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Managing dynamic epidemiological risks through trade

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Abstract

There is growing concern that trade, by connecting geographically isolated regions, unintentionally facilitates the spread of invasive pathogens and pests – forms of biological pollution that pose significant risks to ecosystem and human health. We use a bioeconomic framework to examine whether trade always increases private risks, focusing specifically on pathogen risks from live animal trade. When the pathogens have already established and traders bear some private risk, we find two results that run counter to the conventional wisdom on trade. First, uncertainty about the disease status of individual animals held in inventory may *increase* the incentives to trade relative to the disease-free case. Second, trade may facilitate *reduced* long-run disease prevalence among buyers. These results arise because disease risks are endogenous due to dynamic feedback processes involving valuable inventories, and markets facilitate the management of private risks that producers face with or without trade.

Keywords

Bioeconomics; Arbitrage; Infectious disease; Joint determination; Thresholds

1. Introduction

Trade creates value as a coordinating mechanism to exploit comparative advantages and to redistribute private risks. Yet this mechanism may also connect geographically isolated regions, unintentionally facilitating the spread of external risks. Prominent examples are the introduction and spread of pathogens and pests – forms of biological pollution that pose significant risks to ecosystem and human health and valuable economic sectors like agriculture (Perrings et al., 2010; Finnoff et al., 2010; MEA, 2005; The Economist, 2005; McAusland and Costello, 2004).

Concerns over biological risks have led to many costly measures to prevent the spread of pathogens and pests among animals and plants, some of which restrict or ban certain trades. For instance, the Sanitary and Phytosanitary Measures Agreement of the World Trade

Organization allows member nations to ban products from infected/invaded areas to prevent new introductions (WTO, 2014). International and local trade bans have been used to protect against harmful livestock diseases such as foot-and-mouth disease (FMD), bovine tuberculosis (bTB), and bovine spongiform encephalopathy (BSE) (USDA-APHIS, 2013), and they have been applied to trade in plants, exotic pets, or wildlife that may carry invasive pathogens or pests (e.g., USTR, 2014).

Local trade restrictions may also be implemented to reduce the spread of pathogen and pests within endemic (already-infected) areas. For instance, livestock movement (trade) controls have been implemented in response to the FMD outbreaks in the UK and other parts of the EU (Schley et al., 2009). Movement controls have also been imposed within parts of the UK, New Zealand, US, and elsewhere to prevent the spread of bTB (DEFRA, 2014; Barlow et al., 1998; MDARD, 2011; OIE, 2013), and in Canada to prevent spread of BSE (LeRoy et al., 2005).

While there is a growing chorus for more trade sanctions to protect against biological risks (Sanderson, 2012; Liu et al., 2013), there are also calls for relaxing costly trade restrictions while adopting scientifically sound risk management approaches (ECCFMD, 2012; DEFRA, 2014; MDARD, 2011). One such approach, known as risk-based trading, would allow livestock trades within endemic areas after importers have been made aware of the exporting herd's health history (DEFRA, 2014). Similarly, the EU's Progressive Control Pathway (ECCFMD, 2012) suggests various stages of control, including targeted movement controls that depend on risk differences between and within infected areas. The potential economic and epidemiological performance of such programs is unclear, however, because understanding of the relation between trade, epidemiological risks, and private incentives for risk management is limited.

This study examines how endemic biological risks within a sector like agriculture affect the private incentives for trade, and in turn how trade affects the sector's ability to manage biological risks. The problem of trade introducing new biological risks into a region emerges as a limiting case of our model. Livestock disease is used as a motivating example because of the important role of trade in this sector and because livestock diseases pose major risks to agriculture as well as to human and ecosystem health. Almost two-thirds of human diseases are zoonotic (transmitted from animals to people), with livestock often serving as a conduit to humans (Cleaveland et al., 2001), particularly for emerging infectious diseases (The Economist, 2005). Livestock diseases also pose major risks to wildlife, with infectious diseases being a significant driver of biodiversity loss (MEA, 2005). Our bioeconomic model is easily adapted to examine pathogen or pest risks in the trade of other live inventories, such as pets or plants.

We find the incentives to trade may be increased relative to the disease-free case when both buyers and sellers face uncertainty about the health status of individual livestock being traded.¹ This result contrasts with the standard view that uncertain product quality (defined here as animal health status) reduces private incentives for trade, even for symmetric uncertainty among buyers and sellers (Akerlof, 1970). We find greater trade incentives may arise because disease risks are endogenous due to dynamic feedback processes involving

valuable inventories, and markets facilitate producers' management of the inventory risks they face with or without trade. Specifically, livestock trade enables producers to manage the two components of infection risks, herd prevalence and economic costs, by an arbitrage process between buyers and sellers. To the best of our knowledge, the result that infection-driven inventory risks may increase trade incentives is new to the risk and trade literature.²

We also find trade may reduce buyers' long-run herd prevalence compared to autarky, reducing any associated externalities to other producers, ecosystems or human health. This result contrasts with the prevailing notion that trade of potentially infected animals always increases ecological risks, but supports the idea that institutional responses, such as trade bans that fail to account for economic–ecological feedbacks, may increase ecological risks (Bulte et al., 2003; Horan et al., 2011; Finnoff et al., 2010).³ We show trade may increase sellers' herd prevalence, contrary to the notion that trade primarily puts buyers at risk.

Finally, our analysis contributes to the epidemiology and health economics literatures by showing how trade creates an epi-economic geography involving endogenously determined epidemiological and economic relations, extending prior work on endogenous economic–epidemiological systems within a single region (Aadland et al., 2013; Fenichel et al., 2011, 2010).

2. Production and epidemiological model

Our analytic model integrates disease dynamics and an economic model of livestock production and trade, using (beef) cattle as a motivating example.⁴ As a first attempt at such integration, we adopt a parsimonious model characterized by two key industry features: trade is driven by cattle having different production requirements as they mature, and cattle are assets whose value is diminished by infection. Also, we maintain our focus on how infection risks influence and are influenced by trade by setting up the analysis such that producers primarily have incentives to manage infection risks by choosing more or less trade. An alternative specification analysis in Section 3.2.2 examines how the results are affected by additional risk-management options.

¹Asymmetric information could be a problem if traders were willing to pay for animal testing. A small literature (Sheriff and Osgood, 2010; Wang and Hennessy, 2014) examines livestock disease problems where sellers have private information about animal health status. They focus on behavioral dynamics arising from multi-period interactions, and how markets may evolve to induce information disclosure. However, they do not model infection dynamics. Our analysis complements this work. We do not examine asymmetric information, and so we can only analyze pooling equilibria in which there is a single price for animals, regardless of infection status. But this approach allows us to more clearly examine how infection dynamics involving animal inventories affect trade incentives and associated behavioral and epidemiological outcomes over time, not analyzed in prior work.

²More generally, examples of possible non-ecological areas where the theory could be relevant include file sharing where computer viruses pose a risk, and trading players across sports teams that have been infected by or are susceptible to behavioral and morale problems.

³Trade is also an important adaptation mechanism for biological invasions (Finnoff et al., 2005; Perrings, 2005).

⁴A small literature examines livestock trade and disease risk management but does not consider dynamic economic–epidemiological feedbacks or foresighted risk management (Hennessy et al., 2005; Rich and Winter-Nelson, 2007). In particular, Hennessy et al.'s (2005) static model incorporates neither epidemiological dynamics nor differences in infection levels across trading partners. Rather, they assume a single infection index that is an exogenous, increasing function of aggregate production and trade volume. It is common to assume trade exogenously increases infection (Momota et al., 2005), but empirical findings suggest there are reciprocal relationships. For example, Perrings et al. (2010) find trade can increase spread but it can also increase incentives to mitigate spread via biosecurity.

Consider two cattle-producing regions, indexed by the subscript $i \in \{B, F\}$ (subscripts never indicate partial derivatives). We model the decentralized outcomes as if, within each region, a single, aggregate decision-maker having rational expectations operates in competitive markets and makes choices taking prices and external disease impacts as given.⁵ Region B producers (breeders) have a comparative advantage in breeding and producing young livestock. Region F producers (finishers) have a comparative advantage in finishing, i.e., growing small livestock to marketable size. This setup reflects systems where producers may gain from specializing, with breeders producing animals that are sold to finishers. Producers in each region may engage in one or both activities, with autarky arising as a private response to a disease or as a mandate under a trade ban (for a similar treatment, see Hennessy et al., 2005). The regions are assumed geographically distinct, reflecting the comparative advantages or agglomeration economies often driven by access to feed and pasture, which vary geographically.

2.1. Population dynamics

Denote the number of cattle in region i as N_i . Some cattle in each region are infected with a chronic disease that is not vaccine preventable, so that N_i can be partitioned into healthy but susceptible cattle, S_i , and infected cattle, I_i , where $N_i = S_i + I_i$.⁶ The proportion of infected individuals of population i , or disease prevalence, is $\theta_i = I_i/N_i$.

Consider how N_i changes over time. The newly recruited stock is rh_iN_i , where r is a fertility rate and h_i is the fraction of cattle that are bred. Breeders sell (trade) $s_{BF}rh_BN_B$ of these animals to finishers, where s_{BF} is the fraction traded. The rest are added to the breeding stock, N_B . For simplicity, we adopt q_B as the choice variable for supply, with $q_B \leq q_B^{max} = rh_BN_B$. Proposition 1 in Section 3 indicates $h_B = 1$ when $q_B > 0$, which means the relevant constraint is

$$q_B \leq q_B^{max} = rN_B. \quad (1)$$

Finishers separately choose demand, q_F . There is no constraint on this choice. However, market clearing yields $q_B = q_F = q$, so that the supply constraint is an upper bound on equilibrium trade.

Breeders and finishers may each finish cattle and sell them to slaughterhouses. Only cattle not currently being bred, $(1 - h_i)N_i$, can be sold. Sales of this sub-population occur at the rate s_{iy} so that total sales are $s_{iy}(1 - h_i)N_i$. We simplify matters by re-writing slaughterhouse sales as y_iN_i , where $y_i = s_{iy}(1 - h_i)$ is our choice variable that is restricted by h_i

⁵With competitive markets and identical producers within a region, there is no loss from aggregating producers. Lucas (1988) develops a decentralized, rational expectations model of economic growth along these lines when the external impacts involve human capital spillovers. Gersovitz and Hammer (2004) also model an aggregate decision maker with rational expectations, although the externalities in their model are due to an infectious human disease. The assumption of identical producers is strong, but common in the economic and epidemiological literatures.

