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Two-dimensional simulations of displacement accumulation incorporating shear strain

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Abstract

Using ultrasound images to track large tissue deformations usually requires breaking up the deformation into steps and then summing the resulting displacement estimates. The accumulated displacement estimation error therefore depends on the error in each step, but also on the statistical relationships between estimation steps. These relationships have not been thoroughly studied. Building on previous work with one-dimensional simulations, the work reported here measured error variance for single-step and accumulated displacement estimates using two-dimensional numerical simulations of ultrasound echo signals, subjected to both normal and axial shear strain as well as electronic noise. Previous results were confirmed, showing that errors due to electronic noise are negatively correlated between steps and accumulate slowly, while errors due to strain are positively correlated and accumulate quickly. These properties hold for both normal and axial shear strain. A general comparison of tracking performance for tissue under normal and axial shear strain was also performed. Under axial shear strain error variance tends to increase with larger lateral kernel sizes but decrease for larger axial kernel sizes; the opposite relationship holds under normal strain. A combination of these two types of strain limits the practical kernel size in both dimensions.

Keywords

elastography; displacement estimation; simulation; accumulation; multicompression

1 Introduction

Tracking tissue motion using ultrasound echo signals has often used an accumulation or “multicompression” technique, in which displacements are estimated incrementally between consecutive images in a sequence and then summed to form an accumulated displacement map. This type of strategy is essential for tracking large deformations, since the amount of strain that can be tracked in a single estimation step is limited. These large deformations might be desirable to increase strain image quality,^{1,2} are intrinsic to cardiac elastography,³

and are required for estimating the elastic nonlinearity of tissue.^{4,5} The incremental tracking of a displacement sequence, sometimes accompanied by accumulation, is also used in other ultrasound techniques, including Acoustic Radiation Force Impulse (ARFI),⁶ shear wave speed imaging,^{7,8} and the estimation of poroelastic parameters.⁹ The present study focuses on tracking large deformations, but some of its conclusions may also pertain to these other applications.

When applied to large deformations, dividing the tracking process into steps decreases the error induced by tissue strain for each individual estimate.^{1,10} But each estimate also contributes an error due to the imaging system's electronic noise, so a large number of steps leads to a large number of noise-induced errors being summed. Thus the number and size of the tracking steps affect how these two types of errors accumulate, as do various parameters of the estimation algorithm.

The accumulated error will also depend on whether individual errors are correlated between steps. For the sake of simplicity, many authors have assumed that displacement estimation errors in one step are uncorrelated with previous and subsequent steps. The present work will prove that assumption false, at least in a simulated model.

While many studies have made use of the accumulation of displacement estimates, relatively few have addressed the accumulation process itself. A study by O'Donnell et al.,¹¹ for example, was one of the first to propose the accumulation of many small displacement estimates in creating strain images. Their purpose was not to analyze their accumulation method, however, and they assumed uncorrelated displacement steps.

The multicompression technique was analyzed more thoroughly in studies by Varghese, Ophir, and Céspedes^{1,12}, which also included the effects of temporal stretching of echo signals. Combining Walker and Trahey's Cramér-Rao lower bound for time-delay estimation¹³ with the modified Ziv-Zakai lower bound and the Barankin lower bound described by Weinstein and Weiss,¹⁴ they built a theoretical model of the effect of multicompression on strain image SNR. The authors found that a combination of multicompression and temporal stretching, using a particular optimal strain increment between multicompression steps, obtained the best strain SNR. Only limited simulation support for these results was included, however, and like O'Donnell the authors assumed that displacement errors were uncorrelated between steps.

A more recent study on optimizing the multicompression approach by Du et al.¹⁵ used theoretical estimates as well as data from phantoms and in vivo breast scans. This study also sought to compute optimal strain increments, in this case by minimizing displacement estimate variance. The proper increments differed depending on the data being examined. Notably, this study highlighted the importance of error covariance between steps in multicompression techniques. They were unable to characterize this covariance in detail, however.

Bayer and Hall's recent work with one-dimensional simulations, in which strain was applied along the axial dimension of 1D signals,¹⁶ focused specifically on the accumulation process and the correlation between estimates in a sequence. They found that errors due to tissue

strain were positively correlated between consecutive steps. This positive correlation meant that errors at each step reinforced one another, accumulating faster than they would if they were independent. This positive correlation of strain-induced errors tended to balance the single-step dependence on strain such that the total accumulated error, for a fixed final deformation, was not very dependent on the strain increment at all. A wide range of strain increment sizes could produce very similar accumulated variances.

