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Neuron Selection Based on Deflection Coefficient Maximization for the Neural Decoding of Dexterous Finger Movements

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Abstract

Future generations of brain-machine interface (BMI) will require more dexterous motion control such as hand and finger movements. Since a population of neurons in the primary motor cortex (M1) area is correlated with finger movements, neural activities recorded in M1 area are used to reconstruct an intended finger movement. In a BMI system, decoding discrete finger movements from a large number of input neurons does not guarantee a higher decoding accuracy in spite of the increase in computational burden. Hence, we hypothesize that selecting neurons important for coding dexterous flexion/extension of finger movements would improve the BMI performance. In this paper, two metrics are presented to quantitatively measure the importance of each neuron based on Bayes risk minimization and deflection coefficient maximization in a statistical decision problem. Since motor cortical neurons are active with movements of several different fingers, the proposed method is more suitable for a discrete decoding of flexion-extension finger movements than the previous methods for decoding reaching movements. In particular, the proposed metrics yielded high decoding accuracies across all subjects and also in the case of including six combined two-finger movements. While our data acquisition and analysis was done off-line and post processing, our results point to the significance of highly coding neurons in improving BMI performance.

Index Terms

Brain-machine interfaces (BMI); neural decoding; neural prosthesis; neuron selection

I. Introduction

A brain-machine interface (BMI) is a methodology to form a communication pathway between a brain and an external device. It provides an appropriate alternative for people with damaged cognitive or sensory-motor functions [1]. The key point of BMI systems is a neural decoding procedure, which interprets the recorded spike signals from neural population and infers an intended movement. Up to now, many related studies have been conducted, such as a 2-D target tracking task, a closed-loop control of a computer cursor, and a 3-D reaching task [2]–[6]. Currently, to broaden the practical application, dexterous motion control strategies have been proposed, such as hand and finger movements [7]–[16].

To achieve neural decoding for finger movements, activity of multiple neurons must be simultaneously decoded. However, all recorded neurons do not contribute equally to the all movements and several neurons are related weakly or not at all to the specific movements. Hence, simply using as many neurons as possible does not guarantee a higher decoding accuracy, and even may degrade the decoding performance with the extremely small training data sets [17]. In addition, a large number of input neurons put a computational burden on finding an optimal solution especially when the goal is to implement such decoding algorithms in a prosthetic hardware. Hence, in the neural decoding of arm reaching/grasping tasks, researchers have developed many techniques to evaluate neurons' relative importance [19]–[22]. Some techniques have utilized the filter parameters of a trained decoding filter to ascertain the importance of neurons because training a decoding filter assigns weights or coefficients to the corresponding neurons hinging on their relative contribution.

On the basis of this principle, sensitivity analysis and single neuron correlation analysis through a linear Wiener filter were proposed to quantitatively rate the importance of neurons in neural-to-motor mapping [19]. Individual removal error (IRE), which is the change in decoding error following removal of a given neuron, was presented to initially tune a population vector-based system for neural control without arm movements [20]. In another approach, ensemble fractional sensitivity was proposed by calculating the partial derivative of the output with respect to the input activity [21]. On the other hand, several approaches have attempted to evaluate the importance of neurons by analyzing probabilistic distributions of recorded neuron spikes regardless of a decoding model. For a 2-D target-reaching task to move the cursor on a computer screen, a measure called tuning depth, defined as the difference between the maximum and minimum values of the cellular tuning, was proposed [19]. The mutual information between the spike and the delayed linear filter kinematics vector was applied as a metric for evaluating instantaneously neural receptive properties [22].

These previous works were based on the decoding of continuous reaching movement, for which each motor cortical neuron was found to be tuned with respect to the specific direction of reaching movement, and thus a “preferred direction” could be assigned for each neuron [23]. However, it was shown that single motor cortical neurons do not fire specifically only for a particular flexion-extension finger movement but instead are active with movements of different fingers [24]. Although the previous work using the population

vector analysis found that 75% of motor cortical neurons related to finger movements were tuned to specific directions in a 3-D instructed hand movement space and population vectors could be utilized as a predictor of instructed finger movement [24], the decoding method based on the population vector yielded only 50%–60% decoding accuracy [16]–[18]. Thus, several classification schemes have been presented for the decoding of discrete flexion-extension finger movements based on the Skellam-based maximum likelihood estimation [17] and the artificial neural network [18]. These works have focused on the discrete decoding of finger movements and achieved the improved decoding accuracies. In the same context, a neuron selection method considering the characteristics of neural population activity for discrete finger movements is required.

