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Study of blood flow in several benchmark micro-channels using a two-fluid approach

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Abstract

It is known that in a vessel whose characteristic dimension (e.g., its diameter) is in the range of 20 to 500 microns, blood behaves as a non-Newtonian fluid, exhibiting complex phenomena, such as shear-thinning, stress relaxation, and also multi-component behaviors, such as the Fahraeus effect, plasma-skimming, etc. For describing these non-Newtonian and multi-component characteristics of blood, using the framework of mixture theory, a two-fluid model is applied, where the plasma is treated as a Newtonian fluid and the red blood cells (RBCs) are treated as shear-thinning fluid. A computational fluid dynamic (CFD) simulation incorporating the constitutive model was implemented using OpenFOAM® in which benchmark problems including a sudden expansion and various driven slots and crevices were studied numerically. The numerical results exhibited good agreement with the experimental observations with respect to both the velocity field and the volume fraction distribution of RBCs.

Keywords

Blood; micro-channels; two-component flow; mixture theory; shear-thinning; crevices

1. Introduction

Blood-related diseases, including cardiovascular, cerebrovascular diseases and malaria, are the leading causes of death in the world[1]. For increasing the survival rates of the patients and also improving their life quality, numerous medical devices and therapies have been developed. Their safety and efficacy is intimately related to the properties and behavior of flowing blood. Comparing with the costly experimental trial-and-error, especially at the

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initial phase of a project, numerical simulations have been emphasized for their high efficiency and low costs in the design of medical devices[2].

Whole blood is a suspension of red blood cells (RBCs), white blood cells (WBCs) and platelets, in plasma. The volume fraction (hematocrit) of RBCs is about 45%, therefore the properties of whole blood is greatly influenced by their rheological behavior. The most prominent properties are their aggregation and disaggregation as a function of shear rate, their deformability, and their alignment responding to extensional flow. For additional detail, the reader is directed to[3–7]. These properties are manifested on a bulk scale as shear-thinning and stress relaxation[8]. At the micro-scale, for example in a vessel whose diameter in the range of 20 to 500 microns (and shear rates below 100 s^{-1}), these non-Newtonian phenomena blood are responsible for the trafficking of RBCs (hence their distribution of volume fraction)[9–11].

The multi-component features of blood are considered to be important for many blood related phenomena, such as thrombosis and atherogenesis. A thrombus usually refers to a blood clot anchored to damaged vascular walls. The thrombus formation in vivo or in blood wetted medical devices may cause serious health problems or devices malfunction. Several theoretical works related to the modeling of thrombus formation have been done, see [12–16]. Distribution of the RBCs (red blood cells) influencing the distribution of the platelets due to particles collision is closely related to the thrombus formation [17–21]. It is known that the thrombus initiation and growth near the walls of the vessels or the medical devices is strongly influenced by the number density of the platelets near the walls [22]. Through many mesoscale computational studies, the near wall dense concentration of the platelets is believed to be mainly due to the collisions between the RBCs and the platelets [22–24]. For modeling of the RBCs induced platelets transportation, a multi-component model for blood flow is necessary.

In addition, in the past several decades many important multi-component phenomena have been revealed through the various investigations on blood flow in micro-scale channels. Blood flowing in small tubes exhibits a thin layer of pure plasma, which is called the depletion layer of RBCs[25], and as a result, plasma-skimming occurs in the branch vessel downstream of the depletion layer[26]. Because of the depletion of RBCs, this phenomenon may impair transport of oxygen in distal capillaries[25]. The Fahraeus effect occurs when blood flows into a narrow and long vessel from a larger vessel (for vessel diameters ranging from 0.05 to 1.5 mm.) The hematocrit in the narrow tube reduces compared with that of the larger feeding tube[27]. The Fahraeus–Lindqvist effect is a result of Fahraeus effect, which corresponds to a decrease of the viscosity in the narrow tube compared with that of the feeding (larger) tube[28]. In summary, it is evident that blood flow at micro-scales acts as a multi-component material, and exhibits more complex behavior than can be described by a single component model.