Errors due to electronic noise, in contrast, were anti-correlated between consecutive steps. Thus noise errors tended to cancel out under accumulation, and electronic noise had a smaller effect on the accumulated displacement estimates than might have been expected.

These findings challenged the conventional assumption that displacement estimates in a sequence are independent of one another, and argued that optimizing an accumulated displacement estimate is not reducible to optimizing a single-step estimate. But the study was not definitive, because it was unknown whether these observations extended to more realistic models of tissue and deformation.

The present study expands this investigation to a two-dimensional tissue model. Its primary contribution is to investigate the accumulation properties of displacement estimates under 2D models of normal and axial shear strain. An analysis of single-step displacement estimation error for these two types of strain as a function of kernel size in both dimensions, which to the authors' knowledge has not previously appeared in the literature, is also performed.

2 Methods

2.1 Simulation

The simulation methods were a direct extension of our previous one-dimensional work¹⁶ into two dimensions. All computations were performed in MATLAB (2011b, MathWorks, Inc., Natick, MA).

Radiofrequency (RF) ultrasound data were simulated in small independent sections, which will be referred to as RF echo fields. Each field represented a local region of tissue that was tracked over its entire trajectory. A field covered only enough area for a single displacement estimation site, so the size of the field was proportional to the size of the tracking data kernel. This approach proved more flexible than simulating larger RF fields containing many estimation sites. The product of these simulations was thus an ensemble of small speckle patches and individual displacement estimates, rather than entire echo images or displacement maps.

The RF data were produced by convolving a random scattering medium with a simple 2D point-spread function (PSF) that modeled the pulse-echo response of an ultrasound system. The PSF was separable, consisting of a Gaussian-windowed cosine in the axial direction (the direction along a beam-line) and a Gaussian in the lateral direction (perpendicular to a beam-line). Both Gaussian functions were truncated at the 2-sigma point. For the purposes of simulation the wavelength corresponding to the pulse's center frequency was taken as the fundamental unit of distance, applied to both axial and lateral dimensions, and is denoted by

λ . These dimensions may be converted into concrete units of range or lateral distance by applying the proper scaling factors.

The PSF dimensions were designed so that the resulting RF data would have a similar autocorrelation function to data from the Siemens Acuson S2000 Automated Breast Volume Scanner system (Siemens Ultrasound, Mountain View, CA, USA). The goal was not to precisely replicate the performance of this system, but merely to use it as a reference for creating an approximately realistic PSF. Any depth dependence in the real system's PSF was neglected for simplicity. The system's 2D pulse-echo autocorrelation function was measured on RF data from a phantom containing many randomly-placed Rayleigh scatterers. Then the size of the simulation PSF was set so that its autocorrelation function matched the system data's autocorrelation as closely as possible. The consequent size of the simulated PSF, measured from the 2-sigma boundaries, was 3.5λ by 8.5λ , or 0.38 mm by 0.93 mm at the system data's measured center frequency of 7.0 MHz. Measured by the full-width half-maximum (FWHM), the size of the PSF was 2.1λ by 5.0λ . The corresponding relative signal bandwidth was 61%.

The scattering medium over which this PSF was convolved was modeled as a collection of randomly placed point scatterers whose amplitudes were normal random variables. Scatterers were placed at density of about 50 per pulse area, measured by the FWHM boundaries, which was sufficient to produce fully-developed speckle. The area of the collection of scatterers was made large enough to ensure that all RF data points used for computation were the result of a full convolution without edge effects, taking into account the size of the kernel, the displacement search region, and the future deformation of the scatterers.

Figure 1 is a diagram of a scatterer region that illustrates several aspects of the simulation. Overlaid on the scatterer area are the region of valid convolution, the tracking search region, and the tracking data kernel for initial and fully deformed states. As previously described, the simulated scattering region is large enough so that both pre- and post-deformation search regions are always within the region of valid convolution.

The scatterers themselves were generated first as arrays of positions and amplitudes. This format made it easier to deform their positions later. Then, to enable standard digital convolution, the set of scatterers was converted into a scattering function by binning them into a discrete set of locations, with 100 samples per wavelength in the axial dimension and 25 samples per wavelength in the lateral dimension. If a location bin contained two or more scatterers, their amplitudes were summed.

While such a binning scheme is computationally convenient, it is possible that the finite size of bins may introduce inaccuracies for small strains. Under a small strain, scatterers may not be likely enough to change bins to properly simulate the change in RF signal. To ensure that the binning frequencies used were sufficient, some test simulations were performed at an axial binning frequency of 200 bins per wavelength. Single-step variance results were nearly identical. Accumulated variance results, for the noise-free case, differed by 10% or less for strain steps below about 0.7% . Similar test simulations where the lateral binning

frequency was increased to 50 bins per wavelength had even smaller discrepancies. These tests lead us to believe that the simulated binning frequency produces a reliable level of accuracy.

