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Fisher Information Analysis of Depth-of-Interaction Estimation in Double-Sided Strip Detectors

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Abstract

In very-high-spatial-resolution gamma-ray imaging applications, such as preclinical PET and SPECT, estimation of 3D interaction location inside the detector crystal can be used to minimize parallax error in the imaging system. In this work, we investigate the effect of bias voltage setting on depth-of-interaction (DOI) estimates for a semiconductor detector with a double-sided strip geometry. We first examine the statistical properties of the signals and develop expressions for

likelihoods for given gamma-ray interaction positions. We use Fisher Information to quantify how well (in terms of variance) the measured signals can be used for DOI estimation with different bias-voltage settings. We performed measurements of detector response versus 3D position as a function of applied bias voltage by scanning with highly collimated synchrotron radiation at the Advanced Photon Source at Argonne National Laboratory. Experimental and theoretical results show that the optimum bias setting depends on whether or not the estimated event position will include the depth of interaction. We also found that for this detector geometry, the z-resolution changes with depth.

Index Terms

CdTe; CdZnTe; DOI; double-sided strip; Fisher Information; semiconductor detectors

I. Introduction

High-density, high-atomic-number compound semiconductor detectors, such as CdTe and CdZnTe, have gained wide interest as radiation detectors in nuclear medicine due to their many attractive features [1], [2], [3], [4], [5], [6]; high detection efficiency that covers energy ranges of interest for common radionuclides, excellent spectroscopic performance resulting from low statistical variation (Fano factors $\ll 1$) in number of charge carriers generated per deposited energy [7], [8] and high intrinsic spatial resolution (small carrier cloud size). However, it is well known that these detectors suffer from poor hole transport (low mobility-lifetime products for holes), which when coupled with low applied bias voltage, causes induced charge to be depth-dependent. A low energy tail appears in the pulse-height spectrum as a consequence of the incomplete charge collection. The mean drift lengths of carriers are expressed by the product of mobility, carrier lifetime and applied bias voltage divided by detector thickness. Thus mean drift length, and thereby the effect of trapping, can to some degree be controlled by the magnitude of the applied bias voltage.

Fig. 1 below shows a correlation plot between anode and cathode-strip signals when a tightly collimated ($10 \times 10 \mu\text{m}^2$ cross-section), monochromatic beam of 130 keV photons illuminates a single location on the detector at normal incidence. DOI effects due to the different hole and electron mobilities are evident as cathode signals are spread over a range of lower pulse heights due to hole trapping while the anode pulses are mostly concentrated in a single photopeak.

Adjustment of the bias-voltage setting can therefore provide us a means to tune the detector's sensitivity to depth-of-interaction (DOI). Accurate estimation of 3D gamma-ray interaction location can be used to correct for parallax error, a problem that becomes important as PET and SPECT imaging systems are designed for very-high spatial resolution. When depth of interaction is not accounted for, all events are incorrectly assigned to a particular depth in the crystal (such as at the surface). As a result, the reconstruction process begins with incorrect estimates, causing a loss in both spatial and energy resolutions in the final tomographic images.

In this study, we investigate the effect of different bias voltages on energy and depth-of-interaction estimates in a semiconductor detector with a double-sided strip geometry [9] where each strip is connected to its own charge-sensitive transimpedance amplifier followed by a shaper amplifier. A trigger circuit latches the value in each of the shaper waveforms at a time T after a threshold is crossed. Our objective is to find an optimum bias voltage setting, with consideration given to the tradeoffs in the system. We start by investigating the statistical properties of the signals and expressing them as likelihoods for given gamma-ray interaction positions. We consider the dominant intrinsic random effects in the detector to be carrier generation and trapping. We compute the mean induced charges on the anode and cathode read-out strip electrodes using the Shockley-Ramo theorem. We then utilize Fisher Information to quantify how well (in terms of variance) the measured signals can be used for DOI estimation in different bias voltage.

