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RF Pulse Design using Nonlinear Gradient Magnetic Fields

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Abstract

Purpose—An iterative k-space trajectory and radio-frequency (RF) pulse design method is proposed for **Excitation using Nonlinear Gradient Magnetic fields (ENiGMa)**.

Theory and Methods—The spatial encoding functions (SEFs) generated by nonlinear gradient fields (NLGFs) are linearly dependent in Cartesian-coordinates. Left uncorrected, this may lead to flip-angle variations in excitation profiles. In the proposed method, SEFs (k-space samples) are selected using a Matching-Pursuit algorithm, and the RF pulse is designed using a Conjugate-Gradient algorithm. Three variants of the proposed approach are given: the full-algorithm, a computationally-cheaper version, and a third version for designing spoke-based trajectories. The method is demonstrated for various target excitation profiles using simulations and phantom experiments.

Results—The method is compared to other iterative (Matching-Pursuit and Conjugate Gradient) and non-iterative (coordinate-transformation and Jacobian-based) pulse design methods as well as uniform density spiral and EPI trajectories. The results show that the proposed method can increase excitation fidelity significantly.

Conclusion—An iterative method for designing k-space trajectories and RF pulses using nonlinear gradient fields is proposed. The method can either be used for selecting the SEFs individually to guide trajectory design, or can be adapted to design and optimize specific trajectories of interest.

Keywords

nonlinear gradient fields; high order gradient fields; RF pulse design; k-space trajectory design; multi-dimensional excitation

INTRODUCTION

In conventional MRI, gradient magnetic fields have linear spatial variations (1), which allows the excitation process to be expressed as a Fourier transform if the flip-angle is small (2), or as a set of small flip-angle pulses applied in succession if the flip-angle is large (3). Nonlinear gradient fields (NLGFs) have attracted increased attention in the recent years due to the advantages they offer including spatially varying resolution (4–8), faster data acquisition (9–11), reduced radiofrequency power/specific absorption rate (SAR) (12), reduced field-of-view (FOV) imaging (13–22), imaging curved slices (23) and inhomogeneity correction (24). However, the distinctive characteristics of the spatial encoding functions (SEFs) generated using NLGFs require adjustment of the well-known excitation and encoding equations such as using coordinate-transformations (4,5,12) or iterative algorithms applied on predefined k-space trajectories (6,7,9,25).

Because of scan time considerations as well as relaxation effects, it is desirable to keep radiofrequency (RF) pulses reasonably short, and to do this it is essential to design k-space trajectories that rapidly cover the relevant k-space regions. As will be demonstrated, coordinate-transformations may impose redundant constraints whereas previously shown iterative methods use predefined trajectories (25), which may be rendered sub-optimal when NLGFs are used instead of linear gradient fields (LGFs). Therefore, a method that guides k-space trajectory design is needed if NLGFs are to be used during excitation.

In this study, we first investigate the effects of applying NLGFs during RF excitation. To design RF pulses with NLGFs, we propose an iterative algorithm which is named MP-guided-CG (MPgCG) since it uses the Matching-Pursuit (MP) (26) and Conjugate-Gradient (CG) algorithms (27) successively. Then, we present two approaches to RF pulse design based on this method. In the first approach, the SEFs (k-space samples) are selected individually to generate k-space trajectory blueprints, and then user-defined trajectories are designed based on these blueprints. In the second approach, the MPgCG-algorithm is adapted to design specific trajectories of interest, which is demonstrated for a spoke-trajectory. Examples of trajectories designed using both approaches are compared to predefined and non-iteratively designed trajectories. Spoke-based RF pulses are demonstrated experimentally using a 3T scanner and a Z2-coil. As opposed to previous studies in the literature, which have used either coordinate-transformations (12), or predefined k-space trajectories (20,23,25) for RF pulse design using NLGFs, the proposed method works in Cartesian-coordinates and does not require predefined k-space trajectories. The method is demonstrated for two-dimensionally selective excitation profiles in a two-dimensional space assuming small-tip angles, although these are not limitations of the method.

THEORY

In the small-tip-angle regime, the excitation profile obtained by transmitting an RF pulse $B_1(t)$ can be expressed as (2):

$$m_o(\mathbf{x}) = \frac{i\gamma}{2\pi} \int_{-\tau/2}^{\tau/2} B_1(t) e^{-i2\pi\phi(\mathbf{x};t)} dt \quad [1]$$

where $\mathbf{x} = [x \ y \ z]$ denotes spatial coordinates, t denotes time, γ is the gyromagnetic ratio of the spin of interest and $m_o(\mathbf{x})$ is the unitless excitation profile that is obtained. In [1], $\phi(\mathbf{x};t) = \gamma \int_t^T \mathbf{f}(\mathbf{x}) \mathbf{g}(t') dt' / 2\pi$ where $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(t)$ denote the spatial and temporal variation of the perturbation in the static magnetic field B_0 . Hence, $e^{-i2\pi\phi(\mathbf{x};t)}$ denotes the spatial encoding functions (SEFs) generated by the gradient fields. Note that in this study, a global k-space definition is used by separating the spatially invariant part of $\phi(\mathbf{x};t) = -\mathbf{k}(t) \cdot \mathbf{f}(\mathbf{x})$ (12), and the SEFs ($e^{-i2\pi\phi(\mathbf{x};t)}$) correspond to points in the global k-space spanned by the NLGFs. Eq. [1] can be discretized as follows:

$$\mathbf{m}_o = \mathbf{S} \cdot \mathbf{b} \quad [2]$$

where $\mathbf{b}$, $\mathbf{m}_o$ and $\mathbf{S}$ denote the RF pulse, the obtained profile and the SEFs, respectively.