To create strained versions of this signal, normal and shear strains were applied to the scatterer positions in separate experiments. Both deformations made the assumptions of plane strain and volume incompressibility. Strain states were computed as coordinate transformation matrices that could be applied to scatterer coordinates. This final strain state was divided into 180 increments by finding the 180th root of the final transformation matrix. The number of increments, while ultimately arbitrary, was chosen so that it would be evenly divisible by a large number of factors.

For example, in the normal strain experiment, scatterers were compressed axially and expanded laterally up to a total of 20% axial compressive strain. The strain increments then had a size of about 0.12%. This minimum deformation increment will be referred to as an “elementary strain step,” and was repeatedly applied to the scatterers’ coordinates to build up larger amounts of deformation. After each application of strain, the scatterers were again binned into a scattering function and again convolved with the PSF to form strained RF echo fields.

In the shear strain experiment, scatterers were subjected to simple shear strain along the axial direction, as if the ultrasound A-lines were sliding past one another. The quantity of shear strain is given by the magnitude of the proper off-diagonal component of the coordinate transformation matrix; this component is equal to the tangent of the shear angle. Here too scatterers were deformed in 180 increments up to a total of 0.20 axial shear strain, also written as 20% or 11.3° . Because axial shear is a different type of deformation, the matrix root was slightly different than for normal strain and the increment size was about 0.11%, or 0.06° .

A limited number of simulations were also conducted with a combination of normal and axial shear strain to demonstrate that the results of the combined deformation could be approximated by analyzing its separate components. In these simulations, scatterers again underwent 20% total normal strain and either 10% or 20% total axial shear strain. The combined deformation was divided into increments through matrix roots and analyzed in the same manner as the “pure strain” examples.

In all cases, the center point of the scattering region acted as the stationary point of the deformations—the coordinate that remained in place while all other coordinates were deformed—and was also the center point of the data kernel used for tracking. The true displacement of the tissue was defined as the displacement of this center point. Although a rigid translation of the scattering medium could have been simulated, in addition to its deformation, the assumption of a shift-invariant PSF meant that such a translation would be arbitrary and would not be a source of error. Therefore no additional translation was performed: the center-point displacement was always zero.

After undergoing strain, uncorrelated noise was added to the echo fields to simulate the effects of electronic noise. Each echo field received an independent noise realization. The

noise was modeled as Gaussian, with the same power spectrum as the signal. This choice of noise spectrum approximates the results of using bandpass filters to eliminate noise outside the signal bandwidth, and has been used previously in the literature.^{13,16,17} Noise realizations were thus produced by convolving a 2D array of Gaussian random variables with the same PSF described earlier, followed by appropriate scaling of the noise magnitude to achieve the desired SNR. A range of SNRs was simulated.

The entire simulation process was repeated for 400 independently generated scatterer distributions and sets of noise realizations. A summary of simulation and processing parameters is shown in Table 1.

2.2 Displacement estimation

Once the strained RF data was produced, displacement was estimated between each pair in the set of 181 echo fields (the original field plus 180 strained versions). The number of elementary strain increments that had occurred between each pair of fields determined the strain step size for the estimate. Displacement estimation was performed by selecting a rectangular region, or kernel, of data in the field with lower overall strain, called the “pre-deformation” or “reference” frame of data, and computing the value of a matching function with kernels of data from the second, “post-deformation” frame. The matching function used was normalized cross-correlation (NCC). The center of the post-deformation kernel was varied over a search region, and the location with peak correlation then defined the displacement estimate. The difference between this estimate and the known value of zero displacement was the estimation error.

Several aspects of the displacement estimation need to be addressed in more detail. First, the search region for the matching function was constrained to a range of two-thirds of a wavelength on either side of the true axial displacement and 2.8 wavelengths on either side of the true lateral displacement (equal to $\frac{4}{3}\sigma$ relative to the PSF’s Gaussian lateral profile). This limitation sped up the simulation and also purposely eliminated tracking errors outside of the true peak of the matching function. Known as “peak-hop” errors in the ultrasonic motion tracking literature, these large errors arise from a false peak in the matching function. In rejecting these types of errors, we assumed that there was enough prior information to know the true displacement within a certain distance. “Guided search” motion tracking strategies are one means of obtaining such prior information.^{18–21} Therefore, the errors reported here were in the precise localization of the correlation peak, and did not include large-scale tracking failures.