In the sense of Bayesian inference, for the observed feature parameter x of a neuron n we can make a decision minimizing the probability of error, P_e , based on the *a priori* probability, $p_n(m_i)$, and the conditional probabilities, $p_n(x|m_i)$ for $i = 1, \dots, M$ (where m and M denote the movement types and the total number of m). Furthermore, by the feature space analysis in Fig. 1, the probability of error for each particular neuron can be evaluated by analyzing the M conditional PDFs. Intuitively each PDF of an appropriate input neuron should be separated to minimize the probability of error by reducing the overlapped regions between PDFs. The tuning depth provides a way to find a neuron highly modulated for a specific movement direction by measuring the difference between the maximum and minimum values of the tuning curve, which conveys the expected values of each PDF [19]. For a discrete decoding framework, the method of relative importance in [26] gives a straightforward way to check the separation of the conditional PDFs by calculating only the variance of their means. However, since in the training phase the conditional PDFs can be modeled by the observed neuronal spike data, we may try to utilize more information by analyzing the characteristics of PDFs as well as the variance/min-max of their means.

In this respect, we model the underlying PDFs for the neural activity of each neuron and ascertain their *Bayes risk*. The *Bayes risk*, a generalized version of the probability of error, is the cost assigned to the all types of decision errors. Thus, neuron selection in the sense of the minimum *Bayes risk* may provide an optimal way to select the appropriate neurons for neural decoding. However, to calculate the *Bayes risk* the exact analytic expression of each PDF is required. Also, it entails a large computational burden to compute the probability of all types of errors; with the increase in the number of finger movements (M), the total number of error types becomes the square of $(M - 1)$. Therefore, we need an alternative to cope with the neuron selection problem in neural decoding of the finger movements.

Hence, we propose a neuron selection method based on the *deflection coefficient* in a binary hypothesis testing problem such as detecting the desired signal component (e.g., whether it exists or not) [27]. The *deflection coefficient* is defined as $(E[p_1(x)] - E[p_2(x)])^2/\sigma^2$, or the ratio of the difference of means and the variance of the PDFs when the two distributions are independent and identically distributed (*i.i.d.*) with the same variance (σ^2) [24]. Since the *deflection coefficient* is directly related to the area of overlapped region between two PDFs, it totally characterizes the decision performance in a binary hypothesis testing [27]. Also, the *deflection coefficient* can be obtained by only calculating the mean and variance from observed data set without modeling the exact PDFs. Therefore, to decode M types of finger

movements we derive a *deflection coefficient* for the M -ary hypothesis testing problem and then use it to ascertain the importance of each neuron for the neural decoding.

The remainder of the paper is organized as follows. In Section II, the experimental setup and the neuron selection methods, including the *Bayes risk* minimization and the *deflection coefficient* maximization, are described. Section III shows the decoding results of off-line tests, and Section IV presents the conclusion.

II. Criterion For Neuron Selection

A. Experimental Setup

Three male rhesus (*Macaca mulatta*) monkeys—C, G, and K—were trained to perform visually cued movements of the individuated fingers, wrist, and combined, meaning a pair of fingers at a time. The monkey placed its right hand in a pistol-grip manipulandum which separated each finger into a different slot. A ventral or dorsal micro-switch at the end of each slot was closed by flexing or extending each digit a few millimeters. In addition, the pistol grip manipulandum was mounted on an axis allowing flexion and extension of the wrist. The monkey perceived a display on which each digit was represented by a row of five light emitting diodes (LEDs). If the monkey closed the instructed switch within the response time, 700 ms, and held it closed for a hold time of 500 ms, then the trial was considered a success. When a micro-switch was closed, the LEDs provided the feedback to the subject. There were 12 distinct individuated movements: flexion (f) and extension (e) of each finger (1 = thumb through 5 = little) and the wrist (w) and six combined two-finger movements: $f_{12}, f_{23}, f_{45}, e_{12}, e_{23}, e_{45}$.

The monkeys were prepared for single-unit recording by surgically implanting both a head-holding device and a rectangular Lucite recording chamber that permitted access to the area encompassing primary motor cortex (M1) [6]. Single-unit activities were recorded from task-related M1 neurons. Independent trials were repeated six times for each movement, respectively. A detailed description of the methods used to train the monkeys and the actual experimental protocols can be found in [6] and [7].

B. Mathematical Model of the Neural Activity

To infer the instructed stimulus from recorded data, it is needed to appropriately quantify the neural activity and model the underlying probability distribution of a neuron. The neural activity of the instructed finger movement consists of the two states which are the period of rest and movement. Therefore, the change in firing rate before and after finger movements was presented to quantify the neural activity [17]. Since all response times between the time of cue onset and switch closure are different, neuronal spike trains were aligned at the time of the switch closure (the center of each trial data) as shown in Fig. 2. From the aligned spike trains, the neural activation was defined as the difference of firing rates between before and after the instructed finger movement [17]. Each firing rate was calculated by counting and averaging the number of spikes in the divided two windows (from 0 to 800 ms, from 800 to 1100 ms) based on the time of the switch closure as depicted in Fig. 2.