In the past several decades, various multi-component models for blood have been developed. Mesoscale simulations, such as the *Immersed Boundary Method* (IBM) combined with the *Lattice Boltzmann Method* (LBM), are useful methods for investigating the complex behavior of blood or RBCs in micro-scale flow, particularly due to the

deformation, aggregation of the RBCs (see[29–31]). In these studies, the RBC cytoskeleton and membrane are modeled as a network of springs in combination with bending rigidity and constraints for surface-area and volume conservation. The fluid forces experienced by RBCs are then calculated by integrating the pressure and shear stresses along the RBCs surface[32]. Although these mesoscale simulations have been useful in displaying non-Newtonian behavior of blood, they are still prohibitive for industrial and engineering scale simulations due to the high computational cost[33]. An alternative method, the two-fluid approach two component model overcomes the limitation of high computational cost but still provides useful information, such as the volume fraction distribution of the RBCs[34, 35]. For the two-component formulation, two methods have been widely used: the *Averaging Method* (see [36–38]) and *Mixture Theory* (or the Theory of Interacting Continua) which is applied in current paper[39]. The mixture theory is a homogenization approach within the framework of continuum mechanics, first presented by [40], in which the phenomena of diffusion, dissociation, combination, and chemical reaction in the broadest sense can be represented[41]. For the basics of the theory, the historical development and applications, the reader is directed to review articles by Atkin and Craine[42, 43], Bowen[44], Bedford and Drumheller[45], Massoudi[46, 47], and the books by Truesdell[41], and Rajagopal and Tao[39].

A two-fluid model for blood, based on mixture theory, was previously introduced Massoudi and Antaki[48], and further detailed in Massoudi et al.[49], and Wu et al.[50, 51]. The model treats plasma as a Newtonian fluid while the RBCs are treated as a shear-thinning fluid whose viscosity depends on the volume fraction according to experimental observations of Brooks[52]. For studying this two-component system, a three dimensional CFD solver is implemented based on the solvers and libraries in the OpenFOAM®.

2. Methods

2.1 Governing equations

In the absence of thermo-chemical and electromagnetic effects, the governing equations consist of the conservation of mass, linear momentum and angular momentum. The equations for the conservation of mass in the Eulerian form are (See[44]),

$$\frac{\partial \rho_f}{\partial t} + \text{div}(\rho_f \mathbf{v}_f) = 0 \quad (1)$$

$$\frac{\partial \rho_{rbc}}{\partial t} + \text{div}(\rho_{rbc} \mathbf{v}_{rbc}) = 0 \quad (2)$$

where $\frac{\partial}{\partial t}$ is the derivative with respect to time, div is the divergence operator, the subscript ‘f’ refers to the fluid (plasma) component, and ‘rbc’ to the RBCs component, where $\rho_f = (1 - \phi)\rho_{f0}$, $\rho_{rbc} = \phi\rho_{rbc0}$, ρ_{f0} and ρ_{rbc0} are the pure density of the plasma and the RBCs in the reference configuration, respectively, ϕ is the volume fraction (hematocrit) of RBCs and $\mathbf{v}$ is the velocity field. The balance of the linear momentum can be written as,

$$\rho_f \frac{D^f \mathbf{v}_f}{Dt} = \text{div}(\mathbf{T}_f) + \rho_f \mathbf{b}_f + \mathbf{f}_I \quad (3)$$

$$\rho_{rbc} \frac{D^{rbc} \mathbf{v}_{rbc}}{Dt} = \text{div}(\mathbf{T}_{rbc}) + \rho_{rbc} \mathbf{b}_{rbc} - \mathbf{f}_I \quad (4)$$