When LGFs are used, the excitation k-space representation ($M(\mathbf{k})$) of the target profile ($m_t(\mathbf{x})$) can be calculated using

$$M(\mathbf{k}) = \int_{-\infty}^{\infty} m_t(\mathbf{x}) e^{-i2\pi\mathbf{k} \cdot \mathbf{x}} d\mathbf{x}, \quad [3]$$

and sampled to design an RF pulse (2). This is because the SEFs are orthogonal to each other; i.e. the inner-product of modes p and q , denoted by ξ_{pq} and defined as:

$$\xi_{pq} = \int_{-\infty}^{\infty} e^{-i2\pi\mathbf{k}_p \cdot \mathbf{x}} \cdot \left(e^{-i2\pi\mathbf{k}_q \cdot \mathbf{x}} \right)^* d\mathbf{x} \quad [4]$$

where $\mathbf{K}_p$ and $\mathbf{K}_q$ denote spatial frequencies, is unitary if $\mathbf{K}_p = \mathbf{K}_q$ and zero otherwise. The k-space trajectory on which $M(\mathbf{k})$ is sampled can be designed to satisfy design requirements, since such effects can be evaluated through well-characterized Fourier transform identities (2).

When NLGFs are used instead, $\mathbf{f}(\mathbf{x})$ not only becomes a nonlinear function of spatial coordinates, but also has coupled variations along multiple spatial directions (28) such as $\mathbf{f}(\mathbf{x}) = [x^2 - y^2 \ 2yz]$. Thus, the SEFs are no longer plane waves, but have curved patterns. In this case, RF pulses can be designed by using a coordinate-transformation $\mathbf{u} = \mathbf{f}(\mathbf{x})$ to obtain a Fourier-based k-space formulation:

$$M(\mathbf{k}_u) = \int_{-\infty}^{\infty} m_{ncs}(\mathbf{u}) e^{-i2\pi\mathbf{k}_u \cdot \mathbf{u}} d\mathbf{u}, \quad [5]$$

where $m_{ncs}(\mathbf{u}) = m_t(\mathbf{x})$ is the target profile expressed in the nonlinear coordinates [please refer to (12) for details]. Because RF pulses are customarily designed numerically after discretization and a region-of-interest (ROI) cannot be defined in non-iterative methods (29), such a coordinate-transformation may increase the computational cost and may impose redundant constraints on the RF pulse by enlarging the effective-FOV. As a two-

dimensional example, consider a square FOV in the Cartesian-coordinates (actual/Cartesian-FOV, Figure 1a). After a coordinate-transformation $\mathbf{u} = [u_1 \ u_2] = [x^2 - y^2 \ 2yz]$, the Cartesian-FOV corresponds to a curved region in the nonlinear coordinates (Figure 1a). Using a square FOV in the nonlinear coordinates for numerical calculations that encloses the Cartesian-FOV (Figure 1a,b) is equivalent to performing calculations in a larger effective-FOV in Cartesian-coordinates (Figure 1c). Since some samples of $\mathbf{m}_{ncs}$ [discretized version of $m_{ncs}(\mathbf{x})$] are allocated to represent the regions outside the actual-FOV, the effective resolution inside the actual-FOV is reduced, which causes loss-of-detail. For the square-FOV shown in (b), only 33% of the samples represent the actual-FOV. Because the spatial grids in the Cartesian and nonlinear coordinates do not overlap, the profile needs to be re-gridded, which may exacerbate the loss-of-detail. To prevent these loss-of-detail mechanisms, the resolution can be increased in nonlinear coordinates for the former, and in the Cartesian-coordinates for the latter (Figure 1d–e). More importantly, redundant constraints may be imposed on the RF pulse since non-iterative methods control the excitation overall the whole effective-FOV; in this case, the error inside the actual-FOV may be increased while reducing the overall error inside the effective-FOV (Figure 1f). Notice that, uniform Fourier transforms that are separable in two-dimensions are discussed here (in either coordinate system), for which the FOVs are inherently rectangular. Further discussion on using a non-uniform Fourier transform or a curvilinear discretization grid is outside the scope of this paper.

