Published in final edited form as:

Teach Child Math. 2015 February ; 21(6): 354–361.

The Story of Kyle

Nancy I. Dyson, Nancy C. Jordan, and Brenna L. Hassinger-Das

Kyle, a kindergartner from a low-income family, is shown a set of three black dots on a white mat. His teacher then hides the dots with a small box lid and lays down an additional set of two dots. She pushes the two dots under the cover, one at a time. Kyle must now choose the number of dots “hiding” under the box from a set of four different linear dot pictures. He points to the air and counts the dots that he envisions in his mind: “One, two, three [*pausing*] four, five,” and then he points to the picture with five dots and says, “Five.”

Kyle is not asked to state the answer, yet he gives it anyway.

The task that Kyle completed is a “non-verbal” calculation activity. It examines his ability to perform addition and subtraction calculations without the distraction of the words in a story problem or the symbols of a number combination.

Kyle's response shows that he is able to not only hold the original picture of the dots and the addition of two more dots in his mind but also count and connect the final quantity with the number 5 (Bisanz et al. 2005). What happens next, though, is surprising. The teacher reads the following story problem to Kyle:

José has three cookies. Sally gives him two more cookies. How many cookies does José have now?

Kyle is encouraged to use his fingers, paper and pencil, a number list, or any other way he chooses to solve the problem. Kyle, without showing any signs of counting, quickly gives “nine” as his response.

When next asked, “How much is three and two?” Kyle responds, “One.” He does not seem to perceive the relationship between the nonverbal calculation problem and the more conventional story problems and number combinations.

By the time children enter kindergarten, most have developed number knowledge through their informal experiences in the world (Levine et al. 2010). Yet to navigate formal mathematics, children must be able to link their intuitive understanding of quantities to the symbolic representations of number (e.g., arabic numerals) and number operations (e.g., $3 + 2 = 5$) (Gersten, Jordan, and Flojo 2005). Children like Kyle are expected to develop these skills through classroom instruction in kindergarten. Yet some children do not easily make these connections. This is particularly true of children from low-income families, who

typically perform far below their middle-income peers on story problems and number combinations at the start of kindergarten, but less so on nonverbal problems (Jordan et al. 2006; Starkey, Klein, and Wakely 2004; Jordan, Huttenlocher, and Levine 1992).

Although the need for intervention is greatest among children from low-income families, early math is important for children from all socioeconomic backgrounds. Poor math achievement in kindergarten and elementary school has far-reaching consequences (Duncan et al. 2007; Jordan et al. 2009), and math proficiency is a gateway to highly sought-after careers in science, technology, engineering, and mathematics (STEM) (National Mathematics Advisory Panel [NMAP] 2008). Recognizing the needs of children like Kyle, we developed and tested a number sense intervention program (NSI) (Dyson, Jordan, and Glutting 2013; Jordan et al. 2012; Jordan and Dyson 2013) that is appropriate for response to intervention (RtI) for kindergartners designated at risk of failure in mathematics.

Making mathematical connections explicit with the NSI

The NSI program is based on the premise that three whole-number concepts are important for learning mathematics:

1. Number
2. Number relations
3. Number operations

Weaknesses in any of these number concepts can lead to future mathematics difficulties. Fortunately, our research has shown that these skills can be developed through focused instruction. NSI uses consistent representations of number and explicit instruction in the development of counting strategies to address these foundational concepts (see **figs. 1** and **2**). Our research has shown that targeting the skills that we describe in this article helps improve children's number sense, especially in the area of story problems and number combinations (Dyson et al. 2013; Jordan et al. 2012).

NSI consists of twenty-four lessons, each designed to last about thirty minutes. Lessons are made up of fast-paced activities that are often presented in a game format. The first lesson begins with the numbers 0, 1, and 2. Unlike typical kindergarten curricula, which introduce the numbers 1–10 before introducing number relations and operations, NSI introduces children to a wide range of number ideas using just these three numbers. For example, children practice subitizing (quickly recognizing the quantity without counting) the quantities 0, 1, and 2, and making the quantities 1 and 2 quickly on their fingers. Children also practice the recognition of the numerals 0, 1, and 2 through a number-recognition game.

Using a cardinality chart, children see that 2 is one more than 1 (see **fig. 1a**). To emphasize this, the instructor takes one block, says, “This is one block,” puts the block on the cardinality chart above the number 1, points to the numeral 1, and says again, “One block.” The instructor takes another block, adds one more while saying, “One and one more is two.” The two blocks are placed on the cardinality chart above the numeral 2, linking quantity to numeral. Gesturing to the chart (see **fig. 1b**), the instructor says, “See, it's like steps, one and one more is two.”