⁶Even when available, vaccines may not be given to livestock because vaccinated animals may test positive for disease, either lowering the value of the animals in trade or prompting a trade ban (USDA-APHIS, 2002). For instance, DEFRA (2014) estimates the EU would not allow trade of bTB-vaccinated cattle within 10 years of the start of successful trials.

$$y_i \leq y_i^{max} = (1 - h_i). \quad (2)$$

The evolution of N_i is

$$\dot{N}_i = r h_i N_i - \eta_i q_i - y_i N_i - \alpha \theta_i N_i. \quad (3)$$

The term η_i is a dummy variable ($\eta_B = 1$ and $\eta_F = -1$) that enables us to economize on the notation by writing one general expression for breeders and finishers; without this variable, separate economic and dynamical relations would need to be written for breeders and finishers. Parameter α is the disease mortality rate. We ignore natural mortality, for simplicity, because adding such a term does not qualitatively affect the analysis.

2.2. Infection dynamics

Within-region horizontal disease transmission takes the common density-dependent form $\beta_i I_i S_i$, where $\beta_i I_i$ is a susceptible animal's expected rate of infection (or the force of infection) and β_i is a transmission parameter (McCallum et al., 2001). β_i reflects the animal contact rate (including indirect contacts, e.g., from contaminated people and equipment moving across farms, modeled here to vary with animal densities) and the probability an infectious contact causes infection. β_i also implicitly accounts for region-specific environmental factors, including the (fixed) land area used in production (Begon et al., 2002). Assuming finishers operate at greater animal densities than breeders (i.e., on a smaller land area), *ceteris paribus*, then $\beta_F > \beta_B$.⁷

Offspring are infected at the rate of the parent population's disease prevalence, equivalent to vertical disease transmission (i.e., transmission *in utero* or from parent to offspring prior to weaning). For instance, milk is an important source of transmission for many diseases, including FMD, bTB, and brucellosis (Herenda et al., 1994). While our assumed vertical transmission rate of unity is large, we make this assumption because it implies changes in herd prevalence will not depend directly on breeding choices (see Eq. (4) below). This allows us to focus on the role of trade in managing infection risks. The assumption has an added benefit of greatly simplifying the analysis. We explore the implications of lower vertical transmission rates in Section 3.2.2. Apart from Section 3.2.2, the term transmission subsequently refers to horizontal transmission.

Finally, cattle supply, q , carries the breeders' prevalence rate, θ_B . An implied assumption is that supply is non-selective with respect to disease status. That is, each animal's health status is unknown when trade occurs, with no efforts being made by buyers or sellers to ascertain *individual* animal health status. This assumption prevents other non-trade controls from affecting prevalence dynamics, which enhances our focus on how trade affects infection risks. Testing sometimes occurs in practice, although it often has to be mandated due to costs and inaccuracies. For instance, current bTB tests only correctly predict infection

⁷For instance, let β denote the rate of transmissibility on a unit of land area, and assume this is the same in each region. Density-dependent transmission is then given by $\beta(S_i/A_i)(I_i/A_i)$, where A_i is the land area (and hence S_i/A_i and I_i/A_i are animal densities). Assuming A_i is fixed, we can rewrite transmission as $\beta_i S_i I_i$, where $\beta_i = \beta/(A_i A_i)$.

70–90% of the time (Brooks-Pollock et al., 2014). We examine the potential impact of testing in Section 4. A second assumption, made for simplicity, is that there are no additional infections during transport.

We work with prevalence, θ , rather than infected animals, I , as our state variable as this simplifies the mathematical modeling. Our assumptions about disease transmission imply region i 's prevalence dynamics are (see Appendix A for the derivation)

$$\dot{\theta}_i = \theta_i(1 - \theta_i)[\beta_i N_i - \alpha] + q_i(\theta_B - \theta_i)/N_i. \quad (4)$$

Note that the final term vanishes for breeders, as their exports do not affect $\dot{\theta}_B$.

Under autarky ($q_i = 0$), $\dot{\theta}_i$ is independent of the state variables for region $j \neq i$, and the sign of $\dot{\theta}_i$ depends only on the magnitude of N_i . The value of N_i that solves $\dot{\theta}_i = 0$ under autarky is

$\hat{N}_i^A = \alpha / \beta_i$ (superscript A denotes autarky). This value is region i 's host-density threshold for reduced prevalence (Anderson and May 1979), defined as the population value such that $N_i < \hat{N}_i^A$ reduces infectious contacts and hence prevalence, $\dot{\theta}_i < 0$; alternatively, $\dot{\theta}_i < 0$ when $N_i < \hat{N}_i^A$.

Relation $\dot{\theta}_B$ is the same under trade, with breeders' host-density threshold under trade being $\hat{N}_B^T = \hat{N}_B^A$. Relation $\dot{\theta}_F$ changes under trade. The sign of $\dot{\theta}_F$ under trade depends on prevalence in each herd, the size of the finishers' herd, N_F , and the magnitude of trade, q_i . In particular, trade increases prevalence when $\theta_B > \theta_F$, and decreases prevalence otherwise. These results mean $\dot{\theta}_F$ can be managed directly via the trade choice, and also indirectly by managing the relevant states. Accordingly, if a host-density threshold for finishers exists then it will take the general form $\hat{N}_F^T(\theta_F, \theta_B, q_F)$, which is state dependent and conditional on the trade decision.

We emphasize that q_F in Eq. (4), and h_i and y_i in Eq. (3), are time-dependent variables whose values are chosen to maximize discounted profits, which are shown in Section 3 to depend on the state variables. This means the stability properties of Eqs. (3) and (4) cannot be discussed independently of the profit-maximizing conditions presented in Section 3.

3. Economic model

We now describe the economic values that guide cattle producers' decisions. Finishers purchase cattle from breeders at an endogenously determined price of p_q per head. This purchase price is the same for healthy and infected animals because purchases are assumed non-selective. Below we show that the endogenously-determined market price reflects the mix of healthy and infected animals (prevalence). Prevalence is assumed known by all, as might be the case under a risk-based trading program that stresses making this information available.

Breeders and finishers can each sell cattle to slaughterhouses, where inspections (e.g., by the slaughterhouse or a regulatory agency) detect infections and trace them back to individual producers. Accordingly, slaughterhouses may be able to price-discriminate based on an animal's infection status. Even if an infected animal is not detected, it may weigh less than healthy animals and therefore be valued less. Suppose producers receive an exogenously-determined, fixed price of p_y per head of healthy cattle, and a lower price of $(1 - a)p_y$ (with $a > 0$) for infected cattle. This means the average sales price of the finished stock is $p_y(1 - a\theta)$.

Production involves breeding, recruitment, and feeding costs. Breeding costs are $c_{bi}rh_iN_i$, where c_{bi} is the unit cost with $c_{bB} < c_{bF}$ so that breeders have an advantage in breeding. Recruitment costs, which are the initial costs expended so that young animals become part of the recruited stock N_i , are $c_{qi}[rh_iN_i - \eta_iq_i]$. Here, c_{qi} is the unit cost with $c_{qB} > c_{qF}$ so that finishers have an advantage in feeding. Setting $c_{hi} = c_{bi} + c_{qi}$, the sum of breeding and recruitment costs are $c_{hi}rh_iN_i - c_{qi}\eta_iq_i$. Finally, costs of maintaining the recruited stock are $c_{Ni}(N_i)$, with $c_{NB}(N) > c_{NF}(N)$ for a particular N since these costs include feeding. We assume $c_{Ni}'(N_i) > 0$, as maintaining a larger herd must surely be more costly (e.g., more feed is required). We also assume $c_{Ni}''(N_i) > 0$, as increasing marginal costs are required to ensure a finite optimal herd size (since the model is otherwise linear in N_i in the absence of infection or without trade).

Summarizing, instantaneous profits are $\pi_i = p_y(1 - a\theta_i)y_iN_i + \eta_i p_q q_i - c_{hi}rh_iN_i + c_{qi}\eta_iq_i - c_{Ni}(N_i)$. The first term is slaughter-house sales revenue. The second term is trade revenues for breeders or expenditures for finishers. The third term is the sum of breeding and recruitment costs, assuming all offspring are recruited. For breeders, the fourth term is the recruitment cost savings from trading young animals to finishers. For finishers, the fourth term is the additional recruitment costs due to trade. The fifth term is the cost of maintaining the recruited population.

Each producer i maximizes discounted profits, subject to the state equations for N_i and θ_i and constraint (2), taking prices and the external force of infection as given. Breeders also consider constraint (1). Although the force of infection, βI , can have internal (within-herd) and external (cross-herd) components, we initially assume all transmission is external. This implies producers do not have incentives to manage herd densities to affect herd prevalence dynamics (see Eq. (4') below), which leaves finishers with only trade to manage infection risks and enhances our focus on trade's role in risk management. The focus on external risks is consistent with cross-farm disease spread being the principle concern of all major disease control programs (e.g., bTB, BSE, FMD, H5N1). We allow for the internalization of risks in Section 3.2.2.

Denote the external force of infection by $\beta_i = \beta_i \bar{\theta}_i \bar{N}_i$, where the “bar” notation means producers treat $\bar{\theta}_i \bar{N}_i$ as exogenous and fixed when deriving optimality conditions. This means producers will use the following form of (4) when optimizing (see Appendix A)

$$\dot{\theta}_i = \theta_i(1 - \theta_i)[\beta_i \bar{N}_i \bar{\theta}_i / \theta_i - \alpha] + q_i(\theta_B - \theta_i) / N_i. \quad (4')$$

Expression (4') represents prevalence dynamics as perceived by a representative producer. Breeders and autarkic finishers do not perceive their choices will impact θ_i since they take N_i to be fixed (θ_i does affect $\dot{\theta}_i$, but they are unable to control θ_i since their choices do not otherwise affect $\dot{\theta}_i$). Finishers who trade can influence θ_i directly via their trade choice, and indirectly via the impacts of their choices on future values of N_i and θ_i .

After the optimality conditions have been derived, we set $\bar{\theta}_i N_i = \theta_i N_i$ so that Eq. (4) again becomes the relevant dynamic relation. This approach is consistent with how external impacts are modeled in decentralized settings for disease problems (e.g., Gersovitz and Hammer, 2004) and more generally (e.g., Lucas, 1988).

