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Corrugated Waveguide Mode Content Analysis Using Irradiance Moments

Sudheer K. Jawla [Member, IEEE],

Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA
02139 USA

Michael A. Shapiro,

Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA
02139 USA

Hiroshi Idei, and

Advanced Fusion Research Center, Research Institute for Applied Mechanics, Kyushu University,
Kasuga 816-8580, Japan

Richard J. Temkin [Fellow, IEEE]

Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA
02139 USA

Sudheer K. Jawla: sudheer@mit.edu; Michael A. Shapiro: shapiro@psfc.mit.edu; Hiroshi Idei: idei@triam.kyushu-u.ac.jp;
Richard J. Temkin: temkin@mit.edu

Abstract

We present a novel, relatively simple method for determining the mode content of the linearly polarized modes of a corrugated waveguide using the moments of the intensity pattern of the field radiated from the end of the waveguide. This irradiance moment method is based on calculating the low-order irradiance moments, using measured intensity profiles only, of the radiated field from the waveguide aperture. Unlike the phase retrieval method, this method does not use or determine the phase distribution at the waveguide aperture. The new method was benchmarked numerically by comparison with sample mode mixtures. The results predict less than $\pm 0.7\%$ error bar in the retrieval of the mode content. The method was also tested using high-resolution experimental data from beams radiated from 63.5 mm and 19 mm corrugated waveguides at 170 and 250 GHz, respectively. The results showed a very good agreement of the mode content retrieved using the irradiance moment method versus the phase retrieval technique. The irradiance moment method is most suitable for cases where the modal power is primarily in the fundamental HE_{11} mode, with $< 8\%$ of the power in high-order modes.

Index Terms

Corrugated waveguide; irradiance moments and phase retrieval; quasi-optical beam; waveguide modes

I. Introduction

Low-loss oversized corrugated waveguides are used for high-power millimeter-wave transmission in electron cyclotron heating systems of fusion devices [1]–[4]. The 24-MW electron cyclotron resonance system of the International Thermonuclear Experimental Reactor (ITER) utilizes 63.5-mm diameter corrugated waveguides to transmit continuous wave (CW) microwave power over more than 100 m of distance at a frequency of 170 GHz [5], [6]. The JAEA group uses a 170-GHz, 1-MW gyrotron to test the ITER relevant transmission lines [7].

The critical issue in transmitting high-power CW microwaves, generated by the gyrotron, is the coupling of the Gaussian-like quasi-optical beam into the corrugated waveguide. Ideally, one expects the excitation of a pure linearly polarized HE_{11} (or LP_{01}) mode in the waveguide by the Gaussian-like beam because of its very low ohmic losses as the mode propagates along the waveguide [8]–[11]. However, other higher order modes (HOMs) are also generated in the corrugated waveguide because the gyrotron output beam is not an ideal Gaussian beam, and also due to the experimental misalignment in the coupling assembly. Experimental [12]–[14] and theoretical studies [15], [16] have been conducted to optimize the coupling of the gyrotron beam into the corrugated waveguide using a mirror optics unit. This requires an accurate determination of mode contents in the corrugated waveguide. At megawatt power levels, accurate infrared measurements and data analysis of the field radiated at several locations in free space from the corrugated waveguide aperture were performed [12] and the mode contents were retrieved from the intensity patterns at the waveguide aperture. This requires full complex field information, which is retrieved using the phase retrieval technique [17]–[20]. The accuracy of these techniques for high-average-power gyrotrons is limited by the signal-to-noise ratio and spatial resolution of the infrared measurement technique. In cold test, the mode contents are evaluated to higher accuracy using higher resolution data from a scanner [18], [19], [21]–[23], where intensity profiles are measured down to -50 dB or less. Another approach is to measure the mode contents using a monitor installed with the miter bend assembly on the transmission line [24], [25]. These experiments indicate that other than the fundamental HE_{11} mode, the dominant modes excited in the corrugated waveguide due to quasi-optical Gaussian beam coupling are primarily the four next lowest order modes of the waveguide, LP_{11} , LP_{02} , LP_{03} , and LP_{21} [26]. The LP_{11} is excited due to a tilt/offset in the Gaussian beam, and the LP_{02} (also called the HE_{12}) is excited due to the phase curvature mismatch of the beam and HE_{11} mode, and also due to mode conversion in the miter bends installed in the transmission line system [10], [12]. The HE_{11} and LP_{11} mode superposition results in the waveguide field energy center bouncing in the waveguide [26] and the HE_{11} and LP_{02} mode superposition results in the field intensity distribution oscillating along the propagation direction with the period depending on the beat wavelength of the two modes. Recently, an analytical approach has been proposed to correct these modes using simple mirror correctors [27].

