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Root-flipped multiband refocusing pulses

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Abstract

Purpose—To design low peak power multiband refocusing radiofrequency (RF) pulses, with application to simultaneous multislice spin echo MRI.

Theory and Methods—Multiband Shinnar-Le Roux β polynomials were designed using convex optimization. A Monte Carlo algorithm was used to determine patterns of β polynomial root flips that minimized the peak power of the resulting refocusing pulses. Phase-matched multiband excitation pulses were also designed to obtain linear-phase spin echoes. Simulations compared the performance of the root-flipped pulses with time-shifted and phase-optimized pulses. Phantom and in vivo experiments at 7 T validated the function of the root-flipped pulses and compared them to time-shifted spin echo signal profiles.

Results—Averaged across number of slices, time-bandwidth product, and slice separation, the root-flipped pulses have 46% shorter durations than time-shifted pulses with the same peak RF amplitude. Unlike time-shifted and phase-optimized pulses, the root-flipped pulses excitation errors do not increase with decreasing band separation. Experiments showed that the root-flipped pulses excited the desired slices at the target locations, and that for equivalent slice characteristics, the shorter root-flipped pulses allowed shorter echo times, resulting in higher signal than time-shifted pulses.

Conclusion—The proposed root-flipped multiband RF pulse design method produces low peak power pulses for simultaneous multislice spin echo MRI.

Introduction

Simultaneous multislice (SMS) imaging is a scan acceleration technique in which multiple slices are simultaneously excited and read out, and the resulting overlapped slice images are separated in reconstruction using the receive coil sensitivity information [1]. SMS imaging

is of high interest for accelerating whole-brain diffusion tensor imaging and functional MRI [2].

Spin echo SMS imaging requires multiband RF excitation and refocusing pulses. Multiband pulses are most easily generated by summing multiple conventional single-band pulses that are modulated to excite slices at the desired locations. Due to the direct summation, the peak amplitude of a pulse constructed this way increases linearly with the number of excited slices, with its peak amplitude occurring in the middle of its main lobe [3]. Thus, peak RF power and SAR increase with the square of the number of slices. For high slice acceleration factors the peak power demand can quickly reach SAR and RF amplifier power limits, thus limiting the maximum achievable acceleration. These limits are especially problematic for spin echo SMS imaging sequences that use 180° refocusing pulses with inherently high peak RF amplitude demands.

Several methods have been proposed to design multiband pulses with reduced peak power compared to direct summation. One approach is to apply phase shifts to the pulses prior to the summation that can be optimized to achieve a minimal increase in peak RF amplitude over a single-slice pulse [3, 4]. The peak power of multiband pulses constructed this way has been shown to increase with the square root of the number of excited slices, rather than linearly. Another approach is to introduce time shifts between the single-slice pulse waveforms, so that their main lobes do not overlap in time [5]. That approach requires consideration of the different refocusing times of the different slices, but it was shown to achieve significantly lower peak RF amplitudes compared to phase-optimized pulses. Whereas the phase-optimized and time-shifted pulse design methods use the same constant trapezoid gradient waveforms that would be used by a single-slice pulse, the Power Independent of Number of Slices (PINS) pulse design technique constructs multiband pulses by breaking up a single-slice pulse into a train of hard pulses and placing uniform gradient blips between those pulses, whose area is dictated by the desired slice gap [6]. While PINS pulses are robust and simple to construct, in practice their durations can be unacceptably long and are primarily dictated by the gradient system slew rate. Furthermore, the number of excited slices cannot be controlled. To mitigate these drawbacks, a hybrid of PINS and conventional multiband pulses has been proposed, which have shorter pulse durations than PINS pulses and lower peak amplitudes than conventional multiband pulses [7].

Root-flipping is a method that has been extensively applied to other RF pulse design problems to minimize the peak amplitude of a pulse subject to a fixed pulse duration, or equivalently to reduce its duration subject to a fixed peak amplitude [8–12]. It has typically been applied when a high time-bandwidth product is used to obtain very sharp slice or slab profiles for inversion, saturation or refocusing. The procedure requires first the design of a selective RF pulse using the Shinnar-Le Roux (SLR) algorithm [13], followed by the replacement of selected passband roots of its α and/or β polynomials with the inverses of their complex conjugates. The best pattern of flipped roots will lead to a uniform distribution of RF energy in time, imitating a quadratic phase pulse. Root flipping changes the phase responses of the α and β polynomials but does not change their magnitude responses, so the magnitude of the excited magnetization remains unchanged compared to the original pulse. The best pattern of flipped roots can be determined by exhaustive search when the number

of roots is small, or using optimization methods such as genetic or Monte-Carlo algorithms [11, 12] when it is large.