Second, although the simulations set overall displacement to zero, the results are not limited to that case. Because the PSF is approximated as shift-invariant, an overall displacement is arbitrary. Simulations of zero-displacement scatterers would therefore be equivalent to simulations of scatterers undergoing translations, as long as the data kernel always centered on the same piece of scattering medium as it moved.

Such a method of tracking and accumulation operates in “material” or “Lagrangian” coordinates, that is, coordinates that attach to particular pieces of a medium, and follow those pieces along their trajectories of motion.^{3,22} It is also common for tracking algorithms

to operate in “spatial” or “Eulerian” coordinates, which denote particular points in space. A spatial-coordinates tracking algorithm would estimate displacements at a given set of spatial locations while the medium moves around them. Computing accumulated displacements requires a series of coordinate transformations in spatial coordinates,⁵ but only a simple sum of incremental displacements in material coordinates. The material coordinates system is thus the more natural choice for these simulations.

Finally, the sampling of the echo fields should be described. The computed RF fields consist of grids of numerical samples, so the matching function peak can only occur at a sample location. Measuring finer displacements requires methods of sub-sample estimation, which are themselves the subject of a good deal of research.^{17,23} The sub-sample method naturally affects the accuracy of a displacement estimate, though it is less important for higher sampling rates.

The present study is not interested in sub-sample estimation effects, so the simulations were generated with high sampling rates of 100 samples per wavelength axially and 25 samples per wavelength laterally. An additional sub-sample estimation was then performed during displacement estimation using quadratic fitting around the matching function peak, separately in axial and lateral directions.²³ In contrast, the temporal sampling rate for a commercial ultrasound system might be 40 MHz, which for a signal of 7 MHz center frequency would record only about 6 axial samples per wavelength. A typical A-line density, in turn, might be 6 per millimeter, which again for a 7 MHz center frequency would correspond to about 0.7 lateral samples per wavelength’s distance. The simulated sampling rate is thus about 20 and 40 times higher than the axial and lateral rates, respectively, of a hypothetical commercial system. When dealing with RF data from a real system, similarly high sampling rates and sub-sample accuracy could be obtained by interpolating the data. With this high simulated sampling rate, sub-sample estimation effects were considered to be negligible. Future work could analyze the effects of coarser, more realistic sampling.

Displacement estimates were carried out using all combinations of four axial and three lateral kernel sizes. The axial kernel sizes were 1, 2, 4, and 8 wavelengths, and the lateral sizes were 4, 6, and 12 wavelengths. The axial size will also be referred to as “length” and the lateral size as “width.”

2.3 Accumulation

The method of accumulation is identical to our previous work.¹⁶ Since displacement was estimated between each possible pair of signals, accumulating to the maximum strain only required selecting the proper estimates and summing them. The accumulation process is identical for axial and lateral displacements, and for normal and shear strain experiments. To illustrate, let N be the total number of elementary strain steps, and m be the strain step size expressed as a multiple of those elementary steps. Number the simulated RF data frames from 0 to N and denote the displacement estimation error between frames i and j as $\tilde{D}_{ij}$. When N is divisible by m , the accumulated error is then

$$\tilde{D}_{accum}(m) = \tilde{D}_{0,m} + \tilde{D}_{m,2m} + \cdots + \tilde{D}_{N-m,N} \quad (2.1)$$

If N is not divisible by m , then the first step in the accumulation sequence will have a smaller size, equal to the remainder when dividing N by m . The post-deformation signal for one term in the sum is always the pre-deformation signal for the next term.

The preceding equation shows that the accumulated error is just the sum of many single-step errors. As a sum of random variables, its overall variance is therefore equal to the sum of covariances between each pair of single-step errors. Abbreviating the indices above so that $\tilde{D}_i$ indicates “the error in the i th step of an accumulation sequence,” the accumulated variance can be expressed as

$$\text{Var}(\tilde{D}_{accum}) = \sum_{i,j} \text{Cov}(\tilde{D}_i, \tilde{D}_j) \quad (2.2)$$

Single-step and accumulated variances, as well as covariances between estimation steps, were computed over the ensemble of 400 simulated data sets for each kernel size and SNR. We choose to compute displacement variance instead of some other measure of error, like strain SNR, because it makes error accumulation easier to analyze. And while strain imaging is an important application of ultrasonic motion tracking, displacements have other uses besides computing strain. The estimation of nonlinear elasticity parameters and ARFI imaging, for example, both operate directly on displacements.

