Published in final edited form as:

Physiol Meas. 2015 June ; 36(6): 1283–1295. doi:10.1088/0967-3334/36/6/1283.

Estimating a regional ventilation-perfusion index

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Abstract

This is a methods paper, where an approximation to the local ventilation-perfusion ratio is derived. This approximation, called the ventilation-perfusion index since it is not exactly the physiological ventilation-perfusion ratio, is calculated using conductivity reconstructions obtained using electrical impedance tomography. Since computation of the ventilation-perfusion index only requires knowledge of the internal conductivity, any conductivity reconstruction method may be used. The method is explained, and results are presented using conductivities obtained from two EIT systems, one using an iterative method and the other a linearization method.

Keywords

electrical impedance tomography; ventilation-perfusion ratio; adaptive Kaczmarz method; lung function monitoring

1. Introduction

Electrical impedance tomography (EIT) is an imaging modality that allows us to reconstruct the internal conductivity distribution of a body using boundary voltage and current measurements. The governing model is the conductivity equation

$$\nabla \cdot \sigma(x) \nabla u(x) = 0 \quad x \in \Omega \quad (1)$$

where $\sigma(x)$ is the conductivity at point x inside the domain Ω and $u(x)$ is the electric potential at point x . Current and voltage measurements are modeled by

$$\sigma \frac{\partial u}{\partial \nu} \Big|_{\partial \Omega} = j(x) \quad (2)$$

$$\text{and } u|_{\partial\Omega} = v(x) \quad (3)$$

respectively. The goal of EIT is to reconstruct σ from data pairs (v, j) or the Dirichlet- to-Neumann map, $\Lambda_\sigma: v \rightarrow j$. There are many ways to do this reconstruction, which have been studied greatly. For a review of these methods, see Cheney *et al* (1999), Borcea (2002), Uhlmann (2009), and Mueller and Siltanen (2012).

Medical applications of EIT range from cancer detection to heart and lung function monitoring to brain monitoring, for examples see (Holder 2010). Here we focus on thoracic imaging. Once the conductivity has been reconstructed, we can extract information about cardiac and respiratory cycles (Frerichs *et al* 2002, Hoetnik *et al* 2002, Isaacson *et al* 2006). We aim to use these conductivity reconstructions to estimate a clinically relevant metric, particularly for patients using a mechanical ventilator.

The ventilation-perfusion ($V = Q$) ratio is defined as the ratio between the flow of air ventilating the alveoli and the perfusion of blood flowing past the alveoli in pulmonary capillaries (Levitzky 2003). For a healthy person, the regional $V = Q$ ratio should ideally match well with the global $V = Q$ ratio. This ratio determines the efficiency of gas exchange in the lung. Normally, ventilation and perfusion have a range of 4–6 liters per minute for air and blood, respectively, which yields a global $V = Q$ ratio ranging from 0.8 to 1.2 for a healthy person (Levitzky 2003). In a normal upright adult, the ratio may vary from approximately 0.6 near the bases of the lungs to 3.0 near the apices (Guyton 1981). Of greater clinical concern is a non-uniform distribution of regional $V = Q$ ratios, (i.e. regional $V = Q$ ratios differ from the global ratio). This $V = Q$ mismatch occurs in a wide range of pulmonary ailments, acute respiratory distress syndrome for example. Being able to estimate regional $V = Q$ ratios can help influence clinical treatment, since the mismatch can then be localized. Current tests for $V = Q$ mismatch either detect a possible mismatch without localizing it or those that can localize the mismatch usually require invasive measurement techniques, some involving radioactive markers.

One such method used in practice is SPECT imaging. With SPECT, radiotracers are used to image ventilation and perfusion in patients, which can then be used to measure local $V = Q$ ratios. A recognized limitation of SPECT imaging is exposing patients to ionizing radiation, which allows only a limited number of times these measurements can be made for the safety of the patient. Another limitation is the duration of image acquisition, which can take 10–30 minutes (Petersson *et al* 2007). SPECT $V = Q$ studies have also observed the gravitational gradient of decreasing $V = Q$ ratios moving down the lung (Petersson *et al* 2007). An improved method of acquiring $V = Q$ ratios is to combine SPECT with CT imaging, which comes at a cost of increasing radiation exposure (Roach *et al* 2013). None of these methods is widely applicable in critically-ill patients.

