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A Joint Acquisition-Estimation Framework for MR Phase Imaging

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Abstract

Measuring the phase of the MR signal is faced with fundamental challenges such as phase aliasing, noise and unknown offsets of the coil array. There is a paucity of acquisition, reconstruction and estimation methods that rigorously address these challenges. This reduces the reliability of information processing in phase domain. We propose a joint acquisition-processing framework that addresses the challenges of MR phase imaging using a rigorous theoretical treatment. Our proposed solution acquires the multi-coil complex data without any increase in acquisition time. Our corresponding estimation algorithm is applied optimally voxel-per-voxel. Results show that our framework achieves performance gains up to an order of magnitude compared to existing methods.

1 Introduction

The vast majority of MRI methods have historically retained the magnitude of the reconstructed image and ignored its complex phase. The main usage of the MR phase signal has focused on estimating *globally* varying field inhomogeneities [1]. Recently, however, the *local* phase has been showing its promise in quantifying underlying physiology such as blood flow, electro-magnetic tissue property, tissue elasticity, and temperature [1, 3, 5, 11]. While currently utilized MR pulse sequences and algorithms are optimized for magnitude-domain contrast or SNR, current methods for acquisition of the tissue-dependent phase do not guarantee any optimality criteria. Furthermore, existing phase reconstruction methods are inherently limited by ambiguities due to phase-wrapping, phase-noise and parallel channels' phase-offset. We present here a novel framework for MR phase imaging and formulate the phase estimation problem rigorously using a joint acquisition-processing approach. We consider the problem associated with imaging MR phase originating as a response to a Gradient-Echo (GRE) sequence using an array of receive coils. The measurement obtained using such a sequence at channel (receive coil) element c and echo time TE_k could be written as:

$$\mathbf{m}_{k,c}(x, y) = \rho_{k,c}(x, y) \exp \{ i(\phi_{0,c}(x, y) + 2\pi \Delta B(x, y) TE_k) \} + \mathbf{w}(x, y) \quad (1)$$

where $\rho_{k,c}(x, y)$ is the decayed magnitude at echo time TE_k modulated by the sensitivity of channel c , $2\pi \Delta B(x, y) TE_k$ is the underlying tissue phase value and $\phi_{0,c}(x, y)$ is the channel-

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dependent phase offset of channel c . $\phi_{0,c}(x, y)$ varies spatially due to factors such as the distance between the excited spins and the coil, the length of the coil's cables, and electronic delay [9]. Strictly speaking, $\phi_{0,c}(x, y)$ also varies with time due to drifts in frequency synthesizer and/or imperfections in the centering of k -space [5]. The term $B(x, y)$ accounts for all deviations from the main magnetic field at location (x, y) which are due to both, inherent local tissue properties (which induce such local changes), as well as global object coil-loading effects. We refer here to $B(x, y)$ as “tissue frequency.” Finally, the noise term $\mathbf{w}(x, y)$ in each voxel is drawn i.i.d. from a complex Gaussian Random Variable (RV), i.e. $\mathbf{w} = \mathbf{w}_R + i\mathbf{w}_I$, $\mathbf{w}_R, \mathbf{w}_I \sim \mathcal{N}(0, \sigma_w)$. Quantitation from MR phase requires the extraction of the term $B(x, y)$ from the measurements $\mathbf{m}_{k,c}(x, y)$. However, instead of the absolute phase, we are restricted to measuring the numerically computed angle of $\mathbf{m}_{k,c}(x, y)$, namely,

$$\Psi_{k,c}(x, y) = \phi_{0,c}(x, y) + 2\pi \Delta B(x, y) \text{TE}_k + \Omega_{k,c}(x, y) + 2\pi \mathbf{r}_{k,c}(x, y), \quad (2)$$

where $\Omega_{k,c}(x, y)$ is the phase contribution of the additive noise term and $\mathbf{r}_{k,c}(x, y)$ is a phase wrapping integer which forces the sum of the first three terms on the right side of (2) to be in the range $[-\pi, \pi)$. Note that, because the noise's contribution in the phase depends on $\rho_{k,c}(x, y)$, both $\Omega_{k,c}(x, y)$ and $\mathbf{r}_{k,c}(x, y)$ are a function of TE_k and channel c . It is clear from (2) that there are four unknowns in the angle measurements $\Psi_{k,c}(x, y)$: the parameter of interest