Mode content retrieval requires full complex field information, which utilizes the phase retrieval technique. In this paper, we present a simple but effective method to determine the mode contents in a corrugated waveguide using only the intensity information, skipping the process of obtaining phase using the phase retrieval technique. Under the assumption that

the gyrotron Gaussian beam excites only the low-order modes of the guide, the low-order irradiance moments of the radiated intensity can be used to determine the mode content. We show how the mode contents can be determined from the irradiance moments as a function of propagation distance z from the waveguide aperture. The irradiance moments propagation has been previously studied in [28]–[32]. The technique of phase retrieval at the waveguide radiating aperture using irradiance moments was proposed in [33].

The method proposed in this paper can also be used for the mode content determination in overmoded optical waveguides [34]. In this paper, we will assume that the mode entering the waveguide is linearly polarized. We use the formulation of the waveguide modes as a set of LP (LP_{mn}) waves following [26]. In the LP_{mn} mode notation, the lowest mode of the waveguide, the LP_{01} mode is the same as the HE_{11} mode and the LP_{0n} modes are the same as the HE_{1n} modes. We will use the LP_{0n} mode and the HE_{1n} mode notation interchangeably.

This paper is organized as follows. Section II describes the rigorous analytical equations of the superposition of the dominant mode HE_{11} and the HOMs based on their symmetry. In Section III, we reduce these equations to the modes of interest generated in a corrugated waveguide from coupling of a gyrotron quasi-optical beam. Section IV numerically benchmarks the method using combinations of the known modal powers of the dominant mode. In Sections V and VI, we use the experimentally measured data from two different corrugated waveguide setups and compare the results obtained using the proposed method in this paper to the phase retrieval method [23].

II. Superposition of HE_{11} and Higher Order LP_{mn} Modes

HOMs are generated in addition to the fundamental mode HE_{11} when an overmoded corrugated waveguide is powered by a quasi-optical output beam from a gyrotron. In order to determine the power coupled into these HOMs, we investigate the irradiance moments of the field radiating in free space after the waveguide aperture as a function of propagation distance. We analytically study the propagation of these moments based on the superposition of the fundamental mode HE_{11} and HOMs. As we know from [26] and [27], one can predict the behavior of the field intensity propagating in the waveguide based on the HOM symmetry. The LP_{11} mode can be predicted very accurately by analyzing the tilt and offset in the beam, and LP_{0n} ($n \geq 2$) can be predicted by analyzing the field intensity distribution oscillating in the waveguide as a function of distance. LP_{1n} and LP_{0n} modes contribute independently of each other to the intensity propagation and thus can be studied separately by analytically investigating their superposition with the fundamental mode HE_{11} .

A. Superposition of HE_{11} and LP_{1n} ($n \geq 1$) Modes

We assume that the field is linearly polarized, and the electric field has E_x -component. The electric field at the waveguide aperture is represented as a superposition of HE_{11} and LP_{1n} modes

$$E = \sqrt{P_{01}} \exp(i\theta_{01}) u_{01} + \sum_n \sqrt{P_{1n}} \exp(i\theta_{1n}) u_{1n} \quad (1)$$

where the HE_{11} mode field distribution is given by

$$u_{01} = \frac{1}{\sqrt{\pi a}} \frac{J_0\left(\frac{\nu_{01}}{a} r\right)}{J_1(\nu_{01})} \quad (2)$$

and the LP_{1n} mode field distribution is given by

$$u_{1n} = \frac{\sqrt{2}}{\sqrt{\pi a}} \frac{J_1\left(\frac{\nu_{1n}}{a} r\right)}{J_0(\nu_{1n})} \cos\varphi \quad (3)$$

where (r, φ) are the polar coordinates, $\nu_{01} = 2.405$ is the first zero of the Bessel function J_0 , ν_{1n} is the n th zero of J_1 , for example, $\nu_{11} = 3.832$, P_{01} and P_{1n} are the modal powers, and θ_{01} and θ_{1n} are the modal phases.

