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BRACHYTHERAPY DOSIMETRY SPECIAL FEATURE: REVIEW ARTICLE

Current state of the art brachytherapy treatment planning dosimetry algorithms

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ABSTRACT

Following literature contributions delineating the deficiencies introduced by the approximations of conventional brachytherapy dosimetry, different model-based dosimetry algorithms have been incorporated into commercial systems for ^{192}Ir brachytherapy treatment planning. The calculation settings of these algorithms are pre-configured according to criteria established by their developers for optimizing computation speed vs accuracy. Their clinical use is hence straightforward. A basic understanding of these algorithms and their limitations is essential, however, for commissioning; detecting differences from conventional algorithms; explaining their origin; assessing their impact; and maintaining global uniformity of clinical practice.

Conventional, Task Group (TG)43-based¹ dosimetry marked an improvement over prior dose calculation formalisms for brachytherapy treatment planning by advocating the use of a source strength quantity traceable to international standards, the introduction of two-dimensional (2D) source anisotropy, and global uniformity in source characterization as well as clinical dosimetry practice. In the past decade, brachytherapy has progressed from the traditional surgical paradigm to modern three-dimensional (3D) image-based treatment planning systems (TPSs) and dose delivery. The information available through patient imaging, however, had not been fully exploited since TG43-based dosimetry relies on source-specific data pre-calculated in a standard homogeneous water geometry.^{1–3} Hence, it disregards patient-specific radiation scatter conditions and the radiological differences of tissue or applicator materials from water.

In response to literature on the effect of these shortcomings, which has been reviewed in several recent publications,^{4–7} TPSs have become commercially available that include improved dosimetry algorithms, collectively referred to as model-based dosimetry algorithms (MBDCAs). At the time of writing, these include a deterministic solver of the linear Boltzmann transport equation (LBTE)^{8–10} and a collapsed cone superposition (CCS) algorithm^{11–17} for ^{192}Ir high-dose-rate (HDR) applications.

This work reviews the basic features of these algorithms and their clinical implementation and presents illustrative

results of their performance. Monte Carlo (MC) simulation is also briefly discussed since, besides being a candidate MBDCAs for clinical implementation, it is used for obtaining input data for MBDCAs, as well as for their testing.

DOSIMETRY IN THE BRACHYTHERAPY PHOTON ENERGY RANGE

The vast majority of brachytherapy sources can be considered pure photon emitters.^{1–3} Figure 1 readily implies that mainly density heterogeneities affect dosimetry of higher photon energy-emitting sources like ^{192}Ir , since attenuation and energy absorption per unit mass is comparable for all materials except for bone. On the contrary, attenuation and energy absorption differences exist between different tissue compositions for lower photon energies. Literature on the effect of tissue composition heterogeneities in the energy range of ^{125}I and ^{103}Pd has been recently reviewed.^{6,7} Average elemental composition cannot, however, be obtained through routine clinical imaging methods, with promising alternatives such as dual energy or spectral CT requiring further research.⁶

The μ_{en}/μ ratio plotted in the inset of Figure 1 vs energy quantifies the average energy absorbed per interaction. The minimum of μ_{en}/μ at energies around 60–100 keV implies that incoherently scattered radiation will play an important role in dose deposition for sources emitting photons of energy higher than that. This is because a large number of

Figure 1. Mass attenuation and mass energy absorption coefficients plotted as a function of photon energy in the range of brachytherapy for selected media: water, lung, prostate and cortical bone.⁶ The inset presents corresponding ratios of the linear energy absorption to attenuation coefficients in the same energy range.

incoherent interactions of minimal energy transfer are expected for photons accumulated between 60 and 100 keV, and the mean free path of 60 keV photons is significant (about 3.4 cm in water).

Figure 2 presents dose rate distributions for point sources emitting photons of energy equal to the average photon energy of ^{125}I and ^{192}Ir . MC results in this figure, and throughout this work, were calculated using MCNP v. 6.1.¹⁸ The factors determining the dose rate distributions, besides source strength and the inverse square law that are factored out in Figure 2, are attenuation and scatter. In Figure 2 dose rate is separated into its primary and scatter components. Scatter dose is the greatest at points at distances beyond 2.5 and 6.5 cm for the low- and high-energy source, respectively. Dose rate distributions are shown for spherical water geometries of different radii. The increasing lack of backscatter affects points at distances from the edge of the geometry smaller than the mean free path of backscattered photons and makes each bounded geometry unique in terms of dosimetry.¹³ The effect of phantom dimensions,¹⁹ which is a one-dimensional effect in the example of Figure 2, which corresponds to a symmetric geometry,

should not be confused with the effect of placing a source close to a geometry boundary. The latter is position dependent (Figure 3) with the greatest differences between dose rate distributions obtained with a source situated centrally and eccentrically, observed at points where scatter predominates.^{8,20,21}

For photon energies pertinent to brachytherapy, the ranges of secondary electrons are small relative to photon mean free paths (e.g. Carlsson Tedgren et al²²). Charged particle equilibrium (CPE) can be therefore assumed to exist at all points separated from the geometry boundaries or the source by a distance at least equal to the aforementioned range. Under conditions of CPE, dose equals collision kerma in millimetre-sized voxel elements and can be approximated by kerma since radiative energy loss of secondary electrons is negligible in tissue media.⁶ In short, to know the dose distribution one needs to know the energy distribution of fluence, Φ_E ,²³ at all points of a geometry:

$$D(\mathbf{r}) = K(\mathbf{r}) = \int_E E \Phi_E(\mathbf{r}) [\mu_{\text{en}}(E)/\rho] dE \quad (1)$$

Figure 2. Monte Carlo primary, scatter and total dose rate results calculated using a general purpose code plotted against radial distance from a point source centred in a water sphere of radius of 5, 10 and 15 cm, emitting photons with energies of (a) 28 and (b) 355 keV. All results are multiplied by the square of distance from the source.