Assume a discount rate of $\rho < r - \alpha$, where the inequality ensures ranching is profitable even for some level of infection. Type i producers ($i=B, F$) solve

$$V_i(N_B, N_F, \theta_F, \theta_B) = \underset{y_i, q_i, h_i}{\text{Max}} \int_0^\infty [p_y(1-a\theta_i)y_i N_i + \eta_i p_q q_i - c_{hi} r h_i N_i + c_{qi} \eta_i q_i - c_{N_i}(N_i)] e^{-\rho t} dt \quad (5)$$

s. t. (1)–(3), (4') $N_B(0), \theta_B(0), N_F(0), \theta_F(0)$

where V_i depends on N_k and θ_k ($k \neq i$) only in the case of trading. Producers optimize taking p_q as given. After optimization, the market clearing condition $q_B = q_F = q$ is imposed to determine p_q .

The current-value Lagrangian (henceforth, Lagrangian) for problem (5) is

$$L_i = p_y(1-a\theta_i)y_i N_i + \eta_i p_q q_i - c_{hi} r h_i N_i + c_{qi} \eta_i q_i - c_{N_i}(N_i) + \lambda_{N_i} [r h_i N_i - \eta_i q_i - y_i N_i - \alpha \theta_i N_i] + \lambda_{\theta_i} [\theta_i(1-\theta_i)[\beta_i \bar{N}_i \bar{\theta}_i / \theta_i - \alpha] + q_i(\theta_B - \theta_i)/N_i] + \lambda_{y_i} [(1-h_i) - y_i] N_i + \phi_i \lambda_{q_B} [r N_B - q_B] \quad (6)$$

where λ_{ji} is the co-state for variable $j = N, \theta$ (with $\lambda_{\theta} < 0$), λ_{qB} and λ_{yi} are Lagrangian multipliers for constraints (1) and (2) (constraint (2) has been scaled by N_i to simplify the algebra below), and φ_i is a dummy variable with $\varphi_F = 0$ and $\varphi_B = 1$. The necessary conditions for optimality are⁸

$$\frac{\partial L_i}{\partial y_i} = [p_y(1-a\theta_i) - \lambda_{N_i} - \lambda_{y_i}] N_i \quad \begin{cases} = 0 & \text{iff } y_i \in (0, y_i^{max}) \\ < 0 & \text{iff } y_i = 0 \end{cases} \quad (7)$$

$$\frac{\partial L_i}{\partial q_i} = \eta_i [p_q + c_{qi} - \lambda_{N_i}] + \frac{\lambda_{\theta_i}}{N_i} (\theta_B - \theta_i) - \phi_i \lambda_{q_B} \quad \begin{cases} = 0 & \text{iff } q_i \in (0, q_B^{max}) \\ < 0 & \text{iff } q_i = 0 \end{cases}, \quad \lambda_{q_B} [r N_B - q_i] = 0 \quad (8)$$

⁸When $y_i = y_i^{max}$, the Lagrangian multiplier λ_{y_i} becomes positive and adjusts to ensure L_i/y_i is never positive in expression (7). In contrast, the partial derivatives in expressions (8) and (9) may be positive. This is because we have not included a multiplier for the constraint $h_i \leq h_i^{max}$, as such a multiplier is not needed for the analysis and would complicate the notation. Also note that producer i 's problem does include co-states for the state variables N_k and θ_k for $k \neq i$. Foresighted producers consider the evolution of N_k and θ_k for $k \neq i$, and they attribute value to these states, but those values are solved independently of the optimality conditions (7)–(12) (even if the co-states were included in the Lagrangian). Hence, those values do not affect the derivation of the solution. Note, however, that the other producers' values of all co-states are subsumed into the market-clearing price p_q .

$$\frac{\partial L_i}{\partial h_i} = [-c_{hi}r + \lambda_{Ni}r - \lambda_{yi}] N_i \begin{cases} >0 & \text{iff } h_i=1 \\ =0 & \text{iff } h_i=h_i^* \\ <0 & \text{iff } h_i=0 \end{cases} \quad (9)$$

$$\frac{\partial L_i}{\partial \lambda_{yi}} = [(1-h_i) - y_i] N_i \geq 0, \quad \lambda_{yi} [(1-h_i) - y_i] N_i = 0 \quad (10)$$

$$\dot{\lambda}_{Ni} = [\rho + \alpha\theta_i] \lambda_{Ni} - [p_y(1 - \alpha\theta_i) - \lambda_{Ni} - \lambda_{yi}] y_i - [-c_{hi}r + \lambda_{Ni}r - \lambda_{yi}] h_i + c'_{Ni}(N_i) + \lambda_{\theta_i}(\theta_B - \theta_i) q_i / N_i^2 - \lambda_{yi} - \phi_i \lambda_{qi} r \quad (11)$$

$$\dot{\lambda}_{\theta_i} = \rho \lambda_{\theta_i} + \alpha p_y y_i N_i + \lambda_{Ni} \alpha N_i + \lambda_{\theta_i} [\theta_i [\beta_i N_i - \alpha] + (1 - \theta_i) \alpha] - \lambda_{\theta_i} \frac{\partial [q_i(\theta_B - \theta_i) / N_i]}{\partial \theta_i} \quad (12)$$

Conditions (11) and (12) are the adjoint conditions, while conditions (7)–(10) determine the optimal values of the choice variables. Conditions (7) and (10) indicate y_i takes on a singular value, y_i^* , when $y_i < (1 - h_i)$ such that $\lambda_{yi} = 0$ and $L_i / y_i = 0$. Alternatively, $y_i = y_i^{max}$ when $L_i / y_i = 0$ and $\lambda_{yi} > 0$. Conditions (8) and (9) indicate q_F and h_i optimally take on singular values, q_F^* and h_i^* , when L_F / q_F and L_i / h_i vanish. Condition (8) also indicates q_B is singular when $q_B < rN_B$ such that $\lambda_{qB} = 0$, and $q_i = rN_B$ when $\lambda_{qB} > 0$. Condition (8) distinguishes two scenarios: (i) autarky ($q_i = 0$), occurring under a trade ban or possibly even when trade is allowed, and (ii) trade ($q_i > 0$). We organize our discussion around these scenarios.

3.1. Autarky ($q_i = 0$)

We first identify the optimal long-run autarkic strategy, and later examine the short-run transition to this strategy. A sustainably profitable long-run strategy requires sales ($y_i > 0$, h_i

1) and breeding ($h_i > 0$), such that $h_i = h_i^{A*}$ and either $y_i = y_i^{A*}$ or $y_i = y_i^{max}$. With $h_i = h_i^{A*}$, condition (9) yields $\lambda_{Ni} = c_{hi}$. If $y_i = y_i^{A*}$, then $\lambda_{yi} = 0$ and $L_i / y_i = p_y(1 - \alpha\theta_i) - c_{hi}$, which is positive if production is profitable (i.e., if revenues also cover other herd management costs). Condition (7) then requires $y_i = (1 - h_i^{A*})$: all animals not kept for breeding are sold. Note that y_i is not absolutely constrained; y_i changes as h_i^{A*} changes.

The result that h_i is singular and y_i is constrained stems from the multiplicative nature of s_{iy} and $(1 - h_i)$ in the underlying sales relation. The term $(1 - h_i)$ represents the capacity of sales from the herd. When this capacity is chosen as an interior (singular) value, then $s_{iy} = 1$ is optimal so as to fully utilize this capacity. This is akin to Clark et al.'s (1979) results on fishing harvest levels being chosen via investments in fishing capital and the choice of effort applied to this capital. They find capital investment and fishing effort cannot both be singular. There may be excess capacity (non-singular investments, singular effort) in the short run due to irreversible capital stock investments, but that capacity will be fully utilized (singular investments, maximum effort) in the long run. Our results mimic their long run results; their short-run results do not arise here because our capacity variable is a flow rather than a stock.

To determine h_i^{A*} , we proceed by solving conditions (7) and (9) jointly to yield

$$\lambda_{N_i} = [p_y(1 - a\theta_i) + rc_{hi}] / (1 + r) \quad (13)$$

$$\lambda_{y_i} = [p_y(1 - a\theta_i) - c_{hi}] [r / (1 + r)] \quad (14)$$

Condition (13) equates the marginal value of cattle with the sum of the expected sales price and the capitalized breeding cost, discounted by the growth rate r . Next, differentiate (13) with respect to time and use adjoint condition (11) to obtain the following rate of return condition:

$$A_i(N_i, \theta_i) = \frac{r[p_y(1 - a\theta_i) - c_{hi}]}{p_y(1 - a\theta_i) + rc_{hi}} - \frac{c'_{N_i}(N_i)(1 + r)}{p_y(1 - a\theta_i) + rc_{hi}} - \alpha\theta_i - \frac{p_y a\theta_i(1 - \theta_i)(\beta_i N_i - \alpha)}{p_y(1 - a\theta_i) + rc_{hi}} - \rho = 0 \quad (15)$$

Relation $A_i(N_i, \theta_i)$ is the net return on herd investments: the own rate of return from investing in cattle net of the opportunity cost of investing in other forms of capital (ρ). The own rate of return is the return from holding more animals to sell at the margin (the first RHS term, which equals $\lambda_{y_i}/\lambda_{N_i}$), less the rate of increased herd maintenance costs (the second RHS term) and the rate of infection losses (the third RHS term), less the rate of future losses (or gains) from increases (decreases) in infection levels (the rate of change in the disease liability; the fourth RHS term).