We assume that the LP_{1n} modes are even modes, and the superposition results in a shift of the beam energy center in the x -direction. The superposition with the LP_{1n} odd modes causes a shift in the y -direction and can be analyzed similarly. The first-order moment $\langle x \rangle$, which is the x -coordinate of the energy center, can be represented as a function of the propagation distance z , of the radiated microwave beam, from the waveguide aperture [33]

$$\langle x(z) \rangle = \iint x |E|^2 dx dy - \frac{z}{2ik} \iint \left(E^* \frac{\partial E}{\partial x} - E \frac{\partial E^*}{\partial x} \right) dx dy \quad (4)$$

where k is the free space wavenumber and the electric field distribution E at $z = 0$ is integrated over the waveguide aperture of radius a . Using (2)–(4), we obtain

$$\langle x(z) \rangle = 2 \sum_n b_{1n} \sqrt{P_{01} P_{1n}} \cos \psi_{1n} + 2z \sum_n d_{1n} \sqrt{P_{01} P_{1n}} \sin \psi_{1n} \quad (5)$$

where ψ_{1n} is the phase difference between the HE_{11} and LP_{1n} modes given by $\psi_{1n} = \theta_{1n} - \theta_{01}$, and

$$\begin{aligned} b_{1n} &= \sqrt{2} a \int_0^1 \xi^2 \frac{J_1(\nu_{1n} \xi) J_0(\nu_{01} \xi)}{J_0(\nu_{1n}) J_1(\nu_{01})} d\xi \\ &= -\frac{2\sqrt{2} \nu_{01} \nu_{1n} a}{(\nu_{1n}^2 - \nu_{01}^2)^2} \end{aligned} \quad (6)$$

$$d_{1n} = \frac{\sqrt{2}}{ka} \frac{\nu_{01} \nu_{1n}}{\nu_{1n}^2 - \nu_{01}^2} \quad (7)$$

B. Superposition of HE₁₁ and LP_{0n} (n ≥ 2) Modes

The electric field at the waveguide aperture is represented as a superposition of the HE₁₁ mode and the azimuthally symmetric LP_{0n} modes

$$E = \sqrt{P_{01}} \exp(i\theta_{01}) u_{01} + \sum_{n=2}^{\infty} \sqrt{P_{0n}} \exp(i\theta_n) u_{0n} \quad (8)$$

where the LP_{0n} mode field distribution is given by

$$u_{0n} = \frac{1}{\sqrt{\pi}a} \frac{J_0\left(\frac{\nu_{0n}}{a}r\right)}{J_1(\nu_{0n})} \quad (9)$$

where, for example, $\nu_{02} = 5.52$. The variance $\langle r^2 \rangle = \langle x^2 + y^2 \rangle$ can be represented as follows [33]:

$$\langle r^2(z) \rangle = 2\pi \int_0^a r^3 |E|^2 dr - 2\pi \frac{z}{ik} \int_0^a r^2 \left(E^* \frac{\partial E}{\partial r} - E \frac{\partial E^*}{\partial r} \right) dr + 2\pi \frac{z^2}{k^2} \int_0^a r \left| \frac{\partial E}{\partial r} \right|^2 dr. \quad (10)$$

Using (9) and (10), we derive the equation for effective beam waist $w = 2\langle r^2 \rangle$ as

$$\begin{aligned} w^2(z) = & 2F_{11}P_{01} + 2 \sum_{n=2}^{\infty} F_{nn}P_{0n} + 4 \sum_{n=2}^{\infty} B_{1n} \sqrt{P_{01}P_{0n}} \cos\psi_{0n} \\ & + 4 \frac{z}{k} \sum_{n=2}^{\infty} D_{1n} \sqrt{P_{01}P_{0n}} \sin\psi_{0n} \\ & + 2 \frac{z^2}{k^2 a^2} \left(\nu_{01}^2 P_{01} + \sum_{n=2}^{\infty} \nu_{0n}^2 P_{0n} \right) \end{aligned} \quad (11)$$