$B(x, y)$, and 3 additional sources of ambiguity (phase offset, noise and phase-wrapping integer). **Phase Offsets** $\phi_{0,c}$: Existing methods attempt to address this challenge by either (a) referencing $\mathbf{m}_{k,c}$ to a known coil or given location (equalization) [4, 7], (b) estimating then eliminating $\phi_{0,c}$ [6, 8] or (c) inverting the sensitivity profile of the coil-array [10, 12]. Equalization methods are sensitive to coil geometry, often resulting in severe artifacts in areas of low SNR, or areas of B variations. Cancellation methods suffer from an inherent SNR penalty, and require separate “reference” scans which assume that $\phi_{0,c}$ are temporally invariant [5]. Finally, inversion methods suffer from substantial errors/artifacts in areas of noise/poor regularization. **Phase noise and wrapping**: The authors in [1] have shown that imaging the MR phase signal inherently trades off two types of errors: noise and phase wrapping. There, the authors have identified three phase imaging regimes: (I) a regime dominated by phase-wrapping, with reduced levels of noise, (II) a regime dominated by noise, with no phase-wrapping and (III) a regime where the phase signal needs to be disambiguated from both phase-wrapping and noise contributions. Most phase imaging methods operate in Regime III [1] where post-processing is relied on in order to perform phase unwrapping and denoising. As carefully documented in [9], phase unwrapping methods are not robust and often require expert-user intervention. Other methods proposed in [2] use a combination of short (Regime II) and long (Regime I) echo times in order to recover the phase signal using an ML framework. However, as shown in [1], such methods are suboptimal because (a) long echo acquisitions rely on short echo acquisitions (which induces error propagation) and (b) the phase is computed using echo referencing (which induces noise amplification). Furthermore, the method in [2] relies on spatial regularization which biases the estimate.

2 MAGPI: A Maximum-Likelihood Framework

2.1 Problem formulation

The task here is to estimate B from $\Psi_{k,c}$. The ML estimator is popular due to its optimality properties such as efficiency, sufficiency, consistency and invariance. We can show that the likelihood function for our problem, $\mathcal{L}_{k,c}$, is given by:

$$\mathcal{L}_{k,c}(\Delta B) \triangleq \text{pr}(\Psi_{k,c} = \psi_{k,c} / \Delta B) = \text{pr}(\Omega_{k,c} + 2\pi \mathbf{r}_{k,c} = \psi_{k,c} - 2\pi \Delta B \text{TE}_k - \phi_{0,c}) \quad (3)$$

where $\psi_{k,c}$ is a realization of the random variable (RV) $\Psi_{k,c}$. Note that, because the phase wrapping integer $\mathbf{r}_{k,c}$ depends on the phase noise RV $\Omega_{k,c}$, this implies that $\mathbf{r}_{k,c}$ is also a (discrete) RV. Using the total probability theorem, and carrying on derivations not shown here, we can write:

$$\mathcal{L}_{k,c}(\Delta B) = \sum_r P(\mathbf{r}_{k,c} = r) f_{\Omega_{k,c}}(\psi_{k,c} - 2\pi \Delta B \text{TE}_k - 2\pi r - \phi_{0,c}) \quad (4)$$

where $P(\mathbf{r}_{k,c} = r)$ is the probability of obtaining a given wrapping integer r and $f_{\Omega_{k,c}}$ is the PDF of the noise in channel c , at echo time k , each given by:

$$P(\mathbf{r} = r) = \frac{\sigma_{k,c} \sqrt{2}}{4\zeta_{\max}} \left[\frac{e^{-b_2^2} - e^{-a_2^2}}{\sqrt{\pi}} - a_2 \text{erf}(a_2) + b_2 \text{erf}(b_2) - \frac{e^{-b_1^2} - e^{-a_1^2}}{\sqrt{\pi}} + a_1 \text{erf}(a_1) - b_1 \text{erf}(b_1) \right] \quad (5)$$