For each lesson 1–9, this script is repeated, adding one new number to each lesson, until the number 10 is reached and the cardinality chart is full. For example, during lesson 3 the instructor says, “See, it’s like steps; one and one more is two, two and one more is three, three and one more is four” (see **fig. 1b**). This repetition helps to firmly establish the plus-one principle (that each number is one more than the number before it and one less than the number after it, no matter how big the number). Beginning with lesson 10, children use the number list instead of the cardinality chart (see **fig. 2**). The number list shows accumulation of quantity in a different way. Children do plus-one activities on the number list, but this time, they use black counters. Adding one counter at a time to the dots on the list, teachers reinforce the plus-one principle, “One and one more is two, [and so on].” This time, rather than pointing to the graduating sticks of blocks, the instructor circles the counters with his or her finger. Understanding of the plus-one principle allows simple addition and subtraction problems to be solved by moving up and down the number list (Baroody, Eiland, and Thompson 2009; Baroody, Ginsburg, and Waxman 1983). It also helps children compare numerals without seeing actual physical quantities. Children learn that as they move farther up the number list, the numbers represent larger quantities.

The final activity in lesson 1 introduces children to part-part-whole relationships for the number 2, the two family of number sentences ($1 + 1 = 2$, $2 - 1 = 1$, $2 - 2 = 0$), and related story problems. These activities are repeated in subsequent lessons with the number of the day. Connecting part-part-whole understandings to story problems and number combinations is an important aspect of NSI, which provides scaffolding for part-part-whole understanding through the introduction of “partners” (Fuson, Grandau, and Sugiyama 2001, p. 522) and partner cards. Children are shown that any whole number (with the exception of 1) can be broken into parts consisting of other whole numbers. Understanding number as a set made up of smaller sets lays the foundation for future mental problem solving and allows children to be flexible when using decomposition (i.e., 4 is made up of 3 and 1, or 2 and 2) and recomposition (i.e., $3 + 1 = 4$) strategies for addition and subtraction problems (Carpenter, Fennema, and Franke 1996). For numbers 2–5, children find partners for each number by separating a linear dot representation of the number with a pencil. Each way of separating the dots is visually presented on a partner card, which the children match to a number sentence card. Subtraction is demonstrated by covering one set of dots on the partner card.

Focusing on the number of the day and its partners, story problems are demonstrated using a farm scene background and magnetic animals. After being told the story, children are asked to find the matching partner card and combination card. In this situation, the instructor tells the children a story about pigs on the farm, such as the following:

Look! Three pigs are in the pen, and one pig is in the yard. How many pigs are on the farm altogether? Which of our partner cards shows this story? Which of our number sentence cards shows this story?

Developing flexible counting strategies

Explicit instruction in strategy use helps children develop flexible methods for solving problems (Siegler and Svetina 2006). Children who use effective calculation strategies in

kindergarten are more likely to develop arithmetic fluency earlier than kindergartners who do not use such strategies (Jordan et al. 2008). NSI uses targeted strategy instruction to help children be more accurate and efficient in their problem solving. In lessons 1–5, children are encouraged to use their subitizing skills to quickly recognize small quantities related to the numbers in the lesson. The teacher flashes cards with 1–3 dots on them in various configurations to help students rely on this ability rather than counting small quantities. As children learn combinations for numbers 4 and 5, they are flashed cards with four or five dots and are encouraged to “see” the number of dots by recognizing the smaller sets of two and three dots, which make up the larger set. Ten-frame flashcards are used to allow children to subitize quantities 5–10.

Children are also helped to use their fingers effectively. Linked to the number of the day, they learn to make the numbers 1–5 automatically on one hand. Next, they make numbers 6–10 on two hands, learning that they do not need to count the numbers on one hand because there are always five. Children quickly begin to recognize numbers 6–10 as 5 and so much more (Fuson et al. 2001). For small sums, they make one addend on each hand and are encouraged to subitize rather than count whenever possible. For sums greater than five, children are instructed to *count on* to the greater addend using the “in-hand” method (Fuchs et al. 2012). Children begin by actually holding the number of cubes representing the larger addend in one hand and making the other addend on the fingers of the other hand. After successfully counting on while holding cubes, children begin holding imaginary cubes. “Holding” the one addend rather than making it on fingers helps in two ways:

1. Children are prevented from unnecessarily counting the first addend.
2. Children can “hold” an addend greater than five.

Children also model story problems by drawing sticks or circles to represent the objects in the story. As the numbers in the story became larger, children are asked to write the numerals in the story rather than drawing pictures. Children then solve the problem using their fingers as mentioned above. Counting principles (e.g., +1 is the next number on the number list and –1 is the number before) also help children make quick calculations without the need for counting.

Figure 3 shows an excerpt from a lesson (Jordan and Dyson 2013) that focuses on the number 4. Note that the partners 1 and 3, and 3 and 1 have only one partner card. When children are asked to choose the correct partner card for $1 + 3 = 4$ or $3 + 1 = 4$, the partner card may or may not be in the correct orientation. When necessary, children instinctively rotate the card 180 degrees, allowing them to discover the commutative property of addition.