EIT is a non-invasive, non-ionizing, fast and easily portable imaging modality, which makes it suitable for bedside monitoring. Some excellent previous studies using EIT to assess ventilation and perfusion in comparison with known pulmonary physiology as well as with standard imaging techniques are Kunst *et al* (1998), Grant *et al* (2011), and Borges *et al*

(2012). In this paper, we develop an initial approximation to the regional $V = Q$ ratio, which we call a $V = Q$ index, based on conductivities obtained via EIT. The second section describes the data collection methods as well as a brief overview of the conductivity reconstruction method used. The third section relates conductivities to volume fractions of air and blood. The fourth section describes methods of defining a suitable region of interest, since $V = Q$ is relevant to the lungs. The fifth section, then defines the $V = Q$ index, both local and regional, showing results from two subjects. The appendix briefly shows how well the ventilation-perfusion index approximates the ventilation-perfusion ratio with scaling and offset errors.

For ease of reference, table 1 lists notation used in the paper.

2. Methods

2.1. Data collection

2.1.1. ACT 3 data—Respiratory and cardiac cycle data were taken from the same subject on the same day but one after the other. Thirty-one current patterns (k)

$$T_{\ell}^k = \begin{cases} 0.85 \cos(k\theta_{\ell}), & k=1, \dots, 16 \\ 0.85 \sin((k-16)\theta_{\ell}), & k=17, \dots, 31 \end{cases} \quad (4)$$

were applied with a frequency of 28.8 kHz using the ACT 3 (Edic 1995) current source system at RPI at a rate of 18.8 frames per second. Thirty-two electrodes (ℓ) of size 2.9 cm by 2.4 cm were equally spaced around the upright subject's torso near the sixth thoracic vertebrae. The circumference of the subject's chest was 90 cm.

For the cardiac cycle, one hundred frames of data were collected during a breath hold, with no gating or averaging required. For the respiratory cycle, ninety-seven frames of data were collected during a slow exhalation followed by a slow inhalation. Reference frames were chosen from each data set for the sake of making difference images of conductivities. The reference frame for the cardiac data was chosen from the middle of the cardiac cycle. The reference frame for the respiratory data was chosen at end- inspiration. All choices were made based solely on impedance information and knowledge of the breathing manoeuvres during data collection.

2.1.2. GENESIS data—Data from a second subject were taken using the General Electric Non-invasive Electrical impedance Spectroscopy Imaging System (GENESIS) to show repeatability. More specific details about GENESIS instrument may be found in Ashe *et al* (2014), while Amm *et al* (2014) and Kao *et al* (2014) further describe GENESIS and pilot study data. As with the ACT 3 data, respiratory and cardiac data were obtained from this subject on the same day, but one after the other. For this subject, thirty-two electrodes of 3 cm diameter were placed in two parallel rings of sixteen electrodes each. The top ring was placed at the fourth intercostal space and the bottom ring was placed at the sixth intercostal space with a 6 cm vertical spacing between the centers of the rings. Currents with a frequency of 10 kHz were applied at a rate of 20 frames per second.

Again, cardiac data was taken during a breath hold, and one heart beat lasted 18 frames (0.9 seconds). For the respiratory cycle, two breathing manoeuvres were used. The first manoeuvre was normal breathing, for which one breath lasted 100 frames (5 seconds). The second manoeuvre was rapid, deep breathing, for which one breath lasted 70 frames (3.5 seconds).

2.2. Conductivity reconstruction method

Since the work we propose below does not rely on a particular reconstruction method, we only give a brief overview of the iterative method used to compute the conductivities for the $V=Q$ index. The iterative method we use is the adaptive Kaczmarz method developed by Li *et al* (2013). This method combines the block Kaczmarz method (Kaczmarz 1937, Elfving 1980, Eggermont *et al* 1981, Byrne 1996) with the idea of optimal current patterns (Isaacson 1986, Gisser *et al* 1993, Isaacson *et al* 1995). The block Kaczmarz method is an iterative algorithm to minimize the functional