Alternatively, RF pulses can be designed in Cartesian-coordinates. However, the SEFs generated using NLGFs are linearly dependent (Figure 2), and hence, non-orthogonal in Cartesian-coordinates (Figure 3a–b). This causes flip-angle variations in the obtained profile (Figure 3c), similar to the intensity variations observed when such fields are used for acquisition (5). To correct such non-orthogonality effects (in succeeding sections, “non-orthogonality” will imply the non-orthogonality of the SEFs in the Cartesian-coordinates), the differential volume element $d\mathbf{x}$ must be corrected using the Jacobian of $\mathbf{f}(\mathbf{x})$:

$$M(\mathbf{k}) = \int_{-\infty}^{\infty} m_t(\mathbf{x}) e^{-i2\pi\mathbf{k}\cdot\mathbf{f}(\mathbf{x})} |\mathbf{J}| d\mathbf{x}. \quad [6]$$

For two SEFs to be orthogonal (Eq. [4] and the discussion thereof), an integer number of oscillations should occur inside the computation domain for both modes. Therefore, although Eq. [6] is equivalent to a coordinate-transformation in an infinite space, it may be imperfect in a practical finite FOV, unless the boundaries of the FOV conform to the NLGFs (observe the effective-FOV of the fields in Figure 1f). For the separable uniform Fourier transform used here, the FOV is inherently rectangular, in which the correction is imperfect (Figure 3d–f).

Another difference between LGFs and NLGFs is the effect of sampling the k-space on a finite trajectory to design the RF pulse (2). Because such effects conform to the spatial variations of the fields (Figure 4), and therefore, are spatially non-uniform for NLGFs, k-space trajectory design is not as straightforward as LGF cases.

Among the differences between the LGFs and NLGFs, spatially-varying resolution and non-bijectionality of encoding (4) are outcomes of field characteristics, and can only be corrected by using different field distributions (6,9) or multiple receive/transmit channels to enhance spatial encoding (4,5,11,25). However, flip-angle variations due to SEF non-orthogonality and effects of k-space sampling are substantially affected by the selection of the SEFs, and can be improved by selecting different SEFs. For this purpose, we propose an iterative method for selecting the SEFs and designing the RF pulse design, in the following section.

MPgCG: Matching-Pursuit guided Conjugate-Gradient

Matching-Pursuit (MP) (26) and Conjugate-Gradient (CG) (27) algorithms have been previously used in the MRI literature for image reconstruction (6,30) and RF pulse design (7,25,29,31). When NLGFs are used, MP is more effective in selecting SEFs from the k-space whereas CG outperforms MP in optimizing the RF pulse values (please refer to Supporting Information for details). Therefore, we use the two methods successively. The proposed method, named MP-guided-CG (MPgCG), is as follows:

1. *Discretization and initialization:* After discretization, the following are initialized: $\mathbf{m}_t$: target profile; $\mathbf{m}_o = [0 \dots 0]^T$: profile obtained; $\mathbf{m}_r \triangleq \mathbf{m}_t - \mathbf{m}_o$ residual profile; $\mathbf{S}$: matrix that contains the SEFs; $\mathbf{c} \triangleq \mathbf{S}^H \cdot \mathbf{W} \cdot \mathbf{m}_r$ projection of the profile onto the SEFs, where H is the Hermitian operator and $\mathbf{W}$ is the spatially varying weight function which determines the ROI (29). The set of selected SEFs ($\mathbf{S}_{sel}$) is initialized as an empty matrix.
2. *Iteration:* $\mathbf{c}$ is calculated and the SEF that yields the magnitude-wise largest entry in $\mathbf{c}$ (i.e. the highest RF amplitude) is selected (MP), removed from $\mathbf{S}$, and added to. The RF pulse ($\mathbf{b}$) is optimized using CG on $\mathbf{S}_{sel}^H \cdot \mathbf{W} \cdot \mathbf{S}_{sel} \cdot \mathbf{b} = \mathbf{S}_{sel}^H \cdot \mathbf{W} \cdot \mathbf{m}_t$, which is a modified version of Eq. [2] (please refer to (27) for details). Finally, the obtained ($\mathbf{m}_o = \mathbf{S}_{sel} \cdot \mathbf{b}$) and residual profiles are updated. Iterations continue until a predetermined number of SEFs are selected.
3. *Error calculation:* Root-mean-squared-error (RMSE) is calculated with respect to the total target excitation inside the ROI using the L_2 -norm:

$$\text{RMSE} = 100 \times \min_{\alpha} \frac{\|\mathbf{W} \cdot (\mathbf{m}_t - \alpha \mathbf{m}_o)\|_2}{\|\mathbf{W} \cdot \mathbf{m}_t\|_2}, \quad [7]$$

where α is a flip-angle normalization constant.

Since the RF values calculated using MP (entries of $\mathbf{c}$) may be sub-optimal because of SEF non-orthogonality (please refer to Supporting Information), the RF values are optimized using CG in each iteration. However, this may be computationally expensive. An alternative approach is to use MP and CG successively as follows, instead of step-2 above:

- 2.a. *MP-iteration:* $\mathbf{c}$ is calculated, $\mathbf{S}$ and $\mathbf{S}_{sel}$ are updated as given in step-2 above. The value of $\mathbf{c}$ corresponding to the selected SEF is added to $\mathbf{b}$, and $\mathbf{m}_o$ and $\mathbf{m}_r$ are updated. This step is iterated until a predetermined number of SEFs are selected.

2.b. CG-Iteration: $\mathbf{b}$ is optimized using CG similar to step-2 above.

To distinguish between the two approaches, the first one, which is the full solution, will be referred to as **MPgCG-full** and the second approach, which is faster, will be referred to as **MPgCG-fast**.

