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Detection of point landmarks in multidimensional tensor data★

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Abstract

This paper describes a unified approach to the detection of point landmarks—whose neighborhoods convey discriminant information—including multidimensional scalar, vector, and higher-order tensor data. The method is based on the interpretation of generalized correlation matrices derived from the gradient of tensor functions, a probabilistic interpretation of point landmarks, and the application of tensor algebra. Results on both synthetic and real tensor data are presented.

Keywords

Tensor data; Gradient; Correlation; Point landmark; Corner

1. Introduction

Multichannel data demand more general signal processing schemes than those applied to scalar data. Although the most common arrangement of multichannel data is in the form of vector signals, there are more complex cases where signal models based on tensor fields are a natural choice. Tensor analysis is a generalization of the principles of vector analysis driven by physical processes that cannot be adequately described or explained by scalars or vectors alone. Tensors historically have been used in signal processing to represent image features and to control the behavior of algorithms [3,9,10]. Symbolic matrix notation (second-order tensors) as an invariant representation of, for example, linear operators, expectations of dyadic products (correlation matrices) and derivatives of vector functions (Jacobian matrices), also figures prominently in signal processing. However, the mechanism by which tensor data is generated from different physical processes, such as hydrogeon diffusion in magnetic resonance imaging (reported below), must be examined and formalized into an approach that transcends imaging modalities and various clinical settings.

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Moreover, the theoretical elements of all scalar, vector and tensor signal processing must be codified into a single system that is at once reliable and flexible, crossing multiple domains as necessary.

Detection of point landmarks in images has found major application in image processing and related fields because landmarks provide important information about object and scene structures. Several corner detection schemes for 2D and volume scalar images have been proposed for this purpose [1,2,5,8]. Point landmarks are frequently used for registration of multimodal or multispectral images (i.e., in medical image processing, where data from different modalities such as CT, MRI and ultrasound has to be fused for its simultaneous assessment [6]). This paper describes a unified approach to point landmark detection in multidimensional scalar, vector, and higher-order tensor data. In Section 2, we provide an alternative theoretical justification for gradient-based corner detection on scalar data from which the point landmark detectors in vector and tensor data are derived (Section 3). To clarify the approach, some illustrative results are provided, on both synthetic and real data (Section 4).

2. Point landmarks for scalar data

Consider a scalar random field $s(\mathbf{x})$ and the gradient vector random field $\mathbf{g}(\mathbf{x}) = \nabla s(\mathbf{x})$, which can be represented wrt a basis by the component form $g_k(\mathbf{x}) = s_{,k}$, where the *comma* convention has been used ($,k$ is an indexing of the partial derivatives: $s_{,k} = \partial s(\mathbf{x}) / \partial x_k$) and k can vary between 1 and the dimension of the field (2 or 3) [7]. The correlation matrix of $\mathbf{g}(\mathbf{x})$ is defined at each point by Eq. (1) and an estimate for it is given in Eq. (2), where $N(\mathbf{x})$ stands for the neighborhood of $\mathbf{x}$ and V for its number of samples:

$$\begin{aligned} \mathbf{R}_g(\mathbf{x}) &= \\ E \left\{ \mathbf{g}(\mathbf{x}) \mathbf{g}(\mathbf{x})^t \right\} &= E \left\{ g_k(\mathbf{x}) g_l(\mathbf{x}) \right\} \\ &= \\ E \left\{ s_{,k}(\mathbf{x}) s_{,l}(\mathbf{x}) \right\}, \end{aligned} \quad (1)$$

$$\begin{aligned} \hat{\mathbf{R}}_g(\mathbf{x}) &= \\ \frac{1}{V(N(\mathbf{x}))} \sum_{\mathbf{x}_i \in N(\mathbf{x})} g_k(\mathbf{x}_i) g_l(\mathbf{x}_i)^t &= \\ \frac{1}{V(N(\mathbf{x}))} \sum_{\mathbf{x}_i \in N(\mathbf{x})} s_{,k}(\mathbf{x}_i) s_{,l}(\mathbf{x}_i). \end{aligned} \quad (2)$$

An eigenanalysis of the correlation matrix provides information on the manner in which the gradient changes. In particular, small eigenvalues show a lack of variation of the gradient along the associated principal directions. On the contrary, similar eigenvalues of the correlation matrix show a local variation of the gradient field in all directions and indicate candidate locations for landmarks. The determinant of $\mathbf{R}_g$ is the product of all eigenvalues and consequently tends to be small when there is no variation along some direction. The trace of $\mathbf{R}_g$ is the mean square value of the gradient norm, and it should be large for landmarks. Notice that both the trace and the determinant are invariant wrt orthonormal basis changes; thus, any decision based on them does not require an eigendecomposition of

the matrix. In practice, the landmark points can be detected by selecting local optima in thresholded scalar measures based on ratios between the determinant and the trace of $\mathbf{R}_g$. Averaged outer products on gradient fields (Eq. (2)) have previously been used for point landmark detection [5]. However, the probabilistic interpretation presented here (Eq. (1)) has, to the best of our knowledge, not been reported elsewhere. Furthermore, it provides a theoretical background for the extension of the approach to the tensor case.