Since each neuronal spike train can be assumed as a realization of an underlying random variable, we termed $x_{n,a}(m)$ to represent a random variable of firing rate of a neuron n for a given movement type m . In particular, $x_{n,a}(x)$ denotes the baseline activity of the neuron n before the movement of m . Then, to measure how much a neuron, n is activated by the instructed finger movement m , the neural activation is defined as follows [17]:

$$x_n(m) = x_{n,a}(m) - x_{n,b}(m), \quad m=1, \dots, M \quad (1)$$

where M is the number of total finger movement types. The PDF of $x_n(m)$ is represented as $p_n(x|m)$, and the mean of $x_n(m)$ can be estimated by the sample mean of the trial data, that is

$$\hat{\mu}_n(m) = \frac{1}{K} \sum_{k=1}^K x_n(m, k) \quad (2)$$

where $x_n(m, k)$ denotes the k th trial data of the n th neuron by the instructed finger movement m . K is the number of total trials.

C. Bayes Risk Minimization

In the M -ary hypothesis testing, we wish to distinguish between M hypotheses, where M is larger than 2. Each hypothesis in neural decoding means a decision which finger movement evokes the observed neuronal firing spike among the flexion and extension of each finger and the wrist. If the conditional PDFs, $p_n(x|m)$, for each finger movement are completely known, the discrimination problem is relatively simple comparing to the case of the PDFs with unknown parameters. The primary approach to simple hypothesis testing is the Bayesian approach based on the minimum P_e (probability of error) criterion [28]. A probability of error P_e is given by

$$P_e = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} p(H_i|H_j)p(H_j), \quad i \neq j \quad (3)$$

where $p(H_i|H_j)$ is the conditional probability that indicates the probability of deciding H_i when H_j is true. P_e can be generalized to the *Bayes risk* by assigning costs to each type of error [28]. The expected cost or the *Bayes risk* is defined by

$$R = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} C_{ij} p(H_i|H_j)p(H_j), \quad i \neq j \quad (4)$$

where C_{ij} is the cost when we decide H_i but H_j is true. Usually, if no error is made (i is equal to j), a cost is zero, so that $C_{ii} = C_{jj} = 0$. Otherwise, all costs, C_{ij} , are equal to one. So, the *Bayes risk* is the same with the probability of error P_e . However, for ease of explanation we will retain the more general form.

In the neural decoding problem we assume that the cost of each finger movement is identical and *a priori* probability of any hypothesis is uniform, and thus the *Bayes risk* of the n th neuron can be modified as

$$\begin{aligned} R_n &= \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} p_n(H_{m_i}|H_{m_j}) \\ &= \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} p_n(m_i|m_j), \quad i \neq j \end{aligned} \quad (5)$$

where H_{m_i} denotes the hypothesis that the decision of neural decoding is determined by i th finger movement m_i . Since the *Bayes risk* (R_n) of a particular neuron n indicates the probability of error, P_e of that neuron, it can be utilized to ascertain the importance of each neuron and select the input neuron set for the neural decoding.

Although, in a practical neural decoding problem, it is hard to obtain the complete knowledge about the probabilistic structure between the feature parameter and the instructed movement, we can model the conditional PDFs of a neuron from the observed spike trains in the training phase. However, since the total number of the decision error types is equal to $(M - 1)^2$ for the M types of finger movements, a high computational burden is entailed to calculate the summation of each area of the overlapped regions. To cope with this problem, an alternative is needed to quantify and ascertain the importance of each neuron with an acceptable computational burden.

D. Deflection Coefficient Maximization

Fig. 3 shows the relationship between each type of decision error and the overlapped region of the conditional PDFs in the case of a neural decoding with only three movements. As shown in Fig. 3, the total area of overlapped regions is directly related to P_e . So, a neuron whose conditional PDFs tend to be separated to each other may be an appropriate input for the neural decoding procedure. In this respect, to evaluate the importance of a neuron without calculating the *Bayes risk*, we propose a neuron selection method based on the *deflection coefficient* [27].

In the binary hypothesis testing problem such as signal detection determining the presence or absence of a desired signal component, the *deflection coefficient* is a measure for evaluating the detection performance [27]. The two conditional PDFs, $p(x|H_0)$ and $p(x|H_1)$, for two hypotheses H_0 (absence) and H_1 (presence) are assumed to be independent and identically distributed (*i.i.d.*) with the same variance, $\text{Var}(x|H_0) = \text{Var}(x|H_1)$. Then, the *deflection coefficient*, d^2 , is defined by [27], [28]

$$\begin{aligned} d^2 &= \frac{[E(x|H_1) - E(x|H_0)]^2}{\text{Var}(x|H_0)} \\ &= \frac{[E(x|H_1) - E(x|H_0)]^2}{\text{Var}(x|H_1)}. \end{aligned} \quad (6)$$