3 Results

3.1 Comparison with 1D simulation results

The first objective of this work was to determine whether the previous 1D simulation results could be replicated in a 2D tissue model. To that end, Figure 2 compares the accumulated error variance for the normal strain 2D simulations to our previous 1D results. Complementary information on the *single-step* error as a function of strain is contained in the Appendix. For the accumulated variance in Figure 2, magnitudes vary but the trends are the same, even for lateral displacement errors. Without noise, the accumulated variance depends much more strongly on the axial kernel size than on the strain step size. With the addition of electronic noise, error tends to build up at small step sizes. Electronic noise has a relatively small effect, however, unless the SNR is very low or the kernel size is small.

The bias of these simulated estimates was also measured. Because the net displacement of each simulated echo field was zero and both the PSF and the applied strain were symmetric, the displacement estimates should be unbiased. It was found to be typically less than 5% of the corresponding standard deviation and evenly distributed about zero. This result is consistent with unbiased estimates, since the standard error of the mean for an ensemble of 400 realizations would be $1/\sqrt{400}$ of the standard deviation, or 5%.

Figure 3 displays the covariances between displacement steps for 1D and 2D simulations. With no added electronic noise, errors caused by strain alone are highly correlated with the

errors in other steps, with the correlation falling off gradually as the separation between steps increases. Adding electronic noise to the simulated signals superimposes a pattern on the three central diagonals of the matrix, where a bright main diagonal is flanked by depressed super- and sub-diagonals. This pattern indicates that errors induced by electronic noise add variance (the center diagonal) but are anti-correlated between consecutive steps (the side diagonals). This basic pattern is present in the 2D simulations just as in the 1D simulations, for lateral as well as axial displacement estimates.

3.2 The effects of axial shear

Investigating accumulation for a different type of deformation, Figure 4 shows the accumulated variance for displacements under axial shear strain. For axial shear, the *lateral* kernel size is the more important kernel dimension, so results are displayed for different lateral kernel sizes rather than different axial sizes. With that change, the accumulated displacement variance has similar behavior to the variance under normal strain.

The covariance between steps is also very similar, as shown in Figure 5. The main difference is that the correlation of strain-induced errors falls off more quickly in the axial shear matrices, even to the point of becoming slightly anti-correlated for steps with a large separation.

When axial shear is added on top of normal strain, instead of existing in a pure state, the resulting displacement error variances are approximated well by summing results from the two pure strain states. In Figure 6 the single-step and accumulated displacement error variance is shown for two combinations of axial shear and normal strains, for a single kernel size. These results are compared with a predicted curve, computed by adding the previously simulated variances from normal strain alone and axial shear strain alone. The agreement between the full simulation and the prediction is reasonably good, especially for low-noise cases.

3.3 Kernel size

In addition to analyzing the accumulation process, these simulations provide the opportunity to compare the effects of kernel size in both dimensions for the different types of deformation and levels of noise. While the effects of two-dimensional kernels^{24,25} and of axial shear^{26,27} have been addressed in the literature. The bar plot in Figure 7 shows the single-step displacement estimate error variance as a function of the two-dimensional kernel size; single-step error variance as a function of strain is addressed in the Appendix.

For normal strain, an increase in the axial kernel dimension causes an increase in strain-induced error variance but a decrease in noise-induced error variance. An increase in the lateral kernel dimension decreases the variance for both strain-induced and noise-induced error. These observations apply equally to axial and lateral displacement errors. For axial shear strain, in contrast, longer axial kernels decrease error variance for both types of error, while wider lateral kernels increase strain-induced variance but decrease noise-induced variance. Again, this pattern holds true for both axial and lateral displacement estimates.

4 Discussion

4.1 Comparison with 1D simulation results

In these 2D simulations the main results of our previous 1D work have been confirmed. The accumulated error variance for 2D normal strain, as well as for axial shear strain, had very similar trends to the error variance from the 1D simulations, though the magnitudes tended to be lower for the 2D simulations due to using a 2D data kernel. These similarities indicate that the conclusions of our previous work still hold for more realistic and complex tissue motion.

They also hold for lateral displacement estimates as well as axial estimates, which is perhaps surprising given that the axial estimates are much more precise than the lateral estimates. This greater precision is primarily a result of the RF data having phase information (i.e. a carrier frequency) in the axial direction that is not present in the lateral dimension. Correlation peaks are correspondingly narrower in the axial dimension, which leads to more precise displacement estimates. Nevertheless, the accumulation properties prove very similar for the two dimensions.

Among the implications of these results is that the errors at each step in a displacement sequence are not independent of one another, as Figures 3 and 5 show. Because an accumulated error is merely the sum of the step errors, the accumulated variance is given by the sum of all covariances between steps. These covariance matrices determine how the stepwise errors accumulate.