In this work we propose a multiband pulse design method based on root-flipping. Like phase-optimized and time-shifted pulses, the algorithm produces pulses to be played simultaneously with a constant gradient. However, we show that it achieves substantially lower peak RF amplitudes for a given duration, or equivalently, shorter pulse durations for a given peak RF amplitude. The method is based on an extension of the SLR algorithm to directly design multiband pulses based on multiband digital filter designs, and the duration of the pulses is minimized subject to a desired peak RF amplitude by redistributing RF energy in time using multiband root-flipping. Because the designed refocusing pulses will produce non-linear phase profiles across the refocused slices, phase-matched excitation pulses are also designed to obtain linear-phase echoes. Alternatively, the pulses can be used directly in twice-refocused spin echo sequences with conventional linear-phase multiband excitations, since the nonlinear refocusing phase will cancel out [14]. The pulse design method is explained in detail in the Theory section. That section is followed by a comparison of the performance of the root-flipped multiband pulses to phase-optimized [3, 4] and time-shifted [5] pulses in simulations. Finally, phantom and in vivo human experiments were performed at 7 T to validate the root-flipped pulses' function and to compare their signal profiles to that of time-shifted pulses.

Theory

The basic idea of the proposed method is illustrated in Fig. 1. Figure 1a shows how the timing of a pulse's main lobe depends on the position of its β filter's passband roots relative to the unit circle. This suggests that the passband roots of a multiband pulse can be configured so that the bands' main lobes do not coincide in time, and the peak amplitude of the pulse can be reduced compared to direct summation of identical pulses. Figure 1b illustrates the outputs of the two stages of the proposed method. In the first stage a multiband minimum-phase β filter is designed for the refocusing pulse. Minimum-phase filters are chosen because they have sharper transition bands than the linear-phase filters on which most slice-selective refocusing pulses are based [13]. In the second stage an optimized root flipping pattern is determined using a Monte-Carlo algorithm to minimize the maximum magnitude of the resulting pulse. The best pattern of flipped roots will distribute the pulse's energy evenly over its duration. Once the root-flipped refocusing pulse is designed, a phase-matched excitation pulse can be designed to cancel non-linear phase variation in the refocusing profile, or the pulse can directly be used in place of both refocusing pulses in a twice-refocused pulse sequence.

Multiband minimum-phase β polynomial design

In Ref. [13], Pauly et al provide an algorithm to design single-band minimum-phase RF pulses using SLR. Here we extend that approach to design multiband minimum-phase pulses using numerical optimization to obtain the required multiband β filter. We also incorporate smoothly decaying ripple constraints in the stopbands of the target β filter response to minimize spikes at the ends of the final waveforms [15], which appear in conventional minimum-phase SLR pulses derived from uniform ripple-constrained β filter designs.

To design a length- N multiband minimum-phase β filter, a length $2N - 1$ linear-phase filter is first designed whose magnitude response will equal the square of the minimum-phase filter's magnitude response. After its design it will be factored to produce the minimum-phase filter. Since the linear-phase filter will be symmetric, there are only N unique coefficients to design and only half the frequency axis needs to be considered in the optimization. The length- N half-filter $\mathbf{b}_{\frac{1}{2}}$ is designed by solving the following convex optimization problem:

$$\begin{aligned} & \text{minimize} && \delta \\ & \text{subject to} && -\delta \leq \left\{ \mathbf{A} \mathbf{b}_{\frac{1}{2}} \right\}_i - 1 \leq \delta, i \text{ in passbands} \\ & && 0 \leq \left\{ \mathbf{A} \mathbf{b}_{\frac{1}{2}} \right\}_i \leq \delta \frac{d_2^2}{2d_1} w_i, i \text{ in stopbands.} \end{aligned} \quad [1]$$

In this problem, δ is the maximum passband ripple of the linear-phase filter, which will equal twice the minimum-phase filter's passband ripple d_1 , and the maximum stopband ripple of the linearphase filter is the square of the minimum-phase filter's stopband ripple d_2 .