This is trivial for primary photons since:

$$\Phi_E^{\text{prim}} = \frac{Q_{\text{prim}} \exp(-\mu r)}{4\pi r^2} \quad (2)$$

where Q_{prim} is the number of photons emitted from a mono-energetic source. The spectrum of scatter radiation (Figure 4) formed by photon transport in the geometry also needs to be taken into account.

Photon transport in a geometry can be described by the LBTE. Assume a point photon emitting source in a bounded homogeneous geometry of volume V . Let $\Phi_{\Omega,E}(\mathbf{r}, E, \hat{\Omega})$ be the energy distribution of the time-integrated particle radiance commonly referred to as angular fluence ($\Phi_{\Omega,E}$ equals $d\Phi/d\Omega dE$, where $\Phi = dN/dA$ is the particle fluence²³). It is a scalar function of position $\mathbf{r} = (x, y, z)$, energy and direction given by the direction cosines $\hat{\Omega} = (u, v, w)$. These variables are collectively referred to as phase space; the six-dimensional space of photon states (recall $u^2 + v^2 + w^2 = 1$). Hence, $\Phi_{\Omega,E}$ denotes the number of photons in phase space element $(\mathbf{r}, E, \hat{\Omega})$, that is, passing through area dA normal to $\hat{\Omega}$ and located at $\mathbf{r}$, with Ω between Ω and $\Omega + d\Omega$ and E between E and $E + dE$. It follows that $\Phi_{\Omega,E}$ from points outside V is zero.

At any part of the phase space, conservation of energy dictates particle balance so that the streaming of photons through phase space element $(\mathbf{r}, E, \hat{\Omega})$ (i.e. photons of energy E crossing $dV = dA dr$ along $\hat{\Omega}$) equals photons scattered in it from all others plus photons emitted by the source into this phase space

element, reduced by the number of photons that were absorbed or scattered out of this phase space element. This is the time-independent LBTE for photon transport only:

$$\hat{\Omega} \cdot \nabla \Phi_{\Omega,E}(\mathbf{r}, E, \hat{\Omega}) = Q_{\text{sc}}(\mathbf{r}, E, \hat{\Omega}) + \frac{Q_{\text{prim}}(E, \hat{\Omega})}{4\pi} \delta(\mathbf{r} - \mathbf{r}_p) - \sigma_t(\mathbf{r}, E) \Phi_{\Omega,E}(\mathbf{r}, E, \hat{\Omega}) \quad (3)$$

where $Q_{\text{prim}}/4\pi$ is the number of photons per solid angle element emitted from the point source situated at position $\mathbf{r}_p$, σ_t is the macroscopic total cross-section and Q_{sc} is the photon scattering source:

$$Q_{\text{sc}}(\mathbf{r}, E, \hat{\Omega}) = \int_0^\infty \int_{4\pi} \sigma_s(\mathbf{r}, E' \rightarrow E, \hat{\Omega} \cdot \hat{\Omega}') \Phi_{\Omega',E'}(\mathbf{r}, E', \hat{\Omega}') d\hat{\Omega}' dE' \quad (4)$$

where σ_s is the macroscopic differential scattering cross-section.

Since Equation (3) is linear, primary scatter separation (PSS) can be employed: $\Phi_{\Omega,E} = \Phi_{\Omega,E}^{\text{prim}} + \Phi_{\Omega,E}^{\text{sc}}$, where the arguments were discarded for simplicity.

Figure 3. Monte Carlo-calculated isodose rate contours in units of $\text{cGy h}^{-1} \text{U}^{-1}$ around a point source emitting photons with energies of (a) 28 and (b) 355 keV situated 10 cm from the centre of a spherical water phantom of 15-cm radius. Corresponding results obtained assuming 15 cm of water around the source to resemble the geometry commonly used for TG43 dosimetric calculations¹ are also plotted for comparison. An appropriate colour map is used to depict the percentage differences on a pixel-by-pixel basis.

Equation (3) can then be split to its counterparts for primary and scatter radiation:

$$\hat{\Omega} \cdot \nabla \Phi_{\Omega,E}^{\text{prim}} = \frac{Q_{\text{prim}}(E, \hat{\Omega})}{4\pi} \delta(\mathbf{r} - \mathbf{r}_p) - \sigma_t(\mathbf{r}, E) \Phi_{\Omega,E}^{\text{prim}} \quad (5)$$

$$\hat{\Omega} \cdot \nabla \Phi_{\Omega,E}^{\text{sc}} = Q_{\text{sc}} - \sigma_t(\mathbf{r}, E) \Phi_{\Omega,E}^{\text{sc}} \quad (6)$$

The analytical solution of Equation (5) is analogous to that of Equation (2). If the source is polyenergetic, Equation (2) can be applied using an effective attenuation coefficient weighted over the energy distribution of energy fluence for primary photons:

$$\mu_{\text{eff}} = \frac{\sum_i f_i E_i \mu_i}{\sum_i f_i E_i} \quad (7)$$

where f_i is the emission probability of primary photon with E_i and attenuation coefficient μ_i ²⁴ (Figure 5). When considering real brachytherapy sources in a heterogeneous geometry, the effects of source geometry factor, attenuation within the source,

encapsulation and heterogeneities must be taken into account. This requires ray tracing from points within the source to each point in the geometry to define the pathlength traversed in each medium.^{24–26}

Only Equation (6) then remains to be solved. This is, however, an integro-differential equation of six variables that cannot be solved analytically. One has to seek recourse to numerical solutions of either stochastic (MC) or deterministic nature (LBTE solvers and CCS). Semi-empirical dosimetry models characterized by improved accuracy and computational efficiency, but not general applicability, have also been described in the literature (e.g. Beaulieu *et al*⁶ and references therein).