Strain-induced errors are highly correlated between steps because they are caused by the distortion of a tissue's speckle signal as it undergoes strain. The speckle is largely stable between image frames, but undergoes slight distortions with each strain increment. These distortions are partly random, but also partly determined by the strain. Similar increments of strain, as simulated here, will create similar signal distortions and give rise to correlated errors. As strain builds up, however, the speckle signal gradually changes, or "decorrelates", until the errors late in a displacement sequence are no longer very correlated with the errors at the beginning of the sequence. This is the pattern seen in Figure 3.

Because strain-induced errors are correlated, they reinforce one another when summed. If the covariance is nearly constant across the matrix, the accumulated error increases roughly quadratically with the number of steps in the sequence. The single-step error variance, however, increases roughly quadratically with the size of the step (see Figure 8 in the Appendix). For a fixed total strain, in which the number of steps and the size of the steps are inversely related, these two effects balance one another and result in an accumulated variance which is almost constant as a function of step size, as seen in Figure 2.

Adding electronic noise to the simulation superimposes a new pattern onto the covariance matrices, the elevated center diagonal and depressed super- and sub-diagonals. The new pattern appears to be simply added on top of the existing strain-induced covariance, indicating that the two types of errors are uncorrelated with one another. The brighter main

diagonal, then, unsurprisingly shows that electronic noise increases the variance for each displacement estimation step.

The darker side diagonals, in contrast, show that the noise-induced errors of neighboring estimation steps are anti-correlated. Displacement estimates that are neighbors in a sequence share an echo frame; the earlier estimate's *post*-deformation frame is the following estimate's *pre*-deformation frame. Since these estimates share an echo frame, they also share a noise realization. Whatever effect that noise has on the first displacement estimate, it will have the opposite effect on the second, since its position in the estimation process has been reversed.

The effect of this anti-correlation is that consecutive errors tend to cancel one another out. Adding the depressed side diagonals to the main diagonal in the covariance matrix produces a final variance that is much lower than if one simply summed up the main diagonal, as a model assuming independent step errors would do. Therefore electronic noise has a much lower effect on the final accumulated variance than would be expected under a model where all estimates are independent. This is shown in Figure 2, where the addition of electronic noise has almost no effect on the larger kernel sizes. Only for small kernel sizes, very low SNR, and small strain step sizes does electronic noise significantly increase tracking error variance; under these circumstances the anti-correlation is not strong enough to cancel out all effects.

A more mathematical treatment of these arguments can be found in our previous article on 1D accumulation simulations,¹⁶ and so has been omitted here for brevity.

4.2 Effects of axial shear

Similar results for axial shear in Figures 4 and 5 indicate that changing the type of strain does not drastically change the accumulation process. Other than the dependence on kernel size, which will be discussed in the next section, the main difference between the two types of strain is in error magnitude. For example, for the same kernel size, a 1% (or 0.57°) increment of axial shear strain will usually result in a higher-variance single-step displacement estimate than a 1% increment of normal strain. But this difference is not necessarily surprising. The comparison of 20% total normal strain against 20% (or 11.3°) total axial shear strain was somewhat arbitrary, as were the choices of kernel size, so there is no reason to suppose that 1% normal strain and 1% axial shear strain should be equivalent in any meaningful way. Instead, for a given kernel size, the point at which normal and axial shear strains produce the same amount of error variance could be a basis for identifying “equivalent” amounts of the two types of deformation. Table 2 shows the level of axial shear strain required to produce an equal amount of single-step axial error variance as 1% normal strain, for each kernel size tested. Using lateral error variance instead produces somewhat different results, though the trends remain the same.

This simple difference in error scale—how much error is induced per percentage point of strain—also helps explain why the covariance matrices for axial shear (Figure 5) show more decorrelation between steps of large separation than the matrices for normal strain. Since axial shear strain of a certain percentage usually causes larger errors than normal strain, we

can also assume that it causes more decorrelation in the ultrasound echo field. For equal-percentage steps in an image sequence, then, the echo field under axial shear strain will have undergone more decorrelation, making its displacement error more decorrelated as well.

4.3 Kernel size

The effects of kernel size can be very dependent on the type of deformation being applied. Because of distortions in the echo field caused by deformation, pre- and post-deformation echo fields can often match very closely near the center of a kernel, but become less well-matched toward the edges. Larger kernels, because they include more data far from the kernel center, can decrease the ability to match distorted signals and increase strain-induced error. Furthermore, because an ultrasound system's PSF has a phase-encoding character in the axial direction—due to its carrier frequency—but a blurring character in the lateral direction, displacement estimation is much more sensitive to deformations along the axial direction.

