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Statistical LOR estimation for a high-resolution dMiCE PET detector

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Abstract

We develop a statistical line of response (LOR) estimator of the three-dimensional interaction positions of a pair of annihilation photons in a PET detector module with depth of interaction capability. The three-dimensional points of interaction of a coincidence pair of photons within the detector module are estimated by calculation of an expectation of the points of interaction conditioned on the signals measured by the photosensors. This conditional expectation is computed from estimates of the probability density function of the light collection process and a model of the kinetics of photon interactions in the detector module. Our algorithm is capable of handling coincidences where each annihilation photon interacts any number of times within the detector module before being completely absorbed or escaping. In the case of multiple interactions, our algorithm estimates the position of the first interaction for each of the coincidence photons. This LOR estimation algorithm is developed for a high-resolution PET detector capable of providing depth-of-interaction information. Depth of interaction is measured by tailoring the light shared between two adjacent detector elements. These light-sharing crystal pairs are referred to as dMiCE and are being developed in our lab. Each detector element in the prototype system has a $2 \times 2 \text{ mm}^2$ cross section and is directly coupled to a micro-pixel avalanche photodiode (MAPD). In this set-up, the distribution of the ratio of light shared between two adjacent detector elements can be expressed as a function of the depth of interaction. Monte Carlo experiments are performed using our LOR estimation algorithm and compared with Anger logic. We show that our LOR estimation algorithm provides a significant improvement over Anger logic under a variety of parameters.

1. Introduction

Our laboratory is developing a high-resolution PET detector capable of providing depth-of-interaction information (dMiCE (Miyaoka *et al* 1998, Lewellen *et al* 2004, 2007)) by tailoring the light shared between two adjacent detector elements. Each detector element in the prototype system has a $2 \times 2 \text{ mm}^2$ cross section and is directly coupled to a micro-pixel avalanche photodiode (MAPD) developed by Zecotek Photonics (figure 1). In this set-up, the distribution of the ratio of light shared between two adjacent detector elements can be expressed as a function of the depth of interaction.

The motivation for estimating the depth of interaction (DOI) is to avoid the well-known parallax error (Wernick and Aarsvold 2004). Figure 2 provides an illustration of the parallax

error. Several methods for DOI estimation have been evaluated for PET detector modules (Miyaka *et al* 1998, Lewellen *et al* 2004, 2007, Dokhale *et al* 2006, Wang *et al* 2004, Huber *et al* 2001, McElroy *et al* 2005, Saoudi *et al* 1999, Inadama *et al* 2006, Pratz and Levin 2007, Du *et al* 2007, 2008, Yang *et al* 2009), but to our knowledge this use of a Bayesian estimator for the determination of a line of response from coincidence events is unique.

Conventional commercial PET scanners (which do not estimate the DOI) employ versions of Anger logic to position events (Cherry *et al* 2003, Wernick and Aarsvold 2004). Anger logic positions interactions by calculation of the energy-weighted centroid of the signals measured (figures 2 and 3). The depth of interaction is assigned a constant value for all events based on the attenuation coefficient of the detector crystal. Note that multiple interactions cause even further inaccuracies in the estimate of a line of response. Our LOR estimator sets out to overcome the loss of resolution inherent in Anger logic caused by unknown depth of interaction and multiple interactions.

We determine a line of response (LOR) by estimating the three-dimensional interaction positions of a pair of annihilation photons in the detector module. The three-dimensional points of interaction of a coincidence pair of photons within the detector system are estimated by calculation of an expectation of the points of interaction conditioned on the signals measured by the MAPDs (this is known as Bayesian estimation). This conditional expectation is computed from estimates of the probability density functions of the light collection process and a model of the kinetics of photon interactions in the detector module. Our algorithm is capable of handling coincidences where each photon interacts any number of times within the detector module before being completely absorbed or escaping. In the case of multiple interactions, our algorithm estimates the position of the first interaction for each of the coincidence photons.

This paper is outlined as follows. In the following section we describe the methods used. Section 3 outlines the development of our Bayesian LOR estimation algorithm. Section 4 outlines some numerical experiments performed with simulated data, and we conclude with a short discussion of the results in section 5.

2. Methods

For this work we simulate a PET system (figure 1) comprised of a pair of detector modules with blocks of Lutetium Fine Silicate (LFS, Zecotek Photonics, Inc.) of size $200 \times 200 \times 20$ mm³ with dMiCE crystals. The modules are separated by 100 mm. By using a pair of large panel detectors in close proximity we can test the effect of estimating LORs with a large oblique angle.