The linear-phase filter's maximum stopband ripple should therefore equal $\delta \frac{d_2^2}{2d_1}$, which is the upper stopband constraint used in this filter design. The frequency grid is indexed by i , and the stopband ripple weights w_i decay as $1/i^2$ away from each stopband edge. They are generated for each stopband edge, are normalized to a maximum value of 1 at that edge, and are combined across edges by taking the maximum w_i across edges at each index i . Finally, the system matrix $\mathbf{A}$ has elements:

$$a_{ij} = \begin{cases} 2\cos(2\pi i j / N_f) & i=0, \dots, N_f/2 \\ & j=1, \dots, N \\ 1, & i=0, \dots, N_f/2 \\ & j=0. \end{cases} \quad [2]$$

The frequency grid in the design should be oversampled; an oversampling factor of 10 was used in the designs presented here so that $N_f = 10N$. Note that in this formulation the frequency response is strictly positive, so it need not be biased up prior to taking its square root to obtain the corresponding minimum-phase filter's response.

The set of indices i corresponding to passbands and stopbands in the β filter design are determined by the desired time-bandwidth product TB , the passband and stopband ripples δ_1 and δ_2 , and the distance between the centers of the slices, which has units of slice widths. The time-bandwidth product and ripples determine the fractional transition width W as [13]:

$$W = (2TB)^{-1} d_{\infty}(2\delta_1, \delta_2^2/2). \quad [3]$$

where $d_{\infty}(\delta_1, \delta_2)$ is a polynomial defined in Ref. [13]. In the normalized frequency range $(0, N_f/2)$ spanned by i , the passband edges will then be $(1 - W)TB N_f / (4N)$ away from the center of each passband, and the stopband edges will be $(1 + W)TB N_f / (4N)$ away. The centers of the bands will be $TB N_f / (2N)$ apart from each other.

Once $\mathbf{b}_{\frac{1}{2}}$ is designed by solving the optimization problem in Eq. 1, the full linear-phase filter $\mathbf{b}$ is formed by concatenating $\mathbf{b}_{\frac{1}{2}}$ and its last $N - 1$ coefficients and mirror-flipping the former. The minimum-phase filter's magnitude response is then the square root of the magnitude response of $\mathbf{b}$, and its phase response is the Hilbert transform of its log-magnitude response [13]. Once this complex-valued response is formed, the final length- N minimum-phase filter is calculated as its inverse DFT.

Minimization of Peak RF Magnitude by Monte-Carlo Root-Flipping Optimization

The peak amplitude of an RF pulse can be reduced by flipping its passband roots, since flipping the root of a filter changes its phase response but not its magnitude response, and redistributes the filter's energy in time [16]. In the present algorithm, once the minimum-phase multiband β filter is designed, a root-flipping pattern is found that minimizes the resulting multiband RF pulse's peak amplitude. As in the high- TB saturation and inversion pulse design problems to which root-flipping has previously been applied, exhaustive evaluation of the $2^{N_r/2}$ possible root-flipping patterns (where N_r is the total number of passband roots) is computationally infeasible for many multiband refocusing pulse design problems [12]. For this reason, a Monte-Carlo method is used to optimize the pattern because it enables an efficient search across patterns, and has been successfully used in very-high- TB root-flipped saturation pulse design [11]. The first step of the optimization procedure is to compute the filter's complex roots as the eigenvalues of its companion matrix. Then the roots are sorted by their phase angle, and $N_r/2$ roots in the top half of the complex plane and within a fixed distance of the center of each passband become eligible for flipping. That distance is typically set to be a few times greater than half the slice width to ensure that the entire passband is captured. Each eligible index is given a unique probability between 0 and 1, such that the probability of flipping the first root is $2/N_r$ and the probability of flipping the last root is 1. In each Monte Carlo trial, a length- $N_r/2$ vector of uniformly-distributed random numbers is generated, and a root is flipped if its corresponding random number is less than its probability by replacing it with the complex conjugate of its inverse. If a root is flipped, its conjugate root on the negative half of the complex plane is not flipped, and vice versa, to obtain a symmetric pulse. After the roots are flipped, they are multiplied back out to obtain the root-flipped β filter coefficients, and the inverse SLR transform is used to transform these coefficients and the corresponding minimumphase α filter coefficients into the corresponding RF pulse. The RF pulse with the minimum peak magnitude across all Monte Carlo trials is saved as the output of the minimization.

Matched Excitation Pulse Design

Root-flipped refocusing pulses will generally not produce linear-phase refocusing (β^2) profiles, so if the sequence is not twice-refocused then matched excitation pulses must be designed to cancel that non-linear phase and obtain full signal [17–20]. After completing the above refocusing pulse design, a matched excitation pulse is designed to produce a β profile of:

$$\beta_{ex} = \frac{\beta_{ref}^2}{\sqrt{2}}, \quad [4]$$

where the *ex* and *ref* subscripts denote excitation and refocusing. Due to the squared dependence on β_{ref} , the excitation pulse must reach twice the spatial frequency of the refocusing pulse, so β_{ex} must be evaluated on a length- $2N$ grid of frequency locations. This profile and the corresponding minimum-phase α profile are input to the inverse SLR transform to calculate the matched excitation pulse.

