Exponential synchronization for fuzzy cellular neural networks with time-varying delays and nonlinear impulsive effects

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Abstract In this paper, the globally exponential synchronization of delayed fuzzy cellular neural networks with nonlinear impulsive effects are concerned. By utilizing inequality techniques and Lyapunov functional method, some sufficient conditions on the exponential synchronization are obtained based on p -norm. Finally, a simulation example is given to illustrate the effectiveness of the theoretical results.

Keywords Exponential synchronization · Nonlinear impulse · Time-varying delay · Fuzzy cellular neural network

Introduction

In recent years, synchronization of cellular neural networks has been extensively investigated since it was proposed by Chua and Yang (1998a, b) due to its important applications including secure communication, parallel image processing, associative memories, biological systems, quadratic optimization, information science (Zhang et al. 2003). However, in mathematical modeling of real world problems the uncertainty or vagueness is unavoidable. In 1996, Yang and Yang proposed the fuzzy cellular neural networks which integrates fuzzy logic into the structure of a traditional cellular neural networks and maintains local connectedness among cells in order to take vagueness into to consideration (Yang and Yang 1996; Yang et al. 1996a, b). Meanwhile, many studies have been revealed that fuzzy

cellular neural network is an useful paradigm for image processing problems, which is cornerstone in image processing and pattern recognition. Therefore, it is interesting and important to study the fuzzy cellular neural networks both in theory and application.

In implementation of cellular neural networks, time delays especially time-varying delays are often encountered unavoidably due to finite switching speeds of the amplifiers and neurons. The existence of time delays may destroy the networks synchronization and cause sustained oscillations, bifurcation or chaos. Consequently, it is important to investigate the dynamical behaviors of cellular neural networks with delays. As we all know that neural networks with various types of delays have been widely investigated by many authors (Zhang et al. 2005; Long and Xu 2008; Zhou et al. 2004; Jiang and Teng 2004; Gu et al. 2008; Huang and Cao 2009; Kwon et al. 2012; Liu et al. 2008; Mohamad and Gopalsamy 2003; Mohamad et al. 2008; Song and Cao 2006; Wang et al. 2009; Xu and Long 2012; Zhang et al. 2005; Zhou et al. 2004; Cai et al. 2012).

Furthermore, besides delay effects, we often encounter impulsive effects which exist widely in many evolution process and are characterized by the fact that at a certain moment of time they experience a change of state abruptly (Shen 1999; Benbouziane et al. 2008; Luo and Nieto 2009). Impulsive control synchronization, as a type of synchronization, has been developed. It allows synchronization of chaotic systems using only small control impulses to change the states of a system in discrete time. By discussing the stability of error systems, the synchronization of chaotic systems are obtained via impulsive signal. This provide the method for investigating the chaotic synchronization of delayed neural networks with impulsive effects. Up to now many results on impulsive neural networks have been investigated (Ahmada and Stamova 2008;

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Feng et al. 2011; Zhang and Sun 2005; Gopalsamy 2004; Ge and Ge 2006; Mohamad 2007; Zhang 2009; Yu et al. 2012; Ding et al. 2009). For instance, in Feng et al. (2011) the global exponential synchronization of delayed fuzzy cellular neural networks with impulsive effects was studied by constructing a suitable Lyapunov functional and applying some inequality techniques. Ding et al. (2009) studied exponential lag synchronization of delayed fuzzy cellular neural networks with impulses by the Lyapunov–Lasall principle of functional differential equations. Most previous works have been restricted to linear impulsive effects. However, in most case, the impulsive functions are not exactly linear functions. Few authors have considered the nonlinear impulsive effects on the synchronization of neural networks. Consequently, we consider the nonlinear impulsive function, which will more reality.

Motivated by the above discussion, in this paper our main purpose is to study the synchronization of cellular neural networks with time-varying delays and nonlinear impulsive effects. Some novel and useful sufficient conditions on the exponential synchronization are obtained by employing the Lyapunov functional method and inequality technique.

This paper is organized as follows. In “Problem formulation and preliminaries” section, model description and preliminaries are given. In “Main results” section, the addressed cellular neural networks are investigated and some synchronization conditions are provided. In “Numerical simulations” section, the effectiveness of the developed methods is shown by a numerical example.

Problem formulation and preliminaries

Consider a class of fuzzy cellular neural networks with nonlinear impulsive effects described by

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^n a_{ij} f_j(x_j(t)) + \sum_{j=1}^n b_{ij} v_j + u_i \\ \quad + \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t - \tau(t))) + \bigwedge_{j=1}^n T_{ij} v_j + \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^n S_{ij} v_j, \quad t \neq t_k, \quad i \in I, \quad k \in Z^+, \\ x_i(t_k^+) = p_{ik}(x(t_k)), \quad t = t_k, \quad i \in I, \quad k \in Z^+, \end{cases} \quad (1)$$

where $i \in I = \{1, 2, \dots, n\}$, $Z^+ = \{1, 2, \dots\}$, n corresponds to the number of units in a neural network, $x_i(t)$ is the state variable of the i th neuron at time t ; $\alpha_{ij}, \beta_{ij}, T_{ij}$ and

S_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template, fuzzy feed-forward MAX template, respectively; a_{ij} and b_{ij} are elements of feedback and free-forward template; $\bigwedge, \bigvee$ denote the fuzzy AND and fuzzy OR operation; v_i denotes an input of the i th neuron; u_i denotes an input bias of the i th neuron; $f_j(\cdot)$ denote the activation functions of the j th neuron; $0 \leq \tau(t) \leq \tau$ represents transmission delay at time t ; $c_i > 0$ represents the rate with which i th neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; the discrete set $\{t_k | k = 1, 2, \dots\}$ satisfies $0 \leq t_0 < t_1 < t_2 < \dots < t_k < \dots, t_k \rightarrow +\infty$ as $t \rightarrow +\infty$, $x(t_k^+) = \lim_{t \rightarrow t_k^+} x(t_k)$ and $x(t_k^-) = x(t_k)$, $p_{ik}(x) = p_{ik}(x_1, x_2, \dots, x_n) \in (R^n, R)$ is a nonlinear function represents impulsive perturbations of the i th unit at time t_k .