Since d^2 in (6) means the ratio between the difference of means and their variance, it has a large value when each PDF is separated with narrow width and thereby the area of

overlapped regions decreases. In order to apply the *deflection coefficient* to the neuron selection problem, we define the modified *deflection coefficient*, d_n^2 , in the M -ary hypothesis testing for the neural decoding of finger movements, which is given by

$$d_n^2 = \frac{\nu_n^2}{\gamma_n^2} = \frac{\text{Var}[E(x_n|m)]}{E[\text{Var}(x_n|m)]}, \quad m=1, \dots, M \quad (7)$$

where d_n^2 of a particular neuron n is the ratio of ν_n^2 and γ_n^2 , and M is the total number of finger movements. The variance of each mean of PDFs, ν_n^2 , can be estimated by

$$\hat{\nu}_n^2 = \frac{1}{M} \sum_{m=1}^M (\hat{\mu}_n(m) - \bar{\mu}_n)^2 \quad (8)$$

where the mean of the neural activities over the finger movements is given by

$$\bar{\mu}_n = \frac{1}{M} \sum_{m=1}^M \hat{\mu}_n(m). \quad (9)$$

Since the variances of each PDF are generally different in the neural decoding, the mean of variances is used in the denominator in (7) unlike in that of (6). The estimate of γ_n^2 can be calculated from the trial data as follows:

$$\hat{\gamma}_n^2 = \frac{1}{M} \sum_{m=1}^M \hat{\sigma}_n^2(m) \quad (10)$$

where $\hat{\sigma}_n^2(m)$ is the sample variance of the k th trial $x_n(m, k)$ over trials as an estimate of the variance of $x_n(m)$ and is obtained by

$$\hat{\sigma}_n^2(m) = \frac{1}{K} \sum_{k=1}^K (x_n(m, k) - \hat{\mu}_n(m))^2. \quad (11)$$

Since the overlapped region of PDFs is affected by not only the separation but also the width of each PDF, the deflection coefficient d_n^2 is a more appropriate criterion to ascertain the importance of a neuron compared to the method using only ν_n^2 in [26].

In terms of the computational load, the neuron selection based on the *deflection coefficient* maximization is more effective than the *Bayes risk* minimization. To derive the *Bayes risk* of a neuron, the exact analytic expression of each PDF needs to be modeled, and then the area of $(M-1)^2$ over-lapped regions corresponding to each type of decision errors calculated. This computation, the integral of each $p_n(m_i|m_j)$, should be estimated by the summation of the sampled PDFs, thus the computational burden also relates with the number of sample points N_s . The total number of multiplications is $N_s \times (M-1)^2$ to derive the *Bayes risk* of a

neuron. On the other hand, the *deflection coefficient* requires $2M$ multiplication to calculate $\hat{\nu}_n^2$ and $\hat{\gamma}_n^2$ from the observed data. Hence, the deflection coefficient can be a good alternative of the Bayes risk to quantify the importance of neurons for the statistical inference of finger movements.

E. Individual Removal Error

For comparison purposes, we briefly describe the conventional individual removal error (IRE) method. The IRE is defined as the change in error angle following removal of a given neuron from the ensemble [20] and is given by

$$\text{IRE}(i) = E_{\text{all}} - E_{\text{all}-i} \quad (12)$$

where E_{all} is the average angular error which is the angle between the population vector and a line connecting the cursor to the active target using all available units. $E_{\text{all}-i}$ is the average angular error computed for all available units except the neuron indexed by “ i ”. Originally, even though the IRE is presented for rapid tuning of a population vector-based system for neural control without arm movement [20], it can be employed for various decoding models with an appropriate modification because its basic framework is based on the change of error measurement with and without a particular neuron. However, when the applied classifier achieves a high performance with a small number of input neurons, the decoding accuracy or error measurements cannot be changed by removing any particular single neuron. In this case, the IREs of all neurons are zero, so an alternative way should be considered to quantify the IRE of a neuron.

F. Maximum Likelihood Neural Decoding With the Selected Neurons

In order to compare the effectiveness of the two criteria on decoding a finger movement, we perform offline analysis with the neurons selected by the proposed methods. Several different methods have been presented for decoding finger movements from neural activity [6]–[10], [16]–[18]. Here we use the maximum likelihood (ML) decoding method in [17] which is based on Skellam distribution. In the statistical inference, the ML estimation is asymptotically optimal as the number of training data increases [27]–[29].

ML decoding estimates the instructed finger movement by choosing the argument m that maximizes the log-likelihood, that is

$$\hat{m} = \arg \max_m p(x_1, x_2, \dots, x_N | m) \quad (13)$$

where N is the number of input neurons used for ML decoding. To solve (13) the probability density function of the neuron activity needs to be modeled. The Skellam distribution is presented in [17] and it showed the decoding accuracy of 99% with 25 randomly selected neurons.

The PDF of the Skellam distribution is as follows:

$$s_n(x_n|m) = \alpha_n(m) \left(\frac{E(x_{n,a}|m)}{E(x_{n,b}|m)} \right)^{\frac{x}{2}} \times I_x(2 \sqrt{e(x_{n,a}|m) E(x_{n,b}|m) \Delta t^2}) \quad (14)$$

where $E(x_{n,b}|m)$ and $E(x_{n,a}|m)$ are the means of random variables describing a firing rate of before and after the starting moment, respectively. t is the time interval for observation of neuronal spikes and $I_x(z)$ is the modified Bessel function of the first kind. $\alpha_n(m)$ is defined by

$$\alpha_n(m) = e^{-(E(x_{n,a}|m) + E(x_{n,b}|m)) \Delta t} \quad (15)$$

Assuming that x_i and x_j ($i \neq j$) are independent [8], we get

$$p(x_1, x_2, \dots, x_N|m) = \prod_{n=1}^N s_n(x_n|m) \quad (16)$$

for the Skellam distribution.

