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Algebraic Reconstruction Technique (ART) for parallel imaging reconstruction of undersampled radial data: Application to cardiac cine

Shu Li^{1,2}, Cheong Chan¹, Jason P. Stockmann¹, Hemant Tagare¹, Ganesh Adluru³, Leo K. Tam¹, Gigi Galiana¹, R. Todd Constable¹, Sebastian Kozerke², and Dana C. Peters¹

¹Department of Radiology, Yale Medical School, New Haven, CT ²Institute for Biomedical Engineering, University and ETH Zurich, Switzerland ³Electrical and Computer Engineering Department, University of Utah, Salt Lake City, Utah

Abstract

Purpose—To investigate algebraic reconstruction technique (ART) for parallel imaging reconstruction of radial data, applied to accelerated cardiac cine.

Methods—A GPU-accelerated ART reconstruction was implemented and applied to simulations, point spread functions (PSF) and in twelve subjects imaged with radial cardiac cine acquisitions. Cine images were reconstructed with radial ART at multiple undersampling levels ($192 N_r \times N_p = 96$ to 16). Images were qualitatively and quantitatively analyzed for sharpness and artifacts, and compared to filtered back-projection (FBP), and conjugate gradient SENSE (CG SENSE).

Results—Radial ART provided reduced artifacts and mainly preserved spatial resolution, for both simulations and in vivo data. Artifacts were qualitatively and quantitatively less with ART than FBP using 48, 32, and 24 N_p , although FBP provided quantitatively sharper images at undersampling levels of 48–24 N_p (all $p < 0.05$). Use of undersampled radial data for generating auto-calibrated coil-sensitivity profiles resulted in slightly reduced quality. ART was comparable to CG SENSE. GPU-acceleration increased ART reconstruction speed 15-fold, with little impact on the images.

Conclusion—GPU-accelerated ART is an alternative approach to image reconstruction for parallel radial MR imaging, providing reduced artifacts while mainly maintaining sharpness compared to FBP, as shown by its first application in cardiac studies.

Keywords

radial; projection reconstruction; parallel imaging; algebraic reconstruction technique; cardiac function

Introduction

Undersampled radial k-space trajectories, unlike Cartesian trajectories, have excellent undersampling properties for numerous applications (1–4). Parallel imaging presents computational challenges for radial sampling. Very early on Pruessmann et al. (5) suggested the use of conjugate gradients (CG) to iteratively enforce data consistency, including coil sensitivities (CG SENSE). Other solutions and refinements have also been developed, such as radial kt-SENSE (6), radial GRAPPA (7–9) and further improvements in CG SENSE reconstructions (10, 11). These parallel imaging methods have succeeded in reducing radial streak artifacts in cardiac cine applications with as few as 12 projections.

The outcome of parallel imaging reconstruction of radial data differs from that of Cartesian data. Undersampling artifacts for radial are streak-like, and less dramatic than for undersampled Cartesian. Therefore, improvements from parallel imaging are less clear—because successful implementation results in elimination of diffuse streaks, instead of wholesale replications. In the absence of parallel imaging, Cartesian images with accelerations (R) of $R=2$ to 3 have significant artifacts, while radial images with $R=2-3$ have almost invisible artifacts. (Note that in this work we use R -values referenced to Cartesian sampling for simplicity ($R=N_r/N_p$), while the true undersampling factor for radial is $R_{\text{radial}} = (\pi/2) \cdot (N_r/N_p)$, with N_r the number of readout points, and N_p the number of projections.) Furthermore, iterative radial methods are prone to result in a blurry image, because for undersampled radial, in contrast to what is possible for undersampled Cartesian, artifacts can be reduced by low-pass filtering of k-space (12) (i.e. blurring). Radial imaging's unique sampling pattern results in aliasing of spatial frequencies $k_r > k_{r\text{max}}/R_{\text{radial}}$ (13). Therefore, all aliasing is due to “high spatial frequencies” and the challenge is to simultaneously retain high spatial frequency information (preserve sharpness) and recover missing high spatial frequency data (remove aliasing).