where in general for any scalar β , $\frac{D^\alpha \beta}{Dt} = \frac{\partial \beta}{\partial t} + \mathbf{v}^\alpha \cdot \text{grad} \beta$, $\alpha = f, rbc$, and (for any vector $\mathbf{w}$), $\frac{D^\alpha \mathbf{w}}{Dt} = \frac{\partial \mathbf{w}}{\partial t} + (\text{grad} \mathbf{w}) \mathbf{v}^\alpha$, where ‘grad’ is the gradient operator, $\mathbf{T}_f$ and $\mathbf{T}_{rbc}$ stand for the Cauchy stress tensors for the plasma and the RBCs, respectively, $\mathbf{f}_I$ represents the interaction forces (exchange of momentum) between the components, and $\mathbf{b}_f$ and $\mathbf{b}_{rbc}$ refer to the body forces. The balance of the angular momentum implies that, in the absence of couple stresses, the total Cauchy stress tensor is symmetric. The above relations for ρ_f and ρ_{rbc} follow from the definition of mixture density, i.e., $\rho_{mix} = \rho_1 + \rho_2$ and the ‘volume additivity constraint’. This puts a restriction on the motion of the mixture[53], where it is assumed that the volume of the mixture is the sum of the volumes of the individual constituents in their reference configurations. A mixture stress tensor can be defined as[54]: $\mathbf{T}_m = \mathbf{T}_1 + \mathbf{T}_2$, where, $\mathbf{T}_1 = (1 - \phi)\mathbf{T}_f$ and $\mathbf{T}_2 = \phi\mathbf{T}_{rbc}$, so that the mixture stress tensor reduces to that of a pure fluid as $\phi \rightarrow 1$. Obviously this is an idealization. At a certain ϕ_m called the maximum packing fraction, the particles are in such a close proximity to each other that unless they are crushed or deformed, ϕ can no longer change. Therefore, in the numerical solution to all the practical problems of interest, we have to set certain limits or restrictions on ϕ so that ϕ can never be greater than ϕ_m . To close these equations, constitutive relations are needed for the individual stress tensors and for the interaction force. These are described in the following section.

2.2 Constitutive equations

Mathematically, the purpose of the constitutive relations is to provide connections between kinematic, mechanical, and thermal fields that are compatible with the balance equations and that, in conjunction with them, provide a theory that is useful and solvable for properly posed problems. Deriving constitutive relations for the stress tensors and the interaction forces are among the outstanding issues of research in multi-component flows. The nonlinearities in the constitutive relations give rise to a host of interesting problems from a computational point of view. In general, the constitutive expressions for $\mathbf{T}_f$ and $\mathbf{T}_{rbc}$ depend on the kinematical quantities associated with both the constituents. However, we assume that $\mathbf{T}_{rbc}$ and $\mathbf{T}_f$ depend only on the kinematical quantities associated with the solid and fluid component, respectively. This assumption is sometimes called “the principle of phase separation” and was first put forward by Adkins[55, 56]. In this paper, we assume that the plasma behaves as a linear viscous fluid

$$\mathbf{T}_f = [-p(1 - \phi) + \lambda_f(1 - \phi)\text{tr} \mathbf{D}_f] \mathbf{I} + 2\mu_f(1 - \phi) \mathbf{D}_f \quad (5)$$

where p is the pressure of the mixture, λ_f and μ_f are the (constant) first and second coefficients of viscosity of the pure plasma, where $\mathbf{D}_f = \frac{1}{2} [(\text{grad} \mathbf{v}_f) + (\text{grad} \mathbf{v}_f)^T]$, ‘tr’

stands for the trace of a second order tensor, and $\mathbf{I}$ is the identity tensor. (See [7, 49] for more details). The RBCs are assumed to behave as a viscoelastic shear-thinning fluid, whose viscosity is also a function of the volume fraction,

$$\mathbf{T}_{rbc} = [-p\phi + \beta_{20}(\phi + \phi^2) \text{tr} \mathbf{D}_{rbc}] \mathbf{I} + \mu_{rbc}(\phi, \text{tr} \mathbf{D}_{rbc}) \phi \mathbf{D}_{rbc} \quad (6)$$

$$\mu_{rbc}(\phi, \text{tr} \mathbf{D}_{rbc}) \phi = \left(\mu_{\infty}(\phi) + (\mu_0(\phi) - \mu_{\infty}(\phi)) \frac{1 + \ln(1 + k(\phi)(2 \text{tr} \mathbf{D}_{rbc}^2)^{1/2})}{1 + k(\phi)(2 \text{tr} \mathbf{D}_{rbc}^2)^{1/2}} \right) \quad (7)$$