III. Results and Discussion

The performance of the proposed neuron selection methods was evaluated using leave-one-out cross validation, wherein the data set was partitioned into six folds because the total number of trials is six for each finger movement. From six independent trials' data, a single trial was used for the validation and the remaining trials were used for the training data, i.e., $K = 5$. This procedure was repeated such that each trial in the data set is used once as the validation data, so six independent tests were conducted to examine the performance of the proposed methods.

The offline tests without ordering were performed with randomly chosen N neurons from total 115 neurons (312 and 125 neurons for the monkey C and G, respectively). The random selection was repeated 300 times. On the other hand, for the rank-ordered neurons, if N neurons are used for ML decoding, there is only one input set, consisting of the first-ranked neuron to the N th ranked one. In order to increase the possible neuron sets, we added next-ranked L neurons to highly ranked N neurons, resulting in $(N + L)$ ranked neurons. The value of L was empirically set to $L = 5$ in our testing, then a subset of N neurons was randomly selected from $(N + L)$ neurons. So, the possible number of selections becomes ${}_{N+L}C_N$.

A. Neural Decoding of Individual Finger Movements With Rank-Ordered Neurons

Two classification algorithms, the optimal PV and ML decoding methods [16], [17], were applied to validate the effectiveness of proposed methods and to compare their performances with those of the existing neuron selection methods such as the tuning depth [19] and the IRE [20]. First, we carried out the ML-based neural decoding, which is the most accurate classifier for decoding discrete finger movements [17] of the individuated finger movements (12 flexion/extension movements listed in Section II-A) by using the neuronal spikes recorded from the three monkeys (C, G, and K). Fig. 4(a), (b), and (c) shows the resultant

decoding accuracies according to the number of input neurons. For offline testing, we applied four different neuron selection methods to determine input neuron sets such as the random selection [17], the tuning depth [19], and the individual removal error (IRE) [20], *Bayes risk* minimization, and *deflection coefficient* maximization. Since the ML decoding achieved the accuracy of 100% regardless of which single input neuron is removed, the IRE of all neurons were zero for the ML decoder. So, to derive the IRE of a neuron for the decoding of discrete finger movements, we alternatively employed an abstract movement space in [24] and it allowed that each discrete finger movement could be matched to a particular vector in 3-D space. Then, the IREs of each neuron were calculated by the two neuronal activity models which were based on the regular population vector (PV) [24] and the optimal population vector (OPV) [16]. In the regular PV, the weighting vectors are set *a priori* to the preferred directions while those of OPV are determined in the sense of minimizing the average squared distance between the estimated-PV and the instructed movement vector. In the decoding of discrete finger movements, the OPV achieved a remarkably improved accuracy compared to that of the PV [16]. Note that though the IRE was quantified by using the population vector analysis (OPV- or PV-based IRE), its decoding accuracies were evaluated by the ML decoding in Fig. 4(a), (b), and (c).

Three monkey subjects showed different performances according to their inherent biological characteristics and achievement of training, hence the effects of the proposed neuron selection methods on decoding accuracy differ across three monkey subjects. When we used the randomly selected 15 neurons, each decoding result differed: 89%, 83%, and 92% for the monkeys C, G, and K, respectively. With as few as 15 neurons selected by the proposed two metrics, the decoding accuracies of greater than 98% were achieved in the cases of all monkey subjects. To statistically analyze the performance differences between the proposed methods and the random selection, a statistical t-test was performed on the null hypothesis at the significant level of 0.01 and the number of test data set was 300. Under the null hypothesis, for the three monkeys with 15 neurons, all the p -values were extremely small (almost zero, $p < 10^{-9}$) and less than the predetermined significant level ($p > 0.01$). These results indicate that the performances of the proposed methods are statistically different from those of the random selection.

In the case of the tuning depth, the accuracy of the monkey G stayed around 92% while the performances of other two monkeys, C and K, were enhanced over 96% by using 15 rank-ordered neurons. The tuning depth did not work well for the monkey G because a larger tuning depth does not guarantee wide separation between each conditional PDF in decoding discrete flexion/extension tasks of fingers. In both cases of the PV and OPV, the IRE method showed higher accuracies compared to those of randomly selected neurons for the three subjects, but the OPV-based IRE method yielded more accurate results for all the three monkeys. In particular, the OPV-based IRE method yielded comparable accuracies to those of the tuning depth in the cases of monkeys C and G, respectively, while the tuning depth showed the more accurate performance for the monkey K as shown in Fig. 4. These results imply that the impact of the IRE method would vary with the applied decoding model and its effectiveness could be enhanced by using a more precise model instead of the PV or OPV analysis. On the other hand, the proposed methods achieved the best performance across all

subjects by only analyzing the recorded test data set without any additional consideration of a decoding model.