THE MONTE CARLO SIMULATION METHOD

MC is a numerical method that relies on random sampling to produce observations on which statistical inference can be performed to extract information about a system, based on the law of large numbers and the central limit theorem. It can be used to arrive at a stochastic approximation of the LBTE solution using random sampling from the probability distributions of physical processes underlying photon transport and a set of input data (geometry, materials and cross-sections).²⁷

Figure 4. Monte Carlo results of the energy distribution of photon fluence at 1-, 5- and 10-cm distance per starting particle emitted from a point ^{125}I (top) and ^{192}Ir (bottom) source centred in a 15-cm radius water phantom, multiplied by distance squared.

Details on the method and its application to radiation therapy dosimetry can be found in dedicated textbooks (e.g. Seco and Verhaegen²⁸). MC has played a major role in promoting the accuracy of brachytherapy dosimetry through providing high-quality single source data for use with TG43-based TPS. Several general purpose codes have been benchmarked by various author groups, and methodological recommendations as well as relevant literature can be found in Rivard et al² and Perez-Calatayud et al.³

Despite being considered as a reference dosimetry method for brachytherapy, MC has not been implemented in commercial TPS yet. Traditionally, this has been attributed to its computational efficiency. Besides Type B uncertainty, including code and input data components,^{2,29} as an inherently stochastic method, MC is also subject to Type A uncertainty. This is characterized by the quotient of the variance of the output over the square root of the number of initial photons simulated, N . Achieving a given factor of uncertainty reduction therefore requires an increase of N by the square of this factor, for the same variance. This significantly prolongs computational time and becomes futile beyond a certain point. Obviously, MC is more time consuming for higher photon energy brachytherapy sources owing to multiple scattering (Figures 1, 2 and 4). Variance reduction techniques exist, however, that range from elaborate³⁰ to simple (i.e. energy cut-off, scoring geometry truncation, dose approximation by collision kerma, track-length estimators, analytical calculations for primary dose and use of phase space files for photons exiting a source to preclude simulating

the increased number of interactions within it). Efficiency gains can also be obtained through parallel computations on central as well as graphics processing units.³¹ MC has been shown to be capable of calculation times suitable for clinical implementation. For example, MC calculation times have been reported that range from subminute to a few minutes for low-dose-rate brachytherapy applications,^{32–34} from 2.5 to 17.0 min for 40^3 – 140^3 2-mm voxels, respectively, for an HDR rectal application,³⁵ and from subsecond for a single source to a couple of seconds for an HDR plus shield implant using graphics processing unit implementation.³⁶

LINEAR BOLTZMANN TRANSPORT EQUATION SOLVER BASICS AND PRACTICAL IMPLICATIONS

The finite difference method, an effective iterative method for solving differential equations, can be used to solve Equation (6) numerically. The phase space is discretized to establish a finite difference grid of all variables, hence the commonly used term grid-based Boltzmann solvers (GBBSs). The resulting system of equations is iteratively solved using the source iteration method.³⁷

The scattering source in Equation (4) is numerically integrated by expanding $\Phi_{\Omega,E}(\mathbf{r}, E, \hat{\Omega})$ in spherical harmonics up to a given scattering order L and combining them with cross-sections expanded using Legendre polynomials.^{37,38} Evaluating the integral in a finite number of directions and demanding Equation (6) to hold in the same directions is an angular differencing scheme. It is capable of efficient treatment of anisotropic scattering, and it

Figure 5. Mean mass attenuation (left), effective mass attenuation (centre) and mean mass energy absorption (right) coefficients, plotted against distance from a point ^{125}I (top) and ^{192}Ir (bottom) source centred in a 15-cm radius water phantom. Mean and effective coefficients were calculated by weighting over the separate fluence and energy fluence distribution data of Figure 4.

is commonly referred to as the discrete ordinates method (DOM). The set of directions and corresponding weights is commonly referred to as a quadrature set of order N (corresponding to $N^2 + 2N$ directions). For the discretization of energy, the range from the maximum energy of photons emitted by the source to a minimum energy is divided in groups of intervals that can be variable, with larger ones used for higher energies. A LBTE is constructed for every energy group and each direction. Space is also discretized in volume elements raising the number of equations to be solved to $N_{\text{element}} \times N_{\text{directions}} \times N_{\text{energy groups}}$. After the source iteration method converges to the solution, dose calculations are performed in a post-processing step as the sum of results from Equation (1) applied for each energy group. Heterogeneities are inherently accounted for in Equations (5) and (6). The solution for the primary photon fluence at all points of the geometry can be calculated through ray tracing.

Since a LBTE is solved at every voxel of the geometry only for a finite number of directions, an artificial build-up of particle fluence can occur along these directions. This gives rise to a discretization artefact in the form of rays in the calculated dose distribution. Ray effects are most evident for high gradients of scatter fluence, at points where the relative importance of primary dose is small (*i.e.* at relatively increased distances from a point source). Employing PSS mitigates ray effects. A further benefit can be obtained from separating the scatter source to a first scatter source originating from primary photons and a multiple scatter one originating from higher orders of scattering. This is an iterative procedure referred to as successive

scattering, beginning with the use of the primary fluence in the scattering integral of Equation (4) to determine the first scatter source. The increase of quadrature set order and voxel size would also mitigate ray effects, increasing, however, calculation time and potentially introducing volume averaging uncertainties.^{39,40}

