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Joint reconstruction of activity and attenuation map using LM SPECT emission data

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Abstract

Attenuation and scatter correction in single photon emission computed tomography (SPECT) imaging often requires a computed tomography (CT) scan to compute the attenuation map of the patient. This results in increased radiation dose for the patient, and also has other disadvantages such as increased costs and hardware complexity. Attenuation in SPECT is a direct consequence of Compton scattering, and therefore, if the scattered photon data can give information about the attenuation map, then the CT scan may not be required. In this paper, we investigate the possibility of joint reconstruction of the activity and attenuation map using list-mode (LM) SPECT emission data, including the scattered-photon data. We propose a path-based formalism to process scattered-photon data. Following this, we derive analytic expressions to compute the Cramér-Rao bound (CRB) of the activity and attenuation map estimates, using which, we can explore the fundamental limit of information-retrieval capacity from LM SPECT emission data. We then suggest a maximum-likelihood (ML) scheme that uses the LM emission data to jointly reconstruct the activity and attenuation map. We also propose an expectation-maximization (EM) algorithm to compute the ML solution.

Keywords

SPECT; joint reconstruction; list-mode data; activity and attenuation map; maximum likelihood expectation maximization; Fisher information; Cramér-Rao bound

1. INTRODUCTION

The phenomenon of scattering and attenuation affect image quality in single photon emission computed tomography (SPECT).^{1, 2} Thus, various techniques have been devised to perform attenuation and scatter correction in SPECT. However, most of these techniques require an attenuation map, for which, an additional X Ray computed tomography (CT) scan must be performed on the patient.¹⁻⁹ These CT-based-attenuation-correction (CTAC) schemes result in increased radiation dose, which is a cause of concern for the patients health.¹⁰⁻¹² Recently, a New England Journal of Medicine article concluded that as many as 1 in 50 cancers in the USA may be attributable to the radiation from CT studies.¹³ Acquiring the extra CT scan also increases the costs and the acquisition time. Combined SPECT/CT imaging may also introduce some contamination of emission photons with transmission photons. There is also a possibility of mis-registration between the transmission dataset with

the emission dataset.¹⁴ Often, these CTAC schemes are implemented using dual-modality systems.^{4, 5} Such a dual-modality system may have a reduced field of view due to the use of a fan or cone-beam collimation for transmission imaging. Designing such a dual-modality system also considerably complicates both the scanner design and the data acquisition and processing protocols.¹⁵ Due to these reasons, the possibility to perform attenuation and scatter correction without the need for transmission CT data is important and can have significant and immediate clinical impact.²

Research in transmission-less image reconstruction in SPECT has had a long history, starting with the pioneering work of Censor *et al.*¹⁶ The techniques that have been proposed since can be divided in two classes, as shown in Fig. 1. The first class of algorithms attempt to estimate the attenuation coefficients directly from the emission data. These algorithms either try to perform iterative inversion of the forward mathematical model,^{16–19} or exploit the consistency conditions that are based on the forward model.^{15, 20–22} However, these methods are considerably slow, and mostly use only the data around the photopeak events neglecting the scattered photons. The techniques have met with only limited success.² The second class of algorithms use scattered data to reconstruct the attenuation map, since attenuation is a direct consequence of Compton scattering. Scattered data is used to highlight the boundaries between the different tissues⁹ to determine a segmented attenuation map, to which predefined attenuation coefficients are assigned.^{9, 23} This method works well when activity is widely distributed,^{24–28} but it has limitations when the activity is focal.² There are also some preliminary studies on estimating the attenuation map by inverting the models used to correct for scatter,^{29, 30} but the methods are still slow.

A major issue with these methods is that they have less information to begin with, since either they don't use the scattered photon data, or if they do, then they "bin" the data, which leads to information loss. At SPECT imaging energies, the dominant photon-interaction mechanism within the tissue is Compton scattering. Thus the scattered-photon data should provide information about the attenuation map of the tissue. Also, if the data is acquired in list-mode (LM) format, then the loss in information due to the binning process is avoided. An important question then is the possibility to perform joint reconstruction of the activity and attenuation map using the scattered and photopeak photon data acquired in LM format. If this is possible, then it provides an avenue to design methods to perform attenuation and scatter correction using only the LM emission data, thus negating the requirement of an extra CT scan to obtain the attenuation map. In this paper, we investigate this possibility. We first use Fisher information analysis to study the information content of LM SPECT emission data. We derive the terms of the Fisher information matrix (FIM) with respect to the activity and attenuation map coefficients. The inverse of the FIM gives the Cramér-Rao bound (CRB), which serves as the lower bound on the variance of any unbiased estimator of the activity and attenuation map coefficients that processes the LM data. Therefore, it helps us determine if the LM data contains information to reconstruct the activity and attenuation map. We then suggest a maximum-likelihood (ML) scheme that performs attenuation and scatter correction in SPECT using only the LM emission data, including the scattered-photon data. We develop an expectation-maximization (EM) routine to implement this ML