$$E(\sigma) = \|V - U(\sigma)\|_2^2 \quad (5)$$

where $\|\cdot\|_2$ is the L_2 norm, σ is the piecewise conductivity distribution, $U(\sigma)$ are the voltages predicted by a forward model (1) with an appropriate electrode model (Cheng *et al* 1989, Somersalo *et al* 1992), and V are the measured voltages. Each row submatrix of the system (which corresponds to multiple current patterns in EIT) is recursively solved when using the block Kaczmarz method. One iteration is defined as a single execution that passes through all applied currents and resulting voltages. We start with an initial estimate for the voltages and a proper partition scheme that divides the Jacobian matrix, J , into n subsets. The solution update for the k th iteration of the i th subset is

$$\sigma_{k,i} = \sigma_{k,i-1} + a_{k,i} J_{k,i}^T (J_{k,i} J_{k,i}^T + \lambda_{k,i} I)^{-1} [V_i - U_i(\sigma_{k,i-1})] \quad (6)$$

where $a_{k,i}$ is the step length at iteration k for subset i , $J_{k,i}$ is the row submatrix of the Jacobian obtained at iteration k corresponding to the i th subset, V_i and $U_i(\sigma_{k,i-1})$ are the measured and predicted voltages corresponding to the i th subset given the conductivity distribution $\sigma_{k,i-1}$. More details on the adaptive Kaczmarz method can be found in (Li *et al* 2013). Here, we are reconstructing a piecewise constant conductivity in three-dimensional voxels bounded above and below by parallel planes. The top plane is at the top of the electrodes and perpendicular to them, and the bottom plane is at the bottom of the electrodes, whose vertical heights are 2.9 cm. The label p for a voxel is defined as its center of mass.

For the GENESIS data, the ToDLER algorithm was used to reconstruct the conductivities. The ToDLER algorithm is a three-dimensional linearized inverse solver for EIT and full details may be found in Blue *et al* (2000). Since most of the results we show come from the adaptive Kaczmarz method, the reader is referred to Blue *et al* (2000), Amm *et al* (2014), and Kao *et al* (2014) for in depth usage of the ToDLER algorithm.

3. Volume fractions of air and blood

To calculate local volume fractions of blood and air, we use the ACT 3 data sets collected during a cardiac and respiratory cycle, respectively. We assume that during the cardiac cycle the changes in volume fractions of blood will contribute the most to the changes in conductivity at a given voxel. It is assumed that an increase in volume of blood will result in an increase in conductivity. Under this assumption, we decompose the conductivity per voxel during the cardiac cycle as

$$\sigma_b(p, t) = \sigma_{bM} f_b(p, t) + \sigma_{bm} (1 - f_b(p, t)) \quad (7)$$

where $f_b(p, t)$ is the impedance derived approximation to the volume fraction of blood in voxel p at time t , σ_{bM} is the maximum conductivity over all voxels over the whole cardiac cycle, and σ_{bm} is the minimum conductivity over all voxels over the whole cardiac cycle. From this, we get an estimate for the volume fraction of blood

$$f_b(p, t) = \frac{\sigma_b(p, t) - \sigma_{bm}}{\sigma_{bM} - \sigma_{bm}} \quad (8)$$

where $\sigma_b(p, t)$ is the conductivity at voxel p and time t found using EIT reconstructions using the cardiac cycle data set.

Using the ACT 3 respiratory cycle data set, we can likewise compute the local volume fraction of air. In this case we assume that changes in the volume fractions of air will contribute the most to the conductivity changes at a given voxel. It is assumed that an increase in volume of air will decrease the conductivity. This yields the decomposition of conductivity during the respiratory cycle

$$\sigma_a(p, t) = \sigma_{aM} (1 - f_a(p, t)) + \sigma_{am} f_a(p, t) \quad (9)$$

where $f_a(p, t)$ is the impedance derived approximation to the volume fraction of air in voxel p at time t and σ_{aM} and σ_{am} are, respectively, the maximum and minimum conductivities over all voxels during the respiratory cycle. Now, our estimate for the local volume fraction of air is

$$f_a(p, t) = \frac{\sigma_a(p, t) - \sigma_{aM}}{\sigma_{am} - \sigma_{aM}}. \quad (10)$$

Notice that both volume fractions are estimated using only conductivity, which can be obtained via various EIT reconstruction methods. Also notice that by definition, the volume fractions for both air (10) and blood (8) range from 0 to 1. Figure 1 shows the volume fraction of blood for several frames during the cardiac cycle, while figure 2 shows the volume fraction of air for several frames during the respiratory cycle. Both figures 1 and 2 show calculations in a region of interest defined by a threshold criterion (11) defined in the

next section. Clearly, this criterion does not isolate the heart from the lungs, so the next section will discuss how to do that before computing the $V = Q$ index.