Assuming that the electrode signals result from statistically independent motions of electrons and holes, we model the likelihood of the induced signals as a multivariate normal. We also derive analytical expressions for the Fisher Information for the specified detector geometry to gain more insight on its dependency on the parameters. Finally, we present our experimental findings and discuss selection criteria for an optimum bias setting.

II. Induced Charge on Electrodes

The extraction of gamma-ray event information from semiconductors is an estimation problem. The signals are governed by multiple random effects associated with charge-carrier generation, such as location-of-interaction, interaction type, and number of generated carriers; as well as random effects associated with charge-carrier transport, such as trapping, and spread of the charge cloud by thermal diffusion, drift, and Coulomb repulsion. There are also various noise types in the acquisition electronics. We can expect to achieve optimum spatial and energy resolution only through the use of appropriate estimators that incorporate accurate statistical models of the detector signals.

In this study, we focus on two of the dominant detector effects: charge generation and trapping. We model the distribution of the number of electron-hole carriers produced, by a Gaussian as in (1),

$$\Pr(N_{eh}) \cong \left[2\pi F \bar{N}_{eh}\right]^{-1/2} \exp \left[\frac{-(N_{eh} - \bar{N}_{eh})^2}{2F \bar{N}_{eh}} \right] \quad (1)$$

where F is Fano factor and $\bar{N}_{eh}$ is the mean number of electron-hole pairs. This is a highly peaked function for CdTe and CdZnTe as their Fano factors have been reported to be around 0.16 [10] and 0.14 [11], respectively. We also assume that the entire photon energy is deposited in a small local volume. The theoretical energy resolution at $E = 130$ keV is $\Delta E_{stat} = 2.35 \sqrt{F \times E \times W} = 699$ eV for an ionization energy of $W = 4.5$ eV for CdTe [12], [13].

The instantaneous current induced on electrodes by a moving charge can be found via use of the Shockley-Ramo theorem. First, a weighting potential is determined by solving Poisson's

equation, assuming the strip electrode of interest is held at unit potential and the remaining strips are at ground potential. After employing a separation of variables assumption and expanding the potential in terms of functions with initially arbitrary co-efficients [14], we invoke the boundary conditions to obtain the weighting potential. The boundary condition in the gap between the anode strips is assumed to be a linear interpolation between the two strip potentials [15].

$$\Phi_k(x, y, z) = \sum_{n,m=1}^{\infty} A_{nm}^k \sin(\alpha_n x) \sin(\beta_m y) \sinh(\gamma_{mn} z) \quad (2)$$

where

$$A_{nm}^k = \frac{4}{ab \sinh(\gamma_{mn} L) \alpha_n^2 d} \times [\sin(\alpha_n ((k-1)w + kd)) - \sin(\alpha_n ((k-1)w + (k-1)d)) - \sin(\alpha_n (kw + (k+1)d)) + \sin(\alpha_n (k(w+d)))] \times \frac{\cos(\beta_m b) - 1}{\beta_m} \quad (3)$$

In this expression ‘a’ and ‘b’ denote the detector dimensions as shown in Fig. 2, ‘w’ is defined as the strip width and ‘d’ is the gap width and coefficients are defined via $\alpha_n = n\pi/a$, $\beta_m = m\pi/b$ and $\gamma_{mn} = \pi \sqrt{(n/a)^2 + (m/b)^2}$. Anode and cathode strips are indexed by k and k' , respectively. Thus, the weighting potential for the cathode strips can be expressed as

$$\Phi_{k'}(x, y, z) = \sum_{n,m=1}^{\infty} A_{nm}^{k'} \sin(\alpha_n x) \sin(\beta_m y) \sinh(\gamma_{mn} (L-z)) \quad (4)$$

where

$$A_{nm}^{k'} = \frac{4}{ab \sinh(\gamma_{mn} L) \beta_m^2 d} \times [\sin(\beta_m ((k-1)w + kd)) - \sin(\beta_m ((k-1)w + (k-1)d)) - \sin(\beta_m (kw + (k+1)d)) + \sin(\beta_m (k(w+d)))] \times \frac{\cos(\alpha_n a) - 1}{\alpha_n} \quad (5)$$