Regarding energy discretization, the multigroup approximation is realistic only if cross-sections do not vary considerably within each group. This is not the case at energies and materials where photoelectric absorption probability is considerable. The ^{192}Ir spectrum is quite forgiving in this regard. Besides accuracy, time is of the essence for clinical implementation, and the impact of energy groups for brachytherapy sources on both has been studied in the literature.⁴⁰ Multiple scattering with minimal energy transfer that occurs in the ^{192}Ir energy range (Figures 1 and 6),⁴¹ as well as coherent scattering that is significant at low energies and high Z materials, delay convergence of the source iteration. Diffusion synthetic acceleration algorithms have been developed to reduce calculation time in problems where scattering predominates.³⁸ The scattering order used in the expansion of the scattering source is also related to the photon energy spectrum and computation time increases with increasing order. Cross-sections recast in energy groups, and Legendre polynomial expanded format suitable for scattering source definition are required. These are obtained using custom software like the CEPXS.⁴²

Since LBTEs include the flow of photons through each element surface besides photon fluence in a volume element, an approximate expression is required to relate volume and surface

Figure 6. Primary, first scatter, multiple scatter and total dose rate results⁴¹ (multiplied by distance from the source squared) plotted against distance along the transversal bisector of the VariSource™ VS2000 (Varian Medical Systems, Palo Alto, CA) (a) and the microSelectron® mHDR-v2 (Nucletron, Veenendaal, Netherlands) (b) ¹⁹²Ir high-dose-rate brachytherapy sources centred in a water phantom of 15-cm radius.

averaged fluence and reduce the number of unknowns. Various methods are used, a common approach being the use of linear-discontinuous finite element methods (DFEMs).³⁷ These methods allow for the calculation of fluence everywhere in each element. Spatial discretization artefacts could, however, ensue in the form of local solution over- or underestimations owing to conflicts of the approximate character of the expression used for the spatial variation of fluence with particle balance.^{43,44}

LINEAR BOLTZMANN TRANSPORT EQUATION SOLVER CLINICAL IMPLEMENTATION

GBBS algorithms are not new. The DOM for angular discretization that is most commonly used in medical physics applications^{39,40,45–50} was introduced in the 1950s. Since GBBS algorithms were used primarily for neutron transport and shielding problems to enhance calculation efficiency in view of increased absorption, they did not become particularly popular within the radiotherapy (RT) community.

The first GBBS algorithm to be incorporated in a commercially available ¹⁹²Ir brachytherapy TPS was Acuros™ [BrachyVision™ (BV); Varian® Medical Systems, Palo Alto, CA].^{8–10,51} Acuros is a version of the Attila GBBS developed at Los Alamos National Laboratory (Los Alamos, NM)³⁸ optimized for brachytherapy, and later also for external beam therapy.^{52,53}

The BV-Acuros algorithm is proprietary. Its basic features⁴³ have also been reviewed elsewhere.⁴⁴ In short, Acuros uses an energy discretization scheme of 37 groups for primary photons (primary is used here in the sense of photons that emerge from an ¹⁹²Ir source, see below). An efficiency gain is obtained by collapsing this energy grouping scheme for the scatter radiation. A scattering order $L = 3$ is used for scattering within energy groups (scatter events that change direction with minimal energy transfer) and $L = 2$ for scattering outside energy groups to resolve the anisotropy of scatter radiation for the ¹⁹²Ir energies. Triangular-Chebyshev quadrature sets are used for angular discretization and the integration for the generation of the scattering source. Order ranges from $N = 4$ to 30 (24–960 discrete angles) both within an energy group and between energy groups.

Sources supported by BV-Acuros are represented by effective sources based on pre-calculated data on energy and angle-dependent intensity of photons. Ray tracing is performed for the definition of primary fluence in the geometry using a limited number of points representing each source. Successive scattering is employed wherein a first scatter source is calculated from the primary fluence.

For spatial discretization, Acuros uses a Cartesian spatially variable computational grid where element size is adapted based

on scatter fluence gradient to promote time efficiency while preserving accuracy and mitigating ray effects. Primary fluence is mapped at several points within each computational element through ray tracing from source positions, and the Galerkin linear DFEM is used.

Material properties are derived from X-ray CT imaging, except for applicators that are overlaid in the geometry using an applicator model library containing geometry and material information that are taken into account in the ray tracing for the definition of the primary fluence. Individual voxel densities are derived from a CT calibration, and density is assumed to be constant within computational grid elements. Material elemental composition is drawn from a density look-up table, including lung, adipose, skeletal muscle, cartilage and bone tissues with compositions from International Commission on Radiation Protection Publication 23,⁵⁴ as well as air, aluminium, titanium and stainless steel. Cross-sections are prepared using CEPXS⁴² that includes all photon interactions except for Rayleigh scatter that is of low relative importance for the ¹⁹²Ir energies except when applicators with high *Z* materials are included in the geometry.

All Acuros calculation settings are preset, and the user only needs to specify a dose output grid and its resolution. The output grid extent affects calculation time. Its resolution only affects accuracy since the calculation grid is defined independently using the spatially variable discretization scheme discussed above. The dose calculation grid is automatically defined as the dose output grid plus 10 cm in all directions, unless the CT image boundary is reached, to account for back scatter.

Dose to any medium can be calculated using macroscopic kerma cross-sections. BV-Acuros v. 10.0.33 used for calculating results presented in the following sections calculates dose to water in the heterogeneous geometry. A following version incorporated the option to calculate dose to voxel medium in the heterogeneous geometry to comply with TG186 recommendations.⁶ Acuros dosimetry remains slow for results to be used in dose optimization (from a couple of minutes for single-source calculations over a 30-cm diameter water sphere with 1-mm resolution, up to 7 min for a multicatheter breast implant). Dose prescription is performed using TG43 in accordance with TG186 recommendations.⁶

COLLAPSED CONE SUPERPOSITION BASICS AND PRACTICAL IMPLICATIONS

The superposition principle has been successfully used in external beam therapy for decades employing different kernel representations and implementation schemes (e.g. Ahnesjö and Aspradakis⁵⁵ and references therein). Readers interested in the details of adaptation of the method in brachytherapy dosimetry are referred to a corresponding series of publications.^{11–16}