Although we used an alternative way to utilize the IRE criterion for the ML decoding framework, basically the IRE value of a neuron is required to be quantified and evaluated under the same decoding algorithm. So, additional off-line tests have been conducted to compare the decoding accuracies of each neuron selection method by employing the OPV-based decoding algorithm instead of the ML classifier. These tests allowed a fair and objective evaluation of performance difference between the proposed methods and the IRE criterion. As shown in Fig. 4(d), the overall decoding performances were degraded due to the relatively low performance of the OPV-based decoding method compared to that of the ML decoder, but the proposed methods still achieved the highest accuracies.

In Fig. 5, the conditional PDFs of first-ranked neurons by the three different metrics (the tuning depth, Bayes risk, and deflection coefficient) are presented for the selected seven movements (f1, f2, f5, e1, e3, e4, and ew) from 12 individual finger movements of monkey G. The neurons G0283n were ranked first by the tuning depth and conditional PDFs are shown in Fig. 5(a). Those of two first-ranked neurons (G0242n and G0240n) by the Bayes risk and the deflection coefficient are plotted in Fig. 5(b) and (c), respectively. In Fig. 5(a), the conditional PDFs of neuron G0283n tend to be overlapped around neural activity $x(m) = 5$ although it has the largest difference between the maximum and minimum values in neural activation. On the other hand, in the cases of first-ranked neurons by the Bayes risk and the deflection coefficient, G0242n and G0240n, their conditional PDFs tend to be separated. This means that the proposed two metrics could be appropriate to quantify and analyze the distribution of conditional PDFs to evaluate the importance of each neuron for decoding discrete finger movements.

For a more robust BMI system the neural decoding should yield a desired performance regardless of the condition of subjects. For this reason, we compared the decoding performance across the different neuron selection algorithms according to the monkey subjects in Table I, in which the results were reported as accuracy and standard deviation. These results show that the proposed methods could achieve consistently the higher accuracies for all monkey subjects over the OPV-based IRE and tuning depth. Especially, even when the subject achieves relatively low decoding accuracy with randomly selected neurons (monkey G), the level of performance improvement was higher than those of monkeys C and G by employing the proposed methods. On the other hand, the decoding accuracies of the tuning depth were degraded in the case of monkey G while those of monkeys C and K are relatively high, and this tendency of the performance variation did not change even when the number of input increases to 15 neurons. In other words, the proposed methods worked well across all subjects compared to the tuning depth and the IRE and will be helpful in realistic environment wherein a reliable performance should be achieved regardless of the condition of subjects.

With only five neurons, the accuracy of 83%, 75%, and 83% was achieved for the monkeys C, G, and K when the input neurons were selected based on the Bayes risk (82%, 72%, and 83% by the deflection coefficient), and these decoding accuracies were approximately 9%,

13%, and 2% better than those of the tuning depth, respectively (7%, 10%, and 2% for the deflection coefficient). In the cases of monkeys C and G, p -values were extremely small (almost zero, $p < 10^{-9}$). However, for the well-performed subject (monkey K), p -values of the t-test were larger than the predetermined significance level (0.01) in both cases of Bayes risk ($p = 0.038$) and the deflection coefficient ($p = 0.059$). These results mean that the performance of the tuning depth and the proposed methods are statistically similar in the case of monkey K. Compared to the OPV-based IRE method, the decoding accuracy of Bayes risk was approximately 10%, 7%, and 9%, respectively (8%, 4%, and 9% for the deflection coefficient). In addition, for the three monkeys, all the p -values were almost zero ($p < 10^{-9}$), so these results indicate that the performance difference between the proposed methods and the OPV-based IRE method are statistically significant.

In these results, the decoding accuracy by using the input neurons selected by the proposed two methods achieved the comparable decoding accuracy. Therefore, since the deflection coefficient is derived by only calculating the mean and variance from the neuronal spike data, it can be an appropriate criterion for the neural decoding problem when the goal is to implement such decoding algorithms in a prosthetic hardware.