scheme. Using this EM routine, we obtain update equations using which the activity and attenuation map of the tissue can be simultaneously reconstructed.

2. THEORY

2.1 Likelihood of observed data and a path-based formalism to analyze scattered-photon data

In a SPECT imaging system, when the gamma ray emitted from the object interacts with the scintillator, then, using the measured output of the detector, the energy and position of interaction of the gamma ray photon with the crystal are estimated and recorded. Instead of binning this data, which results in large memory requirements and information loss due to discretization,^{31, 32} we store these attributes of each event in a LM format.^{31, 33–35} LM storage treats the data as a point in continuous measurement space, rather than as a count to a position and energy bin, and allows us to account for detector noise characteristics.^{36, 37}

To establish the notation, we describe our object $f(\mathbf{r})$ using the set of functions $\{\lambda(\mathbf{r}), \mu(\mathbf{r})\}$, where $\lambda(\mathbf{r})$ and $\mu(\mathbf{r})$ are functions that describe the activity and attenuation maps, respectively. We discretize the object in a voxel basis into M voxels, so that the unknown activity and attenuation maps are now a set of M -dimensional vectors $\{\boldsymbol{\lambda}, \boldsymbol{\mu}\}$. We denote the M unknown elements of the activity and attenuation map, by $\{\lambda_1, \dots, \lambda_M\}$ and $\{\mu_1, \dots, \mu_M\}$ respectively, where λ_i is the mean number of photons emitted from the i^{th} voxel per unit time, and μ_i is the attenuation coefficient in the i^{th} voxel. We assume that our system is preset-time so that the measurement time T is fixed, while the number of events J is a random variable. We would like to mention that our analysis can be easily extended to a preset-count system. We denote the estimated attributes of the j^{th} detected event, which includes the estimated energy and the estimated position of interaction of the gamma-ray photon with the scintillation crystal, by the attribute vector $\hat{A}_j$. Also, we denote the full LM dataset of estimated attributes as $\mathcal{A} = \{\hat{A}_j, j = 1, 2, \dots, J\}$. The likelihood of the observed data is given by

$$\text{pr}(\hat{\mathcal{A}}, J | \boldsymbol{\lambda}, \boldsymbol{\mu}) = \text{pr}(J | \boldsymbol{\lambda}, \boldsymbol{\mu}) \prod_{j=1}^J \text{pr}(\hat{A}_j | \boldsymbol{\lambda}, \boldsymbol{\mu}), \quad (1)$$

since the J detected events are independent of each other. Taking the logarithm on both sides of the resulting equation, we obtain the log-likelihood of the observed LM data, which we denote by $\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \hat{\mathcal{A}}, T)$, to be

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \hat{\mathcal{A}}, J) = \sum_{j=1}^J \log \text{pr}(\hat{A}_j | \boldsymbol{\lambda}, \boldsymbol{\mu}) + \log \text{pr}(J | \boldsymbol{\lambda}, \boldsymbol{\mu}). \quad (2)$$

Although we now have an expression for the log-likelihood, to compute the CRB, we should be able to differentiate this expression with respect to the activity and attenuation map coefficient values. This requires obtaining an analytic expression for $\text{pr}(\hat{A}_j | \boldsymbol{\lambda}, \boldsymbol{\mu})$ and $\text{pr}(J | \boldsymbol{\lambda}, \boldsymbol{\mu})$.