Ripple Relationships

Matched Excitation and Refocusing—To determine the relationship between δ_1 , δ_2 , and the magnetization ripples $\delta_{1,e}$ and $\delta_{2,e}$ after refocusing, we analyze the combined excitation and refocusing profile (assuming crushing), given by:

$$2\alpha_{ex}\beta_{ex}^*\beta_{ref}^2. \quad [5]$$

To obtain a linear-phase magnetization profile after the refocusing pulse, the excitation pulse's β response is related to the refocusing pulse's β profile as:

$$\beta_{ex} = \beta_{ref}^2 / \sqrt{2}. \quad [6]$$

Substituting this and taking the magnitude, we obtain:

$$\sqrt{2}|\beta_{ref}|^4 \sqrt{1 - |\beta_{ref}|^2} \quad [7]$$

Based on a truncated Taylor series expansion of this expression about $\beta_{ref} = 0$, the β stopband ripple δ_2 is related to the M_{xy} stopband ripple $\delta_{2,e}$ by:

$$\delta_2 = \left(\frac{\delta_{2,e}}{\sqrt{2}} \right)^{\frac{1}{4}}. \quad [8]$$

The relationship between the β passband ripple δ_1 and the M_{xy} passband ripple $\delta_{1,e}$ is similarly derived from a truncated Taylor series expansion to be:

$$\delta_1 = \frac{\delta_{1,e}}{4}. \quad [9]$$

Twice-Refocused—Assuming crushing, the combined twice-refocused profile is $|\beta_{ref}|^4$. Following the same steps as the analysis of crushed spin echo pulses in [13], the β stopband ripple δ_2 can be related to the M_{xy} stopband ripple $\delta_{2,e}$ as:

$$\delta_2 = \delta_{2,e}^{\frac{1}{4}}.$$

The relationship between the β passband ripple δ_1 and the M_{xy} passband ripple $\delta_{1,e}$ is approximated as:

$$\delta_1 = \frac{\delta_{1,e}}{8}.$$

Methods

Algorithm Implementation

All pulses were computed in MATLAB R2013a (Mathworks, Natick, MA, USA). Effective ripples of $\delta_{1,e} = \delta_{2,e} = 0.01$ were used in all designs. The time-bandwidth products of all root-flipped designs were scaled so that the pulses had the same transition widths as the linear-phase pulses, by determining the minimum-phase time-bandwidth product as

$$TB_{\min - phase} = TB \frac{d_{\infty}(2d_1, d_2^2/2)}{2d_{\infty}(d_1, d_2)}. \quad [10]$$

Applying this scaling ensured that the slice profiles were as similar as possible between pulses designed by the proposed method and previous methods that are based on conventional linear-phase refocusing pulses. All time-bandwidth products reported in this work refer to TB in the above equation. The minimum-phase multiband filter optimization problem (Eq. 1) was solved using CVX, a MATLAB-based software package for specifying and solving convex programs [21]. The A matrices used in those optimizations were built explicitly with an oversampling factor of 10. The function `fmp()` from the `rf_tools` software package (<http://rsl.stanford.edu/research/software.html>) was then used to compute the multiband minimum-phase β filter from the optimized linear-phase filter. Then MATLAB's `roots()` function was used to calculate the roots of the minimum-phase filter. The roots within 1.5 slice widths of the center of each band were deemed eligible for flipping in the Monte Carlo optimizations, which used $80N_b \lceil TB \rceil$ trials, where N_b is the number of bands. In this paper, the maximum number of trials was 8,000 for a 10-band, TB 10 pulse. After root-flipping, a version of MATLAB's `leja()` function that was optimized for speed was used to order the roots to obtain numerically-accurate results when multiplying them out using the `poly()` function. All RF pulses were computed from their β filters using the `b2rf()` function from `rf_tools` package, which determines the corresponding minimum-phase α filter and invokes an inverse SLR transform. Code to design the pulses is available at <http://www.vuiis.vanderbilt.edu/~grissowa/>. For scientists without access to MATLAB, the algorithm could be ported to the free alternative Octave (<http://octave.org>) since the SDPT3 solver on which CVX is based is available on Octave (<https://github.com/sqlp>), the `roots()` and `poly()` functions have direct equivalents in

Octave, the `rf_tools` software package is fully open-source, and our improved `leja()` function is included with our code.