where the phase difference $\psi_{0n} = \theta_{0n} - \theta_{01}$, and

$$F_{nn} = 2a^2 \int_0^1 \xi^3 \frac{J_0^2(\nu_{0n}\xi)}{J_1^2(\nu_{0n})} d\xi = \frac{\nu_{0n}^2 - 2}{3\nu_{0n}^2} a^2 \quad (12)$$

$$B_{1n} = 2a^2 \int_0^1 \xi^3 \frac{J_0(\nu_{01}\xi)J_0(\nu_{0n}\xi)}{J_1(\nu_{01})J_1(\nu_{0n})} d\xi = \frac{8\nu_{01}\nu_{0n}}{(\nu_{0n}^2 - \nu_{01}^2)} a^2 \quad (13)$$

and

$$\begin{aligned} D_{1n} = & 2 \int_0^1 \xi^2 \frac{\nu_{0n} J_0(\nu_{01}\xi) J_1(\nu_{0n}\xi) - \nu_{01} J_0(\nu_{0n}\xi) J_1(\nu_{01}\xi)}{J_1(\nu_{01}) J_1(\nu_{0n})} d\xi \\ = & -\frac{4\nu_{01}\nu_{0n}}{\nu_{0n}^2 - \nu_{01}^2}. \end{aligned} \quad (14)$$

C. Superposition of HE₁₁ and LP_{2n} (n ≥ 1) Modes

The electric field at the waveguide aperture is represented as the following superposition:

$$E = \sqrt{P_{01}} \exp(i\theta_{01}) u_{01} + \sum_n \sqrt{P_{2n}} \exp(i\theta_{2n}) u_{2n} \quad (15)$$

where the LP_{2n} mode field distribution is

$$u_{2n} = \frac{\sqrt{2}}{\sqrt{\pi}a} \frac{J_2\left(\frac{\nu_{2n}}{a}r\right)}{J_1(\nu_{2n})} \cos 2\varphi \quad (16)$$

where, for example, $\nu_{21} = 5.1356$. The second-order moment $\langle x^2 - y^2 \rangle$ can be represented as follows [33]:

$$\begin{aligned} \langle x^2(z) - y^2(z) \rangle = & \iint (x^2 - y^2) |E|^2 dx dy \\ & - \frac{z}{ik} \iint \left[x \left(E^* \frac{\partial E}{\partial x} - E \frac{\partial E^*}{\partial x} \right) - y \left(E^* \frac{\partial E}{\partial y} - E \frac{\partial E^*}{\partial y} \right) \right] dx dy \\ & + \frac{z^2}{k^2} \iint \left[\left| \frac{\partial E}{\partial x} \right|^2 - \left| \frac{\partial E}{\partial y} \right|^2 \right] dx dy. \end{aligned} \quad (17)$$

Using (16) and (17), we derive

$$\langle x^2(z) - y^2(z) \rangle = \sum_n B_{2n} \sqrt{P_{01} P_{2n}} \cos \psi_{2n} + 2 \frac{z}{k} \sum_n D_{2n} \sqrt{P_{01} P_{2n}} \sin \psi_{2n} + \frac{z^2}{k^2 a^2} \sum_n \frac{4 \sqrt{2} \nu_{01} \nu_{2n}}{\nu_{2n}^2 - \nu_{01}^2} \sqrt{P_{01} P_{2n}} \cos \psi_{2n} \quad (18)$$

where the phase difference $\psi_{2n} = \theta_{2n} - \theta_{01}$, and

$$\begin{aligned} B_{2n} = & 2 \sqrt{2} a^2 \int_0^1 \xi^3 \frac{J_0(\nu_{01}\xi) J_2(\nu_{2n}\xi)}{J_1(\nu_{01}) J_1(\nu_{2n})} d\xi \\ = & - \frac{8 \sqrt{2} \nu_{01} \nu_{2n}}{(\nu_{2n}^2 - \nu_{01}^2)^3} (4 + \nu_{2n}^2 - \nu_{01}^2) a^2 \end{aligned} \quad (19)$$

$$\begin{aligned} D_{2n} = & - \sqrt{2} \int_0^1 \xi^2 \frac{\nu_{2n} J_0(\nu_{01}\xi) J_1(\nu_{2n}\xi) + \nu_{01} J_1(\nu_{01}\xi) J_2(\nu_{2n}\xi)}{J_1(\nu_{01}) J_1(\nu_{2n})} d\xi \\ = & \frac{2 \sqrt{2} \nu_{01} \nu_{2n}}{(\nu_{2n}^2 - \nu_{01}^2)^2} (4 + \nu_{2n}^2 - \nu_{01}^2). \end{aligned} \quad (20)$$