Fig. 3 shows the result of computing the weighting potential for a few strips on one side of the detector. In Fig. 4, the small pixel effect [16], [17], [18] for the strip geometry is illustrated; the majority of the signal induction occurs in the volume near the electrode. It is observed to be a weaker effect in strips compared to pixel electrodes [19]. We use the following notation: $\mathbf{r}$ is the spatial position vector in 3D, $\mathbf{r}$ is the corresponding 2D vector for (x,y), the subscript ‘int’, as in $\mathbf{r}_{int}$, defines the interaction position inside the detector.

$$q(\mathbf{r}, \Delta T) = \sum_{i=1}^{N_{eh}} \delta(\mathbf{r} - \mathbf{r}_i) \quad (6)$$

$$= \begin{cases} \delta(\mathbf{r}-\mathbf{r}_{\text{int}}) \times \left[-\frac{1}{\lambda_h} \exp\left(-\frac{z_{\text{int}}-z}{\lambda_h}\right) \text{step}(z_{\text{int}}-z) - \exp\left(-\frac{z-z_{\text{int}}}{\lambda_h}\right) \delta(z) \right], & 0 \leq z \leq z_{\text{int}} \\ \delta(\mathbf{r}-\mathbf{r}_{\text{int}}) \times \left[-\frac{1}{\lambda_e} \exp\left(-\frac{z-z_{\text{int}}}{\lambda_e}\right) \text{step}(z-z_{\text{int}}) + \exp\left(-\frac{L-z_{\text{int}}}{\lambda_e}\right) \delta(z-L) \right], & z_{\text{int}} \leq z \leq L \end{cases} \quad (7)$$

We model the random trapping of charge carriers inside detector bulk as a non-stationary point process [20], shown in (6)–(7) at the bottom of the page.

We assume the time interval, denoted as T in (6), is from $t = 0$ to a shaper peaking time, $t = T$. If each trap has a low probability of being filled, and the trapping is statistically independent of trapping in any other non-overlapping volume, then the probability density on the charge carriers' spatial position can be expressed as in (7), where λ_e and λ_h are the mean drift lengths of electrons and holes, respectively. In this view, $q(\mathbf{r}, t)$ is interpreted as the expectation value (mean) of the random process and the probability density on the spatial position is a normalized version of the mean process. Trapped carriers will eventually be detrapped by thermal excitation; however, here we make the assumption that detrapping process is longer than our integration time T (on the order of microseconds) and can be therefore be neglected.

Induced charge on electrodes can be computed by integrating the charge over the volume of the detector for a given time interval. Assuming there is no net charge in the volume at $t = 0$, the total induced charge on the k th strip electrode at time $t = T$ will be given by (8).

$$g_k = \iint \Phi_k(\mathbf{r}) q(\mathbf{r}, t) d^3\mathbf{r} dt \quad (8)$$

For convenience, we will drop the temporal variable in the signal calculations; signals g_k will be understood to mean the total induced charge in one integration period during which only one gamma-ray interaction has occurred. By inserting the expressions for the weighting potential into (8), we can find the average induced charge;

$$\begin{aligned} \bar{g}_k(\mathbf{r}_{\text{int}}) = & -e\bar{N}_{\text{eh}} \left[\exp\left(-\frac{L-z_{\text{int}}}{\lambda_e}\right) \Phi_k(\mathbf{r}_{\text{int}}, L) \right. \\ & + \exp\left(-\frac{z_{\text{int}}}{\lambda_h}\right) \Phi_k(\mathbf{r}_{\text{int}}, 0) \\ & + \frac{1}{\lambda_e} \int_{z_{\text{int}}}^L \exp\left(-\frac{z-z_{\text{int}}}{\lambda_e}\right) \Phi_k(\mathbf{r}_{\text{int}}, z) dz \\ & \left. - \frac{1}{\lambda_h} \int_0^{z_{\text{int}}} \exp\left(-\frac{z_{\text{int}}-z}{\lambda_h}\right) \Phi_k(\mathbf{r}_{\text{int}}, z) dz \right]. \end{aligned} \quad (9)$$

In the first two terms, $\Phi_k(\mathbf{r}_{\text{int}}, L) = 1$ for anode strips and 0 for cathode strips and $\Phi_k(\mathbf{r}_{\text{int}}, 0) = 0$ for anode strips and 1 for cathode strips.