Condition (15) ensures producers are indifferent between investing in their herd or elsewhere, implicitly defining the optimal value of N_i for a given prevalence level, i.e., $N_i^A(\theta_i)$. This singular herd size is not a steady state value, but rather changes as prevalence changes. It is easily verified that $dN_i^A(\theta_i)/d\theta_i < 0$, and so herd size is optimally reduced in response to greater prevalence that reduces the return to investing in the herd. The singular path or response curve $N_i^A(\theta_i)$ is pursued using the singular control $h_i^{A*} = [(dN_i^A(\theta_i)/dt)|_{N_i=N_i^A(\theta_i)} + (1 + \alpha\theta_i)N_i^A(\theta_i)] / [(1 + r)N_i^A(\theta_i)]$. Both $N_i^A(\theta_i)$ and h_i^{A*} are in a steady state when $\dot{\theta}_i = 0$.⁹

Relation $N_i^A(\theta_i)$ is depicted in Fig. 1a for the specification indicated in the caption.¹⁰ The $\dot{\theta}_i = 0$ isocline is given by the host-density threshold $\hat{N}_i^A = \alpha/\beta_i$, with prevalence increasing (decreasing) to the right (left) of N_i^A . In the short run, if the initial states lie off the singular

⁹Singular solutions involving a nonlinear adjustment path may arise for imperfectly controlled systems, such as when one control affects multiple states (as using the control to affect one state has spillovers onto the other state), or as here where one state is not controlled. Both cases arise in Fenichel and Horan (2007). The intuition is that the imperfect controls effectively restrict one's ability to move quickly to the steady state, and so slower adjustment becomes optimal. This has analogies to second-best problems where some choices are restricted or unavailable.

¹⁰Average prices are from USDA-NASS (2013). Breeding costs are based on cow-calf budgets from North Carolina State University modified to reflect 2012 prices (Benson, 2008). Feeding costs are based on Kansas State University budgets for finishing beef, again modified to 2012 prices (Dhuyvetter and Tonsor, 2012). For simplicity, herd levels are scaled to small values such that the autarkic host-density threshold is 50. Herd levels are easily scaled to larger, more realistic values by dividing β and the herd management cost parameter by the desired herd size scaling factor.

path $N_i^A(\theta_i)$, producers optimally follow a most rapid approach path (MRAP) to $N_i^A(\theta_i)$ as indicated by the trajectories off $N_i^A(\theta_i)$ in Fig. 1a. Once on the singular path $N_i^A(\theta_i)$, movement is indicated by the arrows on this curve, with vertical movements governed by $\dot{\theta}_i$ and horizontal movements governed by a desire to remain on the curve. For instance, starting on the singular path with $N_i^A(\theta_i) > \hat{N}_i^A$, prevalence will increase due to the large N_i . This induces a reduction in N_i , which in turn reduces the number of infectious contacts and hence $\dot{\theta}_i$. The opposite is true when starting at $N_i^A(\theta_i) < \hat{N}_i^A$. In each case, the system moves to a stable steady state $(N_i^{A\#}, \theta_i^{A\#})$, where $N_i^A(\theta_i)$ and $\hat{N}_i^A$ intersect: $N_i^{A\#} = N_i^A(\theta_i^{A\#}) = \hat{N}_i^A$. Note the superscript # denotes steady state values.

The existence of an endemic steady state requires the pre-infected singular herd level, $N_i(0)$, to exceed the host-density threshold, i.e., $N_i^A(0) > \hat{N}_i^A$. The assumptions of Fig. 1 imply finishers will exhibit higher equilibrium prevalence, $\theta_F^{A\#} > \theta_B^{A\#}$, as depicted in Fig. 1b.

3.1.1. Incentives to switch from autarky to trade—Fig. 1a is based on the assumption that autarky is either mandated or everywhere privately optimal, but this may not be the case: if not mandated, autarky may be locally or globally sub-optimal. Proposition 2 below indicates the optimality of different strategies across the state space. For now, we examine condition (8), with $\varphi_B = 0$, to determine when individuals have incentives to switch from the singular autarkic strategy to a trading strategy.

Suppose producers are currently operating under a singular autarkic strategy such that λ_{Ni} is defined by condition (13). Along this trajectory, define the value of p_q that satisfies $L_i/q_B = 0$ from (8) as the breeders' marginal willingness to accept payment ($MWTA_B$) for young animals:

$$MWTA_B = [p_y(1 - a\theta_B) + rc_{hB}] / (1 + r) - c_{qB}. \quad (16)$$

The value of p_q that satisfies $L_i/q_F = 0$ from (8) is the finishers' marginal willingness to pay ($MWTP_F$) for young animals:

$$MWTP_F = [p_y(1 - a\theta_F) + rc_{hF}] / (1 + r) - c_{qF} + \lambda_{\theta F}(\theta_B - \theta_F) / N_F. \quad (17)$$

We emphasize these measures are for a single good exhibiting an unknown quality attribute (healthy or infected), since the market will not price these unobservable attributes separately. Thus, the market clearing price must reflect average quality, defined here as the rate that the purchased animals are expected to be infected.

There are incentives to abandon the singular autarkic strategy for trade when:

$$MWTTP_F - MWTB_B = \underbrace{\frac{r(c_{bF} - c_{bB}) + (c_{qB} - c_{qF})}{1+r}}_G + \underbrace{\left[\frac{p_y a}{1+r} - \frac{-\lambda_{\theta F}}{N_F} \right]}_{\Phi} [\theta_B - \theta_F] \geq 0. \quad (18)$$

The first RHS term in expression (18), G , is the disease-free marginal net gains from trade, arising from comparative advantages that generate cost efficiencies. Our modeling assumptions indicate $G > 0$, so that trade will always occur absent disease.

The second RHS term in (18), Φ , is the disease-related marginal net gains ($\Phi > 0$) or costs ($\Phi < 0$) from trade. Trade is incentivized (dis-incentivized) when the market facilitates (impedes) risk management, which occurs when the bracketed RHS terms are of the same (opposite) sign such that $\Phi > 0$ ($\Phi < 0$). Specifically, the two bracketed terms in Φ reflect relative differences in each of the two components of infection risk: infection costs and prevalence. Consider the economic (cost) component. The term $p_y a / (1 + r)$ is the marginal loss to breeders from finishing an additional infected animal, whereas $-\lambda_{\theta F} / N_F > 0$ is the finishers' marginal cost of an increase in the infected stock, I_F , holding N_F constant.¹¹ If a particular traded animal was known to be infected, then such a trade would produce a net gain when the breeders' marginal loss from keeping the animal exceeded finishers' marginal infection cost of acquiring the animal. Yet, these economic pressures are not independent of prevalence. As animal health status remains unknown, the relative rates of infection (akin to 'probabilities' of acquiring an infected animal, although the model is deterministic) matter in addition to the relative costs.

When $\theta_B < \theta_F$, finishers trade to acquire lower-risk animals and reduce their average prevalence. This will only be mutually beneficial if the gains to finishers (measured by their marginal infection costs) exceed the marginal infection losses to breeders. The reason is that finishers will be willing to pay breeders a premium in this case, as we show in the next section. We refer to this use of trade to dissipate risk (reduce herd prevalence) as *prevalence arbitrage*.

When $\theta_B > \theta_F$, then breeders may wish to trade animals to finishers to avoid future infections within their herd (due to vertical transmission) and the associated losses at the slaughterhouse. Accordingly, breeders are willing to accept a lower price from finishers, up to the expected avoided infection losses. Finishers will accept the increased risk provided the trade lowers their production costs by more than their marginal infection costs. In keeping with the finishers' perspective, we refer to this use of trade to reduce costs as *cost arbitrage*.

¹¹Let $V_F(N_F, I_F, N_B, I_B)$ be the value function that arises from optimizing V_F subject to dI/dt rather than $d\theta/dt$. Then the marginal cost of infection is $\lambda_{IF} = V_F(N_F, I_F, \cdot) / I_F$. Using $\theta = I/N$, $V_F(N_F, I_F, \cdot) = V_F(N_F, N_F \theta_F, \cdot) = V_F(N_F, \theta_F, \cdot)$, where $V_F(N_F, \theta_F, \cdot)$ is the present value of our approach. The relation $V_F(N_F, I_F, \cdot) = V_F(N_F, N_F \theta_F, \cdot)$ indicates that $\lambda_{IF} = V_F(N_F, I_F, \cdot) / I_F = V_F(N_F, N_F \theta_F, \cdot) / (N_F \theta_F)$. The relation $V_F(N_F, N_F \theta_F, \cdot) = V_F(N_F, \theta_F, \cdot)$ indicates that $\lambda_{\theta F} = V_F(N_F, \theta_F, \cdot) / \theta_F = [V_F(N_F, N_F \theta_F, \cdot) / (N_F \theta_F)] N_F$. Combining these results yields $\lambda_{\theta F} = \lambda_{IF} N_F$, or $\lambda_{IF} = \lambda_{\theta F} / N_F$. Finally, note the double negative in (18), $-(-\lambda_{\theta F} / N_F)$, is intentional to indicate the difference between two positive values.

The next section illustrates that $\Phi = 0$ under trade, and that trade is preferred to autarky. Accordingly, Eq. (18) indicates that trade incentives are non-decreasing under a disease outbreak, with the gains from trade being larger for larger absolute differences in the economic components of risk among buyers and sellers, $|p_y a / (1 + r) - (-\lambda_{\theta F} / N_F)|$ and $|\theta_B - \theta_F|$. Larger risk differences among buyers and sellers imply *more* risk arbitrage is possible via trade, thereby generating larger gains from trade; relative risk matters. As these risk-related gains ultimately stem from the uncertain health status of individual animals (otherwise, such animals would not be traded), this means uncertainty increases the incentives to trade. This result differs from Akerlof's (1970) result, for his model of trade under symmetric uncertainty (which most closely resembles our model), that uncertain product quality reduces trade incentives. The difference here arises because all producers are at risk even without trade, since the goods being traded are held in inventories and poor quality goods can damage those inventories. Trade provides a mechanism for endogenously managing these inventory risks.

3.2. Trade ($q_i = q > 0$)

We now assume a trading strategy exists that satisfies the optimality conditions, and we explore how this strategy relates to the sign and magnitude of Φ . As above, we begin by focusing on the long-run (not necessarily steady state) strategy. There are three possible trade scenarios: (a) breeders have comparatively large operations, breeding and selling animals ($h_B, y_B > 0$) while finishers specialize ($h_F = 0, y_F > 0$); (b) finishers have comparatively large operations, breeding and selling animals ($h_F, y_F > 0$) while breeders specialize ($h_B > 0, y_B = 0$); (c) finishers and breeders specialize ($h_F = y_B = 0, h_B, y_F > 0$). For simplicity, we assume trade only occurs when both parties strictly prefer trading (i.e., no indifference). Proposition 1, which identifies the feasible options and associated strategies, only rules out option (a) in this regard:

Proposition 1: Trading scenario (a) is ruled out. Potentially feasible trading strategies under scenarios (b) and (c) involve breeders specializing with $h_B = 1, y_B = 0$, and finishers choosing $y_F^{T*} = 1 - h_F$, where $h_F = h_F^{T*}$ under scenario (b) or $h_F = 0$ under scenario (c).