B. Neural Decoding of Complex Two-Finger and Total Finger Movements With Rank-Ordered Neurons

Furthermore, we performed the neural decoding of other types of finger movements: the combined two-finger movements and the total finger movements. The decoding results were plotted in Fig. 6 and the averaged accuracies were summarized in Tables II and III, respectively. These neuronal spike data were recorded from the monkey K which showed the highest accuracies in decoding the individuated finger movements. The combined two-finger movement means that the monkey moves its two fingers at the same time, and the total finger movements includes the individuated and combined two-finger movements. In the case of the combined two-finger movements, the neurons selected by the proposed methods achieved almost 96% accuracies with five neurons as depicted in Fig. 6(a), while the random selection and the tuning depth resulted in the accuracies of 71% and 82%, respectively. Also, the corresponding p -values were almost zero ($p < 10^{-12}$). On the other hand, the total finger movement including the individuated ($M = 12$) and the combined-two finger movements ($M = 6$) showed the degraded accuracy for all the selection methods because of the increase in uncertainty and ambiguity between the related tasks such as f_{45} and f_5 [26]. In spite of this problem, Fig. 6(b) shows the enhanced accuracy of total finger movements when we used the neurons selected by the proposed methods. With 20 neurons selected by the proposed methods, average 96% accuracies of neural decoding were achieved, respectively, while the random selection and the tuning depth resulted in the accuracy of 89% and 92%, respectively. In both cases, the p -values ($p < 10^{-12}$) were less than the significant level ($p < 0.01$), so the null hypothesis was rejected. These results showed that the performance enhancement by employing the proposed methods is statistically meaningful compared to the random selection and the tuning depth in the case of decoding the total finger movements.

To check all the types of decision errors, the confusion matrices are presented in Fig. 7. Twenty input neurons were selected by the tuning depth and the deflection coefficient. In Fig. 7(a) and (b), most of the decision errors occurred between the individual finger movements and the combined two-finger movements regardless of the neuron selection method. In other words, a particular single-finger movement tends to be classified into the two-finger movement which includes itself and vice versa. Although these false decisions can be reduced by employing the proposed neuron selection method as shown in Fig. 7, it is a challenging problem to implement realistic hand prosthetics since real hand motions consist of more dexterous movements such as flexion/extension of multiple fingers, simultaneously.

As a final discussion, the relationship between the decoding performance and the number of input neurons needs to be addressed. The decoding results presented in our work tend to increase monotonically according to the number of input neuron data. This might be because the Skellam conditional PDFs could be estimated appropriately with five training sets as suggested by [17]. Referring to (13) and (16), the decoding performance was evaluated based on the ML estimation, wherein the joint probabilities in (16) are derived by multiplying conditional PDFs of a neuron. If a nonstationary N th input neuron is extremely dominant enough to cause a change in the existing joint probabilities determined by the previous $N - 1$ neurons, the inclusion of this neuron can result in the degradation of decoding accuracy. The previous study on ML decoding of the finger movements already showed that the additional inclusion of new input neurons does not guarantee the higher accuracy when the number of training data sets is extremely small [17]. Also, in the neural decoding based on a nonlinear decoder such as the artificial neural network (ANN), the decoding accuracies were not monotonically increased according to the number of input neurons [18]. Therefore, although the decoding results presented in our work tend to increase monotonically according to the number of input neuron data, the problem of performance degradation caused by the non-stationary neuron should be carefully considered. In this sense, the rejection of these undesired neurons as well as the selection of highly contributing neurons can be helpful for improving the performance of neural decoding. The related issue was also dealt with in [20], where the authors presented the change in decoding error following the removal of a given neuron.

IV. Conclusion

For the neural decoding of dexterous finger movements, we proposed new metrics for the neuron selection: the *Bayes risk* minimization and the *deflection coefficient* maximization. The proposed criteria, the *deflection coefficient* and the *Bayes risk*, have as a trade-off a slight degradation in performance for reduced computational complexity. When the neurons were selected by the *Bayes risk* minimization criterion, the more accurate decoding performance was achieved. However, this method suffers from the computational load because it needs to calculate the probability of all decision errors based on the exact analytic expression of the probabilistic distribution. On the other hand, the *deflection coefficient* minimization can be an appropriate alternative because it shows the comparable performance to the *Bayes risk* maximization with much smaller computational complexity.

The use of the proposed metrics improved the decoding accuracy compared with the random selection, the individual removal error, and the tuning depth. Also, the decoding performances were robust across the different subjects (the monkeys C, G, and K), so the proposed methods will be helpful in achieving a desirable accuracy by compensating the subject variability. In particular, the decoding accuracy was more improved in the decoding of the total finger movements compared to the case of individual finger movement of the monkey K. Since the real hand motions are composed of the individuated and combined finger movements, this performance enhancement will be of particular importance to develop a BMI device for multi-fingered hand control.

The results of these experiments and offline tests suggest that, in a closed-loop system, an appropriate neuron selection for the initial tuning can be expected to alleviate the computational burden and raise the robustness across a variety of different subjects. Also, when a BMI system requires retraining because of unexpected and undesirable changing factors, such as movements of electrode arrays and noise induced by degradation at the electrode tissue interface, the proposed neuron selection methods will be more helpful and effective. However, since our experimental model and study focus on decoding discrete finger movements when the task-related time schedule is already known on the basis of open loop and offline analysis framework, considerable works, such as a continuous decoding of varying degrees of flexion/extension of multiple fingers and continuous state decoding of dexterous finger and hand motion, remain to eventually design and implement a real-time dexterous prosthetic hand system operating in closed loop environments.

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Fig. 1.