System (1) is supplemented with initial values given by

$$x_i(s) = \phi_i(s), \quad s \in [-\tau, 0], \quad i \in I,$$

where $\phi(s) = (\phi_1(s), \phi_2(s), \dots, \phi_n(s))^T \in C = ([-\tau, 0], R^n)$ denotes all continuous functions of the Banach space, which mapping $[-\tau, 0]$ into R^n with p -norm ($p \geq 1$ is a positive integer) defined by the following form

$$\|\phi\|_p = \left[\sup_{s \in [-\tau, 0]} \sum_{i=1}^n |\phi_i(s)|^p \right]^{\frac{1}{p}}. \quad (2)$$

Throughout this paper, for systems (1), we always assume that

(H_1) There exist real constants $\underline{F}_j$ and positive real numbers $\overline{F}_j$ such that

$$\underline{F}_j \leq \frac{f_j(u) - f_j(v)}{u - v} \leq \overline{F}_j \quad (3)$$

for all $u, v \in R$, $u \neq v$ and $j \in I$.

We consider system (1) as the drive system. For the purpose of synchronization, we introduce a response system described by

$$\begin{cases} \dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^n a_{ij} f_j(y_j(t)) + \sum_{j=1}^n b_{ij} v_j + u_i \\ \quad + \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j(t - \tau(t))) + \bigwedge_{j=1}^n T_{ij} v_j + \bigvee_{j=1}^n \beta_{ij} f_j(y_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^n S_{ij} v_j - K_i(t), \quad t \neq t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, v \\ y_i(t_k^+) = p_{ik}(y(t_k)), \quad t = t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \end{cases} \quad (4)$$

where $K_i(t)$ indicates the external control input given by

$$K_i(t) = \sum_{j=1}^n k_{ij}(x_j(t) - y_j(t)), \quad (5)$$

where each k_{ij} is a constant called control gain and $k_{ii} < 0 (i \in I)$.

The initial conditions for response system (4) are given in the following form

$$y_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \quad i \in I, \quad (6)$$

where $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T \in C([-\tau, 0], \mathbb{R}^n)$.

Define the synchronization error as $e_i(t) = x_i(t) - y_i(t)$ for $i \in I$. From system (1) and (4), the error system can be expressed as

$$\begin{cases} \dot{e}_i(t) = -c_i e_i(t) + \sum_{j=1}^n a_{ij}(f_j(x_j(t)) - f_j(y_j(t))) + \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t - \tau(t))) \\ \quad - \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j(t - \tau(t))) + \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t - \tau(t))) \\ \quad - \bigvee_{j=1}^n \beta_{ij} f_j(y_j(t - \tau(t))) + \sum_{j=1}^n k_{ij} e_j(t), \quad t \neq t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \\ e_i(t_k^+) = p_{ik}(x(t_k)) - p_{ik}(y(t_k)), \quad t = t_k, \quad i \in I, \quad k \in \mathbb{Z}^+. \end{cases} \quad (7)$$

Definition 1 The drive system (1) and response system (4) are said to be globally exponentially synchronized, if there exist constants $M \geq 1$ and $\varepsilon > 0$ such that

$$\|x(t) - y(t)\| \leq M \|\phi - \varphi\| e^{-\varepsilon t} \quad (8)$$

for $t \geq 0$, where $\|x(t) - y(t)\| = (\sum_{i=1}^n |x_i(t) - y_i(t)|^p)^{\frac{1}{p}}$, $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ and $y(t) = (y_1(t), y_2(t), \dots, y_n(t))^T$ are solutions of drive system (1) and response system (4) beginning with different initial values $\phi, \varphi \in C([-\tau, 0], \mathbb{R}^n)$.

To obtain the main results, we need to introduce the following lemmas.

Lemma 1 (see Yang and Yang 1996) For any $\alpha_{ij}, \beta_{ij} \in \mathbb{R}$, and $i, j \in I$, the following estimations are true

$$\left| \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t)) - \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j(t)) \right| \leq \sum_{j=1}^n |\alpha_{ij}| |f_j(x_j(t)) - f_j(y_j(t))|$$

and

$$\left| \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t)) - \bigvee_{j=1}^n \beta_{ij} f_j(y_j(t)) \right| \leq \sum_{j=1}^n |\beta_{ij}| |f_j(x_j(t)) - f_j(y_j(t))|.$$