Here we explore a method recently presented for non-linear encoding MRI referred to as algebraic reconstruction technique (ART) (14, 15). ART is a very early iterative image-domain method which also solves the data consistency equation, similar to other iterative approaches (5, 6, 11, 16–18). One distinction lies in not using gridding, inverse gridding, FFTs or non-uniform FFTs during reconstruction of non-Cartesian data. This is a unique quality, although Wajer et al (19) proposed a CG SENSE speed-up which partly reduced the use of gridding. This distinction significantly simplifies the conceptualization and implementation of ART vs. iterative gridding-based methods. A second distinction is that the projection onto convex sets (POCS) method is used (20, 21) to find the optimal solution, instead of conjugate gradients. ART has been extensively employed for reconstructing CT projection data. There exist only a few reports applying ART to MRI (spiral and EPI) (22–24), and these did not employ it for parallel imaging reconstruction or include coils within the ART formulation.

The use of ART with coil-map constraints for parallel MRI has only recently been described (25). In that work, Stockmann et al. used ART to reconstruct simulated and true 8-channel undersampled radial data, including in excellent vivo images of a human hand (26). They found that incorporating the coil information improves the reconstruction. ART enforces

data consistency including coil-sensitivities which serve as part of the MR encodings. Therefore our goal is to demonstrate its promise as a parallel imaging reconstruction method for in vivo radial imaging and characterize its performance in simulations and cardiac imaging studies. We apply the reconstruction methods to point spread function (PSF) analysis, simulated data, and in vivo undersampled radial imaging of cardiac function. ART reconstructions were compared with filtered back projection (FBP) in order to assess the improvements in image quality with ART vs. conventional radial reconstructions performed without coil information, for cine cardiac imaging. We demonstrate similar parallel imaging capabilities of ART compared to CG SENSE.

Methods

ART Reconstruction

ART refers to a large class of image-domain reconstruction methods, of which our study used Kaczmarz (15), while the GPU-accelerated version described below is similar to SART (Simultaneous Algebraic Reconstruction Technique) (27, 28). It has been previously outlined (2) for MRI and is shown schematically in Figure 1.

For radial, the measured data consists of N_c (number of coils) $\times$ N_r (number of readout points) $\times$ N_p (number of projections) raw k-space points. During the spatial encoding process, the true image, $m(x,y)$, is modulated by each encoding matrix. For multi-channel radial imaging, the encoding matrix is (see Figure 1, top panel):

$$E(x,y)_{n,k_r,\theta} = C(x,y)_n \cdot \exp(i2\pi k_r (\cos(\theta) \cdot x + \sin(\theta) \cdot y)) \quad [1]$$

Where $C(x,y)_n$ is the coil-sensitivity for the n^{th} coil, the projection angle is θ , and k_r is the readout k-space value. The corresponding measured signal is the k-space signal for that encoding (here shown as summation instead of integral):

$$S_{n,k_r,\theta} = \sum_{x,y} E(x,y)_{n,k_r,\theta} \cdot m(x,y) \quad [2]$$

Equation 1 expresses the spatial modulation of the MR signal during collection of a single k-space point, due to gradient and coil encoding. Equation 2 expresses the resulting signal acquired during MR measurement. The total number of both encoding matrices applied and raw k-space signals collected is $N_c \times N_p \times N_r$. If i denotes a particular combination of n , k_r and θ (i.e. a particular k-space point, from a particular applied encoding, as measured by a certain coil), then we can denote the encoding matrix as $E_i = E(x,y)_i$, and corresponding k-space data s_i . Equation 2 shows how each E_i is associated with a single s_i . Both vary with n , k_r and θ . E_i is a matrix defined on the image FOV ($N_x \times N_y$), while s_i is a scalar.