Basic statistics of the stochastic process of high-energy photon interactions in a detector module were explored. We simulated a beam of 511 keV photons normally incident on the center of one of the detector modules with our Monte Carlo-based INTERACT software (Ling 2007).

Let $\mathbf{M} = (M_a, M_b)^T$ be a random variable for the signals measured by a dMiCE crystal pair, where

$$\mathbf{M} = \begin{cases} (M_{\text{primary}}, M_{\text{secondary}})^T, & \text{if } M_a \text{ is primary signal} \\ (M_{\text{secondary}}, M_{\text{primary}})^T, & \text{otherwise.} \end{cases}$$

The primary detector pixel is the one in which the interaction occurred. To characterize the light response, a collimated beta source is directed at a single crystal pair. The light response is then recorded for depths varying over the entire long axis of the crystal pair. This experimental process will enable us to characterize the distribution of the signals given a certain depth (determined from the interaction position, x , and the position of the detector modules) and energy deposited (λ), i.e., we characterize the distribution of

$$\mathbf{M}|x, \lambda.$$

The dMiCE crystal pairs are currently under evaluation. Based on initial testing results, we will model $\mathbf{M}|x, \lambda$ as a joint Gaussian whose mean and variance are described as follows. The mean light response is modeled to vary linearly with depth in a slope–intercept parameterization. Let r_p and b_p be the slope and intercept of the light response in the primary detector pixel, and r_s and b_s be the slope and intercept of the light response in the secondary detector pixel. Let d be the energy resolution and z be the depth of interaction which is determined from x , the position of interaction. The model is then given by

$$\begin{aligned} E[M_{\text{primary}}|z, \lambda] &= (r_p z + b_p) \lambda \\ \text{Var}[M_{\text{primary}}|z, \lambda] &= (d E[M_{\text{primary}}|z, \lambda])^2 \\ E[M_{\text{secondary}}|z, \lambda] &= (r_s z + b_s) \lambda \\ \text{Var}[M_{\text{secondary}}|z, \lambda] &= (d E[M_{\text{secondary}}|z, \lambda])^2. \end{aligned}$$

Figures 4 and 5 illustrate the characteristics of these distributions. For convenience, we let $H = 20$ mm denote the thickness of the detector crystal.

Four different light response schemes (table 1) were simulated and tested with Anger and Bayesian LOR estimation. Other than the depth of interaction, the detector signals do not tell us where in the crystal the interaction occurred. Thus, the Bayesian LOR estimator places estimates in the center of the plane perpendicular to the long axis of a detector pixel.

We also estimated the LORs with what we call the *ideal estimator*. This is a hypothetical estimator that always gives the correct order of interactions and the exact depth of interaction. Just like the Bayesian LOR estimator, the ideal estimator places the estimate in the center of the plane perpendicular to the long axis of the detector pixel. The purpose of the ideal estimator is to provide an upper limit on the performance of any LOR estimator on a PET system with dMiCE crystal pairs.

The LOR estimation algorithms were evaluated with data simulated from our test PET system (figure 1). A point source was placed halfway between the two detector panels, and one hundred thousand coincidences with an isotropic distribution (limited to the 60° acceptance angle) were simulated. Photon interactions within the detector modules were simulated with our Monte Carlo-based INTERACT software (Ling 2007). The output of this software was fed through a program that simulated the noisy detector signals. All coincidences were processed by our estimation algorithms, i.e., no LORs were filtered out due to the total amount of energy deposited in the detector module.

Our Bayesian LOR estimation algorithm estimates the first points of interaction of a photon coincidence pair by calculation of equation (6), and the signal pair order is estimated by (5). All integrals are computed using the trapezoid rule.

Performance of the various estimators was measured based on its ability to correctly position the first point of interaction and its overall effect at estimating an LOR.

The bias and variance of the estimated depth of interaction were measured by the sample mean and variance of the difference of the true and estimated depth of the first interaction. In the case of multiple interactions, our Bayesian LOR estimator not only estimates the depth of interaction, but also which interaction was first. We define the *misorder rate* to be the percentage of events in which the estimator incorrectly determined which interaction was first. Thus if $\hat{\mathbf{i}}$ (a permutation) is the estimated ordering of the detector signal pairs and $\mathbf{i}$ is the correct ordering of the signal pairs, then the misorder rate is defined as $P(\hat{\mathbf{i}}_1 \neq \mathbf{i}_1 | N \geq 2)$, where N is a random variable for the number of interactions within a detector module. The overall estimation error was measured by the sample mean of the difference between the true and estimated positions of the first interaction.