The material parameters, μ_0 , μ_{∞} and k are assumed to depend on ϕ in the form of polynomial function and β_{20} is a constant. At low shear rates, the RBCs aggregate and form rod-shaped stacks due to the fibrinogen and large globulins, which increases the blood apparent viscosity. This aggregation of the RBCs is however reversed when the shear rate increases, causing their disaggregation, see [13, 57, 58]. The viscoelastic behavior of blood has been indicated by Thurston [59, 60]. It also has been pointed out that for cases where the shear rate is low the stress relaxation is more significant. The model used in our paper was proposed by Yeleswarapu et. al [61, 62]. This model is also able to capture the shear-thinning behavior of blood over a wide range of shear rates and is a generalization of a three constant Oldroyd-B fluid, which indicates that in some sense, the model is a viscoelastic shear-thinning fluid model.

The mixture viscosity is assumed to be the weighted sum:

$$\mu_{mix} = \mu_f(1 - \phi) + \mu_{rbc}\phi \quad (8)$$

These equations are calibrated based on experimental measurements of viscosity of suspensions of RBCs in ACD-plasma reported by Brooks [52] (See Figure 1).

$$\mu_0 = 537.002\phi^3 + 55.006\phi^2 - 0.129\phi \quad (9a)$$

$$\mu_{\infty} = 27.873\phi^3 - 21.218\phi^2 + 14.439\phi \quad (9b)$$

$$k = 11 \quad (9c)$$

It is noted that equations (9a) and (9b) imply that the viscosity of RBCs approaches zero when the hematocrit approaches zero. The value of the k is suggested by Yeleswarapu et al. [62]. Compared with the correlations used by Jung and Hassanein [35] and Wu et al. [50], considering the larger set of experimental data used here, this correlation is better for studying the problem when the hematocrit is relatively low, for example blood flow in micro-scale vessel where the hematocrit is about 20%.

For the interaction force between the two components, we only consider terms which would correspond to the Stokes drag force and Saffman's shear- lift force. (See[63, 64].)

$$\mathbf{f}_I = \frac{9\mu_f}{2a^2} f(\phi)(\mathbf{v}_{rbc} - \mathbf{v}_f) + \frac{3(6.46)(\rho_f \mu_f)^{1/2}}{4\pi a} \phi(2tr \mathbf{D}_f^2)^{-1/4} \mathbf{D}_f(\mathbf{v}_{rbc} - \mathbf{v}_f) \quad (10)$$

where a is the diameter of RBCs, here assumed as $8\mu m$, and $f(\phi) = \exp(2.68\phi) + \phi^{0.43}$ is the drag model (hindrance function) measured by Rusche and Issa[65]. For more discussions on the interaction force, see Wu et al.[51], Massoudi and Antaki[48].

3. Results and Discussion

In this section, several problems are studied numerically. Comparisons between the numerical and the experimental data are presented for the velocity and the volume fraction profiles. The velocity and the volume fraction fields are, in general, given by:

$$\begin{cases} \mathbf{v}_f = v_{fx}(x, y, z; t)\mathbf{e}_x + v_{fy}(x, y, z; t)\mathbf{e}_y + v_{fz}(x, y, z; t)\mathbf{e}_z \\ \mathbf{v}_{rbc} = v_{rbcx}(x, y, z; t)\mathbf{e}_x + v_{rbcy}(x, y, z; t)\mathbf{e}_y + v_{rbcz}(x, y, z; t)\mathbf{e}_z \\ \phi = \phi(x, y, z; t) \end{cases} \quad (11)$$