Instead of selecting the SEFs one-by-one, MPgCG can be used to design trajectories with pre-defined shapes and constraints as well. As an example implementation, consider optimization of a spoke-trajectory. The algorithm, named **MPgCG-spokes**, has the same first and third steps as MPgCG-full. Since multiple SEFs exist on a spoke, the second step is modified to maximize the performance of the optimization:

1. *2-sp*: Let I denote the number of candidate spokes to choose from. Using the SEFs on the i^{th} -spoke (S_i) and the set of selected-SEFs, the RF pulse ($\mathbf{b}_i$) is designed using CG on $[S_{\text{sel}} S_i]^H \cdot \mathbf{W} \cdot [S_{\text{sel}} S_i] \cdot \mathbf{b}_i = [S_{\text{sel}} S_i]^H \cdot \mathbf{W} \cdot \mathbf{m}_t$. After the obtained profile and RMSE error are calculated for every candidate spoke $i = 1, 2, \dots, I$; the pulse that yields the lowest RMSE is selected using MP. The SEFs on the selected spoke are added to S_{sel} and removed from S , and the obtained profile is updated.

METHODS

Simulations were performed using Matlab (The Mathworks Inc., Natick, MA, USA) for four excitation profiles specified on the xy -plane with a $20 \times 20 \text{ cm}^2$ FOV (Figure 5a–d). The spatial and k-space matrix sizes were 40×40 for the checkerboard profile (Figure 5a) and 80×80 for the other profiles. The ROI covered the whole-FOV for Figure 5a–c whereas it was selected to be a disc with a radius of 8 cm for Figure 5d. In the simulations, x , y , Z^2 (normalized to $x^2 + y^2$ on the xy -plane), $C2$ ($x^2 - y^2$) and $S2$ ($2xy$) field distributions were used (Figure 5f–j). k-Space sampling distances were $K_{LGF} = 1/(20\text{cm})$ for the LGFs and $K_{NLGF} = 1/(20\text{cm})^2$ for the NLGFs so that the maximum angular encoding generated by the NLGFs inside a circular region (radius: $10\sqrt{2}\text{cm}$) was the same as that caused by the LGFs inside the FOV. Obtained profiles were simulated using Eq. [2].

Comparison of pulse design methods

Using the C2-S2 fields and the checkerboard target profile (Figure 5), MPgCG-full and MPgCG-fast methods were compared to:

- Method 1 – Jacobian correction: M was calculated using a discretized version of Eq. [6]. A predefined number of SEFs that yield the magnitude-wise highest RF values were selected.
- Method 2 – Coordinate-transformation method (CTM): The target profile was mapped to the C2-S2 domain ($\mathbf{m}_{\text{ncs}}$), and its Fourier transform was calculated (discretized version of Eq. [5]). Then, SEFs were selected as in Method 1.
- Method 3 – CTM after up-sampling: The target profile was generated on a 160×160 grid and then mapped to a 160×160 C2-S2 domain. Eq. [5] was discretized at the same 1600 SEF locations as the previous cases, and SEFs were selected as in Method 1.

- Method 4 – CTM with CG: SEFs were selected as in Method 2. Selected SEFs were calculated in Cartesian-coordinates and the RF pulse was optimized using CG.
- Method 5 – CG-guided-CG (CGgCG): $S^H \cdot W \cdot S \cdot b = S^H \cdot W \cdot m_t$ was solved for b using CG. The SEFs were selected similar to Method 1. Using the selected SEFs, the RF pulse was further optimized using CG on $S_{sel}^H \cdot W \cdot S_{sel} \cdot b = S_{sel}^H \cdot W \cdot m_t$
- Method 6 – MP: A predefined number of SEFs were selected and RF pulse was designed using the MP method, as in ‘step-2a’ of the MPgCG-fast method.

For all methods, number of selected SEFs was varied between 1 and 1600, and RMSE was calculated using Eq. [7]. Number of CG-iterations was set to 25000. Increasing this number further did not change RMSE.

Comparison of gradient fields, excitation profiles and ROIs

Profiles were simulated for three target shapes (Figure 5b–d) using five field pairs (x - y , x - $Z2$, x - $C2$, $Z2$ - $C2$, $C2$ - $S2$). The first target profile was rectangular (Figure 5b), which is arguably the most important excitation profile when LGFs are used for encoding. The second profile was an off-center rectangular profile (Figure 5c) and the centers of the NLGFs were shifted to the center of the profile by using the required LGFs in parallel to demonstrate that a combination of NLGFs with lower-order fields may be used for exciting profiles with certain asymmetry without parallel RF transmission. To demonstrate that an ROI can be defined for the MPgCG-algorithm, the third profile was selected to be circular inside an off-center circular ROI with a radius of 8 cm and centered at $x = 5/3$ cm (Figure 5d). The number of SEFs and CG-iterations were 400, and the MPgCG-fast algorithm was used for all cases.