In brief, under conditions of CPE and using PSS,¹⁶ the dose at point *r* can be calculated by integrating the contribution of energy released from all other points, *r'*, according to:

$$\frac{D(\mathbf{r})}{R} = \frac{D_{\text{prim}}(\mathbf{r})}{R} + \frac{D_{\text{sc}}(\mathbf{r})}{R} = \frac{D_{\text{prim}}(\mathbf{r})}{R} + \iiint_V \frac{S(\mathbf{r}')}{R} k(|\mathbf{r} - \mathbf{r}'|, \theta) d^3 r' \quad (8)$$

where *R* is the radiant energy of the source,²³ *S* is the scatter energy released per unit mass from a volume element at *r'* (commonly referred to as SCERMA),¹⁶ and *k* is the point energy deposition kernel expressed as the fraction of SCERMA released from the volume element at point *r'* that is deposited per volume element at *r*.

Primary dose can be easily calculated, and, for a monoenergetic point source, SCERMA can be calculated from the primary dose as:

$$S(\mathbf{r}) = \left(\frac{\mu - \mu_{\text{en}}}{\mu_{\text{en}}} \right) D_{\text{prim}}(\mathbf{r}) \quad (9)$$

The point kernel, *k*, can be calculated analytically using effective linear absorption and attenuation coefficients provided the energy spectrum of scattered photons as a function of direction *θ* relative to the direction of primary photons is known. In practice, however, MC simulation is used to generate *k* in water in tabular format. Results are then parameterized using exponential functions to reduce computer storage requirements, mitigate the effects of MC simulation type A uncertainties and facilitate efficient superposition methods. This results to a set of angle-dependent parameters for water. In order to account for heterogeneities, ray tracing is employed with appropriate factors to scale the water kernel for the difference in energy release in a medium relative to water and attenuation of energy in different media along *r*, using effective energy absorption and attenuation coefficients weighted over the spectrum of scattered photons.^{11–16}

Direct superposition according to Equation (8) for every pair of voxels in the geometry is impractical owing to the time required for the *N*² operations when heterogeneity correction is employed for a volume partitioned to *N*³ voxels. Since the scaled kernel is not spatially invariant,⁵⁶ convolution techniques and fast Fourier transform algorithms of reduced calculation complexity cannot be used. The CCS method is an efficient alternative.^{11–15} Its main assumption is that all the energy released into coaxial cones of a given solid angle is transported, attenuated and deposited in elements on that axis. For implementation in the discretized Cartesian coordinates inherent to patient imaging, a set of solid angle elements are determined and each of these elements yields a transport direction. A lattice of parallel lines along each transport direction is constructed so that every voxel in the geometry is traversed by at least one such transport line. Ray tracing is performed along transport lines, and dose at a voxel is given by the sum of the contributions of all directions.^{11–13} The complexity of a CCS algorithm is *MN*³, where *M* is the total number of solid angle elements (or equivalent transport directions), which is practically many orders of magnitude less than that of direct superposition.

The uncertainty associated with the main assumption of the CCS method increases with distance from an interaction point since cone surface, or the number of voxels over which scattered energy should be distributed, increases as $r^2\Delta\Omega$. This would be manifested as a star-shaped dose pattern at large distances where the relative importance of primary dose decreases. This uncertainty is small when the kernel values decrease rapidly with distance from a scatter generation point. It is further alleviated when SCERMA does not vary significantly with distance so that the lack of SCERMA deposition at voxels out of a cone direction from voxels upstream is counterbalanced from SCERMA deposition from adjacent voxels. In brachytherapy, and especially ^{192}Ir , the decrease of the kernel is reduced by multiple scatter (Figure 6), and SCERMA decreases rapidly with distance from a source.¹³ Increasing the number of transport directions and the resolution of the transport direction lattice, or the voxel size, would limit this discretization uncertainty at the expense of calculation time and averaging errors, respectively.

The relative importance of multiple scatter in brachytherapy introduces another problem since the field of scattered radiation is assumed to be unidirectional within each fixed solid angle element, and this applies only to once scattered photons. When the contribution of multiply scattered photons to dose is significant, as in the intermediate and high brachytherapy energies (Figures 1, 2 and 6), accuracy deteriorates away from the sources and close to geometry boundaries (Figures 2 and 6). Therefore, a successive scattering superposition approach has been introduced.^{13,14} MC simulation can be used to generate kernels from generations of scattered photons up to any given order. The once scattered photons kernel, $k_{1\text{sc}}$, can be used to calculate dose from these photons, *i.e.* $D_{1\text{sc}}$. The SCERMA transferred to twice scattered photons, $S_{2\text{sc}}$, is calculated from $D_{1\text{sc}}$ using Equation (9), and the method is repeated to calculate the dose from multiply scattered photons, D_{msc} , using the corresponding kernel, k_{msc} . The partitioning of SCERMA also alleviates the problem of the steep total SCERMA gradient discussed above. It increases calculation time, however, making it proportional to the $(M_{1\text{sc}} + M_{\text{msc}})N^3$.

The choice of $M_{1\text{sc}}$ and M_{msc} is the result of optimizing calculation time *vs* the distance, or equivalently the dose level, where discretization artefacts will occur.¹¹ The steeper SCERMA distribution for once scattered photons dictates that $M_{1\text{sc}} > M_{\text{msc}}$.¹¹

Additional refinements are necessary for CCS implementation in brachytherapy dosimetry. These include accounting for the presence of high Z shields,¹² evaluating the angle between the direction of the inherently divergent, primary radiation and each transport direction for every ray-trace step and selecting the appropriate angle-dependent kernel parameters¹³ and ray tracing for the assignment of SCERMA values at voxels close to a source.¹³ Details can be found in the cited literature.^{12,13}

COLLAPSED CONE SUPERPOSITION: CLINICAL IMPLEMENTATION

Although the CCS method has been shown to be accurate for brachytherapy dosimetry, many of the above implications made

it computationally intense. The acceleration factor provided by general purpose parallel computing on graphical processing units^{57–61} has made the method applicable. Following implementation for external beam treatment planning, Oncentra® Brachy v. 4.4 (Nucletron, an Elekta company, Veenendaal, Netherlands) was released in 2013 including the capability for ^{192}Ir dosimetry calculations based on the CCS method, under the commercial name Oncentra-ACE (Advanced Collapsed cone Engine).