$$\text{where } a_1 = \frac{\left(\frac{2r-1}{2\text{TE}_k} - \zeta_{\max}\right)}{\sqrt{2}\sigma_{k,c}}; \quad b_1 = \frac{\left(\frac{2r+1}{2\text{TE}_k} - \zeta_{\max}\right)}{\sqrt{2}\sigma_{k,c}}; \quad a_2 = \frac{\left(\frac{2r-1}{2\text{TE}_k} + \zeta_{\max}\right)}{\sqrt{2}\sigma_{k,c}}; \quad b_2 = \frac{\left(\frac{2r+1}{2\text{TE}_k} + \zeta_{\max}\right)}{\sqrt{2}\sigma_{k,c}} \quad (6)$$

and $\zeta_{\max}$ is the maximum value of $B + (\phi_{0,c}/2\pi\text{TE}_k)$, and,

$$f_{\Omega_{k,c}}(\Omega) = \frac{\exp\left(-\frac{\text{snr}_{k,c}^2}{2}\right)}{2\pi} \left\{ 1 + \text{snr}_{k,c} \sqrt{\frac{2}{\pi}} \cos \Omega \exp\left(\frac{\text{snr}_{k,c}^2 \cos^2 \Omega}{2}\right) \left[1 + \text{erf}\left(\frac{\text{snr}_{k,c} \cos \Omega}{\sqrt{2}}\right) \right] \right\} \quad (7)$$

and $\text{snr}_{k,c}$ is the magnitude-domain SNR in channel c at TE_k . Note that our phase signal model does not assume any specific magnitude-decay model. We validated these theoretical predictions using numerical simulations and observed a close match (not shown here). We focus here on the dependence of the likelihood functions on TE_k , B , T2^* and $\text{SNR}_{0,c}$. We plot in Figure 1(a) two families of likelihood functions obtained at two different values of TE_k . We ignore the channel offsets ($\phi_{0,c} = 0, \forall c$) in this figure. We note the following: First, depending on the choice of TE_k , the likelihood functions exhibit either sharp but multiple maxima (wrapping-dominated regimes), or a broad unimodal peak (noise-dominated regimes). Therefore, the likelihood function captures the inherent trade-offs between noise and phase wrapping with respect to the choice of TE_k . Second, we note that repeated measurements (dashed family of lines) yield randomly shifted $\mathcal{L}_{k,c}(B)$. Measurements at the larger TE_k result in a family of likelihood functions that are more tightly centered around the true B , as compared to the shorter TE_k . We include the effects of the unknown phase offset in Figure 1(b), where we plot example $\mathcal{L}_{k,c}(B)$ for different channels c . Note that likelihoods

in the same voxel are shifted with respect to one another by an unknown amount, $\phi_{0,c}$. This example illustrates that maximizing the likelihood is not trivial due to: (1) the function either having multiple maxima or one maximum whose location is sensitive to noise and (2) the unknown $\phi_{0,c}$.

2.2 Proposed Solution: MAGPI

Our proposed framework, coined MAGPI (**M**aximum **A**mbi**G**uity distance for **P**hase **I**maging), acquires Multi-Echo Gradient Echo (MEGE) measurements from a collection of K echoes, and N_c channels, within a single TR. The estimation step is described using the 3-pass process detailed below.

Pass I: Find the most likely B that explains the angle buildup between echoes—We can show that the angle buildup between any two echoes is:

$$\Delta\Psi_{2:1,c} \triangleq \angle \left\{ \mathbf{m}_{2,c} \mathbf{m}_{1,c}^* \right\} = 2\pi \Delta B \Delta \text{TE}_{2:1} + \Delta\Omega_{2:1,c} + 2\pi \mathbf{r}_{2:1,c}, \quad (8)$$

where $\text{TE}_{2:1} \triangleq \text{TE}_2 - \text{TE}_1$, $\Omega_{2:1,c} \triangleq \Omega_{2,c} - \Omega_{1,c}$ and $\mathbf{r}_{2:1,c}$ is a phase wrapping integer which forces the sum of the first two terms on the right side of (8) to be in the range $[-\pi, \pi)$. We note two differences in (8) as compared to (2). First, the relative phase buildup does not depend on $\phi_{0,c}$. The second important difference is a reduced tissue phase amplification (due to multiplication with a smaller term, $\text{TE}_{2:1}$) accompanied by noise amplification (two contributions from noise RVs). We will address this shortcoming in pass III below. The likelihood function, denoted by $\mathcal{L}_{2:1,c}(B) \triangleq \text{pr}(\Psi_{2:1,c}/B)$, is now given by:

$$\mathcal{L}_{2:1,c}(\Delta B) = \sum_{r=-R}^R P(\mathbf{r}_{2:1,c}=r) f_{\Delta\Omega_{2:1,c}}(\Delta\psi_{2:1,c} - 2\pi \Delta B \Delta \text{TE}_{2:1} - 2\pi r) \quad (9)$$

where the noise and wrapping PDFs $f_{\Omega_{2:1,c}}$ and $P(\mathbf{r}_{2:1,c})$ could be readily obtained in closed-form, similar to (5) and (7). Note that, since there is no $\phi_{0,c}$ ambiguity in this dual-echo likelihood, all the peaks of $\mathcal{L}_{2:1,c}(\Delta B)$ are now exactly co-aligned. We can then seek the maximum of the product of the dual-echo likelihoods over all channels c , $\prod_c \mathcal{L}_{2:1,c}(B)$, as the solution to the ML problem. This product assumes that the angle measurements over all the channels are conditionally independent. This assumption may not be exact, due to noise correlation across channels, but we ignore any inter-channel dependence in this treatment. Nevertheless, the resulting dual-echo likelihood is not guaranteed to have a single maximum: we still face here the same unimodal vs multimodal trade-offs as single-echo likelihoods. To address this inherent limitation, we make use of (at least) one additional echo, $k = 3$, and compute a second set of likelihood functions $\mathcal{L}_{3:1,c}(\Delta B)$. We can then ask the following question: what value of B most-likely explains both angle buildups from echo pairs (1,2) and (1,3)? Approximating both angle build-ups as conditionally independent RVs, we can formally write this question as:

$$\hat{\Delta B}_{(1)} = \arg \max_{\Delta B} \prod_{k,k_0; k \neq k_0} \prod_c \mathcal{L}_{k:k_0,c}(\Delta B). \quad (10)$$

The product of likelihoods $\mathcal{L}_{\delta k}(\mathbf{B}) \triangleq \Pi_{k,k_0} \Pi_c \mathcal{L}_{k:k_0,c}(\mathbf{B})$ is defined as the “system likelihood.” We claim that system likelihoods are not subject to the same multimodal vs unimodal tradeoffs as individual likelihood functions. To illustrate this, we plot in Figure 2 example $\mathcal{L}_{\delta k}(\mathbf{B})$ having a single sharp maximum, despite the underlying individual likelihoods having multimodal peaks (Figure 2(a)) or both unimodal and multimodal peaks (Figure 2(b)).

Nevertheless, not all system likelihoods are of equal value. For a given $\mathbf{B}$, $T2^*$ and $\text{SNR}_{0,c}$ in a voxel, it is obvious that there exists a large number of system likelihood functions (selected by TE_k) that one could choose from, each with its own corresponding performance, given by the asymptotic Minimum Variance Unbiased (MVU) bound. A careful design procedure would pick $\mathcal{L}_{\delta k}(\mathbf{B})$ which achieves the lowest MVU bound amongst all possible bounds. This is the key to our method: the design of acquisition parameters (TE_k) such as the corresponding estimation (MLE) achieves the best (MVU) estimate. We formalize the optimizer below. Although the 1D search of system likelihoods (10) still possesses local maxima, the guaranteed global maximum can be obtained with “brute force” as the likelihoods are given by an analytical expression. Furthermore, it is important to emphasize that this step can be solved one voxel at a time, substantially reducing the complexity.

Pass II: Estimate the channel-dependent phase offsets $\phi_{0,c}$ —After obtaining an ML estimate of $\mathbf{B}$ in Pass I, the remaining data in the original measurements (2) that is “unexplained” by the Pass I model (8) can be attributed to $\phi_{0,c}$ and errors in the $\mathbf{B}$ estimate.