The relationship between electronic noise-induced error and kernel size is more intuitive. The larger the kernel, in either dimension, the more information is incorporated into the estimate and the more the effects of noise are suppressed. Therefore larger kernels tend to decrease error variance in noisy data. These effects explain the results of Figure 7. Under normal strain, increasing the axial kernel size decreases variance for noise-induced error but increases it for strain-induced error, because longer kernels include more distorted data.

Increasing the lateral kernel size (still under normal strain) decreases noise-induced variance as well, as expected. It is interesting, however, that it also decreases the strain-induced error, presumably because a larger speckle area increases the specificity of kernel matching. Although the simulated tissue is expanding laterally as well as compressing axially, the lateral expansion appears to have no effect on variance because of the lower sensitivity to deformations along the lateral direction. We would guess that at some point the lateral kernel dimension could become large enough that lateral strain distortion would become important and performance would degrade. But that does not seem to happen out to 12λ , over twice the FWHM pulse width.

In any case, the presence of axial shear places a stricter limitation on the lateral kernel size than lateral expansion does. Under axial shear strain the roles of axial and lateral dimensions are reversed from normal strain. Larger axial sizes improve the variance for both types of error, while larger lateral sizes increase variance for strain-induced error and decrease it for noise-induced error. Note that even though it is larger *lateral* sizes that increase variance, the deformation is still along the more-sensitive *axial* direction. The wider the kernel, the more the data at its lateral edges have shifted axially compared to the data at its center (see Figure 1c), and the worse the match between kernels will be.

4.4 Limitations and future work

Although this study was designed to extend previous results to a more realistic tissue model, there are still some aspects of the motion tracking problem that have been left out or simplified for the sake of clarity of analysis. We would expect that other types of

deformation in the plane, such as rotation, would have similar effects to the normal strain and axial shear strain tested here. A full 3D model of tissue, however, including the effects of tissue motion in and out of the image plane, could be expected to add new effects.

Out-of-plane motion, in particular, is an additional source of error that is very relevant to the accumulation problem. Over large deformations it becomes increasingly difficult to keep the imaging plane stationary in tissue, and even small movements in elevation can introduce decorrelation.¹⁰ The present study has shown that different sources of error can have different accumulation properties, so it seems likely that errors from out-of-plane motion would behave differently than either noise-induced or strain-induced errors.

The finite sampling rate of real data, and the various subsample estimation techniques used to accommodate it, is another aspect of motion tracking that could introduce additional effects. It was ignored here for the sake of simplicity, and so that the effects of electronic noise and strain could be analyzed apart from other confounding factors.

Finally, more advanced motion tracking methods, such as those using companding,^{30,31} could also add to these results. Since companding, in particular, compensates for how the ultrasound echo field distorts in accord with tissue motion, it would likely change the covariance structure of strain-induced errors. Testing the effects of out-of-plane motion, finite sampling rates, or methods like companding would all be interesting avenues of future work, as would extending these efforts to phantom and tissue data.

5 Conclusion

These 2D simulations have confirmed some statistical properties of accumulated displacement estimates, first demonstrated in simplified 1D simulations. The errors in an accumulation sequence have an important covariance structure that affects the quality of the final accumulated displacement estimate. Optimizing an accumulated estimate is therefore a different problem from optimizing a single-step displacement estimate.

Errors due to electronic noise are anti-correlated with one another, and therefore accumulate slowly, while errors caused by tissue deformation are positively correlated with one another and accumulate quickly. These relationships hold for both normal strain and axial shear strain, and for both axial and lateral displacement estimates. Together these effects make the accumulated estimate less sensitive to strain step size than might be expected.

These statistical properties are present and qualitatively similar for both normal strain and axial shear strain, although these two types of strain have different sensitivities to tracking kernel size. In either case, extending the kernel in the dimension that corresponded most to the deformation increased the error variance, while extending the kernel in the perpendicular dimension decreased it. In clinical situations, a mixture of these two types of deformation (as well as rotation and lateral shear) would be expected, and thus limit the size of the tracking kernel in both dimensions.

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Appendix: Single-step error variance as a function of strain

Error properties for single-step displacement estimation have been frequently addressed in the literature, including generic analyses in terms of signal correlation,^{13,17} but also specifically for both normal strain^{25,28,32} and shear strain.^{26,27} For completeness and to allow comparisons to previous work, Figures 8 and 9 show error variance for single-step displacement estimates. The broad patterns observed here are similar to previous reports. Variance increases nonlinearly with strain, and the rate of increase changes with kernel size. Including electronic noise adds a roughly constant error over all strain step sizes. Larger kernels are less sensitive to noise error. Compared to our previous 1D simulations,¹⁶ the use of a 2D kernel improved variance by approximately a factor of 10, depending on the precise kernels used.