Evaluating the expressions for one cathode and one anode strip at the center of the detector, the normalized induced charge is computed as shown in Fig. 5. Strip widths are $230 \mu\text{m}$, gaps are $20 \mu\text{m}$, and detector thickness is $750 \mu\text{m}$, cathode is shown to be located at $z = 0$ and

anode is at $z = L$. The vertical grid markers show the location of the neighboring strips and the gaps.

In this simulation, $\mu_e \tau_e$ is $5.9 \times 10^{-3} \text{ cm}^2/\text{V}$ and $\mu_h \tau_h$ is $0.008 \times 10^{-3} \text{ cm}^2/\text{V}$. To illustrate the effects of depth-dependence, the bias voltage is varied between 100 V to 400 V.

Negative signals are induced for events occurring in the vicinity of the anode strip (shown as darker colored areas in the image). Depth dependence is more evident for the cathode strip, which is lessened as the bias voltage is increased as shown in plot 5(c).

III. Computing Fisher Information

If we assume that the motions, including the trapping of electrons and holes, are statistically independent while inducing currents on the electrodes, then the set of top and bottom signals, a $2N$ dimensional random vector, can be described by a multivariate normal:

$$\text{pr}(\mathbf{g}|\mathbf{r}_{\text{int}}) = \left[(2\pi)^{2N} \det(\mathbf{K}(\mathbf{r}_{\text{int}})) \right]^{-1/2} \exp \left[-\frac{1}{2} (\mathbf{g} - \bar{\mathbf{g}}(\mathbf{r}_{\text{int}})) \mathbf{K}(\mathbf{r}_{\text{int}})^{-1} (\mathbf{g} - \bar{\mathbf{g}}(\mathbf{r}_{\text{int}})) \right] \quad (10)$$

Where $\bar{\mathbf{g}}(\mathbf{r}_{\text{int}})$ is the position-dependent vector of mean signals, $\mathbf{K}(\mathbf{r}_{\text{int}})$ is the position-dependent covariance matrix of $\mathbf{g}$, and M is the number of strips that have non-negligible signals for a single event. This subset is identified by selecting the cathode and anode strips with the largest signals and then defining a neighborhood of strips, centered on these hottest strips. For example, from Fig. 5, we see that the induced charge on the central strip is negligible for the events that occur more than few strip distances away. Therefore, the footprint of each gamma-ray event spans only a few electrodes.

The Fisher Information Matrix (FIM) can be used to quantify how well the measured signals can be used for estimation of event parameters given our likelihood model in (10). The FIM is defined as in (11) [20].

$$F_{hj} = \left\langle \left[\frac{\partial}{\partial \theta_h} \ln \text{pr}(\mathbf{g}|\boldsymbol{\theta}) \right] \left[\frac{\partial}{\partial \theta_j} \ln \text{pr}(\mathbf{g}|\boldsymbol{\theta}) \right] \right\rangle_{\mathbf{g}|\boldsymbol{\theta}} \quad (11)$$