Simulations

Simulations were performed to compare phase-optimized [3] pulses, time-shifted and phase-optimized [5] pulses, and the proposed multiband refocusing pulses, as a function of the number of bands, time-bandwidth product, and slice separation. Phase-optimized pulses were designed by first designing a single-band SLR refocusing pulse, duplicating and modulating it, and optimizing the phase of each of the bands relative to the first band. The phase optimizations comprised 50 randomly-initialized calls to MATLAB's `fminsearch()` function to minimize the maximum amplitude of the summed pulse. The time-shifted and phase-optimized pulse designs started with the same single-band pulses, looped over a range of integer shifts per band (between 0 and N/N_b), and returned the phase-optimized and summed pulse with minimum normalized duration, measured as the product of the total number of samples in the pulse and its peak amplitude.

Pulses were designed with $N = 1024$ over a range of time-bandwidth products ($TB = 4$ to 10), number of excited bands ($N_b = 2$ to 10), and band separation ($\Delta = 2$ to 40 slice widths). The durations of the pulses were computed subject to a peak $|B_1|$ of $13 \mu\text{T}$, based on a peak $|B_1|$ limit of $13.5 \mu\text{T}$ for the body coil of our Philips 3 T Achieva scanners (Philips Healthcare, Best, Netherlands). A minimum-duration matched excitation pulse with the same peak $|B_1|$ was designed for each root-flipped pulse.

Experiments

Phantom and in vivo experiments were performed on a 7 T Philips Achieva scanner (Philips Healthcare, Cleveland, OH, USA) with a birdcage coil and a 32-channel receive-only head coil (Nova Medical Inc., Wilmington, MA, USA), and with the approval of the Institutional Review Board at Vanderbilt University. Root-flipped refocusing and phase-matched 90 degree excitation pulses were designed subject to a peak $|B_1|$ of $13 \mu\text{T}$. The slice profiles were measured in 3D spin echo scans with the slice-select gradient waveforms played along the frequency-encode direction.

Phantom experiments were performed using a mineral oil bottle phantom to validate the slice profile of the proposed refocusing and excitation pulses. Root-flipped pulses with $TB = 4$ were designed to excite and refocus 3 and 6 slices of thickness 3 mm and slice gaps of 3 and 2 cm, respectively. The 3-slice spin echo images were acquired with TR/TE of 257/17.6 ms and the 6-slice images were acquired with TR/TE of 211/21.4 ms. The field-of-views (FOVs) were $20 \times 20 \times 26$ cm with $1600 \times 1600 \times 26$ matrix size. In another phantom experiment, the signal profile of a 3-band root-flipped pulse was compared to a 3-band aligned-echo time-shifted pulse with the same slice characteristics. The matched excitation pulse for the time-shifted refocusing pulse was designed using the same method as the root-flipped pulse. Both time-shifted and root-flipped pulses were designed with $TB = 4$ to excite 3 slices of thickness 3 mm with 3 cm slice separation. In the spin echo scans, the echo times were set to minimum allowed values, which was 28.3 ms for time-shifted and 17.6 ms for

root-flipped pulses. The TR for both scans was 321 ms with FOV of $20 \times 20 \times 26$ cm and $1600 \times 1600 \times 13$ matrix size.

In vivo experiments were performed on a human subject with $TB = 4$ root-flipped pulses that excited and refocused 3 slices with 3 cm slice gap and 3 mm slice thickness. Spin echo images were acquired with TR/TE of 237/17.6 ms and FOV $24 \times 24 \times 16$ cm with $480 \times 480 \times 16$ matrix size.

Results

Simulations

Figure 2 compares the temporal evolution of three-band time-bandwidth product 4 linear-phase and root-flipped multiband refocusing. Whereas the linear-phase multiband pulse refocuses all slices simultaneously, the root-flipped pulse starts refocusing the leftmost slice first, and then the remaining slices in succession. This is similar to a quadratic phase pulse that sweeps through exciting its passband.

Figure 3 compares pulse shapes obtained by the three design methods for $TB = 4$, $N_b = 5$, and a band separation of 4 slice widths. The phase-optimized pulse has the longest duration of 18 ms, with most of its energy concentrated in its center. The time-shifted and phase-optimized design spread out the main lobes, enabling a significantly shorter duration of 12 ms. However, that design has lower-amplitude sidelobes on its ends that are not present in the root-flipped pulse, which has the shortest duration of 7 ms.