Let us define a parameter β which gives the mean arrival rate of photons on the detector. Then the number of measured counts J is a Poisson-distributed random variable with mean βT , so that

$$\text{pr}(J|\lambda, \mu) = \frac{(\beta T)^J \exp(-\beta T)}{J!}, \quad (3)$$

and we thus have an analytic expression for $\text{pr}(J|\lambda, \mu)$. Computing an analytic expression for $\text{pr}(\hat{A}_j|\lambda, \mu)$ is however much more complicated. To simplify this, we realize that each LM event is a result of a certain path $\mathbb{P}$ that is traveled by the gamma-ray photon. The concept of the path of the photon is illustrated in Fig. 2. A path $\mathbb{P}$ taken by a gamma-ray photon can be defined by a set of voxels $\{k_0, k_1, \dots, k_n\}$, where k_0 is the voxel from which the photon is emitted, and $k_1, k_2, \dots, k_n$ represent the voxels in which the photon suffers from scattering. Given the path of the photon, we can derive expressions for the probability of the LM event given the path, $\text{pr}(\hat{A}_j|\mathbb{P})$, and the probability of the path itself, $\text{Pr}(\mathbb{P}|\lambda, \mu)$. We thus decompose $\text{pr}(\hat{A}_j|\lambda, \mu)$ as a mixture model over all possible paths, where the components of the mixture are the probabilities that a LM event occurs given a path, and the weight of each component is the probability of the considered path:

$$\text{pr}(\hat{A}_j|\lambda, \mu) = \sum_{\mathbb{P}} \text{pr}(\hat{A}_j|\mathbb{P}) \text{Pr}(\mathbb{P}|\lambda, \mu), \quad (4)$$

where we have also used the fact that the probability of a LM event given the path is independent of the activity and attenuation map. Thus, using Eq. (2), we can rewrite the log-likelihood of the data given the activity and attenuation map in terms of this mixture-model decomposition as

$$\mathcal{L}(\lambda, \mu | \hat{\mathcal{A}}, T) = \sum_{j=1}^J \log \sum_{\mathbb{P}} \text{pr}(\hat{A}_j|\mathbb{P}) \text{Pr}(\mathbb{P}|\lambda, \mu) + \log \text{pr}(J|\lambda, \mu). \quad (5)$$

The probability of a path $\text{Pr}(\mathbb{P}|\lambda, \mu, J)$ can be described in terms of the mean rate of photon transmission through the considered path:

$$\text{Pr}(\mathbb{P}|\lambda, \mu) = \frac{\text{Mean rate of photons incident on detector through the considered path}}{\text{Mean rate of photons incident on detector}}. \quad (6)$$

To determine an expression for the mean rate of photon transport along a path, we use the radiative transport equation (RTE).^{38, 39} This equation accounts for the processes of emission of radiation from the source, the attenuation of the radiation as the photon propagates through the path, and the Compton scattering of the photons in the various voxels in the path. The differential cross section for the scattering process is given by the Klein-Nishina formula.² The final expression for the probability of the path $\text{Pr}(\mathbb{P}|\lambda, \mu)$ is quite large and complicated. To simplify the expression, we realize that in our analysis, we only have to consider the dependence of this expression on the emission and the attenuation map. Therefore, the quantities that are not dependent on these parameters in the expression are

separately denoted by $\Lambda(\mathbb{P})$. Using the RTE, the final expression for $\Pr(\mathbb{P}|\boldsymbol{\lambda}, \boldsymbol{\mu})$ can be derived to be

$$\Pr(\mathbb{P}|\boldsymbol{\lambda}, \boldsymbol{\mu}) = \frac{\lambda(\mathbb{P})s_{\text{eff}}(\mathbb{P})}{\beta}, \quad (7)$$

where $\lambda(\mathbb{P})$ denotes the activity in the starting voxel of the path $\mathbb{P}$, and where $s_{\text{eff}}(\mathbb{P})$ is given by

$$s_{\text{eff}}(\mathbb{P}) = \Lambda(\mathbb{P}) \exp \left[- \sum_{m=0}^n \gamma(k_m, k_{m+1}, \boldsymbol{\mu}^t) \right] \left[\prod_{m=1}^{n-1} \mu_{k_m} \right]. \quad (8)$$

The vectors $\mathbf{r}_l$ and $\mathbf{r}_m$ denote the center of the k_l^{th} and k_m^{th} voxels, respectively, and the function $\gamma(\mathbf{r}_l, \mathbf{r}_m, \boldsymbol{\mu})$ denotes the exponential path integral between $\mathbf{r}_l$ and $\mathbf{r}_m$ i.e.