Kyle's story continues

After participating in the NSI program, Kyle successfully drew circles to represent cookies and pennies and drew tallies for story problems about crayons and pencils. For number combinations, Kyle was able to use his fingers, except for $7 + 1$, $10 + 1$, and $9 - 1$, where he applied his plus-one/minus-one strategies to give quick, correct answers. Kyle learned to write combinations for the story problems, followed by a quick, correct answer. He showed mastery of combinations involving totals of five or less. Kyle developed along the desired

trajectory of mathematics performance, moving from counting physical and mental models to fluency-based strategies. The interventions put him in a strong position to be successful in first-grade mathematics.

Moving forward

Children who do not link their developing knowledge of quantities to symbolic representations of number are at risk for math difficulties (Jordan et al. 2009). By helping them make explicit connections among different types of number problems, teachers can move children from their nonverbal understanding of number and operations to mathematical thinking that is more complex. Effective strategy use gives children confidence to attempt novel problems they have not yet mastered.

Catching students' weaknesses in number early and applying evidence-based interventions in number sense are key factors for preventing more serious difficulties down the road. Although the NSI program is designed for small groups of kindergartners, the activities can easily be adapted for one-on-one tutoring even for older children with significant mathematics delays or diagnosed disabilities.

Biography

Nancy I. Dyson, ndyson@udel.edu, is the project coordinator for the kindergarten Number Sense Intervention (NSI) project at the University of Delaware. Her interest is in designing interventions for students at risk for mathematics difficulties.

Nancy C. Jordan, njordan@udel.edu, is a professor of education at the University of Delaware. She does research in children's mathematics development and learning difficulties.

Brenna L. Hassinger-Das, hassinger.das@temple.edu, is a postdoctoral researcher at the Temple University infant and child lab in Ambler, Pennsylvania. She is interested in the intersections of mathematics and language learning in early childhood. The authors' research in kindergarten NSI was supported by Grant No. HD059170 from the Eunice Kennedy Shriver National Institute of Child Health and Human Development.

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Identifying weaknesses in number and applying evidence-based interventions early are key to preventing future difficulties.

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(a) Students can see that 2 is one more than 1.

(b) To link quantity to numeral, the instructor puts a block on the chart above the number 1, points to the number, and says, "One block. See, it's like steps; one and one more is two, two and one more is three, three and one more is four."

FIGURE 1.

For each lesson 1–9, the teacher adds one new number until the cardinality chart is full.

Repetition helps students firmly grasp the plus-one principle.

(a) Students can see that 2 is one more than 1.

(b) To link quantity to numeral, the instructor puts a block on the chart above the number 1, points to the number, and says, "One block. See, it's like steps; one and one more is two, two and one more is three, three and one more is four."

FIGURE 2.

Beginning with lesson 10, instead of the cardinality chart, students use counters on a number list, which shows a different way to accumulate quantity.

1. Finding partners
 - [Put down the dot chart for the number 4.] Say, "Today, we are going to find partners for four. How many dots are here?" [Wait for the response, "Four."]
 - Say, "That's right, there are four dots. We are going to find the partners hiding inside four. Put the pencil down so it separates the dots into two groups."
 - [After the child places the pencil] say, "What partners do we see? How many are on this [gesture to the right] side? How many are on this [gesture to the left] side?"
 - Say, "That's right! See _____ and _____ hiding inside four?" [Insert numbers reflecting the partners created by the pencil placement. Possible partners are 1 and 3, 2 and 2, 3 and 1.]
 - [Put down the partner cards.] Say, "Which of these cards matches these partners of four?" [If a child picks the wrong card, match it to the dots and circle each of the two parts.] Ask, "Does that match?" [Make sure the orientation is correct.]
 - [After the correct partner card is chosen, put down the number sentence cards.] Say, "Which of these cards matches these partners of four?"
2. Making stories
 - Say, "Now let's tell a story for these partners of four. Today we are going to the farm!" [Put the farm background with the pond on the whiteboard.]
 - [Model the story as you tell it.] Say, "There were some ducks at the farm. Two were in the pond, and two were in the grass. How many ducks were there altogether?"
 - Say, "See how this partner card shows this story?" [Circle the ducks and the dots on the partner card as you repeat the story.] "Two were in the pond, and two were in the grass. How many ducks were there altogether?"
 - Say, "See how this plus cards show this story?" [Put the number sentence card above the partner card to show that] "two plus two equals four." [Circle each addend and the total as you say them.]
 - Say, "Let's show that on our fingers. [Model with your fingers as you instruct them to use their fingers.] Two ducks were in the pond. Show me two ducks on one hand. [Make sure everyone holds up two fingers.] Two ducks were on the grass. Show me two ducks on your other hand. [Make sure everyone holds up two fingers on the other hand while keeping the first hand up.] How many ducks were there altogether? How many fingers? Four." [Encourage subitizing not counting.]
 - Say, "That's right. There were four ducks altogether."

FIGURE 3.

For this lesson, use the number list, counters, a pencil, partner cards, and number sentence cards.