To illustrate the performance of each estimator in accurately estimating an LOR, we employed focal plane tomography. In focal plane tomography, the estimated LORs are backprojected onto the plane containing the point source and parallel to the face of the detector modules.

3. Statistical LOR estimation algorithm

In this section, we develop our Bayesian LOR estimation algorithm. All probability density functions (PDFs) will be denoted by $f(\cdot)$. We start by developing a probabilistic model of the photon process.

3.1. The photon process

The process of photon interactions in matter is a well-developed stochastic process. For purposes of speed of computation, we use a simplified model of this process. Our model considers the energy-dependent distribution of photoelectric absorption and Compton scatter. Angular distributions of photons that undergo Compton scatter are modeled by the Klein–Nishina formula (Evans 1955). We do not consider positron range, non-collinearity of positron annihilation, photon polarization or coherent (Rayleigh) scatter. Some basic

statistics of the photon process can be found in figure 6. Random variables will be denoted by capital letters, and realizations of random variables will be denoted by lower case letters.

We start by defining the necessary random variables. Let X_k denote the three-dimensional interaction positions of an annihilation photon. For a coincidence pair, we will add subscripts to X_k s (e.g. X_{Ak} , X_{Bk}) to denote the detector module where the annihilation photon interacts. Thus, X_{A2} denotes the position of the second interaction. Let Γ_k denote the energy (keV) of the photon after the k th interaction and $\Lambda_k = \Gamma_{k-1} - \Gamma_k$ denote the energy (keV) deposited by the k th interaction. The trajectory of the photon after the k th interaction will be denoted by the random variable $R_k = \frac{X_{k+1} - X_k}{\|X_{k+1} - X_k\|}$. The random variable triplets (X_0, Γ_0, R_0) are a special case and represent the position, energy and trajectory of the photon as it first enters the detector module.

Let $\mu_\tau(\gamma)$ be the photo-electric absorption coefficient and $\mu_\sigma(\gamma)$ be the Compton scatter attenuation coefficient of the scintillation material. We also let the total attenuation coefficient be given by $\mu(\gamma) = \mu_\tau(\gamma) + \mu_\sigma(\gamma)$. The probability that a photon of energy γ_{k-1} will have energy γ_k after undergoing Compton scatter is given by

$$f(\gamma_k|\gamma_{k-1}) = \frac{1 + \left(\frac{\gamma_k}{\gamma_{k-1}}\right)^2 + \left(\frac{\gamma_k}{\gamma_{k-1}}\right) \left[\left(\frac{511}{\gamma_{k-1}} - \frac{511}{\gamma_k} + 1\right)^2 - 1 \right]}{\gamma_k \gamma_{k-1}} \times C(\gamma_{k-1}) \begin{cases} 1, & \frac{511\gamma_{k-1}}{511+2\gamma_{k-1}} \leq \gamma_k \leq \gamma_{k-1} \\ 0, & \text{otherwise.} \end{cases}$$

This is known as the Klein–Nishina formula (Evans 1955). The angle of scatter is then given by

$$\theta(\gamma_k, \gamma_{k-1}) = \cos^{-1} \left(\frac{511}{\gamma_{k-1}} - \frac{511}{\gamma_k} + 1 \right).$$

The term $C(\gamma_{k-1})$ is a normalizing factor such that

$$\int_{\mathbb{R}} f(\gamma_k|\gamma_{k-1}) d\gamma_k = 1.$$

Note that

$$\langle r_k, r_{k-1} \rangle = \cos \theta(\gamma_k, \gamma_{k-1}).$$

The probability density function of the photon process of a coincidence pair with k total interactions in detector A and l total interactions in detector B is given by

$$f(\gamma_{Ak}, \mathbf{x}_{Ak}, \dots, \mathbf{x}_{A1}, \gamma_{Bl}, \mathbf{x}_{Bl}, \dots, \mathbf{x}_{B1}) = [\delta(\gamma_{A,k}) \mu_\tau(\gamma_{A,k-1}) + f(\gamma_{A,k}|\gamma_{A,k-1}) \mu_\sigma(\gamma_{A,k-1})] e^{-\mu(\gamma_{A,k-1}) \|\mathbf{x}_{A,k-1} - \mathbf{x}_{A,k}\|} \quad (1)$$