Note that the LP_{2n} modes are of different orientation (even and odd). Here, we characterize the HE_{11} and LP_{2n} even mode mixture by the moment $\langle x^2(z) - y^2(z) \rangle = \langle r^2 \cos(2\varphi) \rangle$. The mixture of the HE_{11} and LP_{2n} odd modes can be characterized by the moment $2 \langle x(z) y(z) \rangle = \langle r^2 \sin(2\varphi) \rangle$.

III. Reduced Equations for Dominant Modes

The equations derived in Section II can be used to determine the mode contents if the powers of HOMs are small (total HOM power of ~8% or less) compared with the fundamental mode HE_{11} . Based on [11]–[14], [23], and [27], it is clear that the dominant modes generated in a corrugated waveguide from coupling of a gyrotron quasi-optical beam are low-index HOMs such as HE_{11} , LP_{11} , LP_{02} , LP_{03} , and LP_{21} . Therefore, we can simplify the equations derived in Section II for these dominant modes only.

A. Calculating the LP₁₁ Even and Odd Mode Content

From the z -dependence of the first-order moment, calculated from the propagated intensity profiles at several locations from the waveguide aperture, we can write

$$\langle x \rangle = \langle x \rangle_{z=0} + \alpha_x z \quad (21)$$

and following (5), the LP₁₁ even mode power, P_{11}^e , can be written as

$$P_{11}^e = \left(\frac{\langle x \rangle_{z=0}}{2b_{11}} \right)^2 + \left(\frac{\alpha_x}{2d_{11}} \right)^2 \quad (22)$$

where $b_{11} = 0.329a$ and $d_{11} = 1.464/(ka)$. Substituting α_x and $\langle x \rangle_{z=0}$ in (22), one can calculate P_{11}^e . Similarly, the power of LP₁₁ odd mode can be calculated from $\langle y \rangle_{z=0}$ and α_y . As a check of this approach, we note that (22) is in agreement with the tilt/offset conservation theorem [26] for a superposition of HE₁₁ and LP₁₁ modes in a corrugated waveguide.

B. Calculating the LP₀₂ and LP₀₃ Mode Content

From the propagated intensity profiles, one can calculate the z -dependence of the effective beam waists using the second-order moments $\langle x^2(z) \rangle$ and $\langle y^2(z) \rangle$ as

$$w^2 = w_0^2 + Z_1 z + \frac{\beta_1^2}{k^2 a^2} z^2 \quad (23)$$

and from (11), the LP₀₂ mode power, P_{02} , can then be determined by neglecting the smaller valued terms as follows:

$$P_{02} = \left(\frac{w_0^2 - 2F_{11}}{4B_{12}} \right)^2 + \left(\frac{kZ_1}{4D_{12}} \right)^2 \quad (24)$$

where $F_{11} = 0.218a^2$, $B_{12} = 0.174a^2$, and $D_{12} = -2.151$ are defined by (12)–(14). Z_1 is the coefficient of the linear term in (23). Once P_{02} is obtained, the quadratic term in (11) and (23) can be used to determine the power, P_{03} , in the HOM LP₀₃ from the following equation using the coefficient β_1^2 of the quadratic term in (23):

$$P_{03} = \frac{\beta_1^2 - 2\nu_{01}^2 - 2\nu_{02}^2 P_{02}}{2\nu_{03}^2}. \quad (25)$$

C. Calculating the LP₂₁ Mode Content

From the z -dependence of the second-order moments $\langle x^2 \rangle$ and $\langle y^2 \rangle$, we can also write

$$\langle x^2(z) - y^2(z) \rangle = \langle x^2 - y^2 \rangle_{z=0} + Z_2 z + \frac{\beta_2^2}{k^2 a^2} z^2 \quad (26)$$

where Z_2 is the coefficient of the linear term. The power in the LP_{21} mode, P_{21} , can be determined using (18) as follows:

$$P_{21} = \left(\frac{\langle x^2 - y^2 \rangle_{z=0}}{B_{21}} \right)^2 + \left(\frac{k Z_2}{2 D_{21}} \right)^2 \quad (27)$$

where $B_{21} = -0.3936a^2$ and $D_{21} = 2.026$ are defined by (19) and (20). The quadratic term in (26) is very small if the high-order mode content is small and can be neglected.