The algorithm is proprietary. Based on information available at the time of writing, it features a multiresolution Cartesian calculation grid. The resolution is 1 mm in a cube containing the source dwell positions, and 2, 5 and 10 mm in three additional cubes outside it that extend up to the geometry boundaries, as the latter is defined through patient imaging. All calculation settings are preset, and the user is only presented with two options denoted as standard and high accuracy levels. These options control the set of margins from the source dwell positions that define the above mentioned four cubes (standard: 1, 8, 20 and 50 cm; and high: 8, 20, 35 and 50 cm). Successive scattering superposition is employed, and the accuracy level choice also controls the number of transport directions defined by spherical tessellation and used for CCS of scatter dose from once and multiply scattered photons (standard: $M_{1\text{sc}} = 320$, $M_{\text{msc}} = 180$ and high: $M_{1\text{sc}} = 720$, $M_{\text{msc}} = 240$). The number of transport directions is also automatically adapted according to the number of source dwell positions since the SCERMA gradient is less steep than in the case of a single source.¹¹

Pre-calculated data obtained from MC simulation are used. These include kernels, spectra for the primary, once scattered, and multiple scattered photons, necessary for the calculation of heterogeneity scaling factors, and primary dose required for the estimation of $S_{1\text{sc}}$. Primary dose is obtained from source-specific MC calculated dose rate distributions in water scored separately for primary and scattered photons.^{15,16} Such PSS data⁴¹ are a source characterization equivalent to that of the TG43 formalism.^{16,22}

Material definitions, and all relevant parameters, are obtained from user-defined regions of interest and consensus data from the TG186 for patient tissues,⁶ as well as an applicator library in case one is included in the geometry. The effect of not using individual voxel densities that can be obtained from patient radiographic CT imaging is expected to be small for the ^{192}Ir energies.⁶² An advantage of the method is that it is not vulnerable to CT artefacts and that imaging modalities other than CT can also be used for dosimetry planning. Nevertheless, CT-derived densities will be used in a future version in compliance with TG186 recommendations.

CCS reports dose to medium in the heterogeneous geometry. The method remains slow for dose calculations to be used in dose optimization (from minutes for a single source to tens of minutes for a multicatheter breast implant using the high-accuracy option). Dose prescription is performed using TG43 in accordance with TG186 recommendations.⁶

clinical brachytherapy where multiple source dwell positions are employed.

Multiple sources and stylized, patient equivalent computational models

In the following, MC, TG43 and MBDCAs are compared in two mathematical equation-based, stylized models resembling an oesophageal and a breast brachytherapy patient.^{21,64} The models were voxelized through their conversion to a series of contiguous CT images in DICOM format to facilitate their import to the TPSs.¹⁰ MC results are considered as reference. Their comparison with TG43 dosimetry results depicts the type B uncertainty introduced by TG43 assumptions, and their comparison with MBDCAs is used to depict how effective these algorithms are.

Results for the BV-TPS in the oesophagus model are presented in Figure 8. Findings are compatible with results presented in the literature using the same MC code without the use of the software for the automated preparation of input files.¹⁰ BV-TG43 calculations are correct at points close to the source catheter owing to the predominance of primary dose (Figure 2). Dose to the spinal cord is, however, overestimated up to 16%, mainly owing to the inability to account for the increased attenuation of photon fluence in the vertebrae. Dose to the trachea and the sternum bone is underestimated for similar reasons. Dose differences in the lungs depend on the relative position of the point of interest relative to the source. Acuros results are in excellent agreement ($\pm 2\%$) with MC results for the majority of points in the model. Differences up to 6% are observed at a limited number of points lying close to the directions defined by primary photon rays tangential to the spine and the trachea structures, possibly owing to ray tracing, the increased scatter fluence gradient in the penumbra of these structures and potential difference in the discretization errors inherently involved in MC and BV-Acuris results. A dose overestimation is also observed at points behind the bone heterogeneities, at increased distance from the source catheter probably owing to imperfect resolving of the angular distribution of scatter.

Results for the Oncentra TPS in the oesophagus model are presented in Figure 9. MC results presented in Figures 8 and 9 correspond to, collision kerma approximated, dose to water and dose to medium, respectively, to match the dose reporting convention of Acuros and ACE in the TPS versions used in this work. The fact that the pattern of differences between MC and TPS TG43 data are similar in Figures 8 and 9 is an indirect confirmation that the adopted dose reporting convention has little effect for the ¹⁹²Ir energies in the large cavity approximation (*i.e.* in terms of dose to millimetre-sized voxels). The issue of dose reporting convention with MBDCAs in brachytherapy is still open until conclusive results are available on the biologically relevant target, the dose descriptor best describing its response and the implementation of cavity theory to derive this dose descriptor from MBDCAs results taking into account source photon energy.^{6,65–67}

In Figure 9, ACE results are in good agreement with the corresponding MC calculations. Except from points away from the

source catheter, increased percentage dose differences are observed only at points within the bone heterogeneities. These are probably owing to the inability of the algorithm to account for the difference in the spectra of scattered photons in bone relative to water.^{12,68}

Results for the BV-TPS in the breast model are presented in Figure 10. Findings are compatible with results presented in the literature using the same MC code without the use of the software for the automated preparation of input files.¹⁰ In short, BV-TG43 calculations overestimate the dose distribution, especially at relatively increased distances, owing to the combined effect of the lung heterogeneity and the finite patient dimensions. A BV-TG43 dose underestimation is observed at points either within the catheters, since these are not excluded in any of the data sets, or close to the sources, probably owing to the extrapolation algorithm employed by the TPS. A slight dose underestimation is also observed in the positive *x* side of the planning target volume that is attributed to the different source drive wire length assumed in the simulations of this work and the simulations for the data used as TG43 input to the TPS. Acuros results correctly account for lung and finite patient dimensions, and major differences with MC results are observed only within the catheters or close to the sources.