MPgCG-based trajectory design

For the shifted-rectangular profile, and the x - $Z2$ field pair, 400 SEFs were selected using MPgCG-fast. Based on this information, a double-spiral trajectory was manually defined, for which, rotationally-invariant gradient waveforms were designed (32). For comparison, a spiral trajectory based on the positions of 400 SEFs selected using the Jacobian-based method (Method 1) and a uniform-density spiral trajectory that covers the computational k -space domain were designed. RF pulses were optimized using CG in 4000 iterations for the MPgCG-case, whereas the number of iterations for the reference methods was increased until the RF power was the same. The slew-rate and gradient-amplitude limits were 14 T/sec and 4 mT inside the FOV for both fields. All trajectories have 2 ms duration with 10 μ s gradient dwell-time, but were interpolated (cubic-interpolation) to simulate a 2 μ s RF dwell-time, having 1000 SEFs each. Two other examples are given in Supporting Information.

Experiments

Experiments were performed using a 3T scanner (Siemens Healthcare, Erlangen, Germany), a $Z2$ -gradient coil (Resonance Research Inc., MA, USA) with an additional gradient controller and amplifier, a Siemens head-coil that fits inside the gradient insert and a

cylindrical contrast phantom (J7239, JM Specialty Parts, San Diego, CA; inner length: 14.8 cm, inner diameter: 19 cm). Similar to the simulations, the RF pulses were designed for 2D-profiles and 3D-imaging was performed to prevent aliasing artifacts in the longitudinal direction. A fast-low-angle-shot (FLASH) gradient-echo sequence (33) obtained from the manufacturer of the scanner was used after replacing the gradient and RF waveforms with the designed counterparts.

The MPgCG-spokes algorithm was used for designing two-dimensional 6-spoke and 10-spoke trajectories for the rectangular (Figure 5b) and semi-circular (Figure 5e) profiles, respectively. The ROIs were the entire FOV. To minimize eddy-current effects, flyback-EPI (34) trajectories were designed with the spokes parallel to the k_x -axis. To compensate for increased waveform duration, spatial and k-space matrix sizes were 40×40 for pulse design. For comparison, uniform-density and Jacobian-based trajectories were designed. For the latter, k-space was calculated using the Jacobian-based approach, and the spokes with the highest energy were selected. For the former, the set of equidistant spokes was selected that provided the maximum k-space coverage while lying on the same rectilinear k-space grid, followed by optimization of the RF pulse using CG. In the experiments, the FOV was $192 \times 192 \times 160 \text{ cm}^2$ and the matrix size $128 \times 128 \times 32$. A third experiment that targets a circular profile is given in Supporting Information to provide comparison to previous publications (13,18,19).

RESULTS

Comparison of pulse design methods

For all methods, RMSE decreased as more SEFs were used, as expected (Figure 6). Although there were minor increases in RMSE, these were due to the post-design flip-angle normalization in Eq. [7]. The MPgCG-full algorithm yielded significantly lower RMSE values: averaged over all cases, the RMSE was lower by 12.2% than the Jacobian-based method, 18.5% than CTM, 4.8% than high-resolution CTM, 12.0% than CTM with CG, 20.6% than CGgCG, and 5.2% than MP. The average error between MPgCG-full and MPgCG-fast was 0.5%.

Comparison of gradient fields, excitation profiles and ROIs

Figure 7 shows the SEFs selected using MPgCG-fast and the obtained profiles for various target profiles, field sets and ROIs, which will be discussed in the next section.

MPgCG-based trajectory design

The trajectory designed using the information provided by the Jacobian-based method (Figure 8f–j), reduced the RMSE substantially, compared to the uniform-density spiral (Figure 8b–e). However, MPgCG-fast revealed the relevance of the SEFs around $k_{z2} = 0$ to the target profile. The MPgCG-based trajectory had 56% lower RMSE than Jacobian-based and 90% lower RMSE than the uniform-density trajectories. In the additional examples given in Supporting Information (Figures SI-1, SI-2), MPgCG-based trajectories yielded 78% and 64% lower RMSE than Jacobian-based and 95% and 45% lower RMSE than the

uniform-density trajectories, highlighting the importance of determining the relevant SEFs prior to trajectory design.

Experiments

Figures 9–10 compare MPgCG-spokes to uniform-density and Jacobian-based EPI trajectories. Similar to Figure 6, the Jacobian-based method favored low-frequencies due to the imperfect non-orthogonality correction, underperforming in encoding the transition regions. Although the uniform-density trajectory provided a reasonable excitation profile with 10 spokes (Figure 10), the larger spoke-separation in the 6-spoke case resulted in artifacts at the edge of the FOV (Figure 9). MPgCG optimized spokes provided a much better approximation of the target profiles with 88% and 85% lower RMSE for the rectangular and 62% and 85% lower RMSE for the semi-circular profiles than the other methods. The ratio of the maximum excitation inside the target region to that outside (in the side lobes) was calculated as 7.9 and 14 in the simulations, and measured as 5.9 and 7 in the experiments for the MPgCG cases in Figure 9 and Figure 10, respectively. Notice that, the side lobes were slightly more pronounced in the experiments, due to B_1 -inhomogeneity (Figure SI-3 in Supporting Information). Still, the simulated and experimental profiles showed good agreement. Although shim currents were re-calibrated before each experiment, some residual field-inhomogeneity remained due to the inner structure of the phantom, causing local distortions at profile boundaries, such as slanted boundaries for the rectangular profile. Off-center slices, shown in Supporting Information, demonstrate this effect, which was stronger for the Jacobian-based pulses (Figure SI-4). Figure SI-5 shows that the pulse designed for a circular profile resembles a *sinc* function, as expected from previous literature (13,18,19).