$$\gamma(\mathbf{r}_l, \mathbf{r}_m, \boldsymbol{\mu}) = \int_0^{|\mathbf{r}_l - \mathbf{r}_m|} dt \mu \left(\mathbf{r}_l - t \frac{\mathbf{r}_l - \mathbf{r}_m}{|\mathbf{r}_l - \mathbf{r}_m|} \right). \quad (9)$$

Finally, the expression for β is given by

$$\beta = \sum_{\mathbb{P}'} \lambda(\mathbb{P}') s_{\text{eff}}(\mathbb{P}'). \quad (10)$$

2.2 Evaluating the Fisher information matrix

The expression for the FIM elements is given by

$$F_{ij} = - \left\langle \frac{\partial^2 \mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \hat{\mathcal{A}}, J)}{\partial \theta_i \partial \theta_j} \right\rangle_{(\hat{\mathcal{A}}, J | \boldsymbol{\lambda}, \boldsymbol{\mu})}, \quad (11)$$

where θ_i and θ_j denote the parameters we intend to estimate, and thus could be the activity and attenuation map coefficients in any of the voxels of the phantom, and where $\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \hat{\mathcal{A}}, J)$ is the log-likelihood of the observed LM data given by Eq. (5). Substituting the expression for $\Pr(\mathbb{P})$ from Eq. (7) into Eq. (5), and further using Eq. (3), we get

$$\mathcal{L}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \hat{\mathcal{A}}, J) = \sum_{j=1}^J \log \left[\sum_{\mathbb{P}} \Pr(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \right] + J \log \beta T - \beta T - \log(J!). \quad (12)$$

Using Eq. (12), we can derive the various terms of the FIM to be

$$\left\langle \frac{\partial^2 \mathcal{L}}{\partial \lambda_{q'} \partial \lambda_q} \right\rangle_{(\hat{\mathcal{A}}, J | \boldsymbol{\lambda}, \boldsymbol{\mu})} = - \left\langle \frac{\sum_{j=1}^J \{ \sum_{\mathbb{P}_q} \Pr(\hat{A}_j | \mathbb{P}) s_{\text{eff}}(\mathbb{P}) \} \{ \sum_{\mathbb{P}_{q'}} \Pr(\hat{A}_j | \mathbb{P}) s_{\text{eff}}(\mathbb{P}) \}}{\{ \sum_{\mathbb{P}} \Pr(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \}^2} \right\rangle_{(\hat{\mathcal{A}}, J | \boldsymbol{\lambda}, \boldsymbol{\mu})}, \quad (13)$$

$$\left\langle \frac{\partial^2 \mathcal{L}}{\partial \mu_{q'} \partial \mu_q} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)} = - \left\langle \sum_{j=1}^J \frac{\left\{ \sum_{\mathbb{P}} \text{Pr}(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \left[-\Delta_q(\mathbb{P}) + \frac{\zeta(\mathbb{P})}{\mu_q} \right] \right\} \left\{ \sum_{\mathbb{P}} \text{Pr}(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \left[-\Delta_{q'}(\mathbb{P}) + \frac{\zeta(\mathbb{P})}{\mu_{q'}} \right] \right\}}{\left\{ \sum_{\mathbb{P}} \text{Pr}(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \right\}^2} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)}, \quad (14)$$

$$\left\langle \frac{\partial^2 \mathcal{L}}{\partial \mu_{q'} \partial \lambda_q} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)} = - \left\langle \sum_{j=1}^J \frac{\left\{ \sum_{\mathbb{P}_q} \text{Pr}(\hat{A}_j | \mathbb{P}) s_{\text{eff}}(\mathbb{P}) \right\} \left\{ \sum_{\mathbb{P}} \text{Pr}(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \left[-\Delta_{q'}(\mathbb{P}) + \frac{\zeta(\mathbb{P})}{\mu_{q'}} \right] \right\}}{\left\{ \sum_{\mathbb{P}} \text{Pr}(\hat{A}_j | \mathbb{P}) \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) \right\}^2} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)}, \quad (15)$$

$$\left\langle \frac{\partial^2 \mathcal{L}}{\partial \lambda_{q'} \partial \mu_q} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)} = \left\langle \frac{\partial^2 \mathcal{L}}{\partial \mu_q \partial \lambda_{q'}} \right\rangle_{(\mathcal{A}, J | \lambda, \mu)}, \quad (16)$$