Main results

In this section, we will derive some criteria to guarantee the exponential synchronization of systems (1) and (4). For convenience, we denote

$$\zeta_i = \sum_{j=1}^n \frac{\mu_j}{\mu_i} (|\alpha_{ji}|^{p_{v_{pji}}} + |\beta_{ji}|^{p_{n_{pji}}}) L_j^{p_{\delta_{pji}}}, \quad (9)$$

$$\omega_i = p k_{ii} + \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |k_{ij}|^{p_{h_{lij}}} + \sum_{j=1, j \neq i}^n \frac{\mu_j}{\mu_i} |k_{ji}|^{p_{h_{pji}}}, \quad (10)$$

$$\begin{aligned} \lambda_i = & p c_i - p H_i - \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |\alpha_{ij}|^{p_{\gamma_{lij}}} L_j^{p_{\delta_{lij}}} - \sum_{j=1}^n \sum_{l=1}^{p-1} (|\alpha_{ij}|^{p_{v_{lij}}} \\ & + |\beta_{ij}|^{p_{n_{lij}}}) L_j^{p_{\delta_{lij}}} - \sum_{j=1, j \neq i}^n \frac{\mu_j}{\mu_i} |a_{ji}|^{p_{\gamma_{pji}}} L_i^{p_{\delta_{pji}}}, \end{aligned} \quad (11)$$

where each μ_i ($i \in I$) is a positive constant, $L_i = \max_{1 \leq i \leq n} \{|\underline{F}_i|, |\overline{F}_i|\}$ and

$$H_i = \begin{cases} a_{ii}\overline{F}_i, & a_{ii} \geq 0, \quad i \in I, \\ a_{ii}F_i, & a_{ii} < 0, \quad i \in I, \end{cases} \quad (12)$$

and nonnegative real numbers $\gamma_{lij}, \delta_{lij}, h_{lij}, \eta_{lij}$ and v_{lij} satisfy $\sum_{l=1}^p \gamma_{lij} = 1, \sum_{l=1}^p \delta_{lij} = 1, \sum_{l=1}^p h_{lij} = 1, \sum_{l=1}^p \eta_{lij} = 1$ and $\sum_{l=1}^p v_{lij} = 1$, respectively.

Besides, the following assumptions are necessary to derive the main results.

(H₂) $\tau(t) \leq \hat{\varepsilon} < 1$ for all $t \geq 0$, where $\hat{\varepsilon}$ is a constant.

(H₃) $\lambda_i - \omega_i - \frac{1}{1-\hat{\varepsilon}} \xi_i > 0$ for any $i \in I$.

(H₄) There exist nonnegative matrixes $P_k = (p_{ij}^k)_{n \times n}$ such that

$$|p_{ik}(u_1, u_2, \dots, u_n) - p_{ik}(v_1, v_2, \dots, v_n)|^p \leq \sum_{j=1}^n p_{ij}^k |u_j - v_j|^p$$

for any $(u_1, u_2, \dots, u_n) \in R^n, (v_1, v_2, \dots, v_n) \in R^n, i \in I$ and $k \in Z^+$.

(H₅) There exists a constant $\alpha \geq 0$ such that

$$\frac{\ln \rho_k}{t_k - t_{k-1}} \leq \alpha < \varepsilon^*, \quad k \in Z^+, \quad (13)$$

where $\rho_k = \max_{1 \leq i \leq n} \{1, \sum_{j=1}^n \{\frac{\mu_i}{\mu_j} p_{ji}^k\}\}$, $\varepsilon^* = \min_{1 \leq i \leq n} \{\varepsilon_i\}$, ε_i is the unique positive solution of the following equation

$$\lambda_i - \omega_i - \frac{1}{1-\hat{\varepsilon}} e^{\varepsilon_i \tau} \xi_i - \varepsilon_i = 0, \quad i \in I. \quad (14)$$

Evidently, the existenced and uniqueness of ε_i are ensure by the assumption (H₃).

The following results are established to ensure the drive system (1) and response system (4) are globally exponentially synchronized under nonlinear impulsive effects base on p -norm.

Theorem 1 Suppose that (H₁)–(H₅) are satisfied, then drive system (1) and response system (4) are globally exponentially synchronized.

Proof Construct a Lyapunov functional as the following form

$$V(t) = \sum_{i=1}^n e^{\varepsilon^* t} \mu_i |e_i(t)|^p + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \xi_i \times \int_{t-\tau(t)}^t e^{\varepsilon^*(s+\tau)} \mu_i |e_i(s)|^p ds. \quad (15)$$

For $t \neq t_k$, calculating the Dini right hand derivative of $V(t)$ along with system (7), we obtain