DISCUSSION

Comparison of RF pulse design methods

When the same matrix sizes were used, RMSE was higher for CTM than the Jacobian-method due to the loss-of-detail in CTM. CG reduced RMSE substantially, since SEFs were calculated in Cartesian-coordinates with no loss-of-detail and the ROI was the same as the actual-FOV. However, the loss-of-detail due to the coordinate-transformation is still effective during SEF selection.

Figure 6 demonstrates the effectiveness of MP in selecting the SEFs and CG in RF pulse optimization. When less than 25% of the SEFs were selected, MP-based methods yielded significantly lower RMSE. However, when more were selected, RMSE reached a plateau for MP due to the linear dependence of the SEFs; the SEFs selected in late iterations tend to have negligible weights (RF pulse values), and hence, negligible effect on RMSE. However, when CG is applied after MP (MPgCG-methods), the weights were re-adjusted to make use of small differences between the SEFs, reducing RMSE further. RMSE was substantially higher for CGgCG, especially when less than 40% of the SEFs were used, demonstrating that CG underperforms in SEF selection (For a detailed discussion of the CGgCG-method, please refer to Supporting Information).

SEF selection patterns were similar for all non-iterative methods (Figure 6); mostly low-to-middle frequency SEFs were selected, and the patterns curved towards the vertical axis (k_{s2}). Since CG deposits high RF power in most k-space locations (Supporting Information), mostly high-frequency SEFs were retained, whereas MP selected the SEFs more uniformly. Finally, MPgCG-full produced the most uniform SEF selection pattern, and the lowest RMSE.

Field-profile similarity

For the rectangular target profile shown in Figure 5b, it was expected that LGFs would yield the best results due to the similarity between the field contours and the profile. Because the Z2-field varies slowly around the center (where the target profile is spatially invariant) and rapidly around the transition regions of the profile along y ($y = \pm 5$ cm), the Z2-field can simultaneously encode both regions using fewer SEFs. Thus, using x and Z2-fields reduced the RMSE by 11%, demonstrating that NLGFs can be more effective than LGFs, in encoding even rectangular excitation profiles.

In certain cases, a similarity between the profile and the fields can be obtained by translating/rotating the fields. Although the profile in Figure 5c had a 180°-rotational symmetry around its center, it was not symmetric around the center shared by the fields. However, Figure 8 shows that, by using a combination of NLGFs with lower-order fields (LGFs in this case), the profile could be generated without artifacts.

Specific absorption rate

It was previously demonstrated that NLGFs can be used for reducing the SAR (12). In this study, many results showed that using NLGFs may reduce not only the RMSE, but also the RF power, although the RF power was not a pulse design constraint (Figure 7b,l,n). Even though calculating power from discrete k-space samples imply unbounded gradient-amplitudes and slew-rates, the results suggest the possibility of SAR reduction with NLGFs, in agreement with Ref. (12).

The effect of region-of-interest

In non-iterative pulse design methods, the designed pulse tries to control the excitation everywhere inside the computation domain, since an ROI cannot be specified. This puts more constraints on the RF pulse, possibly increasing RF power and reducing excitation fidelity inside the ROI. However, because the proposed method is iterative, spatially varying weights-of-importance can be assigned to different regions (Figure 7k-o), similar to (29).

The effect of field-of-view

Unlike LGFs, the encoding capability of NLGFs changes when the fields are shifted relative to the FOV. Although the fields were shifted with the profile in Figure 7, the FOV was not, which makes encoding harder and therefore, increases RMSE. As an example, consider the x -Z2 case: the NLGF had to suppress the excitation at $x = y = -10$ cm, which lay at a distance of approximately 19.5 cm from the center of the field, whereas for the centered-rectangular profile, the maximum distance was approximately 14.1 cm. Notice that RMSE

was the same for LGFs. Hence, to localize the excitation to another region inside an object, a new trajectory and a new RF pulse should be designed when NLGFs are used.

k-Space trajectory design

When LGFs are used for excitation in the small-tip-angle regime, a reasonably efficient k-space trajectory can be predicted using the properties of the Fourier transform between the excitation profile and the k-space. This is because not only are the spatial variations of LGFs uniform, but LGFs also vary along orthogonal directions. However, the rate-of-change is spatially non-uniform for NLGFs, such fields have coupled variations along multiple directions (28), and more than one field may vary along a given direction. Furthermore, even though orthogonal fields can be selected, the generated SEFs may be non-orthogonal (Figures 2–3). Figure 7 shows that the distribution of relevant SEFs is substantially affected by the choice of gradient fields. Although well-known trajectories such as echo-planar, spiral or rosette can be used, such an approach may be sub-optimal when NLGFs are used (Figure 8–10). Hence, predicting a reasonable trajectory is not as trivial as in the LGF case, especially without evaluating which SEFs are more relevant to the target profile.