$$\times \frac{1}{(2\pi)^{k-1}} \prod_{i=1}^{k-1} f(\gamma_{A,i} | \gamma_{A,i-1}) \mu_{\sigma}(\gamma_{A,i-1}) e^{-\mu(\gamma_{A,i-1}) \|\mathbf{x}_{A,i-1} - \mathbf{x}_{A,i}\|} \quad (2)$$

$$\times [\delta(\gamma_{B,l}) \mu_{\tau}(\gamma_{B,l-1}) + f(\gamma_{B,l} | \gamma_{B,l-1}) \mu_{\sigma}(\gamma_{B,l-1})] e^{-\mu(\gamma_{B,l-1}) \|\mathbf{x}_{B,l-1} - \mathbf{x}_{B,l}\|} \quad (3)$$

$$\times \frac{1}{(2\pi)^{l-1}} \prod_{j=1}^{l-1} f(\gamma_{B,j} | \gamma_{B,j-1}) \mu_{\sigma}(\gamma_{B,j-1}) e^{-\mu(\gamma_{B,j-1}) \|\mathbf{x}_{B,j-1} - \mathbf{x}_{B,j}\|}, \quad (4)$$

where $\delta(\cdot)$ is the Dirac delta function. Note that

$$\gamma_{A,i} = \frac{511}{\frac{511}{\gamma_{A,i-1}} + 1 - \langle r_{A,i-1}, r_{A,i} \rangle}$$

for $i = 1, 2, \dots, k-1$ and

$$r_{A,0} = \frac{x_{A,1} - x_{B,1}}{\|x_{A,1} - x_{B,1}\|}.$$

Similar equations exist for $\gamma_{B,j}$ and $r_{B,0}$. The entrance positions $x_{A,0}$ and $x_{B,0}$ can be calculated from $x_{A,1}$ and $x_{B,1}$ and knowledge of the geometry of the scanner. We will assume that $\gamma_{A,0} = \gamma_{B,0} = 511$.

Equations (2) and (4) account for the first $k-1$ and $l-1$ Compton scattering events. Equations (1) and (3) account for the final interaction which could be a photo-electric or Compton scattering event (in this case the photon escapes the detector). The exponential terms characterize the probability that a photon will travel a certain distance before going through a photo-electric absorption or Compton scattering event. If both photons of the coincidence pair interact exactly once, the above distribution reduces to

$$f(\gamma_{A1}, \mathbf{x}_{A1}, \gamma_{B1}, \mathbf{x}_{B1}) = [\delta(\gamma_{A,1}) \mu_{\tau}(\gamma_{A,0}) + f(\gamma_{A,1} | \gamma_{A,0}) \mu_{\sigma}(\gamma_{A,0})] e^{-\mu(\gamma_{A,0}) \|\mathbf{x}_{A,0} - \mathbf{x}_{A,1}\|} \\ \times [\delta(\gamma_{B,1}) \mu_{\tau}(\gamma_{B,0}) + f(\gamma_{B,1} | \gamma_{B,0}) \mu_{\sigma}(\gamma_{B,0})] e^{-\mu(\gamma_{B,0}) \|\mathbf{x}_{B,0} - \mathbf{x}_{B,1}\|}.$$

3.2. Bayes estimation of line of response

Our Bayesian LOR estimation algorithm is designed to estimate the first point of interaction of each photon in a coincidence pair. We assume that each signal pair was produced by exactly one interaction of a high-energy photon. With this assumption, the order of the interactions is equivalent to the order of the signal pairs produced.

The signal pairs generated by the sequences of high-energy photon interactions of a coincidence pair will be denoted by $\mathbf{m}_{A,1}, \dots, \mathbf{m}_{A,k}$ and $\mathbf{m}_{B,1}, \dots, \mathbf{m}_{B,l}$. The numerical subscripts are arbitrary and do not imply the order of the signals. We must estimate the order of the signal pairs, since this information is unknown. Let $\mathbf{i} = (i_1, i_2, \dots, i_k)$ be a permutation of $(1, 2, \dots, k)$ and $\mathbf{j} = (j_1, j_2, \dots, j_l)$ be a permutation of $(1, 2, \dots, l)$. The order of interaction is estimated by