3. Point landmarks in tensor data

Tensors are mathematical entities that lie on vector spaces. Once a basis is set, tensors are expressed by means of their components that can be stored in multidimensional arrays $S_{i_1 \dots i_n}$. A tensor field is a function that assigns a tensor to each point of the domain. A tensor field can be associated with a multichannel signal, where each channel corresponds to a different component of the tensor field. The *tensor product* [7] is a new tensor formed by all possible multiplications of the components of two tensors: $P_{i_1 \dots i_p} \otimes Q_{j_1 \dots j_q} = P_{i_1 \dots i_p} Q_{j_1 \dots j_q}$. Differential calculus can be extended to tensor fields [7]; in particular, gradients are defined as taking the gradient of each component and increasing by one the number of components of the tensor field. A case in point is a gradient of vectors that adheres to a Jacobian matrix (each row is the gradient of a component of the original column vector). The gradient of the tensor field S is defined in Eq. (3) using the *comma* convention (k is an indexing of the partial derivatives). We define the *correlation tensor* of the gradient of a tensor random field in Eq. (4):

$$\nabla \otimes S(\mathbf{x}) = S_{i_1 \dots i_n, k}(\mathbf{x}), \quad (3)$$

$$\begin{aligned} \mathbf{R}_{\nabla S}(\mathbf{x}) &= \\ E \{ \nabla \otimes S(\mathbf{x}) \otimes \nabla \otimes S(\mathbf{x}) \} &= \\ E \{ S_{i_1 \dots i_n, k}(\mathbf{x}) S_{j_1 \dots j_n, l}(\mathbf{x}) \}. \end{aligned} \quad (4)$$

Contraction [7] is a common operation performed on a single tensor in which two indices are set as equal and summed up. We define the *generalized correlation matrix* of the gradient of a tensor random field as the contraction of all indices in its correlation tensor (Eq. (4)) but those corresponding to the partial derivatives (comma indices):

$$\begin{aligned} \mathbf{R}_{\nabla S}(x) &= \\ E \left\{ \sum_{i_1 \dots i_n} S_{i_1 \dots i_n, k}(\mathbf{x}) S_{i_1 \dots i_n, l}(\mathbf{x}) \right\} &= \\ \sum_{i_1 \dots i_n} \mathbf{R}_g(\mathbf{x}; i_1 \dots i_n), \end{aligned} \quad (5)$$

where we use the correlation matrix $\mathbf{R}_g(\mathbf{x}; i_1 \dots i_n) = E\{S_{i_1 \dots i_n, k}(\mathbf{x}) S_{i_1 \dots i_n, l}(\mathbf{x})\}$ of the gradient field of the scalar component $i_1 \dots i_n$ of $S(\mathbf{x})$. Consequently, the generalized correlation matrix of the gradient of a tensor field is the sum of the correlation matrices of the gradient of each component of the tensor field. The properties of the correlation matrix (Eq. (1)) in Section 2 apply to the generalized correlation matrix (Eq. (5)). Note that

landmarks in tensor data are not only due to the contribution of each component, but to the interaction between them. The generalized correlation matrix (Eq. (5)) accounts for this interaction by calculating its trace as the sum of the mean square values of the gradient norms of each tensor component. Moreover, its principal directions are obtained from contributions from each tensor component. The addition makes the ellipsoids associated with the generalized correlation matrices (Eq. (5)) rounder than the ones associated with the correlation matrices of each component, which will tend to be more elongated, unless the correlation matrices of all tensor components have the same eigendecomposition. The following equation is an alternative expression for the generalized correlation matrix (Eq. (5)) that allows a more intuitive assessment of the contribution of each component based on the eigenstructure of the matrices $\mathbf{R}_g$ (λ_d and $\mathbf{e}_d$ represent the d eigenvalues and eigenvectors):

$$\mathbf{R}_{\nabla S}(\mathbf{x}) = \sum_{i_1 \dots i_n} \sum_d \lambda_d(\mathbf{x}; i_1 \dots i_n) \mathbf{e}_d(\mathbf{x}; i_1 \dots i_n) \mathbf{e}_d^t(\mathbf{x}; i_1 \dots i_n). \quad (6)$$