Using equation (11), substituting equations (5) ~ (10) into equations (3) and (4), the momentum equations in the vectorial form are obtained (here we have assumed that both components are incompressible). For the plasma component,

$$\begin{aligned} (1 - \phi)\rho_f \left[\frac{\partial \mathbf{v}_f}{\partial t} + (\text{grad} \mathbf{v}_f) \mathbf{v}_f \right] \\ = -\text{grad}((1 - \phi)p) + \text{div}(2\mu_f(1 - \phi)\mathbf{D}_f) + \rho_f(1 - \phi)\mathbf{b}_f \\ + \frac{9\mu_f}{2a^2} f(\phi)(\mathbf{v}_{rbc} - \mathbf{v}_f) + \frac{3(6.46)(\rho_f \mu_f)^{1/2}}{4\pi a} \phi(2tr \mathbf{D}_f^2)^{-1/4} \mathbf{D}_f(\mathbf{v}_{rbc} - \mathbf{v}_f) \end{aligned} \quad (12)$$

For the RBCs component,

$$\begin{aligned} \phi\rho_{rbc} \left[\frac{\partial \mathbf{v}_{rbc}}{\partial t} + (\text{grad} \mathbf{v}_{rbc}) \mathbf{v}_{rbc} \right] \\ = -\text{grad}(\phi p) + \text{div} \left(\left(\mu_\infty(\phi) + (\mu_0(\phi) - \mu_\infty(\phi)) \frac{1 + \ln(1 + k(2tr \mathbf{D}_{rbc}^2)^{1/2})}{1 + k(2tr \mathbf{D}_{rbc}^2)^{1/2}} \right) \mathbf{D}_{rbc} \right) \\ + \rho_{rbc}\phi\mathbf{b}_{rbc} - \frac{9\mu_f}{2a^2} f(\phi)(\mathbf{v}_{rbc} - \mathbf{v}_f) - \frac{3(6.46)(\rho_f \mu_f)^{1/2}}{4\pi a} \phi(2tr \mathbf{D}_f^2)^{-1/4} \mathbf{D}_f(\mathbf{v}_{rbc} - \mathbf{v}_f) \end{aligned} \quad (13)$$

A CFD solver for the mathematical model described above, i.e. equations (1), (2), (12), and (13), is developed using the solvers and the libraries of OpenFOAM®. This is a C++ toolbox for the development of customized numerical solvers, and pre-/post-processing utilities for the solution of continuum mechanics problems, including computational fluid dynamics (CFD)[66]. The details of the numerical algorithms dealing with the two-fluid (Eulerian-Eulerian) approach have been discussed in various publications (e.g.[2, 67, 68]).

For all the cases studied in this paper, we use the following values for the material properties: $\mu_f = 0.96\text{cP}$, $\rho_f = 1027\text{kg/m}^3$ and $\rho_{rbc} = 1093\text{kg/m}^3$. The boundary conditions are listed in Table 1 (For details see[67]). The total number of material parameters or constants associated with the constitutive relations is 10; these are μ_f , λ_f , ρ_f , ρ_{rbc} , μ_0 , μ_∞ , a , k , $f(\varphi)$, β_{20} .

For each geometry studied, the domain was discretized as hexahedral meshes using ICEM. In each of the cases, mesh-dependence studies were performed to assure insensitivity. To save computational time and expense, for cases 4.1, 4.3 and 4.4 we have taken advantage of the symmetric characteristics of the problems; therefore the meshes used in the simulations are 1/4 of the geometries for cases 4.1 and 4.3, and 1/2 of the geometries for case 4.4.

