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A Multiresolution Approach to Shear Wave Image Reconstruction

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Abstract

Shear wave imaging techniques build maps of local elasticity estimating the local group velocity of induced mechanical waves. Velocity estimates are formed using the time delay in the motion profile of the medium at two or more points offset from the shear wave source. Because the absolute time-of-flight between any pair of locations scales with the distance between them, there is an inherent trade-off between robustness to time-of-flight errors and lateral spatial resolution based on the number and spacing of the receive points used for each estimate. This work proposes a method of using the time delays measured between all combinations of locations to estimate a noise-robust, high-resolution image. The time-of-flight problem is presented as an overdetermined system of linear equations that can be directly solved with and without spatial regularization terms. Finite element method simulations of acoustic radiation force-induced shear waves are used to illustrate the method, demonstrating superior contrast-to-noise ratio and lateral edge resolution characteristics compared to linear regression of arrival times. This technique may improve shear wave imaging in situations where time-of-flight noise is a limiting factor.

I. Introduction

Measuring the group or phase velocity of an induced transverse mechanical wave propagating through a region of interest (ROI) has become an increasingly popular way to characterize tissue elastic properties for the purposes of diagnosis, staging, and treatment guidance, among others [1]–[12]. Ultrasound has provided a convenient means of both generating and tracking mechanical waves, using high-intensity ultrasonic pulses to deliver momentum to the tissue through the effect of acoustic radiation force [13], and high-framerate ultrasonic imaging to image the propagation of the induced vibrations. This modality is known as “shear wave elasticity imaging” (SWEI). Magnetic resonance imaging (MRI) has also been used to track vibrations induced by an external mechanical vibrator in a modality called “magnetic resonance elastography” (MRE) [14].

It is important to distinguish the task of using the average velocity of a transverse wave to characterize the modulus of a region of tissue (i.e. fibrosis staging), from that of using the local velocity of the wave to generate images of elasticity (i.e. lesion visualization). In the

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The authors are pursuing intellectual property protection around the algorithms described in this work.

former task, it is of interest to use as large of an ROI as possible to average out local variations in imaged wave propagation, feeding as much data into the model as possible. For the task of imaging local propagation velocity, though, it is of interest to use the smallest possible ROI to maximize resolution, so long as accuracy is maintained and noise is sufficiently low. This trade-off makes the optimization problem more challenging and interesting. While a moving linear regression with a kernel size selected to balance stability with resolution has been commonly used, the algorithm presented here seeks to maximize the data fed into our model prior to the specification of spatial regularization and thus minimize the effect of time-delay jitter while maintaining high resolution.

II. Background

Sarvazyan *et al.* [15] proposed the idea of using an ultrasonically-induced mechanical shear wave to measure material properties by characterizing the propagation velocity of the wave, demonstrating that in a semi-infinite, linear elastic medium, the shear velocity c would be proportional to the square root of Young's modulus E :

$$c = \sqrt{\frac{E}{2\rho(1+\nu)}} = \sqrt{\frac{G}{\rho}}, \quad (1)$$

where ρ is the density, ν is Poisson's ratio, and G is the shear modulus. Under assumptions of incompressibility ($\nu \approx 0.5$) and a density close to water ($\rho \approx 1.0$ g/ml), one must only measure the shear velocity to estimate Young's modulus. To estimate shear velocity, Sarvazyan *et al.* proposed finding the time of peak displacement at a number of receive locations along the path of the wave's propagation and fitting the arrival times by linear regression, assuming homogeneity within the ROI. In the subsequent 15 years, researchers have used a variety of techniques to convert imaged tissue motion into an estimate of wave velocity. Nightingale *et al.* [13] used pseudo-direct inversion of the Helmholtz equation to reconstruct shear velocity images from a single excitation, noting the noisiness of the estimates. Rouze *et al.* [16] and Wang *et al.* [17] developed model-based approaches to better estimate shear velocities within homogeneous regions, while Palmeri *et al.* [18] characterized heterogeneous tissues with local linear regressions of the arrival time differences. Rouze *et al.* [19] analyzed various time-of-flight (TOF) methods, looking for the ideal kernel size for local reconstructions. Meanwhile, McAleavey *et al.* [20] used a spatially-modulated source function to estimate shear velocity from a single recording location, and extended the method to create images using a fixed spatial distance between the source functions and the receive location [21], [22], trading tracking waves from a single excitation at multiple track locations for tracking a single location, and using a synthetic shear wave generated from multiple offset excitations. Bercoff *et al.* [1] proposed Helmholtz inversion in the Fourier domain, as well as estimation by cross correlation of pairs of receive lines to create images. McLaughlin *et al.* used level set inversion of arrival times relative to a single point and the Eikonal equation for shear wave speed estimation [23], [24], and Oberai *et al.* used the adjoint method [25].