The task in this step is to extract $\phi_{0,c}$ from the remainder terms: $\phi_{k,c}^{\text{rem}} = \mathcal{L} \left(\mathbf{m}_{k,c} e^{-i\hat{\Delta}B_{(1)}} \right)$. To achieve this, we take advantage of the following distinct features of $\phi_{0,c}$: smooth variation over space and channel dimensions (x, y, c), and invariance with echo times. Using this prior knowledge, we can use various signal-separation techniques (such as PCA, Wavelet decomposition, Spectral Decomposition methods, Fractional-Fourier methods) to extract $\phi_{0,c}$ from $\phi_{k,c}^{\text{rem}}$. We resort here to a simple and fast technique: spatio-temporal linear low-pass filtering using an operator $\hat{\phi}_{0,c} = \mathcal{L} \mathcal{P} \mathcal{F} \left\{ \phi_{k,c}^{\text{rem}} \right\}$. We omit details here on this filter for brevity.

Pass III: Find the most likely $\mathbf{B}$ that explains the angle of all K echoes—We use here the estimate of $e^{i\hat{\phi}_{0,c}}$, obtained from Pass II, to rewrite the single-echo likelihood functions without any channel-offset ambiguity, namely:

$$\mathcal{L}_{k,c}(\Delta B) \triangleq \text{pr} \left(\Psi'_{k,c} = \psi'_{k,c} / \Delta B \right) = \sum_r P(\mathbf{r}_{k,c} = r) f_{\Omega_{k,c}}(\psi'_{k,c} - 2\pi \Delta B \text{TE}_k - 2\pi r) \quad (11)$$

$$\text{where } \psi'_{k,c} = \mathcal{L} m_{k,c} e^{-i\hat{\phi}_{0,c}}. \quad (12)$$

That is, in this pass, we can directly solve the following estimation problem: What is the most likely $\mathbf{B}$ that could explain the angle as given by (12)? Formally, we solve this MLE problem:

$$\hat{\Delta B}_{(\text{III})} = \arg \max_{\Delta B} \prod_{k=1}^K \prod_c \mathcal{L}_{k,c}(\Delta B). \quad (13)$$

We define (another) system likelihood, given by the product of the K individual likelihoods:

$\mathcal{L}(\Delta B) \triangleq \prod_{k=1}^K \prod_c \mathcal{L}_{k,c}(\Delta B)$. Similar to Pass I, the system likelihood of Pass III is not subject to the unimodal vs multimodal trade-offs of individual likelihoods. Furthermore, $\mathcal{L}(\Delta B)$ could be designed (by choice of TE_k) to yield the lowest MVU bound possible. The optimizer thus needs to optimize 2 system likelihoods, $\mathcal{L}_{sk}(\Delta B)$ for Pass I and $\mathcal{L}(\Delta B)$ for Pass III.

2.3 Optimizer

The solution to MLE problems (10) and (13) are not guaranteed to achieve the “best” tissue phase estimate for arbitrary choices of echo times. The quality of the estimation here is measured using the asymptotic MVU bound. We denote $\sigma_I(\Delta B)$ and $\sigma_{\text{III}}(\Delta B)$ the MVUs associated with MLE problems (10) and (13), respectively. Each of these MVUs is given by the inverse of their distribution’s Fisher Information, $I^{-1}(\Delta B)$. In this work, we make use of the closed form of the likelihood functions to obtain an approximate numerical estimate of $I(\Delta B)$. Ideally, we need to minimize both $\sigma_I(\Delta B)$ and $\sigma_{\text{III}}(\Delta B)$, for all possible values of ΔB . Clearly, this may not be possible. Instead, we propose the following alternative: minimize the average $\sigma_{\text{III}}(\Delta B)$ subject to an upper bound constraint on the average $\sigma_I(\Delta B)$. Formally, this optimization problem could be written as:

$$\begin{aligned} [\text{TE}_1^{\text{opt}}, \text{TE}_2^{\text{opt}}, \dots, \text{TE}_K^{\text{opt}}] = & \arg \min_{[\text{TE}_1, \text{TE}_2, \dots, \text{TE}_K]} \mathbb{E}[\sigma_{\text{III}}(\Delta B)]_{\Delta B} \\ \text{such that, } & \mathbb{E}[\sigma_I(\Delta B)]_{\Delta B} \leq \varepsilon_I \text{ \& } [\text{TE}_1, \text{TE}_2, \dots, \text{TE}_K] \in \mathcal{C}_{\text{TE}}. \end{aligned} \quad (14)$$

We make the following notes about this optimizer:

1. $\mathcal{C}_{\text{TE}}$ is the set of echo times that are physically achievable with the pulse sequence of choice. This constraint set takes into account pulse sequence parameters known *a priori* such as the minimum echo time spacing and the minimum or maximum echo times.
2. We assume ΔB to be uniformly distributed in the expectations of (14). The range of the distribution can be easily determined based on the strength of the main magnetic field and the anatomy of interest.
3. Note that both $\sigma_I(\Delta B)$ and $\sigma_{\text{III}}(\Delta B)$ are a function of $\text{SNR}_{0,c}$ and T2^* , both of which are spatially-varying quantities. In order to address this challenge, we only optimize echoes for the minimum expected $\text{SNR}_{0,c}$ and T2^* of interest. The design would thus target the “worst-case imaging scenario.” Since the optimizer is run offline, once, various imaging scenarios corresponding to different $\text{SNR}_{0,\min}$, $\text{T2}_{\min}^*$ and echo time constraint sets $\mathcal{C}_{\text{TE}}$ could be tabulated and used at acquisition time.

3 Results

In all our acquisitions below, the set of echo times $\mathcal{C}_{TE}$ was constrained such that the *acquisition time is no longer* than traditional single-echo GRE. Also, ε_I in (14) was set to 1Hz. In post-processing, the estimation procedures of (10) and (13) were independently solved for every voxel in the image. *No spatial averaging/smoothing* was ever employed with our method.

3.1 Numerical Phantom

We validated the performance of our proposed MAGPI framework on a modified Shepp-Logan phantom which consists of a 128×128 tissue frequency map (Figure 3(a)) and a corresponding magnitude image. The tissue frequency values are randomly chosen between ± 125 Hz, corresponding to the range of values found in the brain at 1.5T. We simulated GRE acquisitions of this complex-domain object using an array of 16-channel receive coils at different SNRs. The resulting angle of the measurements $\psi_{k,c}$ (2) at $TE_k = 40$ ms are shown in Figure 3(f) for an SNR= 27dB. In all the numerical simulations, the object was assumed to have a $T2^*$ of 40ms. Figure 3 shows the estimated B from $\psi_{k,c}$ using a variety of methods, for different values of SNR_0 (SNR at $TE=0$). **Column 1 (PD-40)** is the Phase Difference method [6] which uses an echo pair, $TE=\{36, 40\}$ ms, with phase conjugation (similar to (8)) to cancel $\phi_{0,c}$ and estimate B . One of the echoes is constrained at 40ms for its utility as a magnitude contrast, and the 3.5ms echo step is chosen to be just small enough to avoid phase wrapping. **Column 2 (Phun+ref)** is a single-echo phase unwrapping method ($TE= 40$ ms) [8]. Single-echo methods require knowledge of $\phi_{0,c}$ in order to combine the complex measurements from all the channels. Phun+ref here assumes perfect knowledge of $\phi_{0,c}$. **Column 3 (UMPIRE)** uses a triple-echo method, $TE= \{6.67, 21.67, 40\}$ ms. This recently proposed method [9] was shown to vastly outperform phase unwrapping algorithms, particularly when the spatial pattern of B is complex. The UMPIRE echoes were chosen here according to the prescription in [9], whereby the smallest difference between the echo steps is able to unwrap a maximum tissue frequency buildup of 150Hz. **Column 4** is our proposed **MAGPI**, which uses the following optimized echoes: $TE= \{26.56, 35, 40.91\}$ ms for $SNR_0 = 30$ dB (Figure 3(e)) and $TE= \{16.4, 22.27, 36.83\}$ ms for $SNR_0 = 27$ dB (Figure 3(j)). We note the following from Figure 3: phase unwrapping fails to recover the underlying tissue frequency for this B pattern. PD-40 and UMPIRE are not robust in the presence of noise, and exhibit noise-induced phase wrapping at low SNRs. MAGPI clearly outperforms all these methods and achieves RMSE reductions by a factor of 15.8X over UMPIRE (at 30dB) and 22.37X over PD-40 at 27dB. We repeat this experiment for different random selections of B , $\phi_{0,c}$ and noise and plot in Figure 4(a) the RMSE of the B estimate as a function of SNR_0 , for all methods. We also include an additional PD method (PD-10) with the same echo time step as PD-40, but without the 40ms echo time constraint ($TE = \{6.5, 10\}$ ms). The black lines in the figure constitute theoretical performance bounds. The first bound, 1-Echo-Limit, is the lower bound on the RMSE obtained with a single echo scan which (a) has perfect knowledge of $\phi_{0,c}$ and (b) only operates on values of B that induce no phase wrapping. Thus, the only ambiguity with this method would be due to noise. The second bound, ML-MVU, is the minimum MVU bound as predicted from our theoretical derivations, with $K=3$ echoes. We note the substantial