The goal is to obtain the true image, $m(x,y)$, from the known encoding matrices, $E(x,y)_i$, and corresponding known k-space data, s_i . The initial image $m^0 = m(x,y)^0$ is zero. Each measured k-space value, s_i , is sequentially processed in turn, with the whole data set processed multiple times. In the k^{th} step, the predicted value of the i^{th} k-space data point is

$s_i^k = \sum_{x,y} E(x,y)_i \cdot m(x,y)^k$ (see Figure 1, middle box). This is compared with the true raw signal, s_i . The residual is transformed through multiplication by E_i^* into image space information and added to the current estimate, m^k , to generate a new image estimate, m^{k+1} , which is closer to the true image. Specifically, the updated image is calculated with the following equation (see Figure 1):

$$m^{k+1} = m^k + \lambda \cdot \left[\frac{s_i - s_i^k}{\|E_i\|^2} \right] \cdot E_i^* \quad [3]$$

Where $\|E_i\|^2$ is a normalization factor and λ is the relaxation parameter which slows convergence but improves accuracy in the presence of data inconsistencies. In each iteration, the residual between the estimated k-space value, s_i^k , and the true value s_i , is calculated. If the current image estimate is good, the residual is small, so a small correction is made, while if the image estimate is poor, the residual is large, and a large correction is made. For ART, each encoding is processed N_{Loop} times, typically 8 to 20. Therefore the step of Eq. 3 (indexed by counter k) is performed $N_c \cdot N_r \cdot N_p \cdot N_{Loop}$ times, while the counter i increments from 1 to $N_c \cdot N_r \cdot N_p$, and does so N_{Loop} times. Therefore, in our implementation, $k = i + N_{iter} \cdot (N_c \cdot N_r \cdot N_p)$ and $i = k \% (N_c \cdot N_r \cdot N_p)$, with N_{iter} the current loop counter ranging from 0 to $N_{Loop} - 1$, indicating the current number of loops performed. In this paper, the “number of iterations” refers to N_{Loop} . Theoretically, ART converges for $0 < \lambda < 1$, but our reconstructions used $\lambda = 0.08$, which performed well in our experience (see below).

It's important to note that no gridding, inverse gridding, density compensation, or Fourier transforms are performed in image-based ART. The steps of gridding and FFT are replaced by the weighting each encoding matrix, E_i^* , by its corresponding k-space signal residual and accumulating the result into an image. Note also that it is the inclusion of the coil sensitivity profiles, $C(x,y)_n$, into the encoding matrix that provides the parallel imaging reconstruction.

GPU-accelerated reconstruction

Image reconstruction was performed in Matlab R2012a (Mathworks, Natick, MA USA), and was GPU-accelerated with a Nvidia GeForce 580. The GPU acceleration was achieved by modifying our serial ART method, by simultaneously processing $N_{||}$ k-space points (of the $N_c \cdot N_r \cdot N_p$ k-space data points), without update of m^k after each step. Typically $N_{||} = 384$ data points –that is, a single (2-fold oversampled) frequency-encoding line for a 192 readout. Each data point generates an update to the image estimate, and the updates are added together, with the image itself only updated after processing of $N_{||}$ points (Equation 4).

$$m^{k+N_{||}} = m^k + \sum_{i=i_o}^{i=i_o+N_{||}} \lambda \cdot \frac{s_i - s_i^k}{\|E_i\|^2} \cdot E_i^* \quad [4]$$

Where s_i , E_i and s_i^k were defined above, and k increments in the same way as in Eq. 3, in jumps of $N_{||}$. The summation sign indicates that sections ($N_{||}$ points) of the total acquired raw data are processed in parallel, with a stale image estimate m^k (used in calculating s_i^k). Again

i increments to span $(N_c \cdot N_r \cdot N_p)$, and does so N_{Loop} times, and likewise i_0 spans 1 to $(N_c \cdot N_r \cdot N_p)$, in jumps of $N_{\parallel}$. Equation 4 is identical to Eq. 3, except for this intermittent updating of $\mathbf{m}^k$. After parallel processing, the image is updated. Due to intermittent updates of the image, the method is now similar to SART, which has been shown for CT data to converge to a less noisy solution than the ART (i.e. non-GPU-accelerated) method (27, 29, 30). We compared the outcomes of reconstruction using CPU-based ART, our original method (25) and the GPU-accelerated method, and found that GPU-acceleration did not greatly affect reconstruction results (see below), but greatly decreased reconstruction time.