Figure 4a plots the durations of $TB = 6$ pulses designed by the three methods as a function of the number of bands, averaged across band separations (). Error bars indicate the maximum and minimum duration across band separations, for each N_b . The phase-optimized and time-shifted and phase-optimized pulses all had durations over 10 ms. Figure 4b plots the durations of $N_b = 5$ pulses as a function of TB, again averaged across band separations. The root-flipped pulses had the shortest durations in all designs, and were on average of 46.2% shorter than the time-shifted pulses. The minimum-duration matched excitation pulses had on average 26.1% shorter durations than their corresponding root-flipped refocusing pulses.

Figure 5 compares excitation accuracy as a function of band separation for the three methods, for $N_b = 6$ and $TB = 6$. Figures 5(a,b) compare maximum passband and stopband ripple amplification as a function of band separation, normalized to a separation of 10 slice widths. Due to Bloch equation nonlinearity and interference between stopband ripples and passbands, both the phase-optimized and time-shifted pulses' profiles degraded significantly as the bands were brought closer together. In comparison, the root-flipped pulses' maximum errors were stable across band separations. Figures 5c and 5d compare refocusing profiles for band separations of 4 and 2 slice widths, respectively, illustrating the degree of profile degradation in the phase-optimized and time-shifted designs as the bands are brought closer together. A band separation of 4 is within the range commonly used in SMS imaging.

Figure 6 plots the complex M_{xy} profile of a conventional single-band $TB = 6$ linear-phase excitation and refocusing pulse pair, and a 3-band root-flipped excitation and refocusing pulse pair designed by the proposed method. Most of the magnetization lies along M_x in both profiles, and the M_y components have similar amplitude and shape. The M_x and M_y profiles are also nearly the same for the 3 bands produced by the root-flipped pair. This validates that the spin echo slice profiles produced by the root-flipped pulse pairs do not contain additional nonlinear phase variations that could lead to through-slice signal loss, compared to conventional slice-selective excitation and refocusing pulses. In both single-band linear-phase and 3-band root-flipped profiles, the imaginary component arises from the nonlinear phase roll of the excitation pulses' α profiles.

Figure 7 compares spin echo signals obtained using an excitation and refocusing pulse pair ($N_b = 6$, $TB = 4$, $\Delta = 5$ slice widths) with the excitation pulse duration either set to twice the refocusing pulse duration so that spin echoes form at the same time across bands ('aligned-echo'), or set for minimum duration subject to a $13 \mu\text{T}$ peak RF magnitude constraint. The two pulse sequences were Bloch-simulated over a ± 50 Hz range of off-resonance frequencies, up to and past the approximately 25 ms minimum TE of the aligned-echo pulse pair. The aligned-echo signals re-focused at the same time, as expected (Fig. 7b). The minimum-duration excitation pulse's duration was almost three times shorter than the aligned-echo pulse duration. This led to a dispersion of the spin echo signals in time (Fig. 7c) that may need to be accounted for in the design of the signal readout; echoes resulting from minimum-duration time-shifted excitation and refocusing pulses have this same characteristic. The echoes are distributed symmetrically about the aligned-echo point due to forced symmetry of the roots across the real axis of the complex plane.

Experiments

Figure 8 shows the slice profiles of the 3-slice and 6-slice root-flipped pulses. Both the pulses excite the desired slices at the target locations and with the target slice widths. The slice profile plots show a left-right increase in signal amplitude across slices, due to TE differences between them. This behavior is consistent with simulations that showed that the left-most slices were excited first, followed by each of the remaining slices.

Figure 9 compares simulated and measured signal profiles of time-bandwidth product 4 time-shifted and root-flipped refocusing pulses, which had durations of 9.76 and 5.76 ms, respectively. The shorter duration root-flipped pulse allowed a TE of 17.6 ms compared to TE of 28.3 ms for the time-shift pulse. Hence, for the same time-bandwidth product the root-flipped pulses produced higher signal due to lower T_2 weighting. The simulated profiles on the left show that without the T_2 decay, the pulses would produce the same signal levels and profiles.

The experimental root-flipped slice profiles in Figs. 8 and 9 are somewhat rounded compared to the simulation, and the experimental time-shift slice profiles in Fig. 9 are also somewhat distorted in different ways. This is most likely the result of errors in RF pulse generation due to limited RF amplifier slew, which can be a problem for multiband pulses since their waveforms contain many rapid and large amplitude swings [22]. To address this,

Zhu et al have described a multiband RF preemphasis method [23] that could be applied to the proposed root-flipped pulses or any other multiband pulse to improve excitation fidelity.

Figure 10 shows results from the in vivo experiment. The root-flipped pulse correctly excited the intended three slices with a 3 cm slice separation. Skull fat was uniformly excited outside the water slices. The figure also shows the transverse slice images acquired using the root-flipped pulse, which contain amplitude modulations due to $|B_1^+|$ inhomogeneity but otherwise appear as expected.