III. Time of Flight Estimation

In accordance with the wave equation for a propagating cylindrical wave, for any pair of recording locations located along the axis of propagation away from a shear wave source in a linear elastic homogeneous medium, the signals of tissue motion at each location $u(x, t)$ will be scaled, time-delayed versions of each other. The time delay reflects the shear wave velocity between the recording locations, and the amplitude decreases with distance from the source of the induced wave:

$$u(x_j, t) = \left(\frac{x_i}{x_j} \right) u \left(x_i, t - \left(\frac{x_j - x_i}{c} \right) \right), \quad (2)$$

where u is the axial displacement, x is the distance from the excitation, t is the time after excitation, and c is the shear velocity. In a heterogeneous medium, the propagation of the wave is affected by the local stiffness, so c becomes $c(x)$ and Equation 2 becomes:

$$u(x_j, t) = \left(\frac{x_i}{x_j} \right) u \left(x_i, t - \left(\int_{x_i}^{x_j} \frac{1}{c(x)} dx \right) \right). \quad (3)$$

The time delay is dependent on the *averaged* compliance between the recorded locations. This assumes that the derivative of $c(x)$ is small relative to the wavelength such that there is negligible reflection. In the case of a single, significant reflection, we can readily split $u(x, t)$ into its transmitted and reflected components $u_T(x, t)$ and $u_R(x, t)$, which are scaled by transmission and reflection coefficients, respectively. For multiple reflections, additional wave components must be added. In practice, a directional filter can be used to suppress the reflected wave components in post-processing [26], and time delay estimators can be made robust to variations in signal amplitude through normalization. There are a number of existing methods to estimate the time delay between the two signals, including sum-of-absolute differences, normalized cross-correlation, and phase-based methods. The “arrival time” of the wave at each particular point can also be estimated directly from some feature of the recorded signal, such as the sample with the greatest displacement from the reference (“time-to-peak” (TTP)), or the greatest velocity (the derivative of displacement through tracking time, “time-to-peak-slope” (TTPS)). When directly estimating arrival time, the velocity can be found from the difference in arrival times between two points or from a set of arrival times such as with a moving linear regression. The algorithm proposed here uses independently-estimated relative time delay estimates, whereas the linear regression method provided for comparison uses directly-estimated arrival times via TTPS.

IV. Algorithm

If we have N receivers, located colinearly along the direction of wave propagation, and the source of the wave is outside of the receiver positions, there are N^2 possible sets of time delay pairs t_{ij} that may be observed as estimates of the true time delays T_{ij} . Each observation can be thought of as the true time delay T plus an error term ε :

$$\Delta t_{ij} = \Delta T_{ij} + \varepsilon_{ij} \quad (4)$$

To form a consistent set, the true delays T obey a set of rules:

1. $T_{ii} = 0$. A signal compared with itself will indicate no time delay,
2. $T_{ij} = -T_{ji}$. The delay is the same regardless of which signal is the reference,
3. $T_{ij} + T_{jk} = T_{ik}$. The delay between two points is the sum of delays along any path that connects the points.