IV. Benchmarking Using Known Mode Contents

To numerically validate the method, we generated an artificial field at the waveguide aperture using several different combinations of the known modal powers of the above mentioned modes and propagated the complex field in free space to several locations using a full diffraction integral approach [35]. We then evaluate the irradiance moments of the radiated field as a function of the propagation distance as follows [33]:

$$M_{pq} \equiv \langle x^p y^q \rangle_z \equiv \frac{\iint (x)^p (y)^q I_z(x, y) dx dy}{\iint I_z(x, y) dx dy}. \quad (28)$$

Here, p and q represent the order of the moment ($p + q$) and $I_z(x, y)$ is the intensity profile at a given z position. The equations described in Section III are then solved for mode content calculation using these irradiance moments. From the formulation of equations in Section II, it is clear that the first-and second-order moments are enough to evaluate the power in the fundamental mode and dominant modes. The calculations done with the reduced set of equations are based on the assumption that the power in the HE_{11} mode is large compared with the power in the other modes. The results of the retrieved mode contents are plotted in Figs. 1–3 showing three different cases. In first case shown in Fig. 1, the known values of HE_{11} and LP_{11} mode powers are varied between 93% and 98% and 5% and 0%, respectively, and the modes LP_{02} and LP_{21} are kept fixed at 0.5% and 1.5%, respectively. For the higher HE_{11} input mode content ($\sim 95\%$), the retrieval error bar is less than $\sim 0.5\%$. A small amount of LP_{03} is also retrieved for lower HE_{11} mode input. It should be noted here that the modes with different symmetry do not affect their retrieval, e.g., the retrieval of LP_{11} mode content is not affected by the presence of LP_{0n} modes and vice versa. They are however, affected by the presence of the next HOMs of same symmetry. It is also important to mention the fact that while the LP_{11} mode is generated mainly due to the experimental errors of misalignment by tilt and offset during the coupling to the corrugated waveguide, the excitation of LP_{02} modes is inherent to the beam itself. If a perfect Gaussian beam couples into a corrugated waveguide, it will excite higher order LP_{0n} modes because it does not match perfectly to the HE_{11} mode. In addition, the gyrotron output beam is never a perfect Gaussian beam. Therefore, the LP_{02} modes will always be excited in the waveguide. In Fig. 2, we have shown the case for symmetric modes only where HE_{11} and LP_{02} are

varied between 94% and 99%, and 5% and 0%, respectively, while LP_{03} is kept fixed at 1%. In this case, the error bar in the retrieval of HE_{11} is slightly higher, $\sim 0.7\%$, compared with the previous case. Another case, shown in Fig. 3, also predicts an error bar of $\sim 0.7\%$.

Detailed investigation based on the presented equations and numerical analysis shows that in the proposed method of irradiance moments, the effect of having different phases between the modes is very small.

V. Experimental Setup and Results

In order to experimentally test the new method, the radiation pattern from the 63.5-mm diameter corrugated waveguide, used for the ITER transmission line setup, was measured at the frequency of 170 GHz using a three-axis scanner. The high-purity $LP\ HE_{11}$ mode in the corrugated waveguide was generated using a combination of waveguide transitions and tapered mode generator designed and built by General Atomics. An Agilent vector network analyzer (VNA) was used to measure the intensity over a $28\text{-cm} \times 28\text{-cm}$ cross section of the measurement plane. The receiving waveguide and the VNA extension head were placed on an automated three-axis scanner to measure the field intensity patterns on a plane parallel to the waveguide aperture at many locations along the waveguide axis. The experimental setup is shown in Fig. 4. As explained in [23], the alignment of the measurement setup is an important issue in correctly evaluating the excited modes of the corrugated waveguide. The total mechanical error induced by the scanner alignment in the measurement plane parallel to the waveguide aperture is estimated to be $<0.15^\circ$ in both the x -direction and y -direction. The offset error of the receiver waveguide with respect to the center of the waveguide aperture is estimated to be <0.5 mm in both the x -direction and y -direction. Measurements were done at several distances between 10 and 70 cm from the waveguide aperture. The measured intensity patterns are shown in Fig. 5. The accuracy of the measurements can be seen from the diffraction rings observed down to -45 dB around the main lobe. The equations derived in Section III are then used to calculate the mode contents of the excited modes in the waveguide using these intensity profiles. Mode content retrieval, which utilizes the phase retrieval method [23] is also employed to calculate the mode contents from the measured intensity patterns. Table I shows the comparison of the two methods, mode contents retrieved using the irradiance moment method versus the mode content retrieval method using phase retrieval technique. A very good agreement can be seen between the two methods and the results are in good agreement within an error bar of less than $\pm 0.4\%$.