In Vivo MR Imaging

Imaging was performed in twelve healthy subjects (4 female, average age=39), providing written informed consent, as approved by our IRB. A cardiac phased-array coil was used with 4–5 elements. Short axis cardiac slices were acquired on a 1.5T Siemens Sonata (Erlangen, Germany), using segmented breath-held 2D radial balanced SSFP with 192 readout points (N_r) (384 acquired), and 192 N_p . Scan parameters included TR/TE/ θ = 2.9ms/1.5ms/60°, 360 cm FOV, 930 Hz/pixel, 1.9 x 1.9 x 8 mm³ spatial resolution. One additional subject was scanned on a 3T scanner (Siemens's Trio) using a 32-element cardiac coil (Invivo Corp., Gainesville, FL), with identical scan parameters, except: 1532 Hz/pixel, TR/TE/ θ = 3.2ms/1.5ms/50°. A center-frequency prescan was used to find the optimal center-frequency.

Images were reconstructed (to a 192 x 192 matrix) using FBP and ART, by subsampling the projections, with accelerations $R=2, 4, 6, 8$, and 12 (corresponding to $N_p=96, 48, 32, 24, 16$). Note that $R_{\text{radial}}=3.1, 6.3, 9.42, 12.4$, and 18.6. FBP was performed with a Ram-Lak filter, and coils were combined with square-root of sum-of-squares. ART was performed with $\lambda=0.08$ and 18 iterations ($N_{\text{Loop}}=18$), for all undersampling levels. Coil sensitivities were calculated using the undersampled radial coil data, except for the 3T data which used coil-maps derived from fully sampled data. Coil maps were calculated by filtered back projections (FBP) of low-pass filtered data. The low-pass filter was a Hanning window with full-width half maximum set to $N_p/2$ points. Then coil sensitivity profiles were generated using the adaptive approach (31, 32). A first order trajectory delay correction was applied by globally shifting each k-space spoke by an optimal subpixel amount, determined by visual assessment of gradient delay artifacts at shifts between 2 to -2, with a 0.25 pixel step-size.

Qualitative and Quantitative Image Analysis

Images were evaluated in randomized order by a single highly experienced reader blinded to the reconstruction type and undersampling level. In all in vivo data sets, the diastolic images reconstructed with each acceleration factor R were subjectively graded on sharpness/details (1–3, 1=obscured details, 2=some details blurred/obscured, 3=sharp) and artifacts (1–5, 0=cardiac structures non-diagnostic due to artifacts, 1=artifacts visible in cardiac anatomy, 2=artifacts visible but not in cardiac structures, 3=artifacts outside of body, 4=no artifacts visible). In all data sets, the SNR was measured as the mean signal of standardized circular ROI (radius=30 pixels) placed in the LV cavity, chosen to exclude papillary muscles, divided by the standard deviation (SD) of signal in this ROI. For a particular subject, the ROI location was constant for each reconstruction, and among subjects the ROI was in a

similar region. This was intended to characterize the increasing SD due to artifacts. In all data sets, sharpness was measured across a line segment drawn across the LV septal wall, with sharpness measured as the signal change in the location of the peak signal change in the corresponding fully sampled image. The statistical significance of the results were determined using two-way ANOVA to test for significance with multiple comparisons, and results of paired student's t-test were Bonferroni corrected by a factor of 4, since 4 tests were performed, ART vs. FBP, at $N_p = 48, 32, 24$ and 16.

Point Spread Function (PSF) Simulations and Analysis

PSFs were simulated, using a noise-free input image with a single bright pixel, with an off-center location ($N/4, N/4$). The input image was then undersampled with multiple undersampling factors, and reconstructed with ART using a single coil, or 8 coils, with a stopping criteria of $< 0.05\%$ change in the norm of image between iterations, or 100 iterations, in order to minimize convergence effects. For input into simulations, coil sensitivities were used as measured on our 8-channel head coil array with circumferentially arranged surface coils (25). PSF's at each undersampling level were analyzed for sharpness and artifact power. Sharpness was defined as the average gradient in signal between the central point location and its 8 nearest neighbors, normalized by the peak signal. Artifact signal was calculated as the ratio of the signal of the central point to the sum of all other signals (artifacts) in the image.