where the sum over $\mathbb{P}_q$ indicates that we are summing up only over those paths that start from voxel q , and where $\zeta_q(\mathbb{P})$ and $\Delta_q(\mathbb{P})$ are the number of scatter events occurring in the q^{th} voxel in the considered path and the distance that the considered path covers in the q^{th} voxel, respectively. Using Eq. (11) and (13)–(16), the elements of the FIM with respect to the activity and attenuation map can be obtained. Since we cannot simplify the resultant expressions further, we use Monte Carlo integration to evaluate these expressions. After we compute all the elements of the FIM, we can take the inverse of this matrix to obtain the CRB on the activity and attenuation map estimates. Thus, using these expressions, we can study the fundamental limits of information-retrieval capacity from LM SPECT emission data with respect to the reconstruction task for different kind of phantoms.

2.3 An EM algorithm to reconstruct the activity and attenuation map

Our reconstruction approach is to estimate those values of activity and attenuation map coefficients that maximizes the probability of the observed LM data, or alternatively maximizes the objective functional given by Eq. (2). Since the activity and attenuation maps are high-dimensional vectors, a optimization-based technique will be considerably slow. However, using the concept of the path of a photon defined earlier, we can derive an EM approach to maximize this objective functional. Using the path-based formalism defined in Sec. 2.1, each LM event is a result of a certain path that is traveled by the gamma-ray photon. However, we do not know this path, so for each LM event and each possible path, we define a hidden random variable $z_{\mathbb{P},j}$ that quantifies whether the path $\mathbb{P}$ was taken for the j^{th} LM event. This hidden variable for the j^{th} LM event and path $\mathbb{P}$ is given by

$$z_{\mathbb{P},j} = \begin{cases} 1, & \text{if event } j \text{ originated in voxel } k_0 \text{ and scattered in voxels } k_1, \dots, k_n \text{ in that order.} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

For notational simplicity, let us denote all possible paths by the vector $\mathcal{Z}$. In the EM approach, we treat the variable $z_{\mathbb{P},j}$ as the unobserved data corresponding to the j^{th} observed

LM event. Our objective then is to maximize the probability of both the observed and the unobserved data, i.e. the log-likelihood function $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \mathcal{A}, J, \mathcal{Z})$. Using Eq. (3), (4) and (17) we can derive the expression for this log-likelihood to be

$$\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \mathcal{A}, J, \mathcal{Z}) = \sum_{j=1}^J \left\{ \sum_{\mathbb{P}} z_{\mathbb{P},j} \left[\log \text{pr}(\hat{A}_j | \mathbb{P}) + \log \text{pr}(\mathbb{P} | \boldsymbol{\lambda}, \boldsymbol{\mu}, J) \right] \right\} + J \log \beta T - \beta T - \log(J!). \quad (18)$$

In the expectation step, we find the expected value of the likelihood $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \mathcal{A}, \mathcal{Z}, T)$, averaged over the hidden variables $z_{\mathbb{P},j}$ given the estimated attributes $\mathcal{A}$, number of counts J , and for a fixed $\{\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t\}$. Conventionally, this expected likelihood is denoted by $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)$. Since Eq. (18) is linear in $z_{\mathbb{P},j}$, the averaging is equivalent to replacing $z_{\mathbb{P},j}$ by its expected value $\bar{z}_{\mathbb{P},j}$. This expected value can be derived to be

$$\bar{z}_{\mathbb{P},j}(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t) = \frac{\text{pr}(\hat{A}_j | \mathbb{P}, \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t) \text{Pr}(\mathbb{P} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)}{\sum_{\mathbb{P}'} \text{pr}(\hat{A}_j | \mathbb{P}', \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t) \text{Pr}(\mathbb{P}' | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)}. \quad (19)$$

Using Eqs. (7) and (18), the function $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)$ can be derived to be

$$\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t) = \sum_{j=1}^J \left\{ \sum_{\mathbb{P}} \bar{z}_{\mathbb{P},j} \left[\log \text{pr}(\hat{A}_j | \mathbb{P}) + \log \lambda(\mathbb{P}) - \sum_{m=0}^n \gamma(k_m, k_{m+1}, \boldsymbol{\mu}) + \sum_{m=1}^{n-1} \mu_{k_m} + \log \Lambda(\mathbb{P}) \right] \right\} + J \log T - \beta T - \log(J!). \quad (20)$$