The proposed methods

Of the variants of the method, MPgCG-full uses both MP and CG at each iteration, whereas MPgCG-fast uses the algorithms sequentially. Since the RF pulse is optimized at each step, MPgCG-full yields a lower RMSE, but is slower. When either of these methods is used, the SEFs are selected individually. At this point, the user has three alternative ways to design the k-space trajectory, first of which is to design a trajectory that traces the selected SEFs. In this case, ordering of the SEFs on the trajectory can be determined using algorithms such as Travelling Salesman, although such a trajectory may not be feasible considering slew-rate and gradient-amplitude limitations. Alternatively, the knowledge of these SEF locations can be used as a guide to choose and design a trajectory, such as shown in Figure 8. Finally, the MPgCG-method can be adapted to optimize the trajectory of choice. Note that, the third approach can be utilized without selecting the SEFs individually as well (Figures 9–10), although this may reduce performance.

Previous studies on excitation using NLGFs either featured nonlinear coordinate-transformations (12), or employed predefined k-space trajectories (20,23,25). The MPgCG-algorithm proposed in this study does not require any predefined trajectories as the SEFs are selected individually, or in small groups (MPgCG-spokes). Furthermore, RF pulses are designed without any coordinate-transformations. Although more time-consuming than CTM, MPgCG yields better excitation fidelity. While the excitation fidelity of CTM can be increased by using a higher spatial resolution and/or employing a CG optimization afterwards, then CTM may lose its advantage of computational cheapness.

Limitations and future studies

The performance of an RF pulse can be evaluated in terms of various parameters including RMSE, SAR, pulse duration and off-resonance effects. Since the primary goal of this paper is to demonstrate the effect of SEF non-orthogonality on excitation fidelity, RMSE was used as the only design constraint. Still, the proposed MPgCG approach can be adapted to utilize

other constraints such as gradient amplitude/slew-rate limits, off-resonance behavior, SAR and trajectory shape, of which the latter was demonstrated with the MPgCG-spokes method. Although MPgCG shows significant improvement over the other methods, selecting SEFs using MP may actually be sub-optimal, since MP is a greedy algorithm. The proposed method was applied in the small-tip-angle regime, and for two-dimensional cases. However, these are not limitations of the method and the method can be extended to the large-tip-angles and three-dimensional spatial coordinates. Furthermore, the method is applicable to higher-dimensional excitation schemes that use more gradient fields than the number of dimensions (35), and can be used for designing parallel-excitation pulses that employ multiple RF transmit channels as well (36).

CONCLUSION

In this study, we investigated the effect of using nonlinear gradient fields on the excitation process, and proposed a k-space trajectory and RF pulse design algorithm. Trajectory design plays an important role in excitation fidelity since it determines which SEFs are available. When NLGFs are used, the selection of SEFs varies significantly depending upon the fields used, the target excitation profile, the field-of-view and the region-of-interest, and hence, predefined trajectories may be sub-optimal. On the other hand, calculating k-space may require modification of the well-known formulations because of SEF non-orthogonality. Even though the effects of non-orthogonality can be reduced using coordinate-transformations or Jacobian-based methods, such methods are not as effective as iterative methods. We investigated the Conjugate-Gradient (CG) and Matching-Pursuit (MP) algorithms when non-orthogonal SEFs are used and showed that CG can be used for RF pulse design for known trajectories but not for designing trajectories and that MP can be used for trajectory design but not for RF pulse optimization. Based on these findings, we proposed an iterative k-space trajectory and RF pulse design method for excitation with NLGFs. We presented three variants of the method, the full-solution, a faster-version, and an adaptation for designing spoke-trajectories. It was shown that the proposed methods improve excitation fidelity and yield lower RMSE values than the aforementioned iterative and non-iterative approaches. Since the MPgCG-algorithm is iterative, no additional non-orthogonality correction is necessary and a region-of-interest can be defined to reduce pulse design constraints (29).

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Figure 1.

Effect of coordinate-transformation on FOV and resolution. A square FOV ($20 \times 20 \text{ cm}^2$) in the Cartesian-coordinates **(a)** corresponds to a curvilinear FOV in the C2-S2 (field profiles: Figure 5i–j) coordinate system **(b)**. **(c)** A square computation domain in the C2-S2 coordinates is equivalent to a larger effective-FOV in Cartesian-coordinates (dashed contour). **(d–f)** Profiles simulated using the coordinate-transformation approach (utilizing discretized version of Eq. [5] and Eq. [2]) for a checkerboard target pattern (Figure 5a) are shown on the xy -plane. **(d)** Profile obtained when spatial and k -space matrix sizes were 40×40 , and the distance between k -space samples $k = 1/(20\text{cm})^2$. **(e)** Profile obtained when the spatial matrix sizes in both Cartesian and nonlinear coordinates were increased to 160×160 . **(f)** Plotting the profile obtained in (d) in a larger-FOV demonstrates that the non-iterative coordinate-transformation method attempts to control the excitation everywhere in the effective-FOV shown in (c).

Figure 2.

SEFs generated using NLGFs are linearly dependent in Cartesian-coordinates. An arbitrarily selected target SEF (SEF_t) was approximated using one (second row), three (third row) and five (fourth row) other SEFs. The error in the obtained SEF (SEF_o) reduced as more SEFs were used (top right). The error was calculated using the L2-norm in: $RMSE = 100 \times \frac{\|SEF_t - SEF_o\|_2}{\|SEF_t\|_2}$. The weights of the SEFs were calculated using CG. The SEFs were generated using C2 and S2-fields (Figure 5i–j). Simulation parameters were; resolution: 40×40 on xy -plane, FOV: 20×20 cm², k-space resolution: 10×10 , k-space sampling distance: $k_{NLGF} = 1/(20\text{cm})^2$.