Conceptual procedure of a neuron selection method consists of four steps. Firstly, a feature parameter x is extracted from the observed data sets and its probability density functions (PDFs) for each neuron are modeled in a training phase. By analyzing the characteristics of these conditional PDFs of a neuron, called feature space analysis, the importance of a neuron can be evaluated, and then input neurons are selected for decoding of finger movements.

Fig. 2.

Neural activity induced by a movement is defined as the difference of an averaged firing rate before $x_{n,b}(m)$ and after $x_{n,a}(m = e_2)$ the instructed finger movement. All trials were aligned such that switch closure occurred at 1 s. The baseline activity was obtained by averaging the number of spikes from 0 and 800 ms. The neural activation after finger movements was obtained by calculating the firing rate of spikes between 800 and 1100 ms.

Fig. 3.

Relationship between four types of decision errors and the overlapped region of the conditional PDFs in the case of a neural decoding with only three movements (m_1 , m_2 , and m_3).

Fig. 4.

Performance of the neural decoding of the individuated finger movements are plotted according to the different monkeys (C, G, and K) and the classification algorithms (the ML classifier and the optimal population vector method). First three subplots show the decoding results of the ML decoding for the three monkeys such as: (a) C, (b) G, and (c) K.

Performance of the optimal population vector decoding for the monkey C is shown in (d) and the number of input neurons is expanded to 80 populations.

Fig. 5.

Neurons of G0283n, G0283n, and G0283n are ranked first by three different neuron selection methods for the monkey G. Their probability density functions of the selected seven different finger movements are plotted (f1, f2, f5, e1, e3, e4, and ew). The 12 finger movements consist of flexion (f) and extension (e) of each finger (1 = thumb through 5 = little) and the wrist (w): (a) First-ranked neurons by the tuning depth, (b) Bayes risk, and (c) Deflection coefficient.

Fig. 6.

Performance of the ML decoding for finger movements of the monkey K: (a) Decoding accuracy of the combined two-finger movements and (b) decoding accuracy of total finger movements including 12 individuated and six combined two-finger movements.

(a)

(b)

Fig. 7.

Confusion matrices show the decoding results of total finger movements including 12 individuated and 6 combined two-finger movements when the rank-ordered 20 neurons are selected among total 115 neurons of the Monkey K. (a) Tuning depth and (b) the deflection coefficient.

TABLE I

Summary of Decoding Results for Individual Finger Movements

Monkey	Method	5 Neurons (%)	10 Neurons (%)	15 Neurons (%)
C	RS	57.2±8.4	79.2±6.4	89.5±4.1
	BR	83.7±4.6	96.2±2.4	98.7±1.0
	DF	81.9±5.3	96.7±2.3	99.3±0.9
	TD	74.4±5.4	92.1±3.5	96.4±1.9
	IRE(OPV)	73.1±5.7	90.2±3.0	96.5±1.7
	IRE(PV)	60.5±6.3	83.4±4.3	91.6±2.1
G	RS	49.3±9.5	72.1±7.5	82.1±5.4
	BR	75.1±6.1	91.9±3.6	98.3±1.9
	DF	72.3±5.6	89.2±3.9	98.1±2.2
	TD	62.9±7.5	83.1±4.6	92.1±2.0
	IRE(OPV)	68.8±7.1	84.3±5.6	92.2±3.2
	IRE(PV)	59.1±8.0	78.3±5.8	83.2±3.8
K	RS	62.3±7.6	87.2±5.1	92.9±2.5
	BR	83.5±4.1	97.9±1.8	99.6±0.4
	DF	83.8±3.6	97.0±2.0	99.4±0.7
	TD	81.6±3.9	95.5±2.4	97.9±1.3
	IRE(OPV)	74.1±5.3	91.8±3.1	97.4±1.5
	IRE(PV)	68.3±6.1	88.6±4.4	93.2±2.8

* RS: Random selection, BR: Bayes risk, DF: deflection coefficient, TD: tuning depth, IRE: individual removal error, OPV: optimal population vector

TABLE II

Summary of Decoding Results for Combined Finger Movements

Monkey	Method	5 Neurons (%)	10 Neurons (%)	15 Neurons (%)
K	RS	72.3±6.9	88.7±3.7	93.5±2.1
	BR	95.7±3.2	98.4±1.6	99.3±0.3
	DF	96.7±3.4	99.3±1.2	99.8±0.2
	TD	82.2±3.1	94.3±1.4	97.4±0.8

* RS: Random selection, BR: Bayes risk, DF: deflection coefficient, TD: tuning depth, IRE: individual removal error

TABLE III

Summary of Decoding Results for Total Finger Movements

Monkey	Method	10 Neurons (%)	15 Neurons (%)	20 Neurons (%)
K	RS	76.3±7.2	83.2±6.1	89.6±4.8
	BR	90.7±6.4	96.2±4.1	97.2±2.9
	DF	87.4±6.6	94.2±3.9	96.2±2.2
	TD	80.1±7.6	87.8±5.0	92.8±4.1

* RS: Random selection, BR: Bayes risk, DF: deflection coefficient, TD: tuning depth, IRE: individual removal error