In the maximization step of the EM algorithm, we find those values of $(\boldsymbol{\lambda}, \boldsymbol{\mu})$ that maximize the function $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)$. To find the q^{th} element of the activity map, we take the derivative of $\mathcal{Q}(\boldsymbol{\lambda}, \boldsymbol{\mu} | \boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)$ with respect to λ_q . Equating the differentiation to zero gives the iterative update equation for λ_q :

$$\lambda_q^{t+1} = \frac{\sum_j \sum_{\mathbb{P}_q} \bar{z}_{\mathbb{P},j}(\boldsymbol{\lambda}^t, \boldsymbol{\mu}^t)}{T \sum_{\mathbb{P}_q} s_{\text{eff}}(\mathbb{P})}. \quad (21)$$

where the summation in Eq. (21) is only over the paths that start from voxel q , which we denote by $\mathbb{P}_q$. We now present a physical interpretation of this equation. The sum over paths in the numerator in the update equation is the probability that the j^{th} LM event originated from voxel q . Since the j LM events are independent of each other, summing up these probabilities over j gives a measure of the number of times the event from voxel q contributed to the total LM output. The sum in the denominator in the update equation is the effective sensitivity function for all possible paths that start from voxel q , and it considers the attenuation that each of these paths have to suffer. The numerator term when divided by the total measurement time T gives the mean rate of contribution of the q^{th} voxel to the total output. This term when further divided by $s_{\text{eff}}(\mathbb{P})$ accounts for the sensitivity to give a measure of emission rate occurring from the q^{th} voxel.

If we assume that the medium has zero attenuation and the system has the same sensitivity s_q to all the paths that start from voxel q , then the update equation for λ_q is obtained by simplifying Eq. (21) to be

$$\lambda_q^{t+1} = \frac{\sum_j \sum_{\mathbb{P}_q} \bar{z}_{\mathbb{P},j}(\lambda^t, \mu^t)}{T s_q}. \quad (22)$$

This is the same as the update equation that was obtained under the assumption of a zero-attenuation medium in Parra et al.³³ Thus our general expression simplifies to the well-known specific expression when there is no attenuation.

Similarly, to find the update equation for the attenuation coefficient in the q^{th} voxel, we maximize the expected log-likelihood $\mathcal{Q}(\lambda, \mu | \lambda^t, \mu^t)$ given by Eq. (20) with respect to μ_q , which leads to:

$$\mu_q^{t+1} = \frac{\sum_{\mathbb{P}} [T \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) - \sum_j \bar{z}_{\mathbb{P},j}] \zeta_q(\mathbb{P})}{\sum_{\mathbb{P}} [T \lambda(\mathbb{P}) s_{\text{eff}}(\mathbb{P}) - \sum_j \bar{z}_{\mathbb{P},j}] \Delta_q(\mathbb{P})}. \quad (23)$$

With these updated activity and attenuation map coefficients, the expectation step is repeated. This iterative process is continued until convergence is achieved.

3. CONCLUSIONS

In this paper, we have studied the problem of joint reconstruction of activity and attenuation map using only the SPECT emission data acquired in LM format. Our study attempts to retrieve information from the often-discarded scattered-photon data in SPECT imaging. We have proposed a path-based formalism to study the information content of LM scattered photon data. We have then derived analytic expressions for the FIM, and thus, can compute the CRB of the activity and attenuation map coefficients that can be estimated using LM SPECT emission data. We have also developed a ML scheme to perform the joint reconstruction of activity and attenuation map using LM SPECT emission data, and followed it with developing an EM algorithm to execute the ML scheme. The EM algorithm considers the path taken by the photon as unobserved data and attempts to maximize the likelihood for a combination of the observed and unobserved data. The algorithm leads to a set of update equations for the activity and attenuation coefficients of the different voxels in the phantom. The method, due to its very design, performs simultaneous attenuation and scatter correction. We have developed software to compute the CRB and to implement the MLEM algorithm for a 2-dimensional (2-D) SPECT imaging system. We are currently evaluating the CRB for different kind of 2-D phantoms. In future, we are interested in developing the software further to study 3-D phantoms. We are also interested in exploring the use of prior information to perform the attenuation map reconstruction.

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Figure 1.

Classification of the current approaches for transmission-less SPECT image reconstruction

Figure 2.

A schematic showing the various paths that a emitted gamma-ray photon can take.