Figure 3.

Effect of SEF non-orthogonality on the excitation profile. **(a–b)** Orthogonality of the SEFs was investigated for C2- and S2-fields (Figure 5i–j) using the inner-product (Eq. [4]) in a logarithmic scale: $10 \log_{10} \xi_{pq}$. Axes denote mode numbers $p, q = k / k$. For orthonormal SEFs, $\xi_{i,i}$ in (a) and $\xi_{0,0}$ in (b) should be 1 while rest are 0. **(c)** The non-orthogonality of the SEFs caused flip-angle variations in the obtained profile (calculated using discretized version of Eq. [6] without the Jacobian, and Eq. [2]). Simulation parameters were; FOV, $20 \times 20 \text{ cm}^2$; spatial and k-space matrix sizes, 40×40 ; k-space sampling distance, $k_{NLGF} = 1 / (20 \text{ cm})^2$; target profile, checkerboard pattern (Figure 5a). **(d–e)** Correcting the differential element using the Jacobian in Eq. [4] reduced the inner-product of the SEFs, although this correction was not perfect since the boundaries of the computation region do not conform to the fields. **(f)** Using the Jacobian reduced the flip-angle variation over the image.

Figure 4.

Effect of undersampling the excitation k-space on the excitation profile is demonstrated for the C2 and S2 fields (Figure 5i–j). **(a)** k-Space samples lying at odd multiples of $k_{NLGF} = 1/(20cm)^2$ along the k_{s2} direction were discarded. **(b)** Because the S2-field varies rapidly and the C2-field varies slowly around the diagonals ($x = \pm y$), resolution around the diagonals was generated mainly by the S2-field. Hence, discarding half of the encoding steps of this field reduced excitation fidelity around the diagonals significantly, leaving the regions around the axes unaltered. **(c)** Larger FOV showed that the aliasing artifacts conform to the field patterns. Profiles are shown on the xy -plane.

Figure 5.

The target excitation profiles and the field distributions used in this study are shown on the xy -plane. **(a)** Checkerboard profile, matrix size: 40×40 . **(b)** Centered-rectangular profile, matrix size: 80×80 . **(c)** Shifted-rectangular profile, matrix size: 80×80 . **(d)** Centered circular profile, matrix size: 80×80 . The ROI used in the simulations is indicated with a dashed contour. **(e)** Semi-circular profile, matrix size: 40×40 . **(f)** x -gradient field. **(g)** y -gradient field. **(h)** Z2-field: $x^2 + y^2 - 2z^2$. **(i)** C2-field: $x^2 - y^2$. **(j)** S2-field: $2xy$. All fields are shown in the same scale with arbitrary units.

Figure 6.

RF pulse design methods are compared in terms of root-mean-squared-error (RMSE), simulated excitation profiles and k-space locations of selected SEFs. (a) The RMSE in the obtained profiles is plotted with respect to the number of selected SEFs for all design methods. The number of available SEFs (100%) was 1600. (b–i) For the simulated profiles (1st and 3rd columns), 600 SEFs were selected in the k-space (2nd and 4th columns). Spatial FOV was $20 \times 20 \text{ cm}^2$, and the k-space limits were $\pm 500 \text{ m}^{-2}$ in both directions. The target profile is shown in Figure 5a. All profiles are shown on the xy -plane.

Figure 7.

Simulated excitation profiles (1st, 3rd, 5th columns) and k-space locations of selected SEFs (2nd, 4th, 6th columns), obtained with the MPgCG-fast method for the excitation profiles shown in Figure 5b–d. All profiles are shown on the xy-plane. k-Space limits are $\pm 200 \text{ m}^{-1}$ for the LGFs and $\pm 1000 \text{ m}^{-2}$ for the NLGFs, and the FOV is $20 \times 20 \text{ cm}^2$ for all images. Power values are calculated as a summation over taken samples: $P = \sum_n |b[n]|^2$ and normalized with respect to the respective LGF cases of all profiles.

Figure 8.

Trajectories designed using the x - Z_2 field pair for the shifted-rectangular profile (a) are compared. (b–e) The uniform-density spiral trajectory (b) designed without any prior information on the distribution of relevant SEFs covered the computational k -space domain. (f–j) The spiral trajectory (g) designed to trace the region that contains the SEFs selected using the Jacobian-based method (f). (k–o) The double-spiral trajectory (l) designed to trace the region that contains the SEFs selected using the MPgCG-fast algorithm (k). The

difference images (e,j,o) between the obtained and the target profiles are given in the same scale for easier comparison. RMSE values are (e) 21.6%, (j) 5.0% and (o) 2.2%.

Figure 9. 6-spoke flyback-EPI trajectories designed to excite the centered-rectangular profile (Figure 5b) are given together with simulated and experimental profiles.

Figure 10.

10-spoke flyback-EPI trajectories designed to excite the semi-circular profile (Figure 5e) are given together with simulated and experimental profiles.