$$\begin{aligned} D^+ V(t) &= \sum_{i=1}^n (e^{\varepsilon^* t} \varepsilon^* \mu_i |e_i(t)|^p + p e^{\varepsilon^* t} \mu_i |e_i(t)|^{p-1} \\ &\quad \times \text{sign } e_i(t) \dot{e}_i(t)) \\ &\quad + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \xi_i e^{\varepsilon^*(t+\tau)} \mu_i |e_i(t)|^p \\ &\quad - \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \mu_i \xi_i (1 - \dot{\tau}(t)) e^{\varepsilon^*(t-\tau(t)+\tau)} \\ &\quad \times |e_i(t-\tau(t))|^p \\ &= \sum_{i=1}^n \mu_i e^{\varepsilon^* t} [\varepsilon^* |e_i(t)|^p + p |e_i(t)|^{p-1} \\ &\quad \times \text{sign } e_i(t) (-c_i e_i(t)) \\ &\quad + \sum_{j=1}^n a_{ij} (f_j(x_j(t)) - f_j(y_j(t))) \\ &\quad + \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t-\tau(t))) - \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j(t-\tau(t))) \\ &\quad + \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t-\tau(t))) - \bigvee_{j=1}^n \beta_{ij} f_j(y_j(t-\tau(t))) \\ &\quad + \sum_{j=1}^n k_{ij} e_j(t)] + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \xi_i e^{\varepsilon^*(t+\tau)} \mu_i |e_i(t)|^p \\ &\quad - \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \mu_i \xi_i (1 - \dot{\tau}(t)) e^{\varepsilon^*(t-\tau(t)+\tau)} \\ &\quad \times |e_i(t-\tau(t))|^p \\ &\leq \sum_{i=1}^n \mu_i e^{\varepsilon^* t} [\varepsilon^* |e_i(t)|^p - p c_i |e_i(t)|^p + p H_i |e_i(t)|^p \\ &\quad + p \sum_{j=1, j \neq i}^n |a_{ij}| L_j |e_j(t)| |e_i(t)|^{p-1} \\ &\quad + p \sum_{j=1}^n |\alpha_{ij}| |f_j(x_j(t-\tau(t))) - f_j(y_j(t-\tau(t)))| \\ &\quad \times |e_i(t)|^{p-1} \\ &\quad + p \sum_{j=1}^n |\beta_{ij}| |f_j(x_j(t-\tau(t))) - f_j(y_j(t-\tau(t)))| \\ &\quad \times |e_i(t)|^{p-1} \\ &\quad + p k_{ii} |e_i(t)|^p + p \sum_{j=1, j \neq i}^n |k_{ij}| |e_j(t)| |e_i(t)|^{p-1}] \\ &\quad + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \mu_i e^{\varepsilon^* t} \xi_i e^{\varepsilon^* \tau} |e_i(t)|^p - \sum_{i=1}^n \xi_i \mu_i e^{\varepsilon^* t} \\ &\quad \times |e_i(t-\tau(t))|^p \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^n \mu_i e^{\varepsilon^* t} [(\varepsilon^* - pc_i + pH_i + pk_{ii}) |e_i(t)|^p \\
&\quad + p \sum_{j=1, j \neq i}^n |a_{ij}| L_j |e_j(t)| |e_i(t)|^{p-1} \\
&\quad + p \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|) L_j |e_j(t - \tau(t))| |e_i(t)|^{p-1} \\
&\quad + p \sum_{j=1, j \neq i}^n |k_{ij}| |e_j(t)| |e_i(t)|^{p-1} \\
&\quad + \frac{1}{1 - \hat{\varepsilon}} \zeta_i e^{\varepsilon^* \tau} |e_i(t)|^p - \zeta_i |e_i(t - \tau(t))|^p].
\end{aligned}$$

For a positive integer $p \geq 1$ and constants $a_i > 0$ ($i = 1, 2, \dots, p$), evidently,

$$pa_1 a_2 \cdots a_p \leq \sum_{i=1}^p a_i^p,$$

then

$$\begin{aligned}
&p \sum_{j=1, j \neq i}^n |a_{ij}| L_j |e_j(t)| |e_i(t)|^{p-1} \\
&= \sum_{j=1, j \neq i}^n \left[\left(p \prod_{l=1}^{p-1} |a_{ij}|^{\gamma_{lij}} L_j^{\delta_{lij}} |e_i(t)| \right) \right. \\
&\quad \times (|a_{ij}|^{\gamma_{pij}} L_j^{\delta_{pij}} |e_j(t)|) \\
&\leq \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |a_{ij}|^{p\gamma_{lij}} L_j^{p\delta_{lij}} |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n |a_{ij}|^{p\gamma_{pij}} L_j^{p\delta_{pij}} |e_j(t)|^p,
\end{aligned} \quad (16)$$

and

$$\begin{aligned}
&p \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|) L_j |e_j(t - \tau(t))| |e_i(t)|^{p-1} \\
&\leq \sum_{j=1}^n \sum_{l=1}^{p-1} (|\alpha_{ij}|^{p\gamma_{lij}} + |\beta_{ij}|^{p\gamma_{lij}}) L_j^{p\delta_{lij}} |e_i(t)|^p \\
&\quad + \sum_{j=1}^n (|\alpha_{ij}|^{p\gamma_{pij}} + |\beta_{ij}|^{p\gamma_{pij}}) L_j^{p\delta_{pij}} |e_j(t - \tau(t))|^p,
\end{aligned} \quad (17)$$

and

$$\begin{aligned}
&p \sum_{j=1, j \neq i}^n |k_{ij}| |e_j(t)| |e_i(t)|^{p-1} \\
&\leq \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |k_{ij}|^{ph_{lij}} |e_i(t)|^p + \sum_{j=1, j \neq i}^n |k_{ij}|^{ph_{pij}} |e_j(t)|^p.
\end{aligned} \quad (18)$$

Let

$$F_i(\varepsilon_i) = \lambda_i - \omega_i - e^{\varepsilon_i \tau} \zeta_i \frac{1}{1 - \hat{\varepsilon}} - \varepsilon_i, \quad i \in I.$$

From (H_3) , we have

$$\begin{aligned}
F_i(0) &= \lambda_i - \omega_i - \zeta_i \frac{1}{1 - \hat{\varepsilon}} > 0, \quad i \in I. \\
\dot{F}_i(\varepsilon_i) &= -\tau e^{\varepsilon_i \tau} \zeta_i \frac{1}{1 - \hat{\varepsilon}} - 1 < 0, \quad \varepsilon_i \geq 0, \quad i \in I.
\end{aligned}$$

Consequently, we obtain

$$\lambda_i - \omega_i - e^{\varepsilon^* \tau} \zeta_i \frac{1}{1 - \hat{\varepsilon}} - \varepsilon^* \geq 0, \quad i \in I. \quad (19)$$

From (16)–(19),

$$\begin{aligned}
D^+ V(t) &\leq \sum_{i=1}^n \mu_i e^{\varepsilon^* t} [(\varepsilon^* - pc_i + pH_i + pk_{ii}) |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |a_{ij}|^{p\gamma_{lij}} L_j^{p\delta_{lij}} |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n |a_{ij}|^{p\gamma_{pij}} L_j^{p\delta_{pij}} |e_j(t)|^p \\
&\quad + \sum_{j=1}^n \sum_{l=1}^{p-1} (|\alpha_{ij}|^{p\gamma_{lij}} + |\beta_{ij}|^{p\gamma_{lij}}) L_j^{p\delta_{lij}} |e_i(t)|^p \\
&\quad + \sum_{j=1}^n (|\alpha_{ij}|^{p\gamma_{pij}} + |\beta_{ij}|^{p\gamma_{pij}}) L_j^{p\delta_{pij}} |e_j(t - \tau(t))|^p \\
&\quad + \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} |k_{ij}|^{ph_{lij}} |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n |k_{ij}|^{ph_{pij}} |e_j(t)|^p + \frac{1}{1 - \hat{\varepsilon}} \zeta_i e^{\varepsilon^* \tau} |e_i(t)|^p \\
&\quad - \sum_{j=1}^n \frac{\mu_j}{\mu_i} (|\alpha_{ji}|^{p\gamma_{pji}} + |\beta_{ji}|^{p\gamma_{pji}}) L_j^{p\delta_{pji}} |e_j(t - \tau(t))|^p] \\
&= \sum_{i=1}^n \mu_i e^{\varepsilon^* t} [(\varepsilon^* - pc_i + pH_i + pk_{ii}) |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n \sum_{l=1}^{p-1} (|a_{ij}|^{p\gamma_{lij}} L_j^{p\delta_{lij}} |k_{ij}|^{ph_{lij}}) |e_i(t)|^p \\
&\quad + \sum_{j=1, j \neq i}^n \frac{\mu_j}{\mu_i} (|a_{ji}|^{p\gamma_{pji}} L_i^{p\delta_{pji}} + |k_{ji}|^{ph_{pji}}) |e_i(t)|^p \\
&\quad + \sum_{j=1}^n \sum_{l=1}^{p-1} (|\alpha_{ij}|^{p\gamma_{lij}} + |\beta_{ij}|^{p\gamma_{lij}}) L_j^{p\delta_{lij}} |e_i(t)|^p \\
&\quad + \frac{1}{1 - \hat{\varepsilon}} \zeta_i e^{\varepsilon^* \tau} |e_i(t)|^p]
\end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^n \mu_i e^{\varepsilon^* t} |e_i(t)|^p \left[\left(\varepsilon^* + pH_i - pC_i \right. \right. \\
&\quad + \sum_{j=1}^n \sum_{l=1}^{p-1} |a_{ij}|^{p\gamma_{lij}} L_j^{p\delta_{lij}} \\
&\quad + \sum_{j=1}^n \sum_{l=1}^{p-1} (|\alpha_{ij}|^{p\gamma_{lij}} + |\beta_{ij}|^{p\eta_{lij}}) L_j^{p\delta_{lij}} \\
&\quad + \sum_{j=1}^n \frac{\mu_j}{\mu_i} |a_{ji}|^{p\gamma_{pji}} L_i^{p\delta_{pji}} \Big) \\
&\quad + \left(\sum_{j=1}^n \sum_{l=1}^{p-1} |k_{ij}|^{ph_{ij}} + \sum_{j=1}^n \frac{\mu_j}{\mu_i} |k_{ji}|^{ph_{pji}} + pk_{ii} \right) \\
&\quad + \left. \left(\frac{1}{1-\hat{\varepsilon}} \sum_{j=1}^n \frac{\mu_j}{\mu_i} (|\alpha_{ji}|^{p\gamma_{pji}} + |\beta_{ji}|^{p\eta_{pji}}) L_i^{p\delta_{pji}} e^{\varepsilon^* \tau} \right) \right] \\
&= - \sum_{i=1}^n \mu_i e^{\varepsilon^* t} |e_i(t)|^p \left(\lambda_i - \omega_i - \frac{1}{1-\hat{\varepsilon}} \xi_i e^{\varepsilon^* \tau} - \varepsilon^* \right) \leq 0.
\end{aligned} \tag{20}$$

Define

$$u_i(t) = e^{\varepsilon^* t} \mu_i |e_i(t)|^p, \quad i \in I, \tag{21}$$

$$u(t) = \sum_{i=1}^n u_i(t) = e^{\varepsilon^* t} \sum_{i=1}^n \mu_i |e_i(t)|^p, \quad t \geq 0. \tag{22}$$

It follows from (20) that

$$u(t) \leq V(t) \leq V(t_{k-1}^+) \tag{23}$$

for $t \in (t_{k-1}, t_k]$, $k \in \mathbb{Z}^+$, where $V(0^+) = V(0)$.

On the other hand, from (7) and (H₄), for $k \in \mathbb{Z}^+$,

$$\begin{aligned}
u(t_k^+) &= e^{\varepsilon^* t_k^+} \sum_{i=1}^n \mu_i |e_i(t_k^+)|^p \\
&= e^{\varepsilon^* t_k} \sum_{i=1}^n \mu_i |p_{ik}(x(t_k)) - p_{ik}(y(t_k))|^p \\
&\leq e^{\varepsilon^* t_k} \sum_{i=1}^n \mu_i \sum_{j=1}^n p_{ij}^k |e_j(t_k)|^p \\
&= e^{\varepsilon^* t_k} \sum_{i=1}^n \sum_{j=1}^n \frac{\mu_j}{\mu_i} p_{ji}^k \mu_i |e_i(t_k)|^p \\
&\leq \max_{1 \leq i, j \leq n} \left\{ \sum_{j=1}^n \frac{\mu_j}{\mu_i} p_{ji}^k \right\} \sum_{i=1}^n \mu_i e^{\varepsilon^* t_k} |e_i(t_k)|^p \\
&\leq \rho_k u(t_k),
\end{aligned} \tag{24}$$

this together with (15), we have

$$\begin{aligned}
V(t_k^+) &= u(t_k^+) + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \xi_i \int_{t_k^+ - \tau(t_k^+)}^{t_k^+} e^{\varepsilon^*(s+\tau)} \mu_i |e_i(s)|^p ds \\
&\leq \rho_k u(t_k) + \frac{1}{1-\hat{\varepsilon}} \rho_k \sum_{i=1}^n \xi_i \int_{t_k - \tau(t_k)}^{t_k} e^{\varepsilon^*(s+\tau)} \mu_i |e_i(s)|^p ds \\
&= \rho_k V(t_k).
\end{aligned} \tag{25}$$

From (23) and (25), we have

$$u(t) \leq V(t) \leq V(t_{k-1}^+) \leq \rho_{k-1} V(t_{k-1}) \leq \rho_0 \rho_1 \cdots \rho_{k-1} V(0) \tag{26}$$

for $t \in (t_{k-1}, t_k]$, $k \in \mathbb{Z}^+$, where $\rho_0 = 1$.

From (H₅) we obtain

$$\rho_k \leq e^{\alpha(t_k - t_{k-1})}, \quad k \in \mathbb{Z}^+. \tag{27}$$

It follows from (26) that

$$u(t) \leq e^{\alpha(t_1 - 0)}, \dots, e^{\alpha(t_{k-1} - t_{k-2})} V(0) \leq e^{\alpha t} V(0) \tag{28}$$

for $t \in (t_{k-1}, t_k]$, $k \in \mathbb{Z}^+$.

For $t = 0$, we have

$$\begin{aligned}
V(0) &\leq \sum_{i=1}^n \mu_i |e_i(0)|^p + \frac{1}{1-\hat{\varepsilon}} \sum_{i=1}^n \xi_i \int_{-\tau}^0 e^{\varepsilon^*(s+\tau)} \mu_i |e_i(s)|^p ds \\
&\leq \sum_{i=1}^n \mu_i \left[|\phi_i(0) - \varphi_i(0)|^p + \frac{1}{1-\hat{\varepsilon}} \xi_i e^{\varepsilon^* \tau} \right. \\
&\quad \times \left. \int_{-\tau}^0 e^{\varepsilon^* s} ds \max_{-\tau \leq s \leq 0} |\phi_i(s) - \varphi_i(s)|^p \right] \\
&\leq \max_{1 \leq i \leq n} \{\mu_i\} \sum_{i=1}^n |\phi_i(0) - \varphi_i(0)|^p \\
&\quad + \frac{1}{1-\hat{\varepsilon}} e^{\varepsilon^* \tau} \max_{1 \leq i \leq n} \{\xi_i \mu_i\} \max_{-\tau \leq s \leq 0} \sum_{i=1}^n |\phi_i(s) - \varphi_i(s)|^p \\
&\leq \left(\max_{1 \leq i \leq n} \{\mu_i\} + \frac{1}{1-\hat{\varepsilon}} \tau e^{\varepsilon^* \tau} \max_{1 \leq i \leq n} \{\xi_i \mu_i\} \right) \|\phi - \varphi\|^p,
\end{aligned} \tag{29}$$

which implies that

$$\begin{aligned}
\sum_{i=1}^n |e_i(t)|^p &\leq \frac{1}{\min_{1 \leq i \leq n} \{\mu_i\}} e^{-t(\varepsilon^* - \alpha)} V(0) \\
&\leq \frac{1}{\min_{1 \leq i \leq n} \{\mu_i\}} e^{-t(\varepsilon^* - \alpha)} \left(\max_{1 \leq i \leq n} \{\mu_i\} \right. \\
&\quad \left. + \frac{1}{1-\hat{\varepsilon}} \tau e^{\varepsilon^* \tau} \max_{1 \leq i \leq n} \{\xi_i \mu_i\} \right) \|\phi - \varphi\|^p.
\end{aligned}$$

So we obtain

$$\|x(t) - y(t)\| \leq M e^{\frac{-t(\varepsilon^* - \alpha)}{p}} \|\phi - \varphi\|, \tag{30}$$

where

$$\begin{aligned}
M &= \left\{ \frac{1}{\min_{1 \leq i \leq n} \{\mu_i\}} \left(\max_{1 \leq i \leq n} \{\mu_i\} \right. \right. \\
&\quad \left. \left. + \frac{1}{1-\hat{\varepsilon}} \tau e^{\varepsilon^* \tau} \max_{1 \leq i \leq n} \{\xi_i \mu_i\} \right) \right\}^{\frac{1}{p}} \geq 1,
\end{aligned} \tag{31}$$

which means that the drive system (1) and the response system (4) are globally exponentially synchronized and the proof of Theorem 1 is finished. $\square$

As a special case, consider the following drive system and response system with linear impulsive effects

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^n a_{ij} f_j(x_j(t)) + \sum_{j=1}^n b_{ij} v_j + u_i \\ \quad + \bigwedge_{j=1}^n \alpha_{ij} f_j(x_j(t - \tau(t))) + \bigwedge_{j=1}^n T_{ij} v_j + \bigvee_{j=1}^n \beta_{ij} f_j(x_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^n S_{ij} v_j, \quad t \neq t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \\ x_i(t_k^+) = \theta_{ik} x_i(t_k), \quad t = t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \end{cases} \quad (32)$$

and

$$\begin{cases} \dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^n a_{ij} f_j(y_j(t)) + \sum_{j=1}^n b_{ij} v_j + u_i \\ \quad + \bigwedge_{j=1}^n \alpha_{ij} f_j(y_j(t - \tau(t))) + \bigwedge_{j=1}^n T_{ij} v_j + \bigvee_{j=1}^n \beta_{ij} f_j(y_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^n S_{ij} v_j - \sum_{j=1}^n k_{ij} e_j(t), \quad t \neq t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \\ y_i(t_k^+) = \theta_{ik} y_i(t_k), \quad t = t_k, \quad i \in I, \quad k \in \mathbb{Z}^+, \end{cases} \quad (33)$$

where each θ_{ik} ($i \in I, k \in \mathbb{Z}^+$) is a constant and represents impulsive perturbation of the i th unit at time t_k .

(H_6) There exists a constant $\alpha \geq 0$ such that

$$\frac{\ln \rho'_k}{t_k - t_{k-1}} \leq \alpha < \varepsilon^*, \quad k \in \mathbb{Z}^+,$$

where $\rho'_k = \max_{1 \leq i \leq n} \{1, |\theta_{ik}|^p\}$, $\varepsilon^* = \min_{1 \leq i \leq n} \{\varepsilon_i\}$, ε_i is the unique positive solution of the following equation

$$\lambda_i - \omega_i - \frac{1}{1 - \varepsilon} e^{\varepsilon_i \tau} \xi_i - \varepsilon_i = 0, \quad i \in I.$$

Obviously, assumption (H_4) is true for the linear impulses given in system (32). According to Theorem 1, the following results are directly derived.

Corollary 1 Assume that (H_1)–(H_3) and (H_6) hold, then drive system (32) and response system (33) are globally exponentially synchronized.

Especially, if the linear impulses satisfy $|\theta_{ik}| \leq 1$ for $i \in I$ and $k \in \mathbb{Z}^+$, we have the following results.

Corollary 2 Assume that (H_1)–(H_3) hold, then drive system (32) and response system (33) are globally exponentially synchronized.

Remark 1 Feng et al. (2011) studied the global exponential synchronization of delayed fuzzy cellular neural networks with linear impulsive effects and constant delays,

in which the linear impulsive perturbations are restricted to $|\theta_{ik}| \leq 1$. Different from this, in this paper, the nonlinear impulsive effect is considered which represents a more natural framework for mathematical modeling of many real-world phenomena. Especially, the main results given in Feng et al. (2011) can be easily derived from Corollary 2. Hence, our results are more general.

Remark 2 In Theorem 1, a large number of parameters are introduced in order to improve the generality, integrity, and elegance of the obtained results. In practice, we can choose appropriate parameters to simplify those conditions for convenience. For example, choosing $\gamma_{lij} = \delta_{lij} = h_{lij} = \eta_{lij} = v_{lij} = \frac{1}{p}, \lambda_i, \omega_i$ and ξ_i are reduced to

$$\begin{aligned} \widehat{\lambda}_i &= pc_i - pH_i - (p-1) \sum_{j=1, j \neq i}^n |a_{ij}| L_j \\ &\quad - (p-1) \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|) L_j - \sum_{j=1, j \neq i}^n \frac{\mu_j}{\mu_i} |a_{ji}| L_i, \\ \widehat{\xi}_i &= \sum_{j=1}^n \frac{\mu_j}{\mu_i} (|\alpha_{ji}| + |\beta_{ji}|) L_j, \\ \widehat{\omega}_i &= pk_{ii} + (p-1) \sum_{j=1, j \neq i}^n |k_{ij}| + \sum_{j=1, j \neq i}^n \frac{\mu_j}{\mu_i} |k_{ji}|. \end{aligned}$$

Correspondingly, (H_3) is replaced by the following assumption:

$$(H_7) \quad \widehat{\lambda}_i - \widehat{\omega}_i - \frac{1}{1-\widehat{\varepsilon}} \widehat{\xi}_i > 0, \quad i \in I.$$

In this case, the following results can be directly derived from Theorem 1.

Corollary 3 Assume that (H_1) – (H_2) and (H_4) – (H_5) and (H_7) hold, then drive system (1) and response system (4) are globally exponentially synchronized.

Remark 3 It is easy to see that the information of all neurons in neural network (1) are required in the con-

where $t_k = 1.4k$, $k \in \mathbb{Z}^+$, $f_1(x) = f_2(x) = \tanh x$, $\tau(t) = \frac{e^t}{1+e^t}$ and

$$p_{1k}(x(t_k)) = 0.42 \sin(x_1(t_k)) - 0.3x_2(t_k), \quad k \in \mathbb{Z}^+, \\ p_{2k}(x(t_k)) = -0.2x_1(t_k) + 0.35 \sin(x_2(t_k)), \quad k \in \mathbb{Z}^+,$$

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix},$$

$$\alpha = \beta = \begin{pmatrix} -1.7 & -0.1 \\ 0.2 & -1.6 \end{pmatrix}.$$

System (35) exists a chaotic behavior as shown in Fig. 1.

The response system is given by

$$\begin{cases} \dot{y}_i(t) = -c_i y_i(t) + \sum_{j=1}^2 a_{ij} f_j(y_j(t)) + \bigwedge_{j=1}^2 \alpha_{ij} f_j(y_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^2 \beta_{ij} f_j(y_j(t - \tau(t))) - \sum_{j=1}^2 k_{ij} e_j(t), \quad t \neq t_k, \quad i = 1, 2, \quad k \in \mathbb{Z}^+, \\ y_i(t_k^+) = p_{ik}(y(t_k)), \quad t = t_k, \quad i = 1, 2, \quad k \in \mathbb{Z}^+. \end{cases} \quad (36)$$

troller (5). It may be difficult even impossible to access the global information of the network when the cell number n is large enough. In this case, we can use the following local feedback controller to replace the original controller (5).

$$K_i(t) = k_i(x_i(t) - y_i(t)), \quad (34)$$

where $k_i < 0$ for $i \in I$. In this case, $\omega_i = pk_i$, $i \in I$ in Theorem 1 and the following can be obtained.

Corollary 4 Assume that (H_1) – (H_2) , (H_4) – (H_5) hold, then drive system (1) and response system (4) are globally exponentially synchronized under controller (34) if $\lambda_i - pk_i - \frac{1}{1-\widehat{\varepsilon}} \widehat{\xi}_i > 0$ for $i \in I$.

Numerical simulations

In this section, an example is given to show the effectiveness of our obtained results in this paper.

Consider the following fuzzy cellular neural networks with time-varying delays and nonlinear impulsive effects

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^2 a_{ij} f_j(x_j(t)) + \bigwedge_{j=1}^2 \alpha_{ij} f_j(x_j(t - \tau(t))) \\ \quad + \bigvee_{j=1}^2 \beta_{ij} f_j(x_j(t - \tau(t))), \quad t \neq t_k, \quad i = 1, 2, \quad k \in \mathbb{Z}^+, \\ x_i(t_k^+) = p_{ik}(x(t_k)), \quad t = t_k, \quad i = 1, 2, \quad k \in \mathbb{Z}^+, \end{cases} \quad (35)$$

Choose $p = 2$, $\mu_i = 1$, $\gamma_{lij} = \delta_{lij} = h_{lij} = \eta_{lij} = v_{lij} = \frac{1}{2}$, $(i, j \in I)$, by calculation that $\varepsilon_1 = 0.3424$, $\varepsilon_2 = 0.1478$, $\underline{F}_j = 0$, $\overline{F}_j = 1$, $j = 1, 2$, $\widehat{\varepsilon} = \frac{1}{4}$ and

$$P_k = \begin{pmatrix} 0.3042 & 0.216 \\ 0.11 & 0.1925 \end{pmatrix},$$

$$\rho_k = \max \left\{ 1, \sum_{j=1}^2 p_{ji}^k \right\} = 1,$$

which implies that (H_5) is satisfied.

Fig. 1 The chaotic attractor of system (35) with the initial values $x_1(s) = 0.1$ and $x_2(s) = 0.2$ for $s \in [-1, 0]$

Take the external control input gain matrix K as follows

$$\begin{aligned}
 K &= \begin{pmatrix} -9 & 0 \\ 0 & -9 \end{pmatrix}, \\
 \hat{\lambda}_1 - \frac{1}{1-\hat{\varepsilon}} \hat{\xi}_1 - \hat{\omega}_1 \\
 &= pc_1 - pH_1 - (p-1) \sum_{j=1, j \neq 1}^2 |a_{1j}| L_j \\
 &\quad - (p-1) \sum_{j=1}^2 (|\alpha_{1j}| + |\beta_{1j}|) L_j - \sum_{j=1, j \neq 1}^2 \frac{\mu_j}{\mu_1} |a_{j1}| L_1 \\
 &\quad - \frac{1}{1-\hat{\varepsilon}} \sum_{j=1}^2 \frac{\mu_j}{\mu_1} (|\alpha_{j1}| + |\beta_{j1}|) L_1 - pk_{11} \\
 &\quad - (p-1) \sum_{j=1, j \neq 1}^2 |k_{1j}| - \sum_{j=1, j \neq 1}^2 \frac{\mu_j}{\mu_1} |k_{j1}| \\
 &= 2.223 > 0, \\
 \hat{\lambda}_2 - \frac{1}{1-\hat{\varepsilon}} \hat{\xi}_2 - \hat{\omega}_2 \\
 &= pc_2 - pH_2 - (p-1) \sum_{j=1, j \neq 2}^2 |a_{2j}| L_j \\
 &\quad - (p-1) \sum_{j=1}^2 (|\alpha_{2j}| + |\beta_{2j}|) L_j - \sum_{j=1, j \neq 2}^2 \frac{\mu_j}{\mu_2} |a_{j2}| L_2 \\
 &\quad - \frac{1}{1-\hat{\varepsilon}} \sum_{j=1}^2 \frac{\mu_j}{\mu_2} (|\alpha_{j2}| + |\beta_{j2}|) L_2 - pk_{22} - (p-1) \\
 &\quad \times \sum_{j=1, j \neq 2}^2 |k_{2j}| - \sum_{j=1, j \neq 2}^2 \frac{\mu_j}{\mu_2} |k_{j2}| \\
 &= 0.7666 > 0.
 \end{aligned}$$

which implies that (H_3) is satisfied.

According to Theorem 1, drive system (35) and response system (36) are synchronized, which are shown in Figs. 2, 3, 4, 5.

Fig. 2 Synchronization error $e_1(t)$ of systems (35) and (36)

Fig. 3 Synchronization error $e_2(t)$ of systems (35) and (36)

Fig. 4 Synchronization of x_1 and y_1 of systems (35) and (36)

Fig. 5 Synchronization of x_2 and y_2 of systems (35) and (36)

Conclusions

In this paper, fuzzy cellular neural networks with nonlinear impulsive effects and time-varying delays are discussed. By constructing a feasible Lyapunov functional and utilizing some inequality techniques, several new and useful synchronization criteria in terms of p -norm are obtained. Compared with the previous works, in this paper, the linear restriction on impulsive functions are removed. Finally, some numerical simulations are provided to verify the effectiveness and feasibility of the presented methods.

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