4. Region of interest

Before estimating the $V = Q$ ratio with the $V = Q$ index, we need to define our region of interest to be the lungs. Pioneering work identifying lung regions using EIT data has been done previously by Leathard *et al* (1994), Hahn *et al* (1996) and Pulletz *et al* (2006) to name a few. Our conductivity reconstructions are cross-sectional images of the human thorax. To isolate voxels within the lungs, from the rest that we image, we assume that the lung regions will have larger conductivity changes during a breathing or cardiac cycle than other regions. Under this assumption, we will designate a voxel, p , to be in the lung region, L , with the criterion

$$p \in L \quad \text{if} \quad |\sigma_M(p) - \sigma_m(p)| > \tau \max_{p \in \Omega} |\sigma_M(p) - \sigma_m(p)| \quad (11)$$

where $\sigma_M(p)$ is the maximum conductivity at voxel p and $\sigma_m(p)$ is the minimum conductivity at voxel p over the cycle. In criterion (11), τ is a threshold chosen to best isolate the lung region. Decreasing τ increases the number of voxels in L and vice versa.

When using conductivities obtained during the cardiac cycle, we are not able to separate the heart from the lungs using the threshold method alone. We propose two methods of further isolating the lungs. The first approach is to remove voxels associated with the heart region by using the knowledge that the respiratory cycle data comes from an exhale followed by an inhale. Thus, we expect less air in the middle of the cycle than at the beginning. Now, we add the criterion

$$p \notin L \quad \text{if} \quad f_a(p, t_0) - f_a(p, \frac{t_f - t_0}{2}) \leq 0 \quad (12)$$

to criterion (11), where t_0 and t_f are the initial and final time points of the respiratory data cycle, respectively. This approach is highly empirical, so we seek another approach. The second approach to refining the lung region is to correlate waveforms per voxel. A typical lung voxel should have an air volume fraction waveform like in figure 3(a), when using respiratory data or a blood volume fraction waveform like in figure 3(b) when using cardiac data. After computing the correlation between the desired waveform and the volume fraction waveform of each voxel, we select $p \in L$ for large enough correlations. When using the correlation approach with the respiratory data, voxels are chosen with a strong ventilation signal, but a weaker perfusion signal and some voxels with poor ventilation may not be selected. Likewise, with the cardiac data, the correlation method picks voxels with a strong perfusion signal and may omit areas with poor perfusion. To keep as many voxels as possible in the lung region, even those with poor signals, we take a union of the two regions derived from ventilation waveforms and perfusion waveforms.

5. Ventilation-perfusion index

The $V = \dot{Q}$ ratio, as mentioned before, is the ratio between ventilation and perfusion, each of which is a measure of flow in liters per minute (or volume per time). We now use our local volume fractions of air and blood to compute ventilation and perfusion estimates, respectively. We estimate ventilation by

$$\dot{V}(p) \approx \frac{f_a(p, t_{aM}) - f_a(p, t_{am})}{\Delta t_a} \cdot vol(p) \quad (13)$$

which we call the ventilation index, where t_{aM} and t_{am} are the times at which $f_a(p, t)$ obtains its maximum and minimum, respectively, t_a is the duration of one breath, and $vol(p)$ is the volume of voxel p as determined by the numerical mesh. Similarly, we define a perfusion index by

$$\dot{Q}(p) \approx \frac{f_b(p, t_{bM}) - f_b(p, t_{bm})}{\Delta t_b} \cdot vol(p) \quad (14)$$

where t_{bM} and t_{bm} are the times at which $f_b(p, t)$ obtains its maximum and minimum, respectively, and t_b is the duration of one heart beat. We use these indices to define our $V = \dot{Q}$ index, $i(p)$,

$$r(p) \equiv \frac{\dot{V}(p)}{\dot{Q}(p)} \approx i(p) = \frac{f_a(p, t_{aM}) - f_a(p, t_{am})}{f_b(p, t_{bM}) - f_b(p, t_{bm})} \cdot \frac{\Delta t_b}{\Delta t_a} \quad (15)$$