By this formulation, we only need to solve for $N - 1$ estimates, the time delays $T_{i(i+1)}$, which are each the denominator term for the corresponding local shear wave velocity:

$$c_{i(i+1)} = \frac{x_{i+1} - x_i}{\Delta T_{i(i+1)}} = \left(\int_{x_i}^{x_{i+1}} \frac{1}{c(x)} dx \right)^{-1}. \quad (5)$$

While the observations $t_{i(i+1)}$ could provide (noisy) estimates of $T_{i(i+1)}$ directly, we can take advantage of overlapping observations to add information to each estimate, reducing noise. To do this, each of the observations t_{ij} is expressed as the linear combination of the parameters $T_{i(i+1)}$:

$$\Delta t_{ij} = \sum_{k=i}^{j-1} \Delta T_{k(k+1)} + \varepsilon_{ij}, \text{ for } j > i \text{ and } \quad (6)$$

$$\Delta t_{ij} = \sum_{k=j}^{i-1} \Delta T_{k(k+1)} - \varepsilon_{ij} \text{ for } j < i. \quad (7)$$

Expressed in matrix form $\mathbf{A} \overrightarrow{\Delta T} + \overrightarrow{\varepsilon} = \overrightarrow{\Delta t}$:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & & 0 \\ 1 & 1 & 1 & & 0 \\ & \vdots & & \ddots & \\ 1 & 1 & 1 & & 1 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & & 1 \end{bmatrix} \begin{bmatrix} \Delta T_{12} \\ \Delta T_{23} \\ \Delta T_{34} \\ \vdots \\ \Delta T_{(N-1)N} \end{bmatrix} + \begin{bmatrix} \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{14} \\ \vdots \\ \varepsilon_{1N} \\ \varepsilon_{23} \\ \varepsilon_{24} \\ \vdots \\ \varepsilon_{(N-1)N} \end{bmatrix} = \begin{bmatrix} \Delta t_{12} \\ \Delta t_{13} \\ \Delta t_{14} \\ \vdots \\ \Delta t_{1N} \\ \Delta t_{23} \\ \Delta t_{24} \\ \vdots \\ \Delta t_{(N-1)N} \end{bmatrix} \quad (8)$$

The problem has been turned into a classical inverse problem with an overdetermined system of equations, and to recover $\overrightarrow{\Delta T}$, we can invoke the linear least-squares solution:

$$\overrightarrow{\Delta T} + \overrightarrow{E} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \overrightarrow{\Delta t}, \quad (9)$$

where $\overrightarrow{E}$ is the combined noise vector:

$$\vec{E} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \vec{\varepsilon}. \quad (10)$$

Notably, the least-squares solution can be expressed as a linear transformation of the observations, and the matrix inversion step can be performed *a priori* on our condition matrix $\mathbf{A}$. Because $\mathbf{A}$ has such a simple structure, we can look at $(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$ explicitly to build an intuition about the solution. As an example, we consider the case where we have three receive locations, giving us three observations t_{12} , t_{23} , and t_{13} :

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta T_{12} \\ \Delta T_{23} \end{bmatrix} + \begin{bmatrix} \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{bmatrix} = \begin{bmatrix} \Delta t_{12} \\ \Delta t_{13} \\ \Delta t_{23} \end{bmatrix}. \quad (11)$$

The solution for the high-resolution estimates T_{12} and T_{23} takes the following form:

$$\Delta T_{12} + E_{12} = \frac{2}{3}(\Delta t_{12}) + \frac{1}{3}(\Delta t_{13} - \Delta t_{23}) \quad (12)$$

$$\Delta T_{23} + E_{23} = \frac{2}{3}(\Delta t_{23}) + \frac{1}{3}(\Delta t_{13} - \Delta t_{12}), \quad (13)$$

where the estimate noise terms E are combinations of the observation noise terms ε , with lower noise power through effective averaging:

$$E_{12} = \frac{2}{3}\varepsilon_{12} + \frac{1}{3}(\varepsilon_{13} - \varepsilon_{23}), \text{ and} \quad (14)$$

$$E_{23} = \frac{2}{3}\varepsilon_{23} + \frac{1}{3}(\varepsilon_{13} - \varepsilon_{12}), \quad (15)$$