$$(\hat{i}_1, \dots, \hat{i}_k, \hat{j}_1, \dots, \hat{j}_l) \equiv \underset{\mathbf{i}, \mathbf{j}}{\operatorname{argmax}} f(\mathbf{m}_{A,i_1}, \dots, \mathbf{m}_{A,i_k}, \mathbf{m}_{B,j_1}, \dots, \mathbf{m}_{B,j_l}), \quad (5)$$

where the max is taken over all permutations of the signal pairs. The estimates of the first signal pairs for each of the coincidence photons are $\mathbf{m}_{\hat{i}_1}$ and $\mathbf{m}_{\hat{j}_1}$. Note that

$$f(\mathbf{m}_{A,2}, \mathbf{m}_{A,3}, \mathbf{m}_{A,1}, \mathbf{m}_{B,1}, \mathbf{m}_{B,2})$$

is the probability that $\mathbf{m}_{A,2}, \mathbf{m}_{A,3}, \mathbf{m}_{A,1}$ are the first, second and third signal pairs produced in detector A, and $\mathbf{m}_{B,1}, \mathbf{m}_{B,2}$ are the first and second signal pairs produced in detector B. Thus, equation (5) is a type of maximum likelihood (ML) estimation because the order of the signals is implicitly assumed (i.e. in the above we implicitly conditioned on the order of the signals).

Consider an arbitrary ordering the signal pairs. For convenience, we let

$$\begin{aligned} \underline{\mathbf{x}}_A &\equiv (\mathbf{x}_{A,k}, \dots, \mathbf{x}_{A,1}), & \underline{\mathbf{x}}_B &\equiv (\mathbf{x}_{B,l}, \dots, \mathbf{x}_{B,1}) \\ \underline{\mathbf{m}}_A &\equiv (\mathbf{m}_{A,k}, \dots, \mathbf{m}_{A,1}), & \underline{\mathbf{m}}_B &\equiv (\mathbf{m}_{B,l}, \dots, \mathbf{m}_{B,1}). \end{aligned}$$

We estimate the points of first interaction of a coincidence pair by Bayesian estimation which is given by

$$\begin{aligned} &\begin{bmatrix} \hat{\mathbf{x}}_{A1} \\ \hat{\mathbf{x}}_{B1} \end{bmatrix} \equiv E[(\mathbf{X}_{A1}, \mathbf{X}_{B1})^T | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B] \\ &\equiv \frac{\int \cdots \int \begin{bmatrix} \mathbf{x}_{A1} \\ \mathbf{x}_{B1} \end{bmatrix} f(\underline{\mathbf{x}}_A, \underline{\mathbf{x}}_B | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) d\underline{\mathbf{x}}_A d\underline{\mathbf{x}}_B}{\int \cdots \int f(\underline{\mathbf{x}}_A, \underline{\mathbf{x}}_B | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) d\underline{\mathbf{x}}_A d\underline{\mathbf{x}}_B} \\ &= \frac{\int \cdots \int \begin{bmatrix} \mathbf{x}_{A1} \\ \mathbf{x}_{B1} \end{bmatrix} f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) d\underline{\mathbf{x}}_A d\gamma_{Ak} d\underline{\mathbf{x}}_B d\gamma_{Bl}}{\int \cdots \int f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) d\underline{\mathbf{x}}_A d\gamma_{Ak} d\underline{\mathbf{x}}_B d\gamma_{Bl}}, \end{aligned} \quad (6)$$

where

$$f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B | \underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) = \frac{f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B) \prod_{i=1}^k f(\mathbf{m}_{Ai} | \mathbf{x}_{Ai}, \lambda_{Ai}) \prod_{j=1}^l f(\mathbf{m}_{Bj} | \mathbf{x}_{Bj}, \lambda_{Bj})}{f(\underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B)} \\ \propto f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B) \prod_{i=1}^k f(\mathbf{m}_{Ai} | \mathbf{x}_{Ai}, \lambda_{Ai}) \prod_{j=1}^l f(\mathbf{m}_{Bj} | \mathbf{x}_{Bj}, \lambda_{Bj}).$$

Note that

$$f(\underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B) = \int \cdots \int f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B) \prod_{i=1}^k f(\mathbf{m}_{Ai} | \mathbf{x}_{Ai}, \lambda_{Ai}) \times \prod_{j=1}^l f(\mathbf{m}_{Bj} | \mathbf{x}_{Bj}, \lambda_{Bj}) d\underline{\mathbf{x}}_A d\gamma_{Ak} d\underline{\mathbf{x}}_B d\gamma_{Bl}.$$

All of the above calculations can be done with knowledge of the photon process and detection process PDFs. In the above, we have implicitly assumed an ordering of the interactions. In practice, we compute (6) for all combinations of the permutations of the signals $\underline{\mathbf{m}}_A$ and $\underline{\mathbf{m}}_B$. We then take the estimate that arose from the ordering that maximized $f(\underline{\mathbf{m}}_A, \underline{\mathbf{m}}_B)$ as in equation (5).