performance gain (Figure 4(b)) obtained with MAGPI over other methods. In particular, MAGPI achieves an RMSE reduction by an order of magnitude (11X) over PD-10 at all SNRs. MAGPI's gain over UMPIRE increases as SNR decreases, from 3.83X at 40dB to 41X at 27dB. Furthermore, MAGPI outperforms the lower bound derived for ideal single echo methods by a factor of 1.81X at 40dB up to 3.9X at 27dB. The RMSE obtained with MAGPI also closely matches the MVU predicted by theory, thereby suggesting the rapid convergence of our ML-based approach.

3.2 Real Phantom

We also validated the performance of MAGPI in real phantom experiments. A water phantom was acquired at 1.5T on a Siemens Aera scanner using a 20-channel head coil and a voxel size of $0.9 \times 0.9 \times 0.9 \text{ mm}^3$ (Figure 5, first column). Figure 5 (top row) shows the resulting B estimates for each of the following methods (along columns): PD-10 $\times 20$ Avgs, Phun + Ref scan, UMPIRE and MAGPI, respectively. The PD-10 method ($TE = \{4, 10\} \text{ ms}$) was averaged over 20 acquisitions in order to form a wrapping-free reduced-noise reference. The Phun+ Ref method uses the phase offsets derived from one of the reference PD-10 scans in order to coherently sum the aligned complex data, then unwrap its phase ($TE = 40 \text{ ms}$, $TR = 46 \text{ ms}$, $TA = 5 \text{ min } 30 \text{ sec}$). UMPIRE ($TR = 46 \text{ ms}$, $TA = 3 \text{ min } 09 \text{ sec}$) and MAGPI ($TR = 42 \text{ ms}$, $TA = 2 \text{ min } 45 \text{ sec}$) used the same echoes as in simulations. MAGPI's echoes were optimized for $SNR_{0, \min} = 28 \text{ dB}$, $T_{2, \min}^* = 40 \text{ ms}$. The inter-echo spacing with the optimizer was constrained to be 5.82 ms , to accommodate flow-compensation and monopolar readout at a bandwidth of 240 Hz/pxl . We note the overall agreement between these methods. We note from the figure the clear phase SNR improvement obtained with MAGPI's estimate, despite having the shortest acquisition time. In order to quantify this phase SNR gain over the other methods, we removed the smooth background phase in each B estimate. Since this is a water-only phantom, the result of such filtering process (bottom row of Figure 5) is mainly the residual noise image. The reduction in noise with MAGPI is clear. We computed the standard deviation in the residual images and noted the following reduction in standard deviation (or gain in phase SNR) with MAGPI: a gain of **3.95X** over the 20-time averaged PD-10, a **3.661X** gain over Phun+Ref and a **7.7X** gain over UMPIRE. These gains are consistent with our simulations (Figure 4(a)), given that SNR_0 in this water phantom was 33 dB .

3.3 in-Vivo

The brain of healthy volunteers was imaged after approval was obtained from our Institutional Review Board and informed consent was given by the subjects. The first set of scans was done on a Siemens Aera 1.5T with a 20-channel head coil: 3D GRE, $FOV = 220(\text{read out}) \times 200 \text{ mm}^2$, $N_x = 448$, $N_y = 406$, slice thickness 2 mm , $FA = 15^\circ$, $BW = 240 \text{ Hz/pxl}$. The voxel size in this scan is thus $0.49 \times 0.49 \times 2 \text{ mm}^3$. Figure 6(a) shows the B image reconstructed by a Siemens product sequence (Adaptive Combine method [12]), from a traditionally utilized single-echo scan ($TE = 40 \text{ ms}$, $TR = 45 \text{ ms}$, $TA = 6 \text{ mins } 10 \text{ s}$). The artifacts in this method originate from phase wrapping errors and phase-offset alignment errors. In order to mitigate these artifacts, a technique known as "Homodyne Filtering" adds the channels' complex data directly in high pass domain (Figure 6(b)) [3]. The high-pass filter is