Intuitively, this makes sense. For either T_{12} or T_{23} , we have the direct observation of our estimate, and two others whose difference must be equal to the first. The direct observation is preferentially weighted over the difference term. Any uncorrelated errors between the observations ε will be partially suppressed through E . If we have more than 3 receiver positions, we can solve the global system of equations, or set a maximum on the distance between receivers that we'd like to use. Setting the maximum spacing defines the kernel size, and makes the multiresolution solution a local one. The kernel size can be defined either as the maximum spacing K (mm) or $n + 1$ receivers:

$$K = n\Delta x \quad (16)$$

As we increase n from the example ($n = 2$), and consider solving for the estimate in the middle of the each local neighborhood, a pattern emerges in the linear combination applied to the nearby observations (the row of $\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$ corresponding to that center estimate). Figure 1 shows the coefficients as a filter that can be applied to the main diagonal of the

matrix indexed by i and j containing all observations t_{ij} for varying neighborhood sizes to estimate T_{ij} .

With no multiresolution kernel ($n = 1$), the estimate $T_{i(i+1)}$ is simply the average of the forward and reverse estimates $t_{i(i+1)}$ and $-1 * t_{(i+1)i}$, which, for cross-correlation, are identical. For larger multiresolution kernels, the solutions include all of the two-observation multipath combinations that yield the estimate with $|i - j| = n$ samples. From this formulation, we can also introduce the idea of spatial regularization, to favor solutions that are more smooth. Using Tikhonov regularization [27], the solution becomes:

$$\overrightarrow{\Delta T} + \overrightarrow{E}_r = (\mathbf{A}^T \mathbf{A} + \lambda \mathbf{\Gamma}^T \mathbf{\Gamma})^{-1} \mathbf{A}^T \overrightarrow{\Delta t} \quad (17)$$

where $\mathbf{\Gamma}$ is a regularization matrix, scaled by the regularization factor λ , and $\overrightarrow{E}_r$ is the regularized combined error. To favor solutions with small first derivatives, we choose the difference operator for $\mathbf{\Gamma}$:

$$\mathbf{\Gamma} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ & & & \ddots & & \\ 0 & \cdots & & 0 & 1 & -1 \end{bmatrix}. \quad (18)$$

As we increase λ , we favor solutions that are increasingly smooth. This can be seen in the filter metaphor, shown in Figure 2.

V. Finite Element Method Simulation

To demonstrate the performance of the algorithm, a finite element method (FEM) simulation of a shear wave generated by acoustic radiation force was created using LS-DYNA. The simulated radiation force excitation and shear wave response in tissue have been previously validated [28]. A Siemens Acuson ER7B linear array transducer was simulated for a 6 MHz excitation, with a transmit focus of 15 mm at F/3. The induced shear wave was recorded at 1,000 fps with lateral spacing of 0.2 mm over 6 mm to the right of the excitation. The elastic material was simulated with a Young's modulus of 4.5 kPa on the left side and 24 kPa on the right side with a vertical layer boundary 3 mm to the right of the excitation. The axial displacements calculated from the FEM simulation were used without simulated ultrasonic tracking.