Computing the integral (6) involves integrating the photon interaction positions over the spatial region that a particular crystal pair occupies. To speed up the computation of this integral, we restrict the integration over the regions with the largest probabilities.

With the detector geometry used here, changes in the depth of interaction of $\mathbf{x}_{B1}$ have little effect on changes of $\mathbf{x}_{A0}$ and $\mathbf{r}_{A0}$. Similarly, changes in the depth of interaction of $\mathbf{x}_{A1}$ have little effect on changes of $\mathbf{x}_{B0}$ and $\mathbf{r}_{B0}$. Thus to improve the computational complexity of the algorithm, the entrance position, $\mathbf{x}_{A0}$, and trajectory, $\mathbf{r}_{A0}$, will be computed from $\mathbf{x}_{A1}$ and the maximum likelihood estimation of $\mathbf{x}_{B1}$ and similarly $\mathbf{x}_{B0}$ and $\mathbf{r}_{B0}$ will be calculated from $\mathbf{x}_{B1}$ and the maximum likelihood estimation of $\mathbf{x}_{A1}$. This will enable us to split the photon process PDF into terms for the photon interactions in each detector module, i.e.,

$$f(\gamma_{Ak}, \underline{\mathbf{x}}_A, \gamma_{Bl}, \underline{\mathbf{x}}_B) \approx f(\gamma_{Ak}, \underline{\mathbf{x}}_A) f(\gamma_{Bl}, \underline{\mathbf{x}}_B),$$

where $f(\gamma_{Ak}, \underline{\mathbf{x}}_A)$ is the product of equations (1) and (2), and $f(\gamma_{Bl}, \underline{\mathbf{x}}_B)$ is the product of equations (3) and (4). This approximation enables us to decouple the integral (6) into a product of an integral over $\mathbf{x}_{A1}, \dots, \mathbf{x}_{Ak}, \gamma_{Ak}$ and an integral over $\mathbf{x}_{B1}, \dots, \mathbf{x}_{Bk}, \gamma_{Bl}$.

4. Results

Some basic statistics of the photon process are shown in figure 6 and table 1. The mean energy deposited by a high-energy photon was 447 keV because approximately 16% of the photons escaped the detector module before depositing all of their energy. Note that the coincidences where each photon only interacts once in the detector module account for approximately $44\% \times 44\% = 19\%$ of the total coincidences.

The results of the Bayesian LOR estimator with the data from schemes I, II, III, IV are denoted by Bayes I, Bayes II, Bayes III, Bayes IV, respectively. Since the results of Anger

estimation did not vary with the data from the different schemes, we only show the results of Anger estimation with scheme I.

Statistics of the performance of the various estimators to correctly position the first point of interaction are shown in table 3. The misorder rates displayed in table 3 may seem high, but if one were to randomly guess which interaction was first, then the misorder rate would be approximately

$$P(\hat{i}_1 \neq i_1 | N \geq 2) = \frac{1}{P(N \geq 2)} \sum_{n=2}^{\infty} P(\hat{i}_1 \neq i_1 | N=n) P(N=n) \approx 57.79\%.$$

In the above, we used the data from figure 6 to estimate $P(N=n)$ and $P(\hat{i}_1 \neq i_1 | N=n) = \frac{n-1}{n}$.

Focal plane tomography histograms using all the LOR estimation techniques are displayed in figure 7. We show (figure 8) the percentage of LORs that get backprojected into square-shaped regions of interest (ROI) centered at the location of the point source to quantify the focal plane tomography results. Note that figure 8 is a plot of the cumulative distribution functions (CDF) of each estimation method.

Our model assumes that each signal pair was produced by exactly one high-energy photon interaction. Approximately 26% of the interaction sequences of the high-energy photons violate this assumption. Signals from two separate interactions within the same light-sharing crystal pair are summed together in our simulation. We will refer to the event where there are multiple interactions in one dMiCE crystal pair as *pair-multiples*.