3.1 Velocity distribution in a tubular sudden expansion channel for low inlet hematocrit

The initial benchmark problem was to reproduce the results of the classic experiment of Karino and Goldsmith[69] which entails of water and 1% hardened RBCs through a tubular sudden expansion channel (See Figure 2). Two flow rates were simulated, corresponding to inlet velocity of 0.0757m/s ($Re=12.2$) and 0.233m/s ($Re=37.8$). Because hardened RBCs were used for this study, the effect of deformability on viscosity was ignored. Figure 3 shows the resulting streamlines and the velocity fields. Figure 4a and Figure 4b show the velocity distribution along the radial line (A–B) from the center of the tube to the wall, crossing the vortex center, illustrating excellent agreement between the simulation and experimental observations.

3.2 Blood flow in a rectangular channel

The second simulation aimed to reproduce the experimental observations of Patrick et al. [70] in a rectangular micro channel. Figure 5 shows the cross section of the computational domain. The inlet velocity as prescribed as $350\text{ }\mu\text{m/s}$ and the inlet hematocrit were 0.45. Figure 6 shows very good agreement between the numerically and experimentally obtained values of the velocity of the RBCs for the conditions of steady and fully developed flow.

3.3 Volume fraction distribution in a rectangular sudden expansion channel

The third benchmark study was aimed to replicate the experiments of Zhao et al.[71] involving flow of blood in a microchannel sudden expansion (See Figure 7). The hematocrit was prescribed to be 0.20 and the inlet velocity was 0.167m/s 0.833m/s. Figure 8 shows the comparison of the volume fraction of the RBCs obtained from the simulation after the flow has reached steady-state (corresponding to 0.15s and 0.035s for low flow rate and high flow rate, respectively.) To account for the finite depth-of-field of the experimental images, the numerical results shown in Figure 8 are averaged in the z-direction corresponding to the dimensions shown in Zhao et al.[71]. Comparison of the simulated RBC volume fraction with the microscopically observed RBC concentration reveals very good qualitative agreement, particularly with respect to the depletion zones. Comparison of simulated streamlines (Figure 3) with the experimental images also indicates similar increase in the size and intensity of the recirculation zone.

Figure 10 shows the results of a mesh sensitivity study for this problem. The average volume fraction profile of the RBCs is plotted as a function of mesh size (number of nodes) at $t=0.035$ s. The profiles illustrate convergence between 68256 and 125544 nodes. Accordingly, 125544 nodes as used for all subsequent simulations to ensure accuracy.

3.4 Volume fraction distribution in a rectangular channel with crevices

Based on the satisfactory performance of the model on the preceding benchmark problems, a final simulation study was performed to simulate flow over a series of driven slots and crevices. This is motivated by a persistent problem involved in most blood wetted devices, namely the seams and joints between component parts of a device or flow circuit which are predisposed to thrombus deposition (See Figure 11). For this case, the inlet hematocrit was prescribed to be 0.40 and the inlet velocity was 0.0238 m/s. Figure 12 shows the comparison of the volume fraction for the RBCs after the flow has reached the steady-state condition (simulated for 7s.). The numerical field was averaged along the z-direction similar to the previous case to account for depth-of-field of the experimental images.

These results are juxtaposed to microscopic images obtained using the method of Zhao et al. [71] in microchannels of the same dimensions, and under similar conditions. Briefly, platelets-sized fluorescent particles (3 μ m, Duke Scientific) was added to a suspension of RBC ghost cells (hematocrit = 40%) in a ratio of approximately 1:10. The sample solution was delivered by syringe pump (Harvard PHD) at the prescribed flow rate, corresponding to the velocity indicated above in the simulations. The fluorescence signal was captured by a sensitive, high-resolution CCD camera (SensiCam-QE, Cooke Corp). The qualitative agreement between simulation and experiment is readily apparent. In particular, both results reveal a deficit of RBCs in the depths of the narrow crevices and slots, see Figure 12 (a) (e) (f) and (g). Counter-rotating recirculation bubbles were also apparent in the crevices of Figure 12 (c) (d) and (h) adjacent to the main flow path. (See also video examples in supplemental materials.) Further detail is provided in Figure 13 which indicates the streamlines and RBCs volume fraction field in the deep rectangular crevice shown in Figure 6c.