Discussion

The simulation and experimental results showed that root-flipped multiband refocusing pulses have shorter durations than phase-optimized and time-shifted pulses, for the same peak RF amplitude. It was shown in a phantom experiment that the shorter duration of the root-flipped pulses resulted in higher spin echo signal than equivalent time-shifted pulses; by extension the root-flipped pulses should also result in higher signal than phase-optimized pulses since their durations are longer than time-shifted pulses, as shown previously by Auerbach et al [5].

Like time-shifted pulses, multiband root-flipped refocusing and phase-matched excitation pulses can be used in 'aligned-echo' mode [5], where the same gradient magnitude is used for both excitation and refocusing pulses but the excitation pulse is approximately twice the duration of the refocusing pulse. In this mode, the slices' signals refocus at the same time, but they have different TEs. This was the mode used in the presented experiments. If the long duration of the excitation pulse or difference in slices' TEs are problematic, the excitation pulse can be shortened since its peak amplitude at full length is lower than that of the refocusing pulse, as was shown in the simulations. This will bring the slices' TEs closer together but their signals will refocus at different times, which may need to be taken into account in readout design as discussed by Auerbach et al for time-shifted pulses [5]. They suggested to tune the excitation pulse duration so that each spin echo occurs in the middle of an EPI readout lobe. Compared to time-shifted pulses, the shorter durations of root-flipped pulses will provide greater flexibility in echo timing, and neglecting the differences in slice refocusing times will result in less disparity between their signal levels. When shortening the excitation pulse of a root-flipped pulse pair, the refocusing time of each slice could be determined by Bloch simulation, as was demonstrated in Fig. 7. More explicit control over the distribution of refocusing and excitation times for each slice could be obtained by flipping roots for bands in pre-specified maximum-, linear-, minimum-phase and other known patterns. This is not an issue for twice-refocused diffusion sequences that use the same root-flipped pulse for both re-focusing pulses, since applying the same refocusing pulse twice results in a zero-phase refocusing profile [24]. In that case, a conventional linear-phase multiband pulse can be used for excitation.

Zhu et al [25] have also described a multiband refocusing pulse design algorithm that uses root-flipping. The primary difference between that method and the proposed method is that instead of root-flipping the full multiband profile, in Zhu's method a single-band minimum-phase refocusing pulse is designed and root-flipped. That pulse is then replicated and

modulated to the target band locations, and inter-band phase optimization is applied prior to summing the pulses across bands. Like the proposed method, matched excitation pulses are required to obtain linear-phase spin echoes. Compared to the proposed method and time-shifted pulses, Zhu's pulses have the advantage that all echoes will appear at the same time with the same TE. However, because the component single-band pulses are identical, the pulse durations can be expected to lie somewhere between time-shifted and phase-optimized pulses for a given peak amplitude.

In the proposed algorithm the excitation pulse's slice profile was set to a scaled version of the refocusing pulse's profile. If the ability to independently adjust excitation pulse parameters such as slice thickness is desired, it should be possible to gain those degrees of freedom while satisfying the phase-matched condition using a method such as [26] to design the excitation pulse.

The current implementation of the algorithm was not optimized for computation time. The design time for a typical root-flipped pulse was 20-30 minutes which is not compatible with online use. There are several ways the pulse computation time could be reduced. Instead of using the CVX [21] package which is optimized for smaller problem sizes and rapid prototyping, a custom constrained optimization problem solver could be implemented using approaches such as the log-barrier and primal-dual methods [27]. The initial linear-phase multiband β filter could also be generated using the method of Cunningham et al [28], though the proposed method gives better control over ripple levels. The Monte Carlo root-flip trials could also be run in parallel, and when the number of bands and the time-bandwidth product is small an exhaustive search may require fewer trials than the number used here. Furthermore, it may be possible to achieve nearly the same performance as the proposed algorithm using an approach in which multiple single-slice β polynomials are jointly root-flipped to minimize the peak amplitude of the resulting summed RF pulses. The individual root-flipped pulses could then be stored in a library and modulated and summed online, as the user adjusts parameters such as slice thickness and separation. Finally, as the proposed algorithm aims to produce pulses that imitate the behavior of quadratic phase excitations in which slices are excited sequentially, it should be possible to achieve the same performance by directly designing multiband quadratic phase pulses, using an extension of a quadratic phase pulse design algorithm to the multiband case [29, 30]. If the appropriate quadratic phase profile can be selected quickly without iteration, such an approach should be considerably faster than the proposed algorithm since root flipping would not be required. Computational acceleration using these approaches will be the focus of future investigations.