The n th diagonal component of the inverse of the FIM defines a lower bound on the variance of an unbiased estimator of the n th parameter θ_n . This inequality is called the Cramér-Rao (CR) lower bound. The FIM expression for the multivariate normal case reduces to the form below [21], [22],

$$F_{hj} = \frac{\partial \bar{\mathbf{g}}^t}{\partial \theta_h} \mathbf{K}^{-1} \frac{\partial \bar{\mathbf{g}}}{\partial \theta_j} + \frac{1}{2} \text{tr} \left(\mathbf{K}^{-1} \frac{\partial \mathbf{K}}{\partial \theta_h} \mathbf{K}^{-1} \frac{\partial \mathbf{K}}{\partial \theta_j} \right) \quad (12)$$

where

$$\frac{\partial \mathbf{K}}{\partial \theta_h} = \begin{bmatrix} \frac{\partial \mathbf{K}_{1,1}}{\partial \theta_h} & \dots & \frac{\partial \mathbf{K}_{1,2N}}{\partial \theta_h} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{K}_{2N,1}}{\partial \theta_h} & \dots & \frac{\partial \mathbf{K}_{2N,2N}}{\partial \theta_h} \end{bmatrix}. \quad (13)$$

The individual terms in the FIM expression provide insight into the physical factors needed for low Cramér-Rao bound. The first term includes derivative of the mean signals along the direction of interest. This indicates that FIM is larger with increased slope in the mean signals. Since we are mainly interested in DOI estimation for the purposes of this study, we compute the derivative of the mean signals along z-direction.

$$\begin{aligned} \frac{\partial \bar{g}_k(r_{int})}{\partial z} = & -e \bar{N}_{eh} \sum_{n,m=1}^{\infty} A^k_{nm} \sin(\alpha_n x_{int}) \sin(\beta_m y_{int}) \\ & \times \left[\frac{1}{\lambda_e} \int_{z_{int}}^L \frac{\partial}{\partial z} \left(\exp \left(-\frac{z-z_{int}}{\lambda_e} \right) \sinh(\gamma_{mn} z) \right) dz - \frac{1}{\lambda_h} \int_0^{z_{int}} \frac{\partial}{\partial z} \left(\exp \left(-\frac{z_{int}-z}{\lambda_h} \right) \sinh(\gamma_{mn} z) \right) dz \right]. \end{aligned} \quad (14)$$

Evaluation of the integrals gives;

$$\begin{aligned} \frac{\partial \bar{g}_k(r_{int})}{\partial z} = & -e \bar{N}_{eh} \sum_{n,m=1}^{\infty} A^k_{nm} \sin(\alpha_n x_{int}) \sin(\beta_m y_{int}) \\ & \times \left[\frac{1}{\lambda_e} \left(\exp \left(-\frac{z_{int}-L}{\lambda_e} \right) \sinh(\gamma_{mn} L) - \sinh(\gamma_{mn} z_{int}) \right) - \frac{\sinh(\gamma_{mn} z_{int})}{\lambda_h} \right]. \end{aligned} \quad (15)$$

For the anode-side strips, we find,

$$\begin{aligned} \frac{\partial \bar{g}_k(r_{int})}{\partial z} = & -e \bar{N}_{eh} \sum_{n,m=1}^{\infty} A^{k'}_{nm} \sin(\alpha_n x_{int}) \sin(\beta_m y_{int}) \\ & \times \left[-\frac{1}{\lambda_h} \left(\exp \left(-\frac{z_{int}-L}{\lambda_h} \right) \sinh(\gamma_{mn} L) - \sinh(\gamma_{mn}(L-z_{int})) \right) - \frac{\sinh(\gamma_{mn}(L-z_{int}))}{\lambda_e} \right]. \end{aligned} \quad (16)$$

We observe that the slope is inversely proportional to the mean drift lengths of electrons and holes. Since the mobility and lifetimes are physical properties characterizing a given detector material, the bias voltage can be used to control the mean signal slope. We can analyze the covariance matrix in the presence of the random distribution of trapped charges and the carrier generation statistics as defined earlier in equations (1) and (7). However, it is not possible to fully describe the covariance without including noise contributions that must be derived from experimental data.

