.108)	38.4 (0.067)	0.2 (0.136)	0.4 (0.249)