A directional filter was applied to the displacements to remove backward-propagating (reflected) components based on their position in the Fourier domain [26], [29]. The time delays between each lateral pair of tracking locations were computed via normalized cross-correlation of the tissue velocity signals at each pixel with the same depth. For the linear regression arrival times, the time delay between the tracking location coincident with the push and each tracking location in the field of view was used. For both linear regression and multiresolution, kernels between 2 points ($n = 1$, $K = 0.2$ mm) and 18 points ($n = 17$, $K = 3.4$ mm) were used, although the estimation of velocity within each of those kernels varies between the two methods. The error term ϵ_{ij} was modeled as normally distributed random

noise with zero mean (no bias) and standard deviations σ_e between 0 and 0.1 ms, applied to each arrival time difference. Sixteen noise realizations were used for each experiment. These error magnitudes were chosen to provide a continuum between completely clean and highly corrupted data. While the distribution of errors in real ultrasound motion data may follow different noise patterns (including correlated bias errors due to the stationary speckle pattern), this simplified model readily allows for demonstration of the proposed algorithm. Characterization of the arrival time noise in shear wave imaging will be left for future work. For regularization of the multiresolution scheme, Tikhonov regularization was applied to the first difference of $T_{i(i+1)}$, with λ values between 0 and 100.

To calculate CNR, the median velocity on each side of the vertical boundary was used, bounded axially about the focus between 10 and 20 mm, and laterally between 1.5 and 2.5 mm to the side of the boundary. CNR was computed as:

$$CNR = \frac{|\mu_L - \mu_R|}{\sqrt{\sigma_L^2 + \sigma_R^2}}, \quad (19)$$

where μ_L and μ_R are the medians of $c(z, x)$ to the left and right of the boundary, and σ_L and σ_R are the respective standard deviations. Edge resolution was calculated as the 20–80% rise distance w for a piecewise linear function fit to $c(x)$ averaged between 10 and 20 mm axially:

$$c(x) = \begin{cases} \mu_L & \text{if } x < -w/2 \\ \frac{\mu_R - \mu_L}{w}x + \frac{\mu_R + \mu_L}{2} & \text{if } |x| \leq w/2 \\ \mu_R & \text{if } x > w/2 \end{cases} \quad (20)$$

VI. Results

Figure 3 shows the axial tissue velocity at the focal depth for a directionally-filtered wave generated on the left side of the ROI ($x = -3$ mm) and propagating to the right, encountering a boundary at $x = 0$ mm. The change in shear wave velocity is apparent in the slope of the line.

Figure 4 shows estimated time delays for every pair of receive locations at the focal depth in the left panel. There is a vertical boundary at $x = 0$ mm. The main diagonal ($x_i = x_j$) has time delays that are identically zero while those in the upper-right ($x_j > x_i$) are positive and those in the lower-left ($x_j < x_i$) are negative. In both directions, the time delay scales with $|x_i - x_j|$ such that estimates closer to the main diagonal are higher-resolution than those in the upper-right and lower-left corners.

Each time delay pair t_{ij} in the left panel of Figure 4 corresponds to a shear velocity estimate, and those estimates are shown in the right panel. The boundary is apparent in the shear velocity estimates in the right panel, as is the degradation in resolution associated with using estimates with greater spacing. When x_i and x_j appear on the same side of the boundary, they indicate the correct shear wave speed for that region. When they appear on

opposite sides of the boundary, the arrival time difference indicates an intermediate shear wave velocity.

Figure 5 shows shear wave images of the simulated phantom, using a moving linear regression of size K to estimate the shear wave velocity from the arrival times. Each column shows an increasing amount of jitter on the arrival time estimates, and each row shows an increasing filter size. Initially, the boundary becomes more clear as noise is suppressed, but as the regression kernel size continues to increase, the edge becomes blurred. Also of note is that the gray levels on the left and right of the image agree with the simulated shear wave speeds (1.22 m/s and 3.83 m/s for the left and right, respectively).

Figure 6 shows the equivalent shear wave images as Figure 5, but uses the multiresolution cross-correlation filter instead of the linear regression filter, combining all combinations of arrival time differences within the kernel. The values of shear wave speeds on the left and right sides of the images are unchanged. Noise is reduced to a lesser extent than with linear regression as the filter size is increased, but without any of the corresponding loss in resolution.