IV. Experiments

In order to investigate charge transport properties and statistics of the strip signals along depth, we conducted experiments at the sector 6 ID-D Beamline at the Advanced Photon Source (APS) at Argonne National Laboratory. Synchrotron light sources are ideal for detector calibration tasks, as they provide highly-collimated x-ray beams at a continuum of energies that can be monochromated to a narrow spectral bandwidth. As shown in Fig. 8, the detector is mounted on a vertically oriented 2D translation stage that allows motions in the plane normal to the horizontal x-ray beam. The entire assembly is mounted on a rotation

stage that enables the setup to rotate about a vertical axis passing roughly through the center of the detector along the direction of the illumination-side cathode strips. The 6-ID-D insertion device and double-Laue crystal monochromator were adjusted to give an x-ray energy of 130 keV at an energy resolution ($\Delta E/E$) of 4×10^{-4} , below the intrinsic energy resolution of the detector. The beam cross-section was adjusted to $10 \mu\text{m}$ by $10 \mu\text{m}$ using a combination of horizontal and vertical slits. The detector was cooled to -5°C and maintained under a dry nitrogen atmosphere.

The double-sided CdTe strip detector used in these experiments is a prototype for the CdTe detectors developed for the Hard X-ray Imager [23], [24], [25]. There are 4 ASICs on each side of the detector, each with 32 channels. The detector crystal is stud-bump bonded to a ceramic fan-out board.

This system uses an exhaustive readout: all channels on both sides are digitized and recorded for every event. Each channel has a charge-sensitive preamplifier followed by a fast shaper for triggering and a slow shaper whose value is clocked into a sample and hold register at a configurable delay time (relative to the trigger). The signals are digitized immediately afterwards using on-ASIC Wilkinson-type ADCs [26].

The first set of measurements consisted of illuminating the detector with a normal incidence beam scanning the detector in a raster pattern, one line of which is shown in Fig. 7. The translation step size was $25 \mu\text{m}$, providing 10 samples over each strip. The measurements at the center of the strips are used for gain correction by adjusting the peak to a common level.

In order to probe the interaction depth underneath a strip, we repeated the scan measurements at rotation angles of 45, 60, 80 and 85 degrees.

In the normal-incidence beam scans, the lateral position of the beam relative to the cathode strips can be easily determined from the mean strip amplitudes with very fine accuracy as evident in Fig. 7. In the slant beam scans we determined depth position by observing the variation of photopeak *area* as a function of translation. As shown in bottom plot in Fig. 9, the photopeak areas vary with the volume the beam intersects under a strip, and the reduction in count rate at the deeper portions due to passing through more detector material prior to reaching the strip. By determining the mean photopeak *location* as a function of translation, we can extract the mean signals at each probed depth (z) for both p and n-side strips for a given x and y , as shown in Fig. 10.

We performed a simultaneous non-linear fitting of p and n side strip signal data to (9) to extract the common parameters of λ_e and λ_h . Assuming a linear electric field, we have found $\mu_h \tau_h 0.243 \times 10^{-3} \text{ cm}^2/\text{V}$ and $\mu_e \tau_e 7.35 \times 10^{-3} \text{ cm}^2/\text{V}$ using this procedure. We have observed that the depth response curves from the neighboring strips were very similar and concluded that the charge transport features of the detector were uniform in the region of interest. The depth response curves can be understood by investigating (9) for *hole-collecting* and *electron-collecting* cases separately. For the p strips, the depth-dependence is dominated by the effects of hole trapping, terms 2 and 4; the electron trapping, terms 1 and 3, contributes negligibly to the signal.

For the n strips, the depth-dependence results from a combination of both electron and hole trapping. Due to hole trapping, the n -strip signal is attenuated when events occur near the anode. The n -strip signal increases as the hole trapping effect is reduced when events occur closer to the cathode. However, very close to the cathode the n -strip signal is again slightly attenuated by the effects of electron trapping.

In order to compute the Fisher Information, in addition to the variation of the mean signals with depth, we also need to account for the sources of randomness and their contributions to the signal variance.