We filter out the coincidences with pair-multiples to quantify their effect on the accuracy of our algorithm. We also compute estimates of the coincidence data where we assume that the correct order of the signals is known. The results of filtering out the coincidences with pair-multiples and the estimates where the signal order is assumed to be known are shown in table 4 and figure 9. In practice, one cannot filter out the pair-multiple events or know the correct order of the signal pairs. We display these results only to help quantify the source of errors in our Bayesian LOR estimation algorithm.

The Bayesian LOR estimation software was written in C++ and experiments were run on a 2.3 GHz PowerPC G5 processor. One million coincidences were processed in about 4 h. No work has been done in optimizing the computational efficiency of the software.

5. Discussion and conclusion

We have developed a statistical LOR estimator for PET systems with dMiCE crystal pairs. The algorithm estimates an LOR by choosing the most likely order of interactions and then the first points of interaction for a pair of annihilation photons are calculated by Bayesian estimation. By estimating both photons of an annihilation pair simultaneously we have reduced the number of unknowns such as the trajectory and entrance position of the photons as they enter the detector modules. Initial numerical experiments illustrate the robustness of our Bayesian LOR estimation algorithm.

Although Anger estimation uses an energy-weighted centroid to position events, the estimator has a semi-discrete nature due to the pixelated detector pixels and the one-on-one readout scheme. This is the source of the peaked distribution in figure 7. One can also see two moderately large peaks to either side of the large central peak. Note that these secondary peaks appear larger on the left and right of the central peak than the peaks in front and back of it. These peaks are pronounced because the light-sharing crystal pairs blur the position of this event.

The distributions for the Anger and Bayesian focal plane tomography results are more sharply peaked than the distribution for the Ideal focal plane tomography results (figure 7). The CDF plots in figure 8 indicate that the Ideal estimate is indeed the best for ROIs of size greater than or equal to 0.5 mm, but is worse than Anger, Bayes I and Bayes III for ROIs smaller than 0.5 mm.

Our Bayesian LOR estimation algorithm was also tested with a more sophisticated PDF of the photon process which included a term that estimated the probability of photon escape in the case where the last interaction was a scattering event. The direction traveled by the photon after the last scattering event is limited to a cone whose aperture is determined by the final energy deposition in the detector. Thus, the probability of escape must be calculated by marginalizing over all the directions contained in this scattering cone. This turned out to be a significant computational burden on the estimation algorithm while providing no significant improvement in accuracy and thus was not considered in the numerical experiments described in this paper.

The model for the distributions of the light response function (the detection process) described in the results section was used in this paper to demonstrate the performance of our Bayesian LOR estimator, but it should be noted that any light response function that varies with depth of interaction could be employed. The model used in our numerical experiments is based on results of some initial testing on the distribution of the light response.

Bayesian estimation is the ideal estimation technique for an L^2 cost function. A more appropriate cost function for a task such as LOR estimation would be L^1 which would lead to an estimation based on a conditional median rather than a conditional expectation which is used with Bayesian estimation. Since the distributions used in our models were nearly symmetric and decayed exponentially, the difference between these two estimation methods is expected to be negligible. An L^1 cost function also leads to a more computationally intensive algorithm.

A major source of error in our Bayesian LOR estimation seems to be the rate at which the order of interaction was incorrectly determined (i.e. the misorder rate). Future advancements will focus on improving this step of the estimation process and improving the computational efficiency of the code.

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Figure 1.

Left: simulated PET system utilized for initial testing of the DOI reconstruction algorithm.

Right: diagram showing the dMiCE light-sharing crystal pair with a triangular-shaped light reflector. Individual crystals are of size $2 \times 2 \times 20 \text{ mm}^3$.

Figure 2.

Diagram showing the parallax error resulting from lack of depth of interaction information for oblique LORs. The solid line represents the true LOR and the dashed line represents the estimated LOR. The vertical lines in detector modules show optically separated detector pixels.

Figure 3.

Anger logic can lead to gross LOR inaccuracies in the case of multiple interactions. Asterisks mark positions where the high-energy photon interacts with the scintillation crystal.

Figure 4.

Model for the distribution of the light response of the dMiCE crystal pair as a function of the depth of interaction. These particular plots show means given by

$E[M_{\text{primary}}|z, \lambda] = (\frac{0.35}{H}z + 0.55)\lambda$ and $E[M_{\text{secondary}}|z, \lambda] = (-\frac{0.40}{H}z + 0.45)\lambda$ and 10% energy resolution (scheme I). For purposes of illustration, the plots have been normalized by λ .