Comparison of ART and CG SENSE

ART was compared directly with NUFFT-based CG SENSE. The CG SENSE software was obtained from an online toolbox (33) (34) and was not further optimized. Pre-whitening and regularization were not performed with CG SENSE (or ART). The toolbox used the LSQR function in Matlab as a "conjugate gradient type" method (35) for minimizing the data fidelity term that includes coil-sensitivities. The raw data, and forward and inverse NUFFT (obtained from the NUFFT online toolbox (36, 37)) were used as inputs. The NUFFT used an oversampling factor of 2, and an interpolation neighborhood of 5, and the default "minmax:kb" interpolator. Simulations were performed in a phantom with 128 N_r (readout) and 8 coils, comparing ART with CG SENSE. Uncorrelated Gaussian noise was added to each simulated line of k-space, for each coil, to achieve a SNR of 42 in the large intermediate intensity region, for the fully sampled case. In-vivo data was also processed, for one subject. Coil sensitivity data for both methods was identical, synthesized using the fully-sampled input data.

Convergence

The convergence behavior of ART was studied, based on the image estimate after each iteration. We calculated true RMS error: the norm of the image estimate and the fully sampled image (5) (38). We also measured convergence error (5): the norm of the difference between predicted and acquired k-space data points. The noise was added, as described above, to generate a 128x 128, 8 coil image with SNR of 42 in the large intermediate intensity region. The convergence of radial ART for different undersampling factors, and different λ values was investigated. The convergence for ART was compared to CG SENSE, for 128 x 32 N_p , and 8 coils.

Results

Cardiac cine imaging

Cardiac cine images from a representative subject are shown in Figure 2. FBP (top) and ART (bottom, $N_{\text{Loop}}=18$ iterations) images are compared at $N_p = 96, 48, 32$ and 24 . ART provides reduced artifacts and clearer delineation of the epicardial free wall than FBP at 48 and $24 N_p$, although blurring is evident at $24 N_p$.

The resulting images for all subjects were analyzed quantitatively (Figure 3A–B) and qualitatively (Figure 3C–D) to assess artifacts and sharpness, showing that ART outperformed FBP in reducing artifacts. SNR (a measure of artifact presence) was significantly higher with ART vs. FBP for $N_p = 48, 32, 24$, and 16 (all $p < 0.05$), after Bonferroni correction. ART provided reduced artifacts, compared qualitatively, for $N_p = 32, 24$ and 16 (all $p < 0.05$, Bonferroni corrected). With FBP, sharpness is relatively unchanged, except for the highest undersampling, where it is erratic, likely due to artifacts. For ART, sharpness trended downward. FBP provided greater sharpness than ART, compared quantitatively, for $N_p = 48, 16$ and 24 ($p < 0.05$, Bonferroni corrected), but with no significant qualitative differences in sharpness.

Figure 4 shows radial ART (96 vs. $16 N_p$) using the 3T 32-channel system, demonstrating very good quality at very high undersampling rates.

Coil-sensitivity maps derived from undersampled data

Even using highly undersampled, low-pass filtered data, coil-sensitivities were reasonably well characterized, as seen in Figure 5, which compares coil sensitivity maps from a fully sampled data set, vs. undersampled data sets ($N_p = 24$). The coil-sensitivity maps are visually similar, providing confidence that undersampled data sets can be used for coil-map generation. Both maps exhibit some errors due to gradient delays, and streaks are seen in the undersampled coil map. The images reconstructed with undersampled coil maps exhibit slightly reduced quality by visual assessment, with a maximal and mean signal difference of 16% and 1.7% . Visual inspection shows that reconstruction utilizing undersampled data for coil maps results in slightly more artifacts and less detail, indicating that coil-sensitivity quality is important. Figure 5F shows the ART reconstruction without use of coil-sensitivities (i.e. no parallel imaging), with each coil data set reconstructed individually without coil estimates, and combined by sum of squares. The result is clearly less sharp, indicating the value of parallel imaging ART.

PSF Simulations

The PSF analysis illustrates the parallel imaging reconstruction capabilities of ART. The PSF for a point located at $(N/4, N/4)$ with 1 uniform coil (Fig. 6A) and an circular array of 8 coils (Fig. 6B) are shown for the ART technique (8 iterations, $R=4$, $32 N_p \times 128 N_r$), demonstrating the reduction in both blurring and artifacts with additional coils. For a point object, ART with coil information reduces the aliasing artifacts generated by the point, for aliasing located sufficiently distant from the point. Note the most proximal artifacts remain, which represents the aliasing of the very highest spatial frequencies. We note that this

residual artifact is progressively removed with increasing iterations. For ART, the sharpness and artifact signal are plotted vs. undersampling level, for 1 and 8 coils (Figure 6C–D). ART provides increasing spatial resolution and reduced artifacts with more coils and higher N_p .