Proof: See Appendix A.

Scenario (a) is ruled out because, by continuing to engage in both activities, breeders neither reduce their costs nor increase their sales relative to autarky, leaving them indifferent to autarky. Only breeders who specialize ($h_B = 1$ and $y_B = 0$) can gain from trade. In contrast, finishers who engage in both activities (scenario b) can pay breeders to produce more young animals than finishers could muster on their own, boosting their sales so that they prefer trade.

We focus on scenario (b) in what follows, for two reasons. First, we find that only scenario (b) satisfies the optimality conditions for the specification presented in Fig. 1. This result is consistent with the fact that many finishing operations in the U.S. are large due to economies of scale, and some breeding often occurs in close proximity to finishing. Second, scenario

(b) generates a richer set of economic tradeoffs, facilitating more general insights, than scenario (c) because complete specialization under scenario (c) constrains all choices but q .

Breeders specialize ($h_B = 1, y_B = 0$) in scenario (b), and so conditions (7) and (9) are inequalities ($L_B/h_B > 0, L_B/y_B < 0$). This means the marginal value of the cattle stock, λ_{NB} , exceeds discounted sales revenues and capitalized breeding costs, $[p_y(1 - \theta_B) + c_{hB}r]/[1 + r]$, and its value is determined by setting $L_B/q_B = 0$ in condition (8):

$$\lambda_{NB} = p_q + c_{qB} - \lambda_{qB} \quad (19)$$

The trade price p_q is determined endogenously from the finishers' optimality conditions since finishers purchase the entire supply from breeders. A moderate trade-inducing price yields $q < rN_B$ and $\lambda_{qB} = 0$. Such an outcome must hold in a steady state with infection, as some surplus breeding is required to replace animals lost to infection. A large price yields $q = rN_B$ and $\lambda_{qB} > 0$.

Finishers do not fully specialize, choosing $h_F = h_F^{T*}$ and $y_F = (1 - h_F^{T*})$. Their optimality conditions yield (13) and (14), while condition (8) yields the market price

$$p_q = [p_y(1 - a\theta_F) + r c_{hF}]/[1 + r] - c_{qF} + (\lambda_{\theta F}/N_F)(\theta_B - \theta_F) = MWT A_B + G + \Phi \quad (20)$$

Other things equal, finishers pay more for animals when their quality is expected to be greater than the quality of the current inventory, $\theta_B < \theta_F$, so that the traded animals dilute finishers' herd prevalence. Alternatively, finishers pay less when trade increases herd prevalence, $\theta_B > \theta_F$.

Taking the time derivative of (13) and using adjoint condition (11), we derive the following modified rate of return condition for N_F :

$$\Lambda_F(N_F, \theta_F) = \Phi[q/N_F]/\lambda_{NF}(\theta_F), \quad (21)$$

where $\lambda_{NF}(\theta_F) > 0$ is as defined in Eq. (13). As with (15), the LHS of (21) is the net rate of return to herd investments, not accounting for trade. The RHS of (21) is the rate at which investments in N_F diminish the benefits of trade: for a given q , an increase in N_F reduces the trade rate, q/N_F , which is valued by the marginal net gains from trade, Φ , and then divided by the marginal value of the herd, λ_{NF} , to convert the RHS to a rate. Alternatively, the RHS is the rate of return, at the margin, to enhancing the finisher inventory via trade.

Condition (21) indicates the pre-trade return to herd investments equals the return from trade, with Φ and Λ_F having the same sign. This means we can partition the state space based on the sign of the known relation $\Lambda_F(N_F, \theta_F)$, and therefore also according to the sign of Φ assuming (21) holds. Recall the autarky response function $N_F^A(\theta_F)$ in Fig. 1 corresponds to $\Lambda_F(N_F, \theta_F) = 0$. As $\Lambda_F/N_F < 0$, both Λ_F and Φ are positive below (negative above) the curve $N_F^A(\theta_F)$. That the marginal gains from trade are larger when finishers have

smaller inventories, ceteris paribus, accords with our results from expression (21) that a larger N_F diminishes the benefits of trade.

The partitioning of the state space leads to the following proposition:

Proposition 2: Finishers prefer trade to autarky when $\Phi > 0$. They otherwise prefer autarky, but only as part of a MRAP to move back to the trade region where $\Phi > 0$. Along an optimal scenario (b) trading path (if it exists), Φ may only vanish for an instant.

Proof: See Appendix A.

Proposition 2 indicates autarky is pursued when $\Phi < 0$ and $A_F(N_F, \theta_F) < 0$. The intuition is that trading produces a negative net rate of return to holding cattle when $N_F > N_F^A(\theta_F)$. Producers do better to adopt an autarky strategy when $\Phi < 0$, which we previously showed involves using bang-bang controls ($y_i = y_i^{max}$ and $h_F = q = 0$) to proceed along a MRAP to the curve $N_F^A(\theta_F)$. Once at $N_F^A(\theta_F)$ the net rates of return under autarky and trade are zero, as $N_F^T(\theta_F) = N_F^A(\theta_F)$ when $\Phi = 0$, but trade is optimal due to the gains from trade G . Producers would do even better to continue to an optimal trade trajectory lying within the region $\Phi > 0$. This is because producers realize additional gains from trade when $\Phi > 0$, as they can trade to arbitrage their relative disease risks. Moreover, as $A_F(N_F, \theta_F) > 0$ when $\Phi > 0$, trading produces a positive rate of return to holding cattle. An optional trade path will not follow $N_F^A(\theta_F)$ for more than an instant.

In addition to increasing the rate of return both to inventory holdings and to trade, a trade strategy with $N_F < N_F^A(\theta_F)$ enhances finishers' disease risk management by reducing infectious contacts. The disease-related benefit of a smaller N_F does not appear in the rate of return for N_F (condition (21)) since we have treated infections as external. However, this benefit does appear in the following rate of return condition for disease control, derived from adjoint condition (12)

$$\rho = \frac{\dot{\lambda}_{\theta_F}}{\lambda_{\theta_F}} + \frac{ap_y(1-h_F^{T*})}{-\lambda_{\theta_F}/N_F} + \frac{\lambda_{N_F}}{-\lambda_{\theta_F}/N_F} \alpha - \beta_F \theta_F N_F - (1-2\theta_F) \alpha - \frac{q_F}{N_F} \quad (22)$$

The RHS of (22) is the rate of return to investing in disease control. After the capital gain/loss term ($\dot{\lambda}_{\theta_F}/\lambda_{\theta_F}$), the return to disease control is increasing in sales-related damages and mortality losses affecting herd size (the second and third RHS terms, respectively): logically, the return to control is greater when damages are greater. The returns to disease control are declining in the force of infection, $\beta_F \theta_F N_F$, as a larger force of infection makes control more of an uphill battle. The returns are also declining in the rate of trade relative to herd size, q_F/N_F , suggesting diminishing returns from trade, the primary control affecting prevalence. Finally, mortality affecting prevalence (the fifth RHS term) has an ambiguous impact on the returns, with this mortality being a substitute (complement) with disease control when prevalence is small (large).

The ability to manage the return to disease control depends on whether q is constrained by rN_B . First, suppose q is unconstrained. In a market equilibrium, the two choices h_F^{T*} and q must manage the rates of return associated with three states: N_F , N_B , and θ_F . But two controls cannot manage all three rates of return perfectly, and so tradeoffs emerge that generate nonlinear responses. Specifically, the rate of return on N_F is managed via q as condition (21) may be solved implicitly for $q(N_F, \theta_F, \theta_B, \lambda_{\theta F})$, which is a feedback decision rule conditional on the co-state $\lambda_{\theta F}$. Expression $q(\cdot)$ is substituted into (19) along with p_q from (20) to yield the implicit relation $\lambda_{NB}(N_F, \theta_F, \theta_B, \lambda_{\theta F})$. Equate the time derivative of this relation with the relevant adjoint condition, $d\lambda_{NB}(N_F, \theta_F, \theta_B, \lambda_{\theta F})/dt = \lambda_{NB}$, and substitute in the equations of motion (3) and (4) with $h_B = 1$, $y_B = 0$, and $y_F^{T*} = 1 - h_F^{T*}$ to solve for the conditional feedback rule $h_F^{T*}(N_F, \theta_F, \theta_B, \lambda_{\theta F})$. Hence, h_F^{T*} manages the rate of return on N_B (adjoint condition (11)) via market interactions. But choices q and h_F^{T*} are not independent: both depend on $\lambda_{\theta F}$, which must follow adjoint relation (12) and hence the rate of return condition for θ_F . The dynamical system for the singular trade solution therefore consists of Eqs. (22) and (3), (4) for $i=B, F$, with $h_B = 1$, $y_B = 0$, $h_F = h_F^{T*}(\cdot)$, $y_F^{T*} = 1 - h_F^{T*}(\cdot)$, and $q = q(\cdot)$. The initial value of $\lambda_{\theta F}$ is the value that places the system on the best path.

Now suppose $q = rN_B$. Rate of return condition (21) is solved for $\lambda_{\theta F}(N_F, \theta_F, \theta_B, q)$ in this case. Equate the time derivative of this relation with the adjoint condition, $d\lambda_{\theta F}(N_F, \theta_F, \theta_B, q)/dt = \lambda_{\theta F}$, and substitute in the equations of motion (3) and (4) with $h_B = 1$, $y_B = 0$, and $y_F^{T*} = 1 - h_F^{T*}$ to solve for the conditional feedback rule $h_F^{T*}(N_F, \theta_F, \theta_B, q)$. This means h_F^{T*} now manages the rate of return on $\lambda_{\theta F}$, which in turn influences the rate of return on N_F . With q fixed, the rate of return condition for N_B is independently satisfied via adjustments in the value of λ_{qB} .