The energy resolution of the detector, can be approximately expressed as comprising three main components (after conversion into common units via appropriate gains);

$$\sigma(r)^2 = \sigma_F^2 + \sigma_e^2 + \sigma_{trap}(r)^2 \quad (17)$$

where the terms represent variation in the number of charge carriers generated (σ_F), electronics noise (σ_e), and the random locations of trapped charge (σ_{trap}), respectively. We found with this detector that electronics noise contribution from common-mode fluctuations can be greatly reduced by subtracting, event-by-event, the mean signal in a small subset of channels near but not immediately adjacent to the location where the interaction occurred.

Fig. 11. shows the standard deviations of p and n -strip signals in terms of normalized ADC values computed from the photopeak data at each translation location. The contribution to signal variance comes from all of the terms in (17), but we assume that the first two terms do not have spatial position dependence. It is considered that the main contribution to variance at each translation location comes from trapping associated with z depth sampling that is set by the angle of the incident beam. The observed trend of higher p and n -side variance for interactions occurring at greater depth, and thereby greater distance from the hole-collecting p -side strips, is consistent with the spatially broader probability law in (7) that describes where holes are trapped.

For the purposes of this study, we are interested in the signals in z -direction for a given x, y location, which was chosen to be the intersection of the center of cathode strip k and anode strip k' . Since we have a very small beam size ($\ll$ strip width), the lateral distribution of interaction locations is much smaller than the strip widths and significant signals are induced only on the anode-cathode pair, rather than extending to the neighboring strips (other than Compton or escape events). In this case the multivariate normal model reduces to a bivariate normal, and the covariance of the signals can be expressed as;

$$\mathbf{K} = \begin{bmatrix} \sigma_k(r)^2 & \rho\sigma_k(r)\sigma_{k'}(r) \\ \rho\sigma_k(r)\sigma_{k'}(r) & \sigma_{k'}(r)^2 \end{bmatrix} \quad (18)$$

where ρ is the correlation coefficient. A small value of .04 was found for the correlation coefficient as can be seen in the nearly isotropic scatter plots between anode and cathode strips in Fig. 12. We observe that the variance terms are very small compared to the gradient

of the mean signals in (12). Therefore, almost all the contribution to the FIM comes from the first term, involving $\bar{g}(z)/z$.

In order to compare the FIM analysis with changing mean drift lengths of electrons and holes, we repeated our slant-beam scanning measurements at varying bias voltages of 75 V, 125 V and 200 V. The results are shown in Fig. 13.

V. Results and Discussions

Inserting the measured mean and variance data of the cathode and anode signals, for a fixed x, y location (at the center) into (12) we find the Cramér-Rao lower bound on the variance of z estimator for applied bias voltage of 200 V as shown in Fig. 14.

We observe that the resolution is around $18 \mu\text{m}$ near the anode, $50 \mu\text{m}$ in the middle of the detector but degrades sharply immediately under the cathode strip. The change in resolution across detector depth can be conceptually interpreted as having different size pixels in z -direction; larger towards cathode and smaller toward anode.

The significance of this analysis can be illustrated by examining the relationship and trade-offs between σ_e and σ_{trap} terms in (17) for different applications.

Higher Trapping

It is the phenomenon of trapping that provides sensitivity to DOI, estimation of which allows parallax error to be minimized in very-high spatial resolution PET and SPECT imaging systems. Allowing more trapping by lowering bias can lead to a higher precision in DOI estimate as suggested by Fig. 14. Lowering the bias voltage might also help decrease the σ_e term by reducing the shot noise associated with leakage current. Although there will be more lower energy tailing appearing in the initial energy spectrum, after DOI estimation this can be corrected for. After the correction, $\sigma_{trap}(r)$ reduces to corrected $\sigma_{c.trap}(r)$. In the case

$$(\sigma_e^2 + \sigma_{c.trap}(r)^2)_{low\ bias} < (\sigma_e^2)_{high\ bias}$$

it may be even be possible to achieve better energy resolution than with no trapping. However, this condition needs to be carefully investigated through complete noise analyses for a given specific detector.