VI. Results From 19-mm Corrugated Waveguide at 250 GHz

The proposed method is also applied to the measured intensity profiles from a 19-mm corrugated waveguide transmission line used at 250 GHz in a dynamic nuclear polarization-nuclear magnetic resonance (DNP-NMR) experiment. The measurements were performed similarly to the 63.5-mm corrugated waveguide and are presented in [23]. The mode content results presented in [23] are compared with the mode contents calculated by the irradiance moment method. The results are summarized in Table II. A very good agreement, at about $\pm 0.5\%$ level, is seen in this case also between the two methods.

VII. Conclusion

The irradiance moment technique allows us to easily calculate the modal power of the dominant HOMs in a corrugated waveguide without requiring a solution for the phase using the phase retrieval technique. The application of the irradiance moment method is based on the assumption that the power in the fundamental mode HE_{11} is large compared with the other modes. The analytical investigation of the superposition of different modes breaks down the method to simple linear equations, which are easier to solve and make the proposed method simpler and faster compared with other methods. Numerical benchmarking of the technique predicts an error of less than $\sim \pm 0.7\%$ in the retrieval of modal power in the dominant modes. The results obtained based on the experimentally measured intensity profiles, in a 63.5-mm diameter ITER-relevant corrugated waveguide and 19-mm waveguide at 250 GHz used in a DNP-NMR experiment, demonstrate a very good agreement between the results obtained by the phase retrieval and mode content retrieval techniques. The technique of mode content retrieval using irradiance moments starts to deviate if there is significant modal content of LP_{03} or other azimuthally symmetric HOMs. In the case of gyrotrons, the output beam is very well optimized to be a Gaussian-like beam by accurately designing the internal quasi-optical helical launcher and the set of beam shaping mirrors and further using an external matching optics unit. Therefore, the modes excited in the corrugated waveguide by this Gaussian-like beam are typically only the low-index LP_{mn} modes, and the power coupled into these modes by the Gaussian-like beam is small. Due to this, the proposed method based on the irradiance moments can be very well applied in such a case.

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Biographies

Sudheer K. Jawla received the M.Sc. degree in physics from IIT Delhi, New Delhi, India, in 2004, and the Ph.D. degree in physics from the Centre de Recherches en Physique des Plasmas, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, in 2010.

He is currently a Research Scientist with the Waves and Beams Division, Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA, USA. His current research interests include vacuum electron devices, high-power gyrotrons, high-frequency gyrotron system for dynamic nuclear polarization/nuclear magnetic resonance applications, overmoded waveguides, and quasi-optical components at millimeter and terahertz wavelengths.

Michael A. Shapiro (M'09) received the Ph.D. degree in radio physics in 1990.

He has been with the Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA, USA, since 1995, where he is currently the Head of the Gyrotron Research Group. His current research interests include vacuum microwave electron devices, high-power gyrotrons, dynamic nuclear polarization spectroscopy, high-gradient linear accelerators, quasi-optical millimeter-wave components, and photonic bandgap structures and metamaterials.

Hiroshi Idei received the B.S. degree in physics from Shizuoka University, Shizuoka, Japan, in 1988, and the M.S. and Ph.D. degrees in physics from Nagoya University, Nagoya, Japan, in 1990 and 1995, respectively.

He was a Research Associate with the Plasma Heating Division, National Institute for Fusion Science, Toki, Japan, from 1993 to 2003. Since 2003, he has served as an Associate Professor with the Advanced Fusion Research Center, Research Institute for Applied Mechanics, Kyushu University, Fukuoka, Japan. His current research interests include plasma confinement research, plasma heating/current drive using micro and millimeter waves, plasma diagnostics using micro and millimeter waves, and micro and millimeter-wave components.