Figure 1.

Diagram of a patch of simulated scatterers. The boundary lines signify, from the outside in: solid gray, the scattering region; solid black, the valid convolution boundary; dotted black, the search region; and solid black, the tracking kernel. The dimensions are those used for a $4\lambda \times 6\lambda$ tracking kernel, and the deformations match 20% total normal or shear strain. Gray dots mark scatterer locations. Note that the undeformed configurations of the normal strain and axial shear experiments would actually be slightly different, since their final deformations would require initial scatterer regions of different sizes.

Figure 2.

Accumulated displacement estimate error variance as a function of normal strain step size. Estimates are for 1D simulations, axial displacement; 2D simulations, axial displacement; and 2D simulations, lateral displacement. Results for different axial kernel sizes are displayed; all plots used a lateral kernel size of 12λ , chosen to make it easier to display a single variance scale for all noise levels.

Figure 3.

Displacement-step covariance matrices for normal strain. Each entry represents the covariance between displacement estimation errors obtained at different steps of an accumulation sequence. Matrices are shown for 1D, 2D axial, and 2D lateral displacement estimates. The 1D results used a kernel length of 12λ , and the 2D results used a kernel size of $8\lambda \times 12\lambda$ (length $\times$ width). The step size used was equal to 10 elementary steps, or about 1.2% strain. Kernel sizes and SNRs were chosen to equalize the dynamic ranges of the noise-free and the noisy results as much as possible.

Figure 4.

Accumulated displacement estimate error variance as a function of shear strain step size.

Under axial shear strain the lateral kernel size is the more important dimension, so this plot shows different lateral kernel sizes instead of axial sizes. All plots used an axial kernel size of 8λ , chosen to make it easier to display a single variance scale for all noise levels.

Figure 5.

Displacement-step covariance matrices for axial shear strain. A kernel size of $8\lambda \times 12\lambda$ was used, as well as a step size of 10 elementary steps, or a shear angle of about 0.6° . Kernel sizes and SNRs were chosen to be consistent with Figure 3.

Figure 6.

Axial displacement error variance for combinations of normal and axial shear strain. All plots are for a kernel size of $2\lambda \times 6\lambda$. Lines labeled #1 have a ratio of normal to axial shear strain of 2:1 (20% total normal strain and 10% total shear strain) and lines labeled #2 have a ratio of 1:1. Solid lines represent full simulations of the combined strain states; dashed lines represent sums of variances from the pure strain states.

Figure 7.

Single-step displacement estimate error variance for normal strain (4%) and axial shear strain (2% or 1.1°) as a function of kernel size, noted by the inset boxes (axial × lateral). Relatively large single-step strains were chosen to make the differences between kernel sizes more apparent. The colored divisions of the bars represent the amount of error attributable either to strain or to electronic noise. These amounts were determined from the simulation results for zero electronic noise and for 13 dB electronic SNR.

Figure 8.

Single-step displacement estimate error variance as a function of normal strain level. Results for different axial kernel sizes are displayed. In order to display a single variance scale for all noise levels, all plots used a large lateral kernel size of 12λ and some lines are excluded on the high-noise plots.

Figure 9.

Single-step displacement estimate error variance as a function of axial shear strain level.

Under axial shear strain the lateral kernel size is the more important dimension, so this plot shows different lateral kernel sizes instead of axial sizes. All plots used a large axial kernel size of 8λ to make it easier to display a single variance scale for all noise levels.

Table 1

Summary of simulation parameters used.

PSF Size	$2.1\lambda \times 5.0\lambda$ FWHM
Sampling rate	$100/\lambda$ axially, $25/\lambda$ laterally
Strains applied	0–20% normal strain in 180 increments 0–20% (0–11.3°) axial shear strain in 180 increments
SNRs	Zero noise, 13 dB, 7 dB
Number of echo field realizations	400
Axial kernel sizes	1λ , 2λ , 4λ , 8λ
Lateral kernel sizes	4λ , 6λ , 12λ

Table 2

Axial shear strain values, in terms of percentage and shear angle, that produce the same single-step error variance as 1% normal strain.

		Lateral size		
		4 λ	6 λ	12 λ
Axial size	1 λ	0.33% · 0.19°	0.24% · 0.14°	0.13% · 0.07°
	2 λ	0.48% · 0.28°	0.35% · 0.20°	0.19% · 0.11°
	4 λ	0.78% · 0.45°	0.59% · 0.34°	0.37% · 0.21°
	8 λ	1.54% · 0.88°	1.32% · 0.76°	0.70% · 0.40°