Results for the Oncentra TPS are similar. The pattern of Oncentra-TG43 differences from MC results in Figure 11b is the same as in Figure 10b besides using MC results of dose to water and dose to medium, respectively. ACE succeeds in taking these effects into account with major differences observed again only within the catheters or close to the sources.

ISSUES ASSOCIATED WITH CLINICAL ADOPTION OF MODEL-BASED DOSIMETRY ALGORITHMS

As already mentioned, all calculation settings are preset in current versions of TPS including an MBDCAs option, according to an optimization scheme of accuracy vs calculation time. User control over algorithm parameters is therefore limited to specifying the extent and/or the resolution of the dose output grid. Calculations are then performed at the push of a button. Caution is needed in that the dose calculation grid to yield results presented in the dose output grid is limited to the volume defined by the image series used for planning. This can compromise the dosimetry accuracy at the external axial planes of the image series and points close to axial image boundaries of cropped image series, since backscatter radiation, which is significant for the ¹⁹²Ir energies (Figure 2), will not be taken into account. The implant will always be included in the image series. The effect of missing backscatter depends on geometry, materials and distance from the implant, and its significance is mitigated by the inverse square law that governs dose deposition in brachytherapy. Indicative results are presented in Figure 12 for the oesophagus and breast models used in this work. CT over scanning in the *z* axis is not warranted unless the importance of accuracy in dose volume histogram data for a parallel organ at risk (OAR) is deemed high. Image cropping is unacceptable.

Tissue segmentation defined as the spatial delineation of different regions of a 3D image data set and assignment of elemental

Figure 8. (a) The central axial image of the voxelized oesophageal model with BrachyVision™ (BV; Varian® Medical Systems, Palo Alto, CA)-TG43, BV-Acuros™ (Varian Medical Systems) and corresponding Monte Carlo (MC)-calculated dose results for an oesophageal brachytherapy plan presented in the form of percentage isodose lines within the extent of the user-defined dose output grid. (b) The spatial distribution of the percentage dose differences between BV-TG43 and MC results [$100 \times (D_{TG43} - D_{MC}) / D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map. (c) The spatial distribution of the percentage dose differences between BV-Acuros and MC results [$100 \times (D_{ACUROS} - D_{MC}) / D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map.

compositions and mass density⁶ is another issue. The TG186 report⁶ critically reviewed the subject with particular focus on low-energy brachytherapy where it is of increased importance (Figure 1). In the absence of a robust method for the direct extraction of interaction coefficients from CT images, a table of

representative tissues was set forth. For the ¹⁹²Ir energies, tissue density is the determining factor for individualized patient dosimetry (Figure 1). Apart from density, all human tissues, except for bone, are almost water equivalent under the large cavity assumption (*i.e.* for millimetre-sized voxels) regardless of dose

Figure 9. (a) The central axial image of the voxelized oesophageal model with Oncentra® Brachy v. 4.4 (Nucletron an Elekta company, Veenendaal, Netherlands)-TG43, Oncentra-advanced collapsed cone engine (ACE) (TG186) and corresponding Monte Carlo (MC) calculated dose results for an oesophageal brachytherapy plan presented in the form of percentage isodose lines within the extent of the user-defined dose output grid. (b) The spatial distribution of the percentage dose differences between Oncentra-TG43 and MC results [$100 \times (D_{TG43} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map. (c) The spatial distribution of the percentage dose differences between Oncentra-ACE and MC results [$100 \times (D_{TG186} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map.

reporting convention, and TPS relying on individual voxel density mapping combined with a limited material composition look-up table are accurate for ^{192}Ir dosimetry.⁶²

The effect of CT calibration uncertainty to ^{192}Ir dosimetry is presented in Figure 13. Assuming the CT uncertainty is, on average, ± 15 HU,⁶⁹ two extreme CT calibrations were prepared.

Figure 10. (a) The central axial image of the voxelized breast model with BrachyVision™ (BV; Varian® Medical Systems, Palo Alto, CA)-TG43, BV-Acuros™ (Varian Medical Systems) and corresponding Monte Carlo (MC) calculated dose results for a breast brachytherapy plan presented in the form of percentage isodose lines within the extent of the user-defined dose output grid. (b) The spatial distribution of the percentage dose differences between BV-TG43 and MC results [$100 \times (D_{TG43} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map. (c) The spatial distribution of the percentage dose differences between BV-Acuros and MC results [$100 \times (D_{ACUROS} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map.

Figure 13 summarizes the percentage differences between BV-Acuros dose calculations performed for a breast and a head and neck case when these two extreme calibrations were used,

assuming a rectangular distribution for Hounsfield units. Dose differences that pile up with distance from the implant are small, affecting only dose to OARs under 2%.