4. Conclusions

In this paper, the multi-component characteristics of blood flow are studied using a two-fluid approach. The mathematical model is developed based on the framework of the mixture theory, and the model is solved numerically by a CFD solver built based on OpenFOAM®. The mathematical model is applied to several benchmark problems and compared favorably with the available experimental data. In particular, the value of the model was illustrated by the accurate prediction of volume fraction distribution of RBCs, which is essential for developing a predictive model for thrombosis in blood-wetted devices. This represents the ongoing and future objectives of this research.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.
RBCs viscosity as a function of the shear rate. Experimental data is by Brooks[52].

Figure 2.

The geometry of the tubular sudden expansion channel. The radius of the tube before and after the sudden expansion is 0.0755mm and 0.252mm, and the length before and after the sudden expansion is 1mm and 2mm.

Figure 3. Streamlines and velocity field following sudden expansion. $Re=12.2$ (a) and 37.8 (b) respectively. The measured positions of the reattachment points are $230\mu m$ for $Re=12.2$ and $715\mu m$ for $Re=37.8$. The scale bars represent the velocity magnitude.

Figure 4.

a. Velocity profile along the A-B line, when $Re=12.2$; b. Velocity profile along the A-B line, when $Re=37.8$ (Experimental data from Karino and Goldsmith[69])

Figure 5.
Schematic of the rectangular micro-channel, with a depth of $100\ \mu\text{m}$ (z direction).

Figure 6.

(Left) Numerical and experimental[70] streamwise (x) velocity distribution of the RBCs as a function of the distance to the wall y at the depths $z=8\ \mu m$ and $z=16\ \mu m$.

Figure 7.
Schematic of the sudden expansion micro-channel, with a depth of 100 μm (z direction).

Figure 8.

Volume fraction of the RBCs obtained from simulation (top) compared to experimental observation of Zhao et al.[71] (bottom). Left: inlet velocity = 0.167 m/s; right: 0.833 m/s. Hematocrit = 0.2. The scale bar represents the volume fraction magnitude of the RBCs.

Figure 9. Streamline of the mixture velocity for inlet velocity = 0.167 m/s(left) and 0.833 m/s (right), at the depth $z = 50\mu\text{m}$, for hematocrit = 0.2. The scale bars represent the velocity magnitude.

Figure 10.

Averaged volume fraction profile of RBCs along y direction from center of the channel to the wall, at $x=1.03\text{mm}$ (the line A-B shown in Figure 8), when the inlet velocity is 0.833 m/s .

Figure 11.

Schematic of the channel with deep gaps, with a depth of $70\ \mu\text{m}$ (z direction).

Figure 12.

Volume fraction of the RBCs obtained from simulation (top) compared with experimental observations of RBC ghosts seeded with 3μm fluorescent particles (bottom). Both simulation and experiment correspond to inlet velocity = 0.0238 m/s and hematocrit = 0.4. Occasionally a portion of RBCs and fluorescent particles (bright points) are observed to deviate from the main stream and migrate into the depths of the crevices, and become trapped. The highly bright regions correspond to concentrated accumulation of fluorescent particles. (Additional details can be seen in the videos provided in the supplemental materials.) The scale bar represents the volume fraction magnitude of the RBCs.

Figure 13. Simulated streamlines (A) and volume fraction (B) in crevice of Figure 6c, at depth $z = 35 \mu\text{m}$. The scale bars represent the velocity magnitude (A) and the volume fraction magnitude of the RBCs (B).

Table 1

Boundary conditions for simulations in this study.[66, 67]

Boundary	Pressure	Velocity	Volume Fraction
Inlet	Fixed flux	Fixed value	Fixed value
Outlet	Fixed value (reference)	Zero gradient	Zero gradient
Wall	Fixed flux	Fixed value (0)	Zero gradient
Symmetric Plane	symmetryPlane	symmetryPlane	symmetryPlane