This index yields a distribution of approximate $V = \dot{Q}$ ratios for one cardiac and respiratory cycle. Appendix A shows a brief analysis of how $i(p)$ holds up to scaling and offset errors, since it is calculated from volume fractions derived from noisy data. For continuous monitoring, trends in this distribution can be found by taking subsequent maximum-minimum pairs over time.

The local $V = \dot{Q}$ index, (15), may be too fine, or larger regions of the lungs may be of interest, so we propose a regional $V = \dot{Q}$ index value for all voxels in a given region of interest, R . We define this value as

$$I(R) = \frac{\sum_{p \in R} \dot{V}(p)}{\sum_{p \in R} \dot{Q}(p)} \quad (16)$$

The examples from the ACT 3 data in this paper use a 2-D reconstruction algorithm to find the conductivity. The algorithm's assumption is that the conductivity does not vary in the vertical direction and we are reconstructing in voxels of height equal to the electrodes' height in the z direction. For this reason, the problem reduces to a 2-D algorithm but the scaling of the data in the codes is by areas of electrodes so that we obtain reconstructed conductivity in voxels with the height of the electrodes. There is also an assumption that no

current flows out of the top and bottom of this volume, within the body, bounded by the electrodes. This is the more serious assumption that allows us to use 2-D algorithms, but the actual currents do go above and below the planes bounding the top and bottom of the electrodes. The reconstruction takes this into account by putting more conductivity inside the voxels we reconstruct. This compensates for and represents the effects of the conductivity above and below the planes within each of the voxels. Also, since the $V = Q$ index can be computed using any conductivity reconstruction method, a 3-D reconstruction method could be used, where conductivity is allowed to vary in the vertical direction. So while we choose the region of interest, R , being a region within a transverse cross-section of the lungs in this paper we could choose regions of interest such as a lobe of the lung, a single lung, or both lungs when using a 3-D reconstruction method, as is done with the GENESIS data.

If you take $R = L$ so that the region is both lungs, then $I(L)$ should approximate the global $V = Q$ ratio. Using the adaptive Kaczmarz method and criterion (11) along with the correlation method of region refinement yields $I(L_a) = 1.37$, $I(L_b) = 0.8$, and $I(L_c) = 0.97$ for the lung regions shown in figures 4(a), 4(b), and 4(c), respectively. The first index value is higher than 1.2, but as we see in 4(a), L_a appears to be smaller than L_b and L_c . The two index values corresponding to 4(b) and 4(c) fall within the range, 0.8 to 1.2, of expected ventilation-perfusion ratios in a healthy adult (Levitzky 2003). With the GENESIS data, we have $V = Q$ indices for the upper lung, lower lung, and total lungs for both normal breathing and deep breathing. For normal breathing, the subject's upper lung regional $V = Q$ index was 1.12 and the lower lung regional $V = Q$ index was 1.01, yielding a global $V = Q$ index of 1.12. For deep breathing, the subject's upper lung regional index was 1.72 and the lower lung regional index was 1.75, yielding a global index of 1.73. Table 2 shows the computed index values side-by-side. All the reconstructed values lie within the range 0.6 to 3.0 reported by Guyton (1981) for the $V = Q$ ratios in a standing individual. Figure 4 shows isogravitational heterogeneity in the $V = Q$ index values. Using SPECT imaging, isogravitational heterogeneity has been observed for perfusion, but the same is not reported regarding ventilation nor $V = Q$ ratios (Petersson *et al* 2007). A comparison with SPECT $V = Q$ measurements would be beneficial, but due to the lack of resources is beyond the scope of this paper and will be the topic of future work.