We cannot fully explore this five dimensional dynamic system analytically, but steady state outcomes (which, recall, require q to be unconstrained) provide some insights. First, consider the steady state outcome for breeders. Solving $\dot{\theta}_B = 0$ yields the same steady state value as under autarky, $N_B^{T\#} = \hat{N}_B^A$. Solving $\dot{N}_B = 0$ yields the implicit steady state relation $\theta_B^{T\#} = [r - (q(\cdot)/\hat{N}_B^A)]/\alpha$. A similar relation arises under autarky, $\theta_B^{A\#} = [r h_B^{A*} - y_B^A]/\alpha$. The difference is

$$\theta_B^{T\#} - \theta_B^{A\#} = [r(1 - h_B^{A*}) - ([q(\cdot)/\hat{N}_B^A] - y_B^A)]/\alpha, \quad (23)$$

where $h_B^{A*} < 1$ and $q(\cdot)/\hat{N}_B^A > h_B^A$; (20) implies $p_q = MWTP_F > MWTA_B$, so that breeders supply more under trade than autarky. Relation (23) therefore implies the trade steady state may exhibit greater or lesser prevalence than the autarkic steady state. Prevalence is relatively greater under trade the greater the breeding rate under trade relative to autarky, $(h_B^{T*} - h_B^{A*}) = (1 - h_B^{A*})$, as breeding causes animals to remain in the herd longer creating a greater risk of exposure. Animal sales/trade has the opposite effect, and so prevalence is

relatively smaller under trade the greater is the rate of trade relative to the autarkic sales rate, $([q(\cdot)/\hat{N}_B^A] - y_B^A)$.

To see which outcome occurs, solve the steady state relation $\dot{\lambda}_{NB} = 0$ for θ_B under both trade and autarky: $\theta_B^j = [(\lambda_{NB}^j - c_{hB})r - c'_{NB}(\hat{N}_B^A)] / [\alpha \lambda_{NB}^j] - \rho / \alpha$ for $j=A, T$, after having used condition (9) to derive the relation $\lambda_{yB}^j = (\lambda_{NB}^j - c_{hB})r$. This expression for θ_B^j can be used to derive

$$\theta_B^{T\#} - \theta_B^{A\#} = [\lambda_{NB}^T - \lambda_{NB}^A][c_{hB}r + c'_{NB}(\hat{N}_B^A)] / [\alpha \lambda_{NB}^T \lambda_{NB}^A] > 0, \quad (24)$$

which is positive because trade will only occur when $\lambda_{NB}^T = p_q + c_{qB} > \lambda_{NB}^A$ (by condition (10)). This result, along with our results from expression (23), implies the prevalence-increasing effect of more breeding under trade outweighs the prevalence-reducing effect of more sales.

The steady state stock level for finishers is determined by solving $\dot{\theta}_F = 0$ to obtain the following implicit relation defining an endogenous, bioeconomic host-density threshold:

$$\hat{N}_F^T = \frac{\alpha}{\beta_F} + \left[\frac{(\theta_F - \theta_B)}{\beta_F \theta_F (1 - \theta_F)} \right] \frac{q(\cdot)}{N_F} = \hat{N}_F^A + \left[\frac{(\theta_F - \theta_B)}{\beta_F \theta_F (1 - \theta_F)} \right] \frac{q(\cdot)}{N_F} \quad (25)$$

The first RHS term is the standard ecological host-density threshold when there is no trade and animals only mix locally. This term can be thought of as nature's risk mitigation efforts (disease mortality) discounted by the transmission rate.

The second RHS term is an adjustment reflecting humans' risk mitigation efforts via trade, which alters how animals mix across the landscape. The endogenous manner of mixing alters traditional epidemiological and economic results. The absolute magnitude of the adjustment (i.e., deviation from autarkic threshold) is larger the larger is the absolute difference in prevalence, $|\theta_F - \theta_B|$, and the larger is the trade rate, $q(\cdot)/N_F$. The sign of the adjustment is the same as the sign of $\theta_F - \theta_B$. When $(\theta_F - \theta_B)$ is positive (negative), the bracketed ratio in the second RHS term represents epidemiological benefits (costs) of herd replacement through trade, discounted by new transmissions per animal. The effective discount factor reflects the fact that imported animals not currently infected will be at risk after importation. Other things equal, larger herd replacement benefits (costs) increase (decrease) the threshold, but the adjustment is small if the benefits are expected to be temporary due to a large likelihood of transmission.

The steady state stock $N_F^{T\#}$ exceeds the autarky host-density threshold $\hat{N}_F^A$ when $\theta_F^{T\#} > \theta_B^{T\#}$ so that finishers are engaging in prevalence arbitrage. Steady state prevalence is less than under autarky, $\theta_F^{T\#} < \theta_F^{A\#}$, in such instances since the steady state lies in the region $\Phi > 0$ and since $N_F^A(\theta_F)$ is downward sloping. The steady state stock $N_F^{T\#}$ is less than the autarky

threshold $\hat{N}_F^A$, so that finishers engage in cost arbitrage, when $\theta_F^{\#} < \theta_B^{\#}$. Trade's impact on prevalence is ambiguous in such instances: it is possible that $\theta_F^{\#} > \theta_F^A$ when $\theta_B^{\#} > \theta_F^A$, but otherwise $\theta_F^{\#} < \theta_F^A$ will occur. Ascertaining whether a steady state outcome optimally occurs (and hence is stable), as well as the relative stock and prevalence levels at that outcome, must be done numerically.

3.2.1. Numerical example—Fig. 2 presents feedback control diagrams for the numerical example of Fig. 1. The singular paths are indicated as solid curves with arrows indicating the trajectory to the steady state point, z_i . Arrows off each singular path indicate the direction of the MRAP to the singular path (i.e., the off-path arrows are *not* phase arrows associated with the singular solution, but rather only pertain to the non-singular MRAP; see Fenichel and Horan (2007) for a similar diagram). To understand the singular path, we begin with the disease-free trade outcome. This is a stable (singular) equilibrium at $N_F^T(0) = N_F^A(0) = 241$ and $N_B^T(0) = 183$, with $q = rN_B = 44$ animals being traded so that q is constrained. Finishers choose the same herd size as they do under autarky, but trade allows them to breed fewer animals so that more animals are available for sale. Breeders increase their herd size relative to autarky in response to the greater value of young animals under trade.

Starting at the disease-free outcome, suppose breeders' herds become infected (e.g., by a random introduction) so that $\theta_B^T > \theta_F^T = 0$ initially. We find finishers prefer to trade and risk infection than switch to autarky. Indeed, their trade incentives have increased, with $\Phi > 0$ due to cost arbitrage: breeders accept a lower price to trade animals out of this high-risk area where animals could become infected or infect new recruits, and the lower price benefits finishers even though trade puts them at risk. That finishers prefer trade, along with breeders having a larger herd size under trade, means trade increases the likelihood of pathogen introduction and spread.

Initially, trade volume continues at $q = q^{max}$. The initial portion of the finishers' optimal path is illustrated in Fig. 2a as being a smooth northwesterly progression along the curve $N_F^A(\theta_F)$, but in reality the system “chatters” along this curve. Chattering here involves rapid switches between the trade and autarky regimes, as each regime moves the system across the $\Phi = 0$ curve separating these regimes (see Fenichel and Horan (2007) for a similar example of chattering in a disease setting). For instance, upon becoming infected, finishers find it optimal to move to a trading path in the region with $\Lambda_F, \Phi > 0$: the pre-trade rate of return on cattle is positive as are the disease-related gains from trade. The trade path with $q = q^{max}$ in this region immediately moves the system back to the curve $\Lambda_F = \Phi = 0$, at which time autarky becomes optimal. The trade path naively moves back to the autarkic region because singular solutions do not account for the fact that a regime switch is optimal when $\Lambda_F, \Phi < 0$. Likewise, once in the autarky regime, finishers again have incentives to move along a MRAP back into the region $\Lambda_F, \Phi > 0$. This chattering process, which is consistent with Proposition 2's result that the system does not remain on the curve $\Phi = 0$ for more than an instant, continues until q is no longer constrained. At this point, the optimal trade path permanently moves into the region $\Lambda_F, \Phi > 0$. This qualitative change occurs as θ_F overtakes

θ_B and switches from cost arbitrage to prevalence arbitrage.¹² The system then moves along a singular path to the saddle point steady state z with $N_F^{T\#}=70$ and $\theta_F^{T\#}=0.47$, as this is the only long-run outcome that satisfies the profit-maximizing conditions.

Finishers' steady state prevalence under trade is 14.5 percent lower than under autarky, confirming that trade can help control spread after pathogen introduction. Equilibrium z_F exhibits prevalence arbitrage, with finishers having larger prevalence than breeders, $\theta_F^{T\#} > \theta_B^{T\#}$, as well as large infection costs, $-\lambda_{\theta_F^{T\#}}/N_F^{T\#} > p_y a/[1+r]$. Finishers trade to replace high-risk animals (which are sold to slaughterhouses) with low risk animals so that the risk to finishers dissipates with trade. This approach to risk management is more beneficial than the autarkic approach of reducing herd densities in response to higher prevalence. Indeed, trade yields a smaller long-run prevalence and a larger equilibrium herd level.

Breeder dynamics are shown in Fig. 2b. The portion of the path where q is constrained, and which involves chattering by finishers, is represented here for simplicity as a smooth curve after an initial impulse sale y reduces the stock from $N_B^T(0)=183$ to $N_B^T=125$. With $h_B^T=1$ and $q=rN_B$ after this impulse, disease mortality will cause the herd size N_B to diminish over time relative to its disease-free value. The optimal path moves northwesterly. Eventually, q becomes unconstrained and the system settles at the steady state point z with $N_B^{T\#}=50$ and $\theta_B^{T\#}=0.45$. Steady state prevalence exceeds the autarkic value of $\theta_B^{A\#}=0.15$, as predicted above, but trade more than compensates breeders for any increased infection losses.

3.2.2. Alternative specifications—Our analysis is based on some specific simplifying assumptions to generate insights into bioeconomic linkages involving disease spread and trade. Using this model as a benchmark, it is possible to examine how some of the insights might change for different model assumptions. We focus here on three model variants: (i) not all transmission risk is external, (ii) the rate of vertical transmission is less than unity, and (iii) when only finishers are at risk of spread.