Figure 5.

Plots of $f(\mathbf{m}|x, \lambda)$ for varying depth of interaction (x varies over the depth) and energy deposited (λ). Left: scheme I light response with $\mathbf{m} = (105.4, 62.9)$. Middle left: scheme I light response with $\mathbf{m} = (141.1, 22.1)$. Middle right: scheme IV light response with $\mathbf{m} = (96.9, 69.7)$. Right: scheme IV light response with $\mathbf{m} = (107.1, 49.3)$.

Figure 6.

Statistics of photon interactions in a detector module. The bottom two plots show a different curve for photons that interact 1, 2, ..., or 8 times in the detector module before being completely absorbed or escaping. These plots show the mean behavior, but we note that the variances are relatively large.

Figure 7. Focal plane tomography results. The top row shows the 2D histograms and the bottom row shows a 1D profile of the histograms. From left to right, we have the Anger, Bayes I, Bayes IV and Ideal estimates.

Figure 8.

The percentage of LORs that get backprojected (using focal plane tomography) into square-shaped regions of interest (ROI) centered at the location of the point source. The plot on the right shows the lower left part of the left plot in detail.

Figure 9.

The percentage of LORs that get backprojected (using focal plane tomography) into square-shaped regions of interest (ROI) centered at the location of the point source. The data labeled *no pair-multiples* show the results of the coincidences without pair-multiples interactions. The data labeled *correct order* assume that the order of the signals is known. The left plot shows the results with Bayes I and the right shows the results with Bayes IV.

Table 1

Parameters of the four light response schemes modeled.

	r_p	b_p	r_s	b_s	d
Scheme I	$\frac{0.35}{H}$	0.55	$-\frac{0.4}{H}$	0.45	$\frac{0.10}{2\sqrt{2\ln 2}}$
Scheme II	$\frac{0.1}{H}$	0.55	$-\frac{0.2}{H}$	0.45	$\frac{0.10}{2\sqrt{2\ln 2}}$
Scheme III	$\frac{0.35}{H}$	0.55	$-\frac{0.4}{H}$	0.45	$\frac{0.15}{2\sqrt{2\ln 2}}$
Scheme IV	$\frac{0.1}{H}$	0.55	$-\frac{0.2}{H}$	0.45	$\frac{0.15}{2\sqrt{2\ln 2}}$

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Table 2

Mean of the total energy deposited for photons that interact 1, 2, ..., or 8 times in the detector module before being completely absorbed or escaping.

Total number of interactions	1	2	3	4	5	6	7	8
Total energy deposited (keV)	401	476	496	503	506	507	508	511

Table 3

Statistics of the estimation performance of various LOR estimation algorithms. The depth bias and variance are the sample mean and variance of the difference of the true and estimated depth of the first interaction. The mean position error is the sample mean of the (3D) euclidean norm of the difference between the true and estimated interaction positions.

	Depth bias	Depth variance	Misorder rate	Mean position error
Anger	-0.15 mm	28.00 mm ²	–	5.40 mm
Bayes I	0.16 mm	2.42 mm ²	23.54%	1.62 mm
Bayes II	0.16 mm	3.94 mm ²	24.22%	2.01 mm
Bayes III	0.15 mm	2.66 mm ²	23.53%	1.70 mm
Bayes IV	0.11 mm	5.29 mm ²	24.82%	2.28 mm
Ideal	0 mm	0 mm ²	0%	0.77 mm

Table 4

Statistics of the estimation performance of the Bayesian LOR estimation algorithm on modified data. The data labeled *no pair-multiples* show the results of the coincidences without pair-multiple interactions. The data labeled *correct order* assume that the order of the signals is known.

	Depth bias	Depth variance	Misorder rate	Mean position error
Bayes I, no pair-multiples	0.02 mm	0.77 mm ²	19.62%	1.36 mm
Bayes I, correct order	0.13 mm	2.49 mm ²	0%	1.29 mm
Bayes I, no pair-multiples and correct order	0.02 mm	0.87 mm ²	0%	1.08 mm
Bayes IV, no pair-multiples	-0.06 mm	4.12 mm ²	22.08%	2.15 mm
Bayes IV, correct order	0.04 mm	5.40 mm ²	0%	1.97 mm
Bayes IV, no pair-multiples and correct order	-0.08 mm	4.14 mm ²	0%	1.85 mm