Lateral variation due to interpolation artifacts and a finite simulation grid appears near the boundary in Figure 6, even without the addition of noise. Within the multiresolution framework, these errors and the noise that persists through the multiresolution filter can be suppressed through regularization at the expense of resolution. Figure 7 shows the effect of Tikhonov regularization for a 1.8 mm multiresolution kernel size. Increasing the regularization scaling λ suppresses the remaining noise and artifacts, but begins to degrade resolution. The effect is similar to that of linear regression, but because regularization begins with the partially suppressed noise, boundaries are better preserved.

To quantify the observations about noise suppression and resolution, Figure 8 shows contrast-to-noise ratio (CNR) as a function of kernel length for a moving linear regression filter of arrival times, on the upper-left, and multiresolution filter length, on the upper-right, at different arrival time noise levels. Error bars represent the standard deviation over sixteen noise realizations. As noise increases, larger kernels are needed to achieve a given CNR. With no regularization, linear regression shows higher CNR than multiresolution. The lower-right panel shows that increasing the scaling of the Tikhonov regularization term λ causes significant increases in CNR, surpassing the linear regression CNR values for the equivalent filter size.

Figure 9 shows the CNR-resolution pairs for different linear regression kernel sizes, under different noise conditions. As the regression kernel size is increased, the points move up and to the right, sacrificing spatial resolution for noise suppression.

To illustrate how the increase in CNR afforded by Tikhonov regularization is accompanied by a corresponding decrease in edge resolution, Figure 10 shows the matched CNR-resolution pairs for each value of λ for the 1.8 mm kernel at each noise level. The CNR values cover a similar range to those in figure 9, but with significantly better resolution.

To compare the trade-off of CNR and resolution in the multiresolution scheme against the same trade-off for linear regression, Figure 11 shows both trade-off curves, using a kernel of 1.8 mm and varying λ for the multiresolution scheme, and varying kernel length for linear regression. The noise level was set to 5 μ s. With no regularization ($\lambda = 0$), multiresolution filtering has lower CNR than linear regression using the same 1.8 mm kernel, but for any given CNR value achieved by linear regression, the multiresolution scheme can achieve the same CNR through regularization with significantly better resolution.

VII. DISCUSSION

A. Comparison to Linear Regression

The regularized multiresolution approach to time-of-flight estimation central to shear wave image reconstruction presented here provides better suppression of uncorrelated time-delay jitter than linear regression with less loss in spatial resolution. Because the quantity of data fed into the model is maximized prior to any spatial regularization, the CNR-resolution tradeoff curve (Figure 11) can maintain higher CNR at a given resolution or better resolution at a given CNR compared to linear regression by appropriate selection of regularization parameter λ . Linear regression successfully suppresses jitter (Figures 5 and 9), but does so at the expense of lateral resolution, seen as a blurring of the boundary. For very noisy arrival time estimates, the visual conspicuity of small structures would be lost either in the noise or in the regression. The multiresolution approach without regularization is equivalent to a weighted average of all combinations of time delays that sum to give the high-resolution time delays (Figure 1). This serves to maximally reduce jitter (Figure 6) while preserving the spatial independence of the estimates. Kernel size selection for multiresolution estimation should be made based on the widest two points that a shear wave can be expected to be tracked between. For additional noise suppression or for dealing with correlated noise, the use of regularization allows controlled use of neighboring information to influence each estimate. Figure 2 visualizes this performance explicitly. As the regularization term λ increases, estimates between points outside of the high-resolution pair t_{ij} are given increasing weight. For large λ , the filter starts to look like a large version of the $n = 1, \lambda = 0$ case, which is just a lower-resolution estimate, more resistant to a given jitter level, since the size of the actual time delay is larger compared to the error. Figure 7 demonstrates the use of regularization, suppressing even high levels of jitter in exchange for a reduction in resolution. Linear regression kernel size more aggressively suppresses noise than multiresolution kernel size, but this is because linear regression is fully regularized, providing the smoothest possible result given the information within the kernel. Tikhonov regularization of the slope is comparable to linear regression in terms of the noise-reduction/resolution trade-off, but the multiresolution form already has the redundant-estimate averaging in place, so starts at a relatively higher CNR. An additional advantage of using regularization is that λ can take on any value, while linear regression must be specified in terms of samples, providing more precise control of the CNR-resolution trade-off.