Conclusion

A method was described to design multiband refocusing and matched excitation pulses. Simulations and experiments demonstrated that the pulses produce the desired slice profiles, and have shorter durations than existing multiband refocusing pulses with the same peak RF amplitude. The proposed pulses will be useful in simultaneous multislice acquisitions such as diffusion tensor imaging and spin echo functional MRI, where multiple slices need to be acquired at a high temporal resolution and short refocusing pulse durations are needed.

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Figure 1.

Illustration of how root flipping can reduce the peak RF amplitude of multiband pulses. (a) The positions of the complex passband roots with respect to the unit circle determines the position of the main lobe within a single-band pulse. To minimize the peak amplitude of a multiband pulse, the passband roots of each band can be configured so that its main lobe does not coincide with the other bands' main lobes. (b) A 3-band pulse designed by the proposed algorithm. The algorithm first designs a minimum-phase multiband pulse (left zoomed-in roots, gray RF amplitude waveform). The peak amplitude of that pulse is minimized using Monte Carlo optimization to determine the best configuration of flipped passband roots. In this 3-band case, the optimization converged on minimum-, linear- and maximum-phase root configurations for the bands, producing three humps in the pulse's amplitude waveform corresponding to the bands' main lobes.

Figure 2.

Comparison of the temporal evolution of three-band time-bandwidth product 4 root-flipped and linear-phase multiband refocusing. $|\beta^2|$ profiles are plotted one-third, two-thirds, and all the way to the end of the pulse.

Figure 3.

Comparison of five-band time-bandwidth product 4 pulse shapes designed by phase optimization, time-shifting, and root-flipping. Each pulse has been scaled to a peak magnitude of $13 \mu\text{T}$.

Figure 4.

Durations of multiband pulses subject to $13 \mu\text{T}$ maximum $|B_1^+|$. (a) Duration versus number of excited bands, for a time-bandwidth product of 6. Error bars indicate maximum and minimum durations across band separations. (b) Duration versus time-bandwidth product for 5 bands.

Figure 5.

Excitation accuracy as a function of band separation for 6-band time-bandwidth product 6 pulses. (a) Amplification of maximum passband ripple as a function of band separation, relative to the maximum passband ripple for a 10-slice width band separation. (b) Amplification of maximum stopband ripple as a function of band separation, relative to the maximum stopband ripple for a 10-slice width band separation. (c) Refocusing $|\beta^2|$ profiles for a 4-slice width band separation. (d) Refocusing $|\beta^2|$ profiles for a 2-slice width band separation.

Figure 6. Comparison of complex M_{xy} profiles between a single-band linear-phase excitation and refocusing pulse pair, and a 3-band root-flipped excitation and refocusing pulse pair.

Figure 7.

Comparison of aligned-echo and minimum-duration excitation pulses. (a) Magnitude plot of matched excitation and root-flipped refocusing RF pulse sequences, illustrating the relative durations of the refocusing pulse and the two excitation pulses (designed for 6-bands, time-bandwidth product 4, 5 slice-width band separation). (b) Spin echo signal profiles for each of the 6 bands when an aligned-echo excitation pulse is used. The spin echoes occur at the same time. (c) Spin echo signal profiles for each of the 6 bands when a minimum-duration excitation pulse is used, which reflect that the spin echoes are dispersed in a symmetric pattern around the aligned-echo point.

Figure 8.

Time-bandwidth product 4 root-flipped pulse profiles measured in a mineral oil phantom at 7 T. (a) Slice profile of pulse designed to excite 3 slices of thickness 3 mm and slice gap of 3 cm. (b) Slice profile of pulse designed to excite 6 slices of thickness 3 mm and slice gap of 2 cm. The pulses excited the desired slices at the target locations.

Figure 9.

Comparison of slice profiles from time-bandwidth product 4 time-shifted and root-flipped pulses. (Left) Bloch simulations showed that without T_2 decay, both time-shifted and root-flipped pulses produce maximum signal at the desired slice locations. (Right) In the experiment, the shorter root-flipped pulse allowed a shorter TE of 17.6 ms compared to 28.3 ms for the time-shifted pulse, resulting in less T_2 weighting and higher signal.

Figure 10.

Time-bandwidth 4 root-flipped slice profile measured in the human head at 7 T. (Top Left) Green lines show the locations of the desired slices overlaid on a scout image. (Top Right) Imaged slice profiles appear at the intended locations. (Bottom) In-plane images of the three excited slices.