There are also other constraints that need to be taken into account when selecting the optimum bias voltage setting. One of the most important factors is the minimum bias voltage that is required for properly providing full depletion in the detector volume. This minimum voltage setting can be confirmed by experiments in the laboratory [27]. Another factor that needs to be considered is the rise time of the induced charge signal. Pulse rise time depends on the slowest carrier and if the transit time is longer than the peaking time, there will be incomplete charge collection known as ballistic deficit. Another important effect that needs to be considered when operating at lower bias voltage is the polarization phenomenon which

is explained as the accumulation of charge that causes non-uniform electric field and shifts of the pulse heights with time.

Lower trapping

For applications where estimation of DOI is not necessary, it is possible to apply very high bias voltage and minimize the σ_{trap} term in (17). The energy spectrum performance is then dominated by the electronic noise, σ_e , which motivates very low noise front-end ASICs and σ_F . For this work's thin and very-low noise detector, energy resolution close to σ_F limit is observed when the detector is operated at high bias. However, for thicker detectors that are required for higher absorption efficiency, it may not be possible to apply high enough bias voltage without significantly increasing noise due to leakage current [28], [29]. In this case a DOI estimation technique can be particularly useful.

Our analysis also suggests that the detector illumination side can be chosen according to the task. In the case of cathode illumination, more events will occur close to the cathode strips, with high signal amplitudes ($z = 0$ in Fig. 10 and 13) but at the expense of lower z -resolution ($z = 0$ in Fig. 14). Therefore, if the task involves estimation of DOI, it might be advantageous to illuminate the detector from the anode side.

In summary, FIM provides a tool to evaluate different detector settings and their relation to expected variance in maximum-likelihood estimation of event parameters. Although the FIM analysis is here carried out specifically for strip geometries, the same procedure can be followed for pixel geometries with substitution of the proper weighting potential in the expressions. Using the results of the FIM analysis, it is possible to find settings for optimum detector performance for a given task of estimating energy, 3D spatial position or both.

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Fig. 1. Event by event correlation plot between anode and cathode pulse heights under a bias voltage of 300 V for a 750 μ m thick CdTe crossed-strip detector.

Fig. 2.
Detector geometry and coordinates.

Fig. 3.
Weighting potential shown for few strips.

Fig. 4.
Weighting potential contour plot.

Fig. 5.

Mean induced charge for normalized charge density, for a center cathode strip electrode (a), center anode strip electrode (b) where cathode strips are located at $z = 0$ m and anode at $z = 0.75$ m. Cuts through z -direction on locations marked as A, B, C and D; the center and in between two strips, for cathode strip (c) and for anode strip (d) shown for different bias voltage settings.

Fig. 6.
Close up pictures of the detector and the detector box.

Fig. 7. Measured mean cathode (hole-collecting) signals from a linear detector scan with a $10\mu\text{ m} \times 10\mu\text{ m}$ beam at 130 keV.

Fig. 8.
Experimental setup at APS Sector 6 ID-D beamline (inside the hutch).

Fig. 9.

(Top) Probing DOI by scanning with the beam slanted at $\theta = 45$ (Bottom) The photopeak areas at each translation point, shown for few neighboring p-side strips, namely; 62, 63 and 64.

Fig. 10.

Measured mean signals (normalized to maximum value) in z-direction for (top) p strip (bottom) n strip, at 200 V bias voltage. A low order spline fit is used for calculating local derivatives in FIM.

Fig. 11. Standard deviations of p-side and n-side strip signals as a function of z (depth) for x, y given by intersection of the centerline of cathode strip 64 with the centerline of anode strip 64.

Fig. 12.

Scatter plot of P and N strip signals at shown for mean depths of 350 μm , 500 μm and 662.5 μm (left to right).

Fig. 13.

Measured mean (top) P side and (bottom) N side signals for applied bias voltage values of 200 V, 125 V and 75 V.

Fig. 14. Cramér-Rao Lower bound on the standard deviation of depth estimated with an unbiased estimator, such as maximum-likelihood estimator (MLE).

Fig. 15.
Concept of pixels in depth (resolution not to scale).