Comparison of ART and CG SENSE

Comparison shows that ART and CG SENSE perform similarly in simulations (Figure 7A–B). For 32 N_p simulated data, the resulting images are similar, although more structure is apparent in the ART image. For In-vivo data, ART appears to perform slightly better, based on visual inspection (Figure 7C–E).

GPU-accelerated reconstruction

Reconstruction time (5 coils, 1 iteration, $192 \times 48 N_p$) for GPU-accelerated ART was about 10 seconds, approximately 15x faster than without GPU-acceleration. Figure 8 shows in vivo images reconstructed with $192 \times 48 N_p$, with both ART (on the CPU) and GPU-accelerated ART, demonstrating minimal differences in the images. The average difference was 0.3%, and the peak difference was 3%, which was on the order of the convergence stopping criteria (1%). Note the good quality at 48 N_p ($R=4$).

Convergence of ART

The true error and convergence error of GPU-accelerated ART for noisy simulation data is shown in Figure 9, for different undersampling levels and different λ values. The plots show that a λ of about 0.05–0.01 is optimal in the presence of noise, and 18 iterations suffice for convergence for each undersampling level. The true RMS error and convergence error for CG SENSE are also shown in Figure 9(A,C), for $N_p=32$. CG SENSE appears to converge more rapidly to the true image, but then diverge, compared to ART, which is stable at lower λ . The minimal value for true error is similar for both methods.

Discussion

ART provides a parallel imaging reconstruction for radial MRI, reducing artifacts, and maintaining sharpness, and this has an impact on visualization of anatomy. Compared to Cartesian imaging, improvements are not as dramatic, but they are noticeable and this is particularly true for high acceleration rates. In vivo imaging demonstrates that at $R=4$, there was a significant reduction in streaks with ART compared with FBP. SNR measurements demonstrate reduced artifacts with ART. However, SNR is spatially dependent in these images due to parallel imaging and due to the influence of artifacts on the SD measurement. Therefore, other regions might show different trends. Sharpness was similar for FBP vs. ART qualitatively, although FBP provided significantly greater sharpness at $N_p=48$ to 24, measured quantitatively. Preserved sharpness is important clinically, and iterative solutions of radial tend to converge to a blurry solution, since this reduces streaking artifacts (12). Indeed, our study finds that ART converges to an unsharp solution at high accelerations (e.g. $R>8$) with 4 coils.

PSF simulations showed that ART was able to preserve sharpness and remove artifacts through parallel imaging. However, these noiseless simulations demonstrate that even under

ideal noise-free conditions, the highest spatial frequency information –for which undersampling factor is greatest--is only synthesized with increasing iterations. Furthermore, the PSF results of radial ART with parallel imaging will depend on the location of the point in the FOV, similar to Cartesian SENSE.

The convergence plots demonstrate that, similar to other iterative approaches (5, 39) convergence depends on undersampling factor. At lower λ , convergence by ART is somewhat slower than CG SENSE but possibly more stable, perhaps because ART provides a POCS-type method of convergence (21). With a high value of λ , a low number of iterations is sufficient for an accurate image, but the image rapidly diverges with increasing iterations.

Iterative reconstruction techniques in general are slow, and this is especially true of ART. Sullivan (40) showed with a step by step analysis that gridding is an $O(N^2 \log(N))$ operation, while filtered back projection is $O(N^3)$, for a $N \times N$ matrix. CG SENSE has been reported to be an $O(N^2 \log(N))$ (5), similar to gridding. Our non-GPU-accelerated ART method had a complexity on the order of $O(N_p \cdot N_c \cdot N^3)$ per iteration, with N -dependence similar to what has been reported by others (41). This $\sim O(N^4)$ complexity (assuming $N_p \sim N$) is likely the major reason that ART has not been commonly applied in radial MRI. However, we have demonstrated a GPU acceleration that makes ART feasible—at least more feasible-- for clinical use. Further speed-ups are likely possible, including use of methods to speed up convergence, and improved use of the GPU.