First, suppose some transmission risk is internalized so that the force of infection becomes $\beta_{i,ext}N_i\theta_i + \beta_{i,int}N_i\theta_i$, where $\beta_{i,ext}$ and $\beta_{i,int}$ represent external and internal transmission. Producers have incentives to manage herd size as an additional method of managing prevalence in this case. Qualitatively, the marginal incentives for trade remain $G+\Phi$, although Φ depends on λ_{θ_F}/N_F which will change quantitatively. We calibrate the numerical model with $\beta_{i,int} = 0.6\beta_i$ and $\beta_{i,ext} = 0.4\beta_i$, such that $\beta_{i,ext} + \beta_{i,int} = \beta_i$, where β_i is the value from our baseline model. Here we find the marginal incentives to trade are smaller than in the baseline case ($\Phi=6.83$ in this case, relative to $\Phi=7.5$ in the baseline case), as are the epidemiological gains that finishers experience from trade (i.e., a four percent reduction in prevalence). These results indicate the economic and ecological benefits of trade are increasing in the level of external risks, which ironically corresponds to the situations where trade is mostly likely to be banned. Trade is a method for controlling prevalence among

¹²We have examined many different parameter combinations and could not find one in which cost arbitrage occurs in a steady state equilibrium. It is unclear whether this is a general result.

finishers, and this control is increasingly important when finishers are more limited in their ability to manage these risks internally.

Next, consider a reduction in vertical transmission, which means more newly bred animals will be healthy. The first implication is that trade has an *immediate*, negative (positive) impact on breeders' (finishers') prevalence, as a disproportionate number of the traded animals are healthy. The effect is to increase both $MWTP_F$ and $MWTA_B$, with ambiguous impacts on the gains from trade. The second implication of reduced vertical transmission is that the marginal value of the herd, λ_{Ni} , is reduced for both breeders and finishers. This is because λ_{Ni} equals the marginal value of sales, and this is reduced as more sales means producers reduce their capacity for *future* prevalence reductions via breeding (as breeding increases the proportion of healthy animals). The smaller λ_{NF} reduces both $MWTP_F$ and $MWTA_B$, with analytically ambiguous impacts on the gains from trade. Numerically, we find non-monotonic incentive effects. For instance, a 10% reduction in the vertical transmission rate increases the disease-related marginal net gains from trade slightly, and yields an 11% reduction in finishers' prevalence relative to autarky. A 20% reduction in the vertical transmission rate reduces the disease-related marginal net gains from trade, but yields a 50% reduction in finishers' prevalence relative to autarky. Note that the disease-related marginal net gains from trade remain positive for all further reductions in the vertical transmission rate.

Finally, suppose only finishers are at risk of spreading infection in the long run. We examine two possibilities. The first is when only finishers are initially infected (e.g., by a random introduction not involving trade). This case is of interest as there have been concerns over importing healthy animals into infected regions. The fear is that these animals are likely to become infected, making it more difficult to eradicate disease in endemic regions. Trade will not lead to infection among breeders, and hence their prevalence remains at zero. In this setting, the host density threshold for finishers as $\theta_F \rightarrow 0$ implicitly solves

$\hat{N}_F^T = \hat{N}_F^A + q^{max} / [\beta_F \hat{N}_F^T]$, which yields $\hat{N}_F^T = 289 > \hat{N}_F^T(0) = 241$ in our baseline model. Trade leads to eradication in this case, whereas autarky would lead to greater infection. A second possibility, yielding similar results, occurs for a smaller β_B even when both types of producers are initially infected. For instance, a 73% reduction in β_B yields

$\hat{N}_F^T = 185 > N_F^T(0) = 183$ so that infection cannot be sustained among breeders. When this happens, $\hat{N}_F^T$ again will equal 289 animals, which exceeds the largest value of N_F under trade. Trade again eventually leads to disease eradication for finishers.

4. Discussion

Markets are the key mechanism for managing risks to portfolios containing combinations of assets and liabilities. Financial markets are well known for allocating and distributing risks. The fact that markets could play important role in reducing risks associated with real assets and liabilities, such as livestock and infectious disease, adds to the robust history in economics of adapting financial markets insights to real asset markets.

Our results show that even unregulated markets may help to reduce endemic biological pollution risks that are typically thought of as trade externalities, provided the externality-generating producers involved in trade bear some of the risks privately. The ability to arbitrage heterogeneous private risks increase producers' incentives to trade, even in the face of uncertain quality of the goods being traded. Moreover, because markets may improve the management of private risks, efficiency may be improved and external risks reduced through trade, even though externalities remain. The results arise because producers both respond to and affect biological risks so that these are dynamically endogenous. Some form of intervention, such as incentives to promote surveillance and biosecurity, will be required to address the externalities. But these interventions need to account for economic and ecological or epidemiological feedbacks. Indeed, our results indicate that institutional responses to biological risks, such as trade bans that generally serve as the default response in most locales, that do not account for such feedbacks may counterproductively reduce efficiency and increase ecological risks (e.g., Bulte et al., 2003).

Our results provide support for proposed risk-based trading of livestock in infected areas (e.g., DEFRA, 2014). The results are especially strong, with trade leading to disease eradication, if infection risks are confined to downstream producers (here finishers). While such an approach may be easily implemented within a single jurisdiction, coordination across multiple jurisdictions might prove more difficult – particularly if the risks within one region outweigh the risks in others. Coordination would likely also be important, and potentially challenging, to implement policy tools to address infection externalities, so as to further increase efficiency and reduce prevalence. Future work is needed to address the socially optimal design of risk-based trading markets along with incentives that may be needed to promote additional investments in other risk management mechanisms such as enhanced surveillance and biosecurity in high-risk areas.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Appendix A. Derivation of Eqs. (4) and (4')

We derive Eq. (4') for finishers ($i=F$; the derivation for $i=B$ is analogous, but simpler). We start by setting $\beta_F I_F = \beta_F F$, since the force of infection is external (see the discussion in Section 3). Given this assumption along with the description of infection dynamics in the main text, finishers' *perceived* equation of motion for infection is $\dot{F} = r I_F h_F + \beta_F F S_F + q_F I_B / N_B - \gamma_F I_F - \alpha F$. Note that the other I_F terms in $\dot{F}$ are not taken to be external. The definitional relation $\theta_F = I_F / N_F$ implies $\theta_F / \theta_F = F / I_F - \dot{N}_F / N_F$. Substituting $\dot{F}$ and $\dot{N}_F$ into this expression yields

$$\begin{aligned} \frac{\dot{\theta}_F}{\theta_F} &= r h_F + \frac{\beta_F \bar{I}_F S_F}{I_F} + \frac{q_F I_B / N_B}{I_F} - \gamma_F - \alpha - \left[r h_F + \frac{q_F}{N_F} - \gamma_F - \alpha \theta_F \right] \\ &= \beta_F \frac{\bar{I}_F}{N_F} \bar{N}_F \frac{S_F / N_F}{I_F / N_F} + \frac{q_F}{N_F} \frac{I_B / N_B}{I_F / N_F} - \alpha (1 - \theta_F) - \frac{q_F}{N_F} \\ &= \beta_F \bar{\theta}_F \bar{N}_F \frac{(1 - \theta_F)}{\theta_F} - \alpha (1 - \theta_F) + \frac{q_F}{N_F} \frac{\theta_B}{\theta_F} - \frac{q_F}{N_F} \\ &= \left[\beta_F \frac{\bar{\theta}_F \bar{N}_F}{\theta_F} - \alpha \right] (1 - \theta_F) + \frac{q_F}{N_F} \left[\frac{\theta_B - \theta_F}{\theta_F} \right]. \end{aligned}$$

Multiplying through by θ_F yields Eq. (4'). Finally, the *actual* equation does not treat the force of infection as fixed. Eq. (4) is therefore derived from (4') by setting $\theta_F N_F = \theta_F N_F$.

Proposition 1

Trading scenario (a) is ruled out. Potentially feasible trading strategies under scenarios (b) and (c) involve breeders specializing with $h_B = 1$, $y_B = 0$, and finishers choosing $y_F^{T*} = 1 - h_F$, where $h_F = h_F^{T*}$ under scenario (b) or $h_F = 0$ under scenario (c).

Proof

First consider breeders, noting that trade requires $h_B > 0$. Consider scenario (a), with $h_B = h_B^{T*}$ and $y_B > 0$. As under autarky, $y_B = (1 - h_B^{T*})$ must hold. This means (13)–(15) also hold, and so the response function $N_B^{A*}(\theta_B)$ is also satisfied under trade. Breeders would be indifferent between autarky and trade, as trade neither reduces their costs nor increases their revenues, and so we say trade will not emerge along these lines. Breeders can only gain from trade if they specialize in breeding ($h_B = 1$ and $y_B = 0$) as in scenarios (b) or (c).

Now consider finishers. Profitable trade requires sales $y_F > 0$, and so $h_F = 1$ is ruled out. Consider scenario (b), with h_F^{T*} . As under autarky, $y_F = (1 - h_F^{T*})$ must hold. We explore this case in the main text. Scenario (c), where finishers specialize with $h_F = 0$ also cannot be ruled out. In this case $y_F = 1$. Otherwise, if $y_F = y_F^{T*}$, then (7) yields $\lambda_{NF} = p_y(1 - a \theta_F)$ and condition (9) becomes $L_F / h_F = -c_{hF} + p_y(1 - a \theta_F) > 0$, implying $h_F > 0$ is optimal – a contradiction.

Lemma 1

In trade scenario (b), breeders are always willing to trade when $\Phi > 0$.

Proof

Condition (20) indicates this scenario's market clearing price is $p_q = MWTAB + G + \Phi$, which clearly exceeds $MWTAB$ when $\Phi > 0$. Hence, breeders will be willing to trade when $\Phi > 0$.

Lemma 2

If $\Phi = 0$ during an interval along an optimal path in trade scenario (b), then during this interval breeders will choose $q = q(\theta_B)$ to maintain $N_B^T(\theta_B)$, where $N_B^T(\theta_B)$ is the herd response function under trade. $N_B^T(\theta_B) > N_B^A(\theta_B)$ if $\dot{\theta} > -[(1+r)(rc_{hB} + c'_N)/[p_y a]]$, and $N_B^T(\theta_B) < N_B^A(\theta_B)$ if the inequality is reversed.