4. Results

Fig. 1a shows the horizontal (top) and vertical (bottom) components of a smoothed noisy synthetic vector field (a first-order tensor field) and Fig. 1b depicts a vector representation of this field, which shows edges along the horizontal and vertical axes. Any attempt to detect the crossing point (*corner*) between both edges must consider the two components of the vector field since neither can independently supply the discriminant information. Hence, the estimated generalized correlation matrices of the vector field gradient (Eq. (5)) leads to the geometric representation shown in the Fig. 1c, which is based on their associated quadratic forms. Notice how the ellipsoids tend to be rounder around the central point, indicating the presence of the corner. Fig. 1d shows a scalar measure of the ellipsoids roundness, here defined by $G(\mathbf{x}) = \det(\mathbf{R}_{\nabla S}(\mathbf{x})) = \text{tr}(\mathbf{R}_{\nabla S}(\mathbf{x}))^{(N-1)}$. Corners are detected as local maxima of this function above a certain threshold (to avoid false maxima due to noise). Notice how the corner is clearly indicated by the brightest point in Fig. 1d.

We next applied the method to a Diffusion Tensor-MRI dataset of the brain. DT-MRI overcomes the inability of conventional MRI to show the structure of the white matter of the brain and its complex arrangement of fiber tracts [4] by providing a second-order symmetric tensor field codification of the water diffusion in each tissue. The measured tensors exhibit anisotropic diffusion along the direction of the tracts, while in other areas the diffusion is essentially isotropic. The particular structure of the sensed tensors allows their identification with symmetric quadratic forms, with a geometric interpretation based on local ellipsoids that are oriented along the fiber tracts. These tend to be rounder as the diffusion becomes more isotropic. Fig. 2a shows part of a sagittal slice obtained with conventional T2-weighted MRI (scalar data showing anatomic information) of the brain with an overlay of all the point landmarks that have been detected from a corresponding DT-MRI slice of the same patient. Figs. 2b–g show the six independent components of a portion of the DT-MRI sagittal slice (top: diagonal components, bottom: off-diagonal), corresponding to the square highlighted in Fig. 2a, with an overlay of the detected landmarks. Note that the diagonal components

provide stronger and more structured signals than the off-diagonal ones and the precision with which the point landmarks find the thin structures in these images, again owing to the cooperation of all the components. In order to obtain the landmarks, we have normalized the tensor field components to fit into the interval $[-1; 1]$ (weaker components do not reach the extrema values) and we have looked for local maxima over 5×5 neighborhoods of the goal function $G(\mathbf{x}) = \det(\mathbf{R}_{\nabla\mathbf{y}}(\mathbf{x})) = \text{tr}(\mathbf{R}_{\nabla\mathbf{y}}(\mathbf{x}))^{N-1}$ where N is the dimension of the domain, 2 or 3. The estimated gradients and generalized correlation matrices have been calculated using 3×3 neighborhoods. Only points whose values are above 0.005 in the goal function are considered to avoid detections due to noise gradients. Finally, some of the detections are discarded if they are below a threshold in a function $T(\mathbf{x}) = \det(\mathbf{R}_{\nabla\mathbf{y}}(\mathbf{x})) / \text{tr}((1/N)\mathbf{R}_{\nabla\mathbf{y}}(\mathbf{x}))^N$ whose range goes from 0 to 1, and where the greater the values the rounder the ellipsoids associated with the generalized correlation matrices (1 means a perfect sphere). This second threshold (set to 0.50 in this experiment) guarantees that the ellipsoids are rounded enough to avoid straight edges. The difficulty in presenting illustrative results from volume data using 2D figures has necessitated using a single DT-MRI slice in this experiment (the tensors here are 3D). Nevertheless, the method is N -dimensional and can be directly applied to volumes of data using the same parameters by adding one more dimension in the definition of the neighborhoods.

5. Conclusions

We have presented a new method for point landmark detection on general scalar, vector and tensor multidimensional data. We have introduced a tensor formulation by which certain scalar corner detection schemes can be extended to generic tensor fields. Moreover, we have provided an alternative theoretical justification for this family of algorithms based on a generalized correlation matrix eigenanalysis and tensor algebra concepts that, to the best of our knowledge, has not been previously reported. This approach is valid for any modality of imaging sensors (i.e., multi-spectral images and medical data). In particular, we are applying it to DT-MRI medical data registration [6] at Brigham & Women's Hospital and we are currently working on practical implementations of the method in routine clinical use.

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Fig. 1.
(a) Horizontal (top) and vertical (bottom) components, (b) Vector field, (c) correlation matrix, (d) roundness function.

Fig. 2.

(a) Sagittal T2W MRI of the brain with point landmarks overlaid. Sagittal DT-MRI components: (b) (1; 1), (c) (2; 2), (d) (3; 3), (e) (1; 2), (f) (1; 3), (g) (2; 3).