B. Limitations

The first limitation of the multiresolution approach to shear wave speed estimation is added computational complexity. For N receive locations, N^2 cross-correlations can be performed,

compared to the N arrival time estimates that must be performed to linear regression. The total number of cross-correlations can be reduced, however, because cross-correlations are symmetric and we know that the peak of the autocorrelation function will be at 0 time delay. We only need to find the time delays t_{ij} for $j > i$, applying the first two continuity constraints ($T_{ii} = 0$ and $T_{ij} = -1 * T_{ji}$, respectively) to the observations to fill in the rest. This reduces the number of cross-correlations to $N(N-1)/2$, but the multiresolution approach is still $\mathcal{O}(N^2)$, compared to $\mathcal{O}(N)$ for linear regression. We reduced the number of necessary cross correlations further by only considering estimates up to $|j-i| = n$ (Figure 1). This reduces the number of correlations to $n(N - (n+1)/2)$, which brings the order down to $\mathcal{O}(N * n)$, but still requires more correlations than linear regression. Choosing a large n provides greater noise reduction but requires more computation, and estimates with very large x_{ij} may have low correlation coefficients and be less useful when estimating $t_{ij} = t_{ik} - t_{jk}$ for $k \gg j$. A trade-off between noise reduction gains and computational complexity therefore exists, and in practice is addressed by selecting an appropriate value of n . The natural extension, and likely source of future work, is to address the varying quality of t_{ij} estimates directly, by introducing weighting to the least-squares solution:

$$\overrightarrow{\Delta T} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \overrightarrow{\Delta t} \quad (21)$$

The weights can be chosen based on peak cross-correlation coefficient and/or *a priori* knowledge (de-weighting estimates with $j \gg i$, for example). If the weighting is based on the data, however, the least-squares solution cannot be precomputed and must be calculated for each estimate. This requires matrix inversion for every sample, further increasing the computational load, but is likely to provide superior results.

The second limitation of the multiresolution approach is that it primarily reduces uncorrelated jitter on the time-delay cross-correlations. Some of this error may be introduced by interpolation noise (as seen in the finite CNR for the no-added noise case in Figure 8), and some may be due to the SNR of the received RF data. It may also reduce reflection artifacts, since errors due to the reflected wave change for different receiver pairs. However, an unregularized multiresolution approach does not assist with the reduction of biased time delay data. An error in receiver position due to the local scattering random walk will be consistent across all estimates, and all multiresolution paths to the estimate will yield the same biased result. This indicates that for conventional SWEI, the noise parameter has a nonzero expectation ($E[\varepsilon] \neq 0$), such that this bias is passed through the algorithm, while only the jitter is suppressed. Depending on the imaging configuration and signal to noise ratio, speckle bias may be the dominant source of error when using multiple tightly-spaced tracking locations. To deal with correlated noise, spatial regularization is necessary, and the regularization term can serve the same purpose as the kernel size in linear regression, while still benefiting from the first step of jitter reduction. This algorithm may provide the greatest benefit to single tracking location (STL) SWEI, which is theoretically free of speckle bias [22], [30], but subject to the same arrival time estimation jitter as conventional, multiple track location (MTL) SWEI. In either case, phrasing the problem in this way provides access to the suite of tools for solving and regularizing inverse problems and opens many possibilities beyond simple linear regression.