Figure 11. (a) The central axial image of the voxelized breast model with Oncentra® Brachy v. 4.4 (Nucletron an Elekta company, Veenendaal, Netherlands)-TG43, Oncentra-advanced collapsed cone engine (ACE) (TG186) and corresponding Monte Carlo (MC)-calculated dose results for a breast brachytherapy plan presented in the form of percentage isodose lines within the extent of the user-defined dose output grid. (b) The spatial distribution of the percentage dose differences between Oncentra-TG43 and MC results [$100 \times (D_{TG43} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map. (c) The spatial distribution of the percentage dose differences between Oncentra-ACE (TG186) and MC results [$100 \times (D_{TG186} - D_{MC})/D_{MC}$] on the central axial plane, presented on a pixel-by-pixel basis using an appropriate colour map.

Figure 12. (a) Percentage dose differences on the central axial plane of the voxelized oesophagus model between BrachyVision™ (BV; Varian® Medical Systems, Palo Alto, CA)-Acuros™ (Varian Medical Systems) calculations performed for the full model and the model symmetrically reduced from 17 to 12 cm in the z axis. (b) Same as (a) on an axial plane 4 cm away from the central axial plane. (c) Percentage dose differences on the central axial plane of the voxelized breast model between BV-Acuros calculations performed for the full model and the model symmetrically reduced from 10 to 8 cm in the z axis.

Figure 13. The dosimetric uncertainty from an uncertainty of ± 15 HU (on average for all Hounsfield units⁶⁹) presented using an appropriate colour map on the central axial plane of (a) a voxelized oesophagus model and (b) a voxelized head and neck model. Results were calculated assuming a rectangular distribution and using BrachyVision™ (Varian® Medical Systems, Palo Alto, CA)-Acuros™ (Varian Medical Systems) for dose calculations.

In view of different methods, or even different implementations of the same method in MBDCA-based TPS, augmenting commissioning procedures is key to its clinical integration without compromising the global uniformity achieved with TG43. The TG186 has proposed a graded approach to the commissioning of MBDCA-based TPS,⁶ similar to the procedure followed in a series of works for the dosimetric benchmarking of Acuros.^{8–10} Verification of the MBDCA to reproduce consensus TG43 data³ for sources supported by the TPS is referred to as Level I commissioning. In the absence of reference data in DICOM-RT dose format for import to the TPS and appropriate TPS tools for 2D comparisons, this could become a tedious process of point calculations that could easily miss localized differences, as shown to have been carried out for TG43 TPS algorithms in the discussion above. Definitive acceptance criteria are also hard to form since the optimization of an MBDCA could lead to significant differences in bounded, single-source geometries (Figure 7). The recommendation to carefully examine differences, understand and document the clinical impact⁵ is therefore much more useful than the criterion of 2% difference adopted from TG43U1.²

Level I commissioning is to be followed by checks of efficiency of the algorithm in accounting for heterogeneities and scatter

conditions (Level II) using reference, MC generated or experimentally determined, dose distributions for particular tests.⁶

A workflow is proposed and a working group has been set up seeking to address limited test case availability, tools and methods. Significant progress has been made by the group in that a theoretical source was prepared that was included in the systems of both vendors of TPSs incorporating MBDCAs.⁷⁰ Example test case plans and reference results, to be made available through a web registry, are under way. It is envisioned that these can be uniform amongst vendors and included in their commissioning test procedures to depict differences between TG43 and MBDCAs.⁶ A similar, yet more flexible approach, would be the preparation of end-user-oriented tools to assist the evaluation of MBDCAs.

THE CLINICAL BENEFIT EXPECTED FROM MODEL-BASED DOSIMETRY ALGORITHMS

The sensitivity of anatomic sites treated with brachytherapy to limitations introduced by TG43 assumptions have been reviewed in the literature.^{4,22} For the ¹⁹²Ir energies where MBDCAs have been introduced, sites potentially affected include breast, head and neck, superficial applications and all applications involving shielded catheters. MBDCAs are expected to improve dosimetric

accuracy, as shown in the literature reviewed, and affirmed by illustrative results of this work.

Regarding the clinical benefit expected from improved dosimetric accuracy, it must be kept in mind that MBDCAs remain slow for plan optimization, and dose prescription is performed using TG43.⁶ Hence the potential of MBDCAs is currently reserved for radiation oncologist assessment. Clinical protocols are not expected to change either, at least not until a sufficient volume of clinical data become available that justify such changes.

Such data could be in the form of planning dosimetry studies for patient cohorts.^{71,72} It is important to understand that dosimetric effects that are currently not taken into account but affect the patient population in the same manner (direction and magnitude) are included, at least to some extent, in clinical protocols through clinical trial data.

The expected benefit of MBDCAs in dosimetry planning is in the amount of reduction it will achieve (through the individualization of patient dosimetry) in the variance of the response of clinical trial populations. This is equivalent to a reduction of uncertainty that must be contrasted, however, to the combined dosimetric uncertainty in brachytherapy from physical and clinical sources.⁵

Apart from that, a clear benefit can be expected from the need to augment commissioning and quality assurance procedures introduced by MBDCAs, and the active involvement of physicists in examining differences and radiation oncologists in assessing their impact.

CONCLUSIONS

Model-based dose calculation algorithms for brachytherapy dosimetry share a lot of similar features: a long heritage of use outside brachytherapy, angular and spatial discretization, ray

tracing, PSS, successive scattering methods, use of pre-calculated data, increase of calculation time with number of sources, use of applicator libraries and pre-fixed calculation settings in their clinical implementation according to vendor optimization schemes, to name but a few.

Presented results confirm that both clinically available model-based dose calculation algorithms achieve a significant dosimetric accuracy improvement compared with TG43-based calculations. The clinical benefit from improved accuracy remains to be evaluated. The explanation of differences observed between model-based and TG43-based algorithms is not trivial and requires a basic understanding of all methods coupled with augmented test procedures and user-oriented tools.

Treatment planning using model-based dose calculation algorithms should not be considered a simple investment on a new TPS mandated by recommendations or current trend, but rather as an opportunity for concerted, interdisciplinary effort to reach improved levels of care quality in the near future.

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