GPU-acceleration increased reconstruction speed by a factor of 15. We employed GPUs using a simple approach which processed many k-space data points in parallel before updating the image estimate. The resulting images were very similar. For CT data, it is known that the ART algorithm, modified to update the image estimate only intermittently (SART), does change the resulting image slightly(29). However, our method does not process rays as in CT, but rather k-space data. Since central k-space contributes most significantly, ART and SART are likely to be very similar in our implementation, since the image is only updated significantly by the central k-space data. GPU-accelerated reconstructions have been presented for other iterative methods (42, 43), especially conjugate gradients (34, 44, 45).

Further investigations and optimization of the method for obtaining coil-sensitivity maps is important. Our preliminary investigation shows that using undersampled data to generate coil-maps does moderately reduce the quality of the resulting images. Multiphase imaging with temporally interleaved projection sets would reduce such effects by providing more k-space data for coil-map generation. Other improvements that would improve auto-calibrated radial coil-maps include reducing gradient delay errors (46–48).

We have shown a comparison of ART with CG SENSE. The results demonstrate that the quality of ART is highly comparable to the gridding method, with slightly poorer phantom reconstructions, and slightly better in vivo imaging reconstructions. The results are encouraging –ART is similar, and potentially has advantages in vivo compared to the gold

standard. We have also shown that the λ slows convergence but stabilizes ART vs. CG SENSE.

Prior work and future applications

Early parallel imaging radial studies include coronary imaging with 3-fold ($126 N_p$) acceleration using a 5-element array (11). Radial GRAPPA (7) has demonstrated good quality with a 12-channel coil and as few as $16 N_p$, applied to real-time cardiac cine, using a highly averaged GRAPPA kernel calculation. Good real-time cine results have also been obtained with a 5 channel coil and as few as $16 N_p$ (6) using radial kt-SENSE which combines parallel imaging with kt-BLAST. Parallel imaging radial has been combined with CS, generating good radial images of the brain with only 32 projections (10) and 12 coils.

Our study demonstrates that radial ART—without CS or temporal filtering—provides good quality at $48\text{--}24 N_p$, and even more promising results with the 32-channel coil. Accelerated radial using the ART reconstruction will excel in interventional MRI, real-time cardiac function, multi-slice perfusion, coronary imaging, and MR angiography (49–53) i.e. any application which requires high resolution, rapid imaging, with high baseline SNR.

The good results obtained here for undersampled radial cardiac cine were tremendously improved by using a higher number of receiver channels (i.e. 32 vs. 5), and further investigation is highly warranted. Temporal undersampling and processing using radial kt-BLAST (6, 17) or radial UNFOLD (44, 53, 54) will further improve the images, and it is possible to incorporate these schemes, as well as CS (52, 55, 56), into ART. Radial ART may be used with these techniques to further improve image quality.

Conclusions

ART is an alternative approach to parallel imaging for radial, which enforces data consistency, including coil sensitivities. The GPU-accelerated method results in a more clinically feasible reconstruction. ART improves the quality of undersampled radial cine images, compared to standard FBP, and provides similar quality compared with CG SENSE, as shown by its first application in cardiac studies.

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Figure 1.

Schematic showing the ART algorithm. The Encoding matrix, $E(x,y)_i$, includes coil-sensitivity profiles and Fourier encoding, with element wise multiplication. ART iterates through all raw data points multiple times ($N_c \times N_r \times N_p \times N_{Loop}$). ART begins with an image estimate, and generates an estimated k-space signal, S_i^k , using the encoding matrix. The residual between the true k-space, S_i , and the estimated k-space is then used as a weight for that encoding matrix, which is added to the image estimate $m(x,y)^k$.

Figure 2.

Direct comparison of FBP (top) and ART (bottom) shows that ART reduces radial undersampling artifacts compared to FBP, at higher undersampling factors (with coils from fully sampled data sets), while mainly maintaining sharpness. Images are shown with $N_p = 96, 48, 32$ and 24 .

Figure 3.