VIII. Conclusion

This work poses the shear wave time-of-flight estimation problem for multiple tracking locations as a classic overdetermined linear systems inverse problem and examines the performance of some basic solution strategies, demonstrating improved CNR and resolution compared to simple linear regression in the presence of jitter on the time-delay estimates. The averaging of noisy data using multiple estimates and the reduction in resolution are separated in the multiresolution approach, compared to linear regression where they are tied to the integer-sample-limited kernel size. Spatial constraints on the multiresolution solution are imposed by Tikhonov regularization for additional noise suppression at the expense of resolution. This work motivates further exploration of the inverse problem approach to shear wave velocity estimation, as it opens the field to a wide array of research, algorithms, and techniques that have been used for years in other applications. As computational power increases and the number of tracking locations imaged for a shear wave rises, new opportunities for leveraging the volume of data arise to improve the quality and diagnostic usefulness of shear wave images.

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Fig. 1.

The least-squares multiresolution solution for a high-resolution estimate T_{ij} , expressed as a filter applied to a localized set of time delays with kernel size $1 \leq n \leq 11$. The upper-right half of the matrix is the forward observations t_{ij} , ($j > i$), and the lower-left half is the reverse observations t_{ij} , ($j < i$), which are negatively symmetric.

Fig. 2.

The least-squares solution for a high-resolution estimate T_{ij} , expressed as a filter applied to neighboring multiresolution observations with kernel size $1 \leq n \leq 11$ and including varying degrees of first-order difference Tikhonov regularization λ . As λ increases, the solution looks increasingly like a low-resolution version of the no-multiresolution filter.

Fig. 3.

Axial tissue velocity as a function of lateral position and time for a wave generated at $x = -3$ mm and tracked as it propagates to the right, encountering a boundary at $x = 0$ mm. A Fourier-domain directional filter has been used to remove the reflected wave.

Fig. 4.

Left Image: All combinations of time delay estimates for a wave excited at $x = -3$ mm, propagating across a boundary at $x = 0$ mm. As $|x_i - x_j|$ gets larger, the measured time delay increases. Right Image: The time delay differences, visualized as velocity estimates. Estimates where x_i and x_j are on the same side of the boundary show the correct value within some interpolation error, while estimates where x_i and x_j are on opposite sides of the boundary show an intermediate shear wave speed.

Fig. 5.

Shear wave images of the simulated phantom using moving linear regression kernels of size K on the arrival times. The linear regression suppresses more noise as the kernel size is increased, but also blurs the vertical boundary.

Fig. 6.

Shear wave images of a simulated phantom using multiresolution kernels of increasing size K . Some of the noise is reduced as kernel size increases, while edge resolution is maintained.

Fig. 7.

Shear wave images for the simulated phantom using a 1.8 mm multiresolution kernel for different levels of noise and regularization. Higher values of λ reduce the noise at the expense of resolution, but preserve the edges more so than in figure 5

Fig. 8.

CNR for different levels of noise, as a function of multiresolution kernel size for linear regression (top left), multiresolution kernel size with no regularization (top right), and regularization parameter λ with a 1.8 mm multiresolution kernel (bottom right). In the bottom right frame, the 1.8 mm linear regression values are shown as thin horizontal lines. For large values of λ , the regularized multiresolution estimate surpasses the linear regression estimate.

Fig. 9.

CNR-resolution regularization trade-off curve for linear regression, as a function of filter size and noise amplitude. Select filter sizes are labeled over the top line. Larger regression kernels increase CNR at the expense of resolution.

Fig. 10.

CNR-resolution regularization trade-off curve for 1.8 mm multiresolution kernel. The lower-left point of each curve (highest resolution, lowest CNR) is $\lambda = 0$, and the upper-right point (lowest resolution, highest CNR) is $\lambda = 10000$. The CNR values are similar to those in figure 9, but are achieved at much finer resolution.

Fig. 11.

CNR-resolution trade-off curves for multiresolution (circles) and linear regression (squares) at the $\sigma_e = 5 \mu s$ noise level. Multiresolution uses a 1.8 mm kernel and varies the regularization term λ , while linear regression directly varies filter size. Because the multiresolution curve lies above and to the left of the linear regression curve, regularized multiresolution shear wave speed estimation can achieve higher CNR, better resolution, or both, when compared to equivalent linear regression.