Richard J. Temkin (M'87–SM'92–F'94) received the B.A. degree in physics from Harvard College, Cambridge, MA, USA, and the Ph.D. degree in physics from the Massachusetts Institute of Technology (MIT), Cambridge.

He was a Post-Doctoral Research Fellow with the Division of Engineering and Applied Physics, Harvard University, Cambridge, from 1971 to 1974. Since 1974, he has been with MIT, first with the Francis Bitter National Magnet Laboratory and then with the Plasma Science and Fusion Center (PSFC) and the Department of Physics. He currently serves as a Senior Scientist with the Department of Physics, an Associate Director of PSFC, and the Head of the PSFC Waves and Beams Division. He has authored or co-authored over 200 published journal articles and book chapters, and has been an Editor of six books and conference proceedings.

Dr. Temkin is a fellow of the American Physical Society and the Institute of Physics, London, U.K. He was a recipient of the IEEE Plasma Science and Applications Award; the Kenneth J. Button Prize and Medal; the Robert L. Woods Award of the Department of Defense; the Exceptional Service Award from the International Society of Infrared, Millimeter, and Terahertz Waves; a Certificate of Merit from the U.S. Department of Energy; and a Certificate of Recognition from the IEEE Electron Device Society.

Fig. 1.

Retrieved mode contents are plotted using the known input mode contents. In this case, the known input values of the HE_{11} and LP_{11} mode contents are varied between 93% and 98% and 5% and 0%, respectively, while the other mode contents are fixed at small input values of $LP_{02} = 0.5\%$, $LP_{03} = 0.0\%$, and $LP_{21} = 1.5\%$. The retrieved mode contents and input values of HE_{11} (red lines), LP_{11} (black lines), LP_{02} (blue lines), LP_{03} (magenta lines), and LP_{21} (green lines) are plotted in solid and dashed lines, respectively.

Fig. 2.

Retrieved mode contents are plotted using the known input mode contents. In this case, the known input values of the HE_{11} and LP_{02} mode contents are varied between 94% and 99% and 5% and 0%, respectively, while LP_{03} is kept at 1%. The retrieved mode contents and input values of HE_{11} (red lines), LP_{02} (black lines), and LP_{03} (blue lines) are plotted in solid and dashed lines, respectively.

Fig. 3.

Retrieved mode contents for benchmarking are plotted using the known input mode contents. In this case, the known input values of the LP_{11} and LP_{02} mode contents are both varied between 0% and 5%, while the HE_{11} is fixed 95% and LP_{03} and LP_{21} are zero. The retrieved mode contents are plotted as solid lines while the input values are plotted as dashed lines, and HE_{11} (red lines), LP_{11} (black lines), LP_{02} (blue lines) and LP_{03} (green lines) are plotted versus the known input mode contents.

Fig. 4.
Experimental setup for measuring the Intensity patterns after the waveguide aperture.

Fig. 5. Measured field intensities after the waveguide aperture at (a) 22.8, (b) 30.8, (c) 39.5, (d) 48.2, (e) 56.8, and (f) 65.5 cm.

TABLE I

Comparison of the Experimental Mode Contents of 63.5-mm Corrugated Waveguide (170 GHz) Calculated Using the Irradiance Moment Method Versus the Phase Retrieval Method

Mode	Calculated from irradiance moments	Calculated from retrieval method
HE ₁₁	94.9	95.0
LP ₁₁ ^o	2.3	1.9
LP ₁₁ ^e	1.9	1.7
LP ₀₂	0.3	0.2
LP ₀₃	0.3	0.1
LP ₂₁	0.2	0.1
Others	0.1	1.0

TABLE II

Comparison of the Mode Contents of 19-mm Corrugated Waveguide (250 GHz) Calculated Using the Irradiance Moment Method Versus the Phase Retrieval Method

Mode	Calculated from irradiance moments (this paper)	Calculated from retrieval method (Ref. [23])
HE ₁₁	94.6	94.2
LP ₁₁ ^o	0.77	0.1
LP ₁₁ ^e	0.27	0.3
LP ₀₂	3.75	4.0
LP ₀₃	0.08	0.3
LP ₂₁	0.53	0.4
Others	0.00	0.7