Image quality assessment of cardiac cine images at each undersampling level A) SNR measured in the LV cavity was significantly higher for ART vs. FBP. B) Sharpness measured on the LV free-wall was higher with FBP for $N_p = 24$. C) Subjective scoring of artifact level (1–5, 5=no artifacts). ART outperforms FBP in artifact reduction. D) Subjective scoring of sharpness (1–3, 3=sharp). FBP outperforms ART in sharpness, but no qualitative differences were significant. $N_p = 96, 48, 32, 24$, and 16 corresponds to $R = 2, 4, 6, 8$ and 12 . Asterisks (*) indicate statistically significant differences.

Figure 4. Radial ART with A) 96 and B) 16 N_p , using a 32 channel coil on a 3T scanner. C) 16 N_p FBP. The quality of the 2-fold and 12-fold undersampled images is very similar. In the FBP image, many artifacts are present, and details of the ventricles are obscured.

Figure 5.

A) Fully sampled individual coil maps, and B) undersampled ($N_p = 24$) coil maps show a similar pattern. The reference input images for coil calculations are also shown in A and B. Artifacts due to gradient delays (arrow, A) and due to undersampling (arrow, B) are observed. ART recons of $N_p = 24$ data show a similar quality with coil maps from fully (C) and undersampled (D) data. Quality is slightly lower using undersampled coil maps, and subtraction shows differences (E), with a maximum difference of 16% and a mean difference of 1.7%. F) A sum-of-squares ART reconstruction, not using parallel imaging shows evident blurring.

Figure 6.

The PSF of a $128 N_r \times 32 N_p$ acquisition, with 1 coil, reconstructed with ART using A) 1 coil and B) 8 coils circularly arranged. Images have been windowed identically (0–10% max) to enhance display of the artifact pattern and central PSF broadening. Note that for the 8 coil image, some artifact—aliasing of the highest spatial frequencies—appears at FOV/R away (with $R=6.3$) from the generating point. The residual artifact is suppressed by a factor of 1.5 compared to the 1 coil image. C) The ratios of peak signal to total artifact signal were also quantified at different undersampling levels ($N_p = 128$ to 16, $R=1-8$) for ART for 1 flat coil and 8 coils. Note that 8 coils provide reduced artifacts compared with a single coil. D) PSF average sharpness vs. N_p . Note that sharpness depends on N_p for 1 coil, but more rapidly improves for 8 coils.

Figure 7.

Comparison of ART (top) with CG SENSE (bottom). A–B) $128 N_r \times 128 N_p \times 8$ coil noisy input data were used, A) $32 N_p$ images (15 iterations for CG SENSE, 18 for ART, with $\lambda=0.05$). B) Difference images between A) and fully sampled images, scaled identically -20% to $+20\%$ peak signal. C–E) In vivo data. C) $192 N_r \times 96 N_p$ images. D) $192 N_r \times 32 N_p$, and E) $192 N_r \times 24 N_p$. For phantoms, the true error with CG SENSE has less structure. For in vivo images, ART appears to retain more details.

Figure 8.

A) $192 N_r \times 48 N_p$, reconstructed with FBP. B) $192 N_r \times 48 N_p$, GPU-accelerated ART. C) $192 N_r \times 48 N_p$, ART without GPU-acceleration. D) Difference image (B-C) (maximum

3%, average 0.3%). Reconstructions were very similar but reconstruction speed with GPU-acceleration was 15 x faster.

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Figure 9.

Plot of normalized error vs. iteration number using noisy input data ($128 N_r$, and 8 coils). A) Residual k-space RMS error for reconstruction with $128 N_r \times 32 N_p \times 8$ coils, for ART with different values of λ , and for CG SENSE. B) Residual k-space RMS error as a function of undersampling factor with $\lambda = 0.05$. C) True RMS error for $128 N_r \times 32 N_p \times 8$ coils, for ART with different λ and CG SENSE. D) True RMS error at multiple undersampling factors with $\lambda = 0.05$. (E) Resulting images ($128 N_r \times 32 N_p \times 8$ coils) for $\lambda = 0.05, 0.10$, and 0.35 (top to bottom), at iteration steps 1, 4, 20 and 40 (left to right). Note that with higher λ , convergence is rapid, but noise increases, while with a lower λ , ART converges more slowly, with less noise.