used here to reduce large values of B , and reduce impact of $\phi_{0,c}$. Note the predominantly noisy image in Figure 6(b), as expected at high resolutions not traditionally utilized at 1.5T. Finally, we applied our MAGPI framework, optimized for $\text{SNR}_{0,\min}=26.5\text{dB}$, $T_{2\min}^*=40\text{ms}$: $\text{TE}=\{16.18, 27.98, 35.37\}\text{ms}$, $\text{TR}=40\text{ms}$, and $\text{TA}=5\text{mins}20\text{secs}$ (ie, 87% of the time of the traditional single echo scan). The resulting images are shown in Figures 6(c) (B) and 6(d) (high-pass filtered B). Consistent with our simulations and phantom results, we note substantially improved phase SNR with our proposed technique. This is particularly apparent in the homodyne domain, where we see venous structures not seen with the traditional method. Finally, we show in Figure 6(e) a tissue frequency image in homodyne domain obtained at 3T using MAGPI, at an impressive in-plane resolution of $390\mu\text{m}$. We can clearly distinguish here microstructures such as perforator veins and laminar pattern of cerebral cortical layers. To the best of our knowledge, such phase images displaying high-contrast high-resolution anatomical details are not commonly seen at 3T.

4 Conclusion

We presented here a unifying framework for MR phase estimation which jointly combines the complex channels' data and estimates the tissue phase values in an ML-optimal fashion. The proposed approach optimizes echo times such that the corresponding MLE achieves the lowest MVU bound possible. Reconstructing a 128×128 phase image took ~ 1 minute on a personal computer. Our results show that our proposed MAGPI framework achieves gains in phase SNR by at least a factor of 3.5X (and up to a factor of 40X), compared to other methods.

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Fig. 1.

(a) Example likelihood functions, with $\phi_{0,c} = 0$, in a voxel where $B = 20\text{Hz}$, $\text{SNR}_0=22$ (27dB) and $T_2^*=30\text{ms}$. The family of blue lines correspond to $\mathcal{L}_{k,c}(B)$ for different $\psi_{k,c}$ realizations, at $\text{TE} = 5\text{ms}$. The red lines are $\mathcal{L}_{k,c}(B)$ obtained at $\text{TE} = 40\text{ms}$. (b) $\mathcal{L}_{k,c}(B)$ for the same voxel as (a), at $\text{TE}=40\text{ms}$, but obtained through 3 channels each with different phase offsets.

Fig. 2.

Example system likelihood functions obtained from individual dual-echo likelihoods. Note that the system likelihoods (red) are not subject to the same noise-phase wrapping trade offs as the dual-echo likelihoods (green and blue).

Fig. 3.

(a) Original B and (f) angle of measurements $\psi_{k,c}$ at SNR=27dB. (Columns 2–4) Example B estimates using a collection of methods for 2 different SNR=30dB (row 1) and SNR = 27dB (row 2). RMSE of each estimate is shown in sub-captions.

Fig. 4.

(a) RMSE (Hz) averaged over random realizations of noise, B and $\phi_{0,c}$. Result in log-scale.

(b) RMSE reduction achieved with MAGPI over other methods.

Fig. 5.

(First Column) Magnitude image combined using standard sum of squares (SSQ) method. (Top Row) B estimate in a water phantom. (Bottom row) Remainder after removing background phase from top row: result measures noise content. All units in Hz. Note that the circular truncation artifacts visible in the SSQ image are only detectable with MAGPI which shows that (i) MAGPI substantially reduces the noise floor and (ii) MAGPI does not use spatial smoothing and preserves pixel resolution.

Fig. 6. *in-vivo* B estimates at 1.5T, at in-plane resolution of 0.49mm, obtained with (a,b) traditional single echo methods and (c,d) MAGPI. Images in (b) and (d) are the high-pass counterparts of (a) and (c). (e) *in-vivo* B at 3T, at in-plane resolution 0.39mm obtained using MAGPI.