6. Conclusion

We have shown some methods to define regions for use in approximating volumes within the lungs where cardiac or respiratory or both changes occur. We have further shown how to compute quantities that we call the local and regional ventilation-perfusion indices within these regions. These $V = Q$ indices are not exactly the physiological $V = Q$ ratio since the lung is a complicated composite for which the relation between conductivity and components is not exactly simple and linear convex combinations as we have used in (7) and (9). However, it is shown, in two subjects, that these indices approximate the values that might be expected from a normal vertical adult subject's ventilation-perfusion ratio for the region of lung surrounded by the electrode array. The standard value of the global $V = Q$ ratio ranges from 0.6 to 3.0 according to Guyton (1981) and we found our total $V = Q$ index

values in table 2 to be within the standard range. Since our $\dot{V} = \dot{Q}$ indices rely only on EIT measurements, they are, in principle, applicable to any patient.

This study suggests the following questions that remain open:

- i. To what extent could these local and/or regional $\dot{V} = \dot{Q}$ indices be used to help diagnose and guide treatment of heart and lung disease?
- ii. How precisely do these $\dot{V} = \dot{Q}$ indices reflect the actual $\dot{V} = \dot{Q}$ ratios determined from detailed physiological measurements?
- iii. To what extent do the EIT system, algorithms, and approximations used influence the clinical usefulness and interpretation of these indices.

Following publication of this paper all the ACT 3 EIT data used in this study will be made available to the public under a creative commons license, on the RPI web site.

Acknowledgments

This publication was made possible by Grant Number 1R01HL109854-01 from the National Institutes of Health. Its contents are solely the responsibility of the authors and do not necessarily represent the official views of the NIH. The authors would like to thank the referees for their comments and literature suggestions.

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Appendix A

We show here why the method for computing the ventilation-perfusion index, $i(p)$, defined in this paper, may be less sensitive to certain types of errors than are the approximations f_a and f_b to the individual volume fractions of inspired air or incoming venous blood, respectively.

Let $F_b(p, t)$ be the volume fraction of venous blood that has come from the pulmonary artery during systole and accumulated in the voxel p at time t . In other words,

$$F_b(p, t) = \frac{\text{volume of venous blood that entered } p \text{ at time } t}{\text{volume of } p}.$$

Let $F_a(p, t)$ be the volume fraction of inspired air that has accumulated in voxel p at time t . In other words,

$$F_a(p, t) = \frac{\text{volume of air that entered } p \text{ at time } t \text{ from inspiration}}{\text{volume of } p}$$

Equations (10) for f_a and (8) for f_b represent approximations to F_a and F_b , respectively. Now we assume f_a and f_b are related to the actual volume fractions F_a and F_b by errors due to scaling, A , and offsets, B , as follows

$$f_s(p, t) = A_s(p)F_s(p, t) + B_s(p), \quad (\text{A.1})$$

where $s = a$ for inspired air or $s = b$ for incoming venous blood.

The definition of the local ventilation-perfusion ($\dot{V} = \dot{Q}$) ratio, $r(p)$ is

$$r(p) = \frac{\left(\frac{\text{volume of inspired air entering voxel } p}{\text{time of one breath}} \right)}{\left(\frac{\text{volume of venous blood entering voxel } p \text{ during systole}}{\text{time of one heart beat}} \right)}.$$

Let t_{bm} denote the time venous blood begins to enter the voxel, t_{bM} denote the time venous blood stops entering the voxel, and t_b denote the duration of the cardiac cycle, i.e. the time of one heart beat. Likewise, let t_{am} denote the time inspired air begins to enter the voxel, t_{aM} denote the time inspired air stops entering the voxel, and t_a denote the duration of the respiratory cycle, i.e. the duration of one breath. Thus, the local ventilation-perfusion ratio is given by

$$r(p) = \frac{\left(\frac{F_a(p, t_{aM}) - F_a(p, t_{am})}{\Delta t_a} \right)}{\left(\frac{F_b(p, t_{bM}) - F_b(p, t_{bm})}{\Delta t_b} \right)} = \frac{F_a(p, t_{aM}) - F_a(p, t_{am})}{F_b(p, t_{bM}) - F_b(p, t_{bm})} \cdot \frac{\Delta t_b}{\Delta t_a}.$$

The local ventilation-perfusion index, $i(p)$ is defined in (15) by

$$i(p) = \frac{f_a(p, t_{aM}) - f_a(p, t_{am})}{f_b(p, t_{bM}) - f_b(p, t_{bm})} \cdot \frac{\Delta t_b}{\Delta t_a}.$$