Proof

Suppose $\Phi = 0$ at some point along an optimal path in trade scenario (b). The market-clearing price (20) becomes $p_q = \lambda_{NB}^A - c_{qB} + G$ where λ_{NB}^A is defined as in (13). Optimality condition

(8) then yields $\lambda_{NB}^T = \lambda_{NB}^A + G$ and hence $\dot{\lambda}_{NB}^T = \dot{\lambda}_{NB}^A = [-ap_y a / (1+r)] \dot{\theta}_B$. Using adjoint condition (11), we obtain a modified form of (15):

$$\Lambda_B^T(N_B, \theta_B) = r - \alpha \theta_B - \frac{c'_{NB}(N_B)(1+r)}{p_y(1-a\theta_B) + rc_{hB} + G} - \frac{p_y a \theta_B (1-\theta_B)(\beta_B N_B - \alpha)}{p_y(1-a\theta_B) + rc_{hB} + G} - \rho = 0 \quad (\text{A.1})$$

Expression (A.1) can be solved for a relation of the form $N_B = N_B^T(\theta_B; G)$, with

$$N_B^T(\theta_B; 0) = N_B^A(\theta_B) \text{ and } \partial N_B^T(\theta_B; G) / \partial G > 0 \text{ when } \dot{\theta} > -[(1+r)(rc_{hB} + c'_N)] / [p_y a].$$

Suppressing the notation for G , this result implies $N_B^T(\theta_B) > N_B^A(\theta_B)$ when

$\dot{\theta} > -[(1+r)(rc_{hB} + c'_N)] / [p_y a]$, given that $G > 0$. The value $N_B^T(\theta_B)$ is pursued by choosing $q = q(\theta_B)$ to satisfy

$$q(\theta_B) = r N_B^T(\theta_B) - \alpha \theta_B N_B^T(\theta_B) - \left[dN_B^T(\theta_B) / dt \right] \Big|_{N_B = N_B^T(\theta_B)} \quad (\text{A.2})$$

Proposition 2

Finishers prefer trade to autarky when $\Phi > 0$. They otherwise prefer autarky, but only as part of a MRAP to move back to the trade region where $\Phi > 0$. Along an optimal scenario (b) trading path (if it exists), Φ may only vanish for an instant.

Proof

Lemma 1 shows breeders are willing to trade when $\Phi > 0$, and so we focus our attention on finishers. We begin by using a contradiction to show finishers' are not willing to trade when $\Phi < 0$. Suppose finishers pursue a trading strategy in which $\Phi < 0$ (above the autarky curve in Fig. 1), and then consider whether the finishers have incentives to abandon this strategy in favor of autarky. The optimal autarky strategy is a MRAP ($h_F = q = 0$ and $y_F = 1$) to the singular autarky curve $N_F^A(\theta_F)$. We compare these strategies by comparing the optimized Lagrangians under each strategy, given the current states, since the Lagrangian is a welfare measure reflecting the net present value of each strategy. The optimized Lagrangians are

$$L_F^A = -c_{NF}(N_F) + \lambda_{NF}^A [-\alpha \theta_F N_F] + \lambda_{\theta_F}^A \theta_F (1 - \theta_F) [\beta_F N_F - \alpha] + \lambda_{y_F}^A N_i \quad (\text{A.3})$$

$$L_F^T = -c_{NF}(N_F) + \lambda_{NF}^T [-\alpha \theta_F N_F] + \lambda_{\theta_F}^T \theta_F (1 - \theta_F) [\beta_F N_F - \alpha] + \lambda_{y_F}^T N_i \quad (\text{A.4})$$

where superscripts A and T refer to autarky and trade. The difference between these measures depends on the relative values of the co-states and the Lagrangian multiplier. The values of λ_{NF}^T and $\lambda_{y_F}^T$ are given by (13) and (14), but to ease comparison we use condition (7) to derive

$$\lambda_{NF}^j = p_y(1 - a\theta_F) - \lambda_{yF}^j, \quad \text{for } j = T, A. \quad (\text{A.5})$$

Using (A.5), the marginal value of breeding in condition (9) is

$$\partial L_F^j / \partial h_F = \left[-c_{hF}r + p_y(1 - a\theta_F)r - (1+r)\lambda_{yF}^j \right] N_i \quad (\text{A.6})$$

Condition (A.6) indicates that λ_{yF} must rise when $h_F = 0$ is chosen under autarky (since the expression vanishes under trade when $h_F > 0$ is chosen); hence $\lambda_{yF}^A > \lambda_{yF}^T$.

We can now write the differences between (A.3) and (A.4) as

$$\begin{aligned} L_F^A - L_F^T &= [\lambda_{NF}^T - \lambda_{NF}^A][\alpha\theta_F N_F] + [\lambda_{\theta F}^A - \lambda_{\theta F}^T]\theta_F(1 - \theta_F)[\beta_F N_F - \alpha] + [\lambda_{yF}^A - \lambda_{yF}^T]N_i \quad (\text{A.7}) \\ &= [\lambda_{yF}^A - \lambda_{yF}^T][\alpha\theta_F N_F] + [\lambda_{\theta F}^A - \lambda_{\theta F}^T]\theta_F(1 - \theta_F)[\beta_F N_F - \alpha] + [\lambda_{\lambda F}^A - \lambda_{\lambda F}^T]N_i \end{aligned}$$

where the second line comes from the results in (A.5). The first and third terms in the second line are positive, given our results above. The sign of the second term depends on $\lambda_{\theta F}^A$. As there is no condition requiring $\lambda_{\theta F}^A$ to change when switching from trading to autarky, $\lambda_{\theta F}^A - \lambda_{\theta F}^T = 0$ and the difference in (A.7) is positive: finishers will have incentives to adopt autarky when $\Phi < 0$. The opposite results hold when $\Phi > 0$: finishers will choose trade over autarky, provided an optimal trade path exists given their current state. When $\Phi = 0$ such that finishers are currently on the curve $N_F^A(\theta_F)$, condition (18) indicates the gains from trade are $G > 0$: finishers and breeders have incentives to trade, provided an optimal trade path exists given the current state.

Now consider whether $\Phi = 0$ may hold for an instant along an optimal path with $h_F^{T*} > 0$. There are three possible ways in which $\Phi = 0$ for at least an instant: (i) $p_y a / (1 + r) = -\lambda_{\theta F} / N_F$ and $d[\lambda_{\theta F} / N_F] / dt = 0$, (ii) $\theta_F = \theta_B$ and $\dot{\theta}_F = \dot{\theta}_B$, and (iii) $\theta_F = \theta_B$ and $p_y a / (1 + r) = -\lambda_{\theta F} / N_F$, with $\dot{\theta}_F \neq \dot{\theta}_B$ and $d[\lambda_{\theta F} / N_F] / dt \neq 0$. Cases (i) and (ii) will only hold for an instant if $\Phi < 0$ prior to and immediately after $\Phi = 0$, with $\Phi = 0$ when $\Phi = 0$; these cases will hold for more than an instant if $\Phi = 0$ persists. Case (iii) can only hold for an instant as both $(\theta_F - \theta_B)$ and $[p_y a / (1 + r) + \lambda_{\theta F} / N_F]$ simultaneously change sign so that $\Phi > 0$ before and after this point.

Consider case (i), where $d[\lambda_{\theta F} / N_F] / dt = 0$ implies $\dot{\lambda}_{\theta F} / \lambda_{\theta F} - \dot{N}_F / N_F = 0$ (and hence $\dot{\Phi} = 0$).

This expression with $-\lambda_{\theta F} / N_F = p_y a / (1 + r)$ and $N_F^A(\theta_F)$ (as $\Phi = 0$ implies $A = 0$) yields

$$\rho - r - [p_y(1 - a\theta_F) + r c_{hF}] \alpha / [a p_y] + \theta_F [\beta_F N_F^A(\theta_F) - \alpha] + \alpha = 0 \quad (\text{A.8})$$

Expression (A.8) only depends on θ_F and can therefore only be satisfied for at most a discrete number of values of θ_F . Such an outcome can only arise at an instant. Otherwise, the value of θ_F that solves (A.8) must be in a steady state. But it is generally not possible to ensure such an outcome since q has already been defined in (A.2).

Consider case (ii), where $\theta_F = \theta_B = \theta$ and $\dot{\theta}_F = \dot{\theta}_B$ implies $\beta_B N_B(\theta) = \beta_F N_F(\theta)$. This relation may only be satisfied for at most a discrete number of values of θ_F . Further, the relation $\dot{\theta}_F = \dot{\theta}_B$ means $\theta_F = \theta_B$ continues to hold in subsequent periods. This means the solution for θ_F must continue to hold as in a steady state, thereby yielding an additional condition $\beta_F N_F(\theta) = \alpha$ that causes the system to be overdetermined. Hence, case (ii) is generally not possible.

Finally, case (iii) with $\theta_F = \theta_B$ and $-\lambda_{\theta F}/N_F = p_y \alpha/(1+r)$ only occurring at an instant, cannot be ruled out. In a trading equilibrium with $\theta_F = \theta_B = \theta$, the condition $A_i = 0$ yields

$$\beta_i N_i = [r p_y (1 - a\theta) - [\alpha\theta + \rho][p_y (1 - a\theta)] - \varpi_i] / [p_y a\theta (1 - \theta)] + \alpha \quad (\text{A.9})$$

where $\varpi_i = c'_{N_i}(N_i)(1+r) + [1 + \alpha\theta_i + \rho]rc_{hi} + [\alpha\theta_i + \rho]\kappa G > 0$ and κ is a dummy variable that equals unity for $i=B$ and zero for $i=F$. The term ϖ_i represents the only element in (A.9) that differs between breeders and finishers. Given that the RHS of (A.9) is decreasing in ϖ_i , it is clear that $\varpi_B > \varpi_F$ implies $\beta_F N_F^T > \beta_B N_B^T > \beta_B N_B^A$ (the final inequality holds from Lemma 2) and hence $\theta_B < \theta_F$. The opposite is true when $\varpi_B < \varpi_F$.

Appendix B. Supporting information

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jedc.2015.02.005>.

Fig. 1.

(a) Dynamics under autarky (for breeders) parameters: $p=1500$, $a=0.3$, $\rho=0.05$, $c_{hB}=950$, $c_{hF}=775$, $c_{qB}=900$, $c_{qF}=700$, $c_{NB}=0.2N_B^2$, $c_{NF}=0.15N_F^2$, $r_i=0.24$, $\beta_i=0.0006$, $\alpha_i=0.03$, $i=B, F$. (b) Response functions for finishers and breeders. Parameter values are the same as in (a).

Fig. 2.

(a) Trading dynamics for finishers. Parameter values are the same as in Fig. 1a. (b) Trading dynamics for breeders. Parameter values are the same as in Fig. 1a.