From equation (A.1), we have that, assuming the errors A and B do not change significantly during the cardiac or respiratory cycle,

$$i(p) = r(p) \frac{A_a(p)}{A_b(p)}.$$

If these position-dependent errors are proportional, that is $\frac{A_a(p)}{A_b(p)} = E$, independent of p , then $i(p) = r(p)E$. Thus the $V = Q$ index is proportional to the $V = Q$ ratio, even if there are large position-dependent scaling and offset errors for both approximate volume fractions, f_a and f_b , which are determined from EIT reconstructions. When the scaling distortions are approximately the same, E is approximately 1 and the $V = Q$ index is approximately equal to the $V = Q$ ratio, $i(p) \approx r(p)$.

For these reasons, there may be sources of large, position-dependent errors in the approximate volume fractions to which the $V = Q$ index is less sensitive in a diagnostically useful manner. Thus, we have defined the $V = Q$ index, shown how to compute it from EIT data, and propose including it in an analysis of a subject's pulmonary function.

Figure 1.
Volume fraction of blood ranging from 0 (lightest) to 1 (darkest), with frame rate 18.8 fps
using ACT 3 data

Figure 2.

Volume fraction of air ranging from 0 (lightest) to 1 (darkest), with frame rate 18.8 fps using ACT 3 data

(a) Air volume fraction in a typical lung voxel over 97 frames, or 5.16 seconds

(b) Blood volume fraction in a typical lung voxel over 100 frames, or 5.32 seconds

Figure 3.

Typical volume fraction waveforms for a lung voxel during ventilation and perfusion, with a frame rate of 18.8 fps using ACT 3 data

(a) $\dot{V}/\dot{Q}$ index map with lung region, L_a , defined by ventilation waveform correlation

(b) $\dot{V}/\dot{Q}$ index map with lung region, L_b , defined by perfusion waveform correlation

(c) $\dot{V}/\dot{Q}$ index map with lung region, L_c , defined by union of ventilation and perfusion correlation regions

(d) $\dot{V}/\dot{Q}$ index scale

Figure 4.
Ventilation-perfusion index maps from ACT 3 data

Table 1

Notation explanation

a	from air (ventilation) data
b	from blood (perfusion) data
s	dummy for a or b
p	label of a voxel in the domain (its center of mass)
t	time in frames (can be other units)
L	lung region of interest
R	region of interest within L
$\sigma_s(p, t)$	conductivity in s cycle at (p, t)
$\sigma_M(p)$	$\max_t \sigma(p, t)$
$\sigma_m(p)$	$\min_t \sigma(p, t)$
σ_{sM}	$\max_{p,t} \sigma_s(p, t)$
σ_{sm}	$\min_{p,t} \sigma_s(p, t)$
$f_s(p, t)$	approximation to the volume fraction of s at (p, t)
$t_{sM}(p)$	time when $f_s(p, t)$ achieves max. value (s stops entering voxel p)
$t_{sm}(p)$	time when $f_s(p, t)$ achieves min. value (s begins entering voxel p)
t_a	time duration of one breath
t_b	time duration of one heart beat
T_ℓ^k	k th current pattern applied to electrode ℓ
θ_ℓ	angle location of electrode ℓ
$vol(p)$	volume of voxel p
$V \dot{(p)}$	ventilation at voxel p , flow of air in $p \in L$ per unit time
$Q \dot{(p)}$	perfusion at voxel p , flow of blood in $p \in L$ per unit time
$r(p) \equiv \frac{\dot{V}(p)}{\dot{Q}(p)}$	local ventilation-perfusion ratio at $p \in L$
$i(p)$	local ventilation-perfusion index at $p \in L$
$I(R)$	regional ventilation-perfusion index value for $R \subset L$

Table 2Global $\dot{V} = \dot{Q}$ indices

$\dot{V} = \dot{Q}$ index	Upper Lung	Lower Lung	Average/Total
ACT 3 ($I(L_c)$)	N/A	0.97	0.97
GENESIS - Normal breath	1.12	1.01	1.12
GENESIS